Powder Technology 320 (2017) 43–50

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Correlations between flow rate parameters and the shape of the grains in a silo discharge

César A. Calderón [a], Marcela C. Villagrán Olivares [b], Rodolfo O. Uñac [b], Ana M. Vidales [b,*]

[a] Facultad de Química, Bioquímica y Farmacia, Universidad Nacional de San Luis, Ejército de los Andes 950, D5700HHW San Luis, Argentina
[b] INFAP, CONICET, Departamento de Física, Facultad de Ciencias Físico Matemáticas y Naturales, Universidad Nacional de San Luis, Ejército de los Andes 950, D5700HHW San Luis, Argentina

ARTICLE INFO

Article history:
Received 16 February 2017
Received in revised form 28 June 2017
Accepted 3 July 2017
Available online 05 July 2017

Keywords:
Flow
Silo
Beverloo's equation

ABSTRACT

This is an experimental study of the discharge of seeds in a silo in a mass flow regime. It demonstrates how the flow parameters involved in Beverloo's equation (C and k) can be correlated with some of the shape parameters belonging to the seed grains. Results show that the concept of empty annulus introduced through the parameter k is dependent on the size and on the specific surface area. Besides, the circularity of the grains, alone, is not enough to predict the value of k in a grain discharging process.

1. Introduction

The physical understanding in the discharge of silos is still a topic of intensive research, given its connection with important industrial operations like the storage and manipulation of granular matter used in the production of food, pharmaceuticals, agrochemicals and construction materials.

Among other features, a safe design of a silo needs determining the pressures exerted on the walls during the filling, storage and discharging time. Even more, before deciding the features of a new discharging set up (or the change of an already existing one), it is necessary to characterize of the granular material to be stored.

Many physical properties of the grains are involved in this characterization and some of the most important ones to be measured are the bulk density, the size and the shape of the particles, the repose angle and the humidity content. Besides, the study of the flow inside silos and hoppers is crucial for the determination of adequate dimensions to avoid the presence of fluctuations and the eventual jamming during the discharge [1,2].

Although the long list of experimental and numerical works dedicated to the topic of the present paper [3–14], there are still open questions to answer, and basic studies have to be conducted in order to understand many features of the outflow properties of granular matter in silos.

As clearly explained by Mellmann et al. [13], few works has been published on the influence of the particle form on the discharge rate in silos. In that paper, the authors show how the particle's form, their flow properties and the outlet geometry can affect the mass flow during the discharge from a silo. They found that, for a bulk material, when the particle form factor and the standard deviation of the form factor distribution function increase, the Froude number is reduced and so does the discharging mass flow rate.

Despite its old formulation, the well-known Beverloo's equation [15] is still a key correlation to predict flow discharge in silos. It has frequently been modified to account for effects do not included in the original model as, for example, the hopper angle, the outlet orifice geometry and the particle size distribution [3,5–7,16].

For example, the parameter k in Beverloo's equation has been related to the particle shape and it has little dependence on the size of the grains, although modifications have been made to take into account the presence of binary mixtures. For instance, the replacement of k by a function of the particle size distribution has been tried by Humby and collaborators [17,18] in order to modify Beverloo's correlation.

On the other hand, the parameter C in that correlation has been associated with the internal friction of the particles and the hopper geometry. Nevertheless, many works find its value limited to a narrow range [19], or vary this parameter in order to account for hopper angle effects or hopper geometry [20,21].

By using Beverloo's equation in 2D silos, a recent study demonstrates that flow increases with the hopper angle while C decreases [22]. On the other hand, the parameter k shows a variation with the hopper angle, while experiments find independence of the results on it.

In simulations performed by Toson and Khinast [23], it is proven that the form factor k varies with particle shape and can be scaled, as a good

* Corresponding author.
E-mail address: avidales@unsl.edu.ar (A.M. Vidales).

http://dx.doi.org/10.1016/j.powtec.2017.07.004
0032-5910/

approximation, by using the squared root of the aspect ratio of the particles. On the other hand, they find that in all their simulations the parameter C is quite constant.

Looking at all the work presented so far, one of the challenges is still to correlate the values of the Beverloo's constants with the shape of the particles involved in the discharge. In this way, one could predict the behavior of the flow when a change in the geometry of the grains is needed in an industrial process [13].

The present work studies experimentally the discharge properties of three different seeds, commonly found in food production, aiming to correlate their different shape features with the parameters entering in the flow equations.

In the next section, we present a brief theoretical background to understand our physical basis in analyzing and measuring the variables of interest. The third section shows the experimental set up and procedure used in the discharge experiments at the laboratory scale. Then, a section dedicated to our results is presented, followed by a proper discussion. The last section is devoted to the main conclusions.

2. Theoretical background and definitions

2.1. Beverloo's equation

It would be difficult, may be unnecessary, to conduct a thorough analysis of all the work done around the Beverloo's correlation and its diverse modifications. Here, we just address some main concepts for our future analysis. Interested readers can refer to the literature presented in [6,7,11,15,24], among many others.

The discharge rate of particles, with a given diameter d, inside a flat-bottomed cylindrical silo with outlet orifice diameter D, is expressed as [15]:

$$W_{flat} = C\rho_a g^{\frac{1}{2}}(D - kd)^{\frac{5}{2}} \tag{1}$$

where g is the gravity acceleration and ρ_a is the bulk density of the packing of grains after filling the silo. The constant k is called the Beverloo's constant and it is expected to depend on the size and shape of the particles. The constant C is found empirically in the range of $0.55 < C < 0.65$ and it is usually related to the hopper geometry.

Some previous works rise the possibility of modifying Eq. (1) by introducing a dependence of the term kd on the particle size distribution [7,18], especially for binary mixtures. Given that the bulk density is modified by the presence of particles with different shapes and sizes, the actual effect of the grain features on k could be difficult to isolate.

In the present work we will consider Eq. (1) as the fundamental correlation between the flow rate and the silo-particle system. Here, the values for the constants C and k are determined through this equation and they are related to the different particles used in each experiment.

Bulk density is considered constant for a given type of grains and, consequently, this assumption affects the values of C determined through our measurements given that, strictly speaking, the bulk density is supposed to vary at different points of the discharging mass.

2.2. Shape factors and specific surface area

There are numerous shape factors that are able for the characterization of a grain [25,26]. Most of them deals with the comparison of a non-regular shaped grain with a regular shaped one, commonly a sphere or a disk, depending on the geometrical quantities to be measured.

Here, we define some shape factors that will help to correlate our results for the discharge of the silo. All these factors are related to the determination of the main three dimensions of the seed grains, i.e., their characteristic thickness, T, width, W_{th}, and length, L. They are also related to the determination of the surface area and volume. Based on the knowledge of these parameters, the equivalent volume diameter, d_v, for a grain with approximate volume V is:

$$d_v = \left(\frac{6V}{\pi}\right)^{1/3} \tag{2}$$

Three different seeds are chosen in our experiments: wheat, soy and corn. To calculate the respective volumes for each seed type, we assume that soy and wheat grains can be considered as perfect revolution ellipsoids, where the principal semi-axes are obtained from W_{th} and L. Finally, corn volume is calculated by multiplying the average projected surface area by the average of T.

The shape factors used in the present work are the circularity, CI, and the sphericity, Ψ. Their calculation follows standard definitions used in other works and are, respectively, the following ones [25,26]:

$$CI = \frac{4\pi A_p}{P^2} \tag{3}$$

$$\Psi = \frac{S_p}{S_C} \tag{4}$$

The circularity expresses the closeness of the shape of the particle projection to that of a perfect circle. Among other possible definitions, it can be set as the square of the rate between the perimeter of a circle with projected area equal to that of the particle, A_p -provided the contribution of gravity will generally ensure that this is the maximal area- and the perimeter, P, of the projected area. Circularity has the advantage of incorporating a measure of the roughness or surface undulations of the particle [25,27]. The sphericity concept used here comes from the ratio between the area of the particle, S_p, and the area of the smallest circle circumscribing the particle, S_c. This shape factor is a measure of the similarity of a particle to a perfect sphere.

In the absence of a direct image analyzer for the measurement of the specific surface area, SSA, we follow a theoretical approximation that can provide a good idea of this quantity for the present purposes. The procedure outline is detailed in the original work of Hunger and Brouwers [28], thus, we only present here some of the main calculations to follow the idea behind the method.

The specific surface area of a single particle (assuming that all particles are ideal spheres) is expressed as:

$$a = \frac{6m}{d\rho V} \tag{5}$$

where m is the mass of the particle, d its diameter, ρ its density, and V its volume. Thus, for each different set of particle size, the total specific surface area, SSA, of a granular material with known particle size distribution can be computed as the weighted sum of terms like the one in Eq. (5) [28]. In our case, we assume mono-sized grains. In this way, SSA is calculated as:

$$SSA = \frac{6M}{d\rho V_T}\xi \tag{6}$$

where M is the total mass of the sample, V_T its volume and ξ is a shape factor that has to be included in order to correct for the non-spherical shape of particles. According to [28], this shape factor is equal to s/ε, i.e., the ratio between the surface area of one grain, s, and the surface area, ε, for an equivalent volume sphere. One can compute s by assuming regular geometrical shapes as close as possible to the real shapes of the grains. On the other hand, considering a sphere with the same volume as that of a grain, the corresponding diameter, d_v, is resolved through Eq. (2) and used to compute ε. Finally, SSA is given by Eq. (6).

3. Experimental set up and procedure

3.1. The silo

The silo used for the discharge of the seeds consists of a cylinder made with a zinc sheet, with a diameter D_S 33 cm and a height, H_S, of 100 cm. The hoppers fitted at the bottom of the silo are either conical or flat-bottomed. The diameter of the orifice at the outlet is varied by means of an interchangeable set of hoppers. A sketch of the silo and the interchangeable hoppers is shown in Fig. 1. The discharging mass is continuously weighted by an electronic balance with a maximum capacity of 30 kg and an accuracy of 10 g.The balance is linked to a PC by a RS 232 interface. In this way, we record dynamically the weight as a function of time, with a maximum temporal resolution of 0.2 s. The mean diameter, D, of each of the outlets used in the experiments is shown in Table 1. The orifice diameters are measured using a digital caliper and averaging over five measurements taken on different equidistant directions at the orifice. All the interchangeable conical hoppers have the same height $h = 21$ cm, but a different angle α, thus varying the diameter of the outlet.

Let remember that the seeds used are wheat, soy and corn. The silo is filled with one kind of seed at a time and the mass flow vs. time is registered for a given outlet D and hopper type.

In each realization, the same amount of material is loaded using the same procedure. Both, the height and the diameter of the silo are large enough to prevent wall effects and to ensure that the flow rate is independent of the height.

Once the silo is filled, the outlet is opened, starting the discharge and the corresponding data recording. For each hopper geometry and size, five repetitions are done to average results.

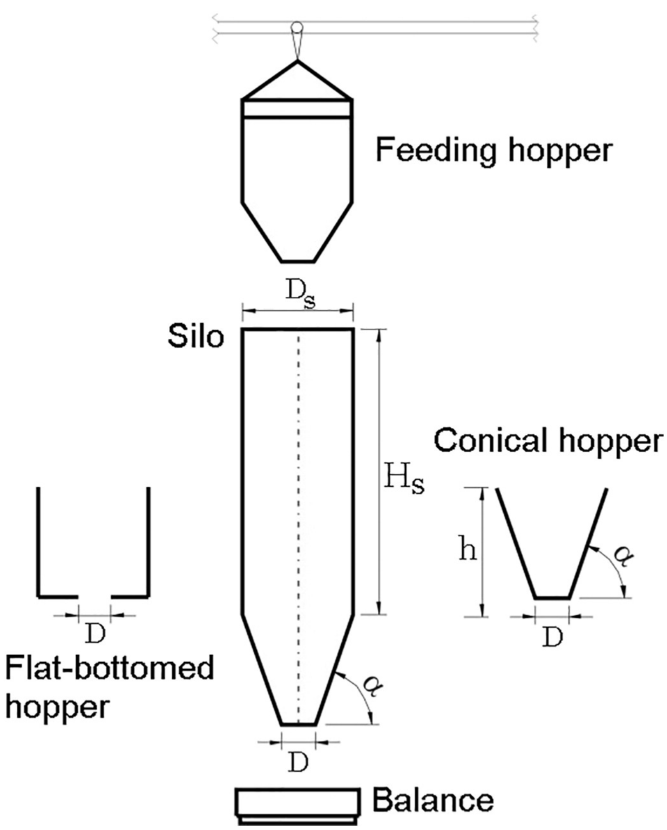

Fig. 1. Sketch of the silo with its interchangeable hoppers. Geometrical features are indicated with the corresponding letters.

Table 1
Values for the mean diameter D of the hoppers and angles for hopper walls.

Flat (cm)	Conical (cm)	Angle for conical hopper, α (°)
4.39	4.89	56.0
4.83	5.41	56.5
5.33	5.95	57.0
5.94	6.42	57.5
6.39	6.95	58.0

3.2. The grains

As said before, three kinds of grains, frequently involved in food production, are used. Their characterization is performed by first measuring their typical length, width and thickness through digital image treatment over a representative number of grains. This technique is also complemented with direct measurements with the aid of a caliper. The average values obtained for L, W_{th} and T, and the regular shapes assumed to represent each type of seed are used to calculate the diameter d_v of the particles.

On the other hand, the quantities needed to calculate CI and Ψ through Eqs. (3) and (4) are obtained by standard image processing and their average values are also shown in Table 2.

The determination of the bulk density, ρ_a, is performed through the weighing of a container with known volume into which one type of seed grains is poured from a short constant height and at a constant flow rate. The apparent or bulk density is determined as the ratio between the weighed mass and the volume. The obtained values are in Table 2.

Finally, the specific surface area for a representative set of grains of each type of seed is calculated following the procedure explained above, with the help of Eqs. (5) and (6). These results are also displayed in Table 2.

From the results of the shape factors and in coincidence with expectation, CI and Ψ increase from left to right in the table. All types of grains show a high value for circularity, even wheat, which also demonstrates to be far from a spherical shape. The values are in agreement with a direct ocular inspection of the grains.

The bulk densities for the three grains are quite comparable as it is typical for these kinds of seeds. Concerning the specific surface area, the results confirm the expectation that the more elongated particles have the largest surface area, while those for corn and soy are comparable.

4. Results and discussion

4.1. Flow and fluctuation measurements

Once a grain type is chosen, the cumulative mass discharge as a function of time is recorded for each of the outlet sizes shown in Table 1. As commonly done, the rate flow is calculated from the slope of the curves and averaged over the five equivalent realizations. Just as an example, Fig. 2 shows a typical plot for the case of mass discharge of corn through

Table 2
Values for equivalent diameter, shape factors, density and surface area, for the three seed grains. Errors are indicated.

	Wheat	Corn	Soy
Equivalent volumen diameter, d_v (cm)	0.40 ± 0.01	0.89 ± 0.01	0.69 ± 0.01
Circularity, CI	0.72 ± 0.02	0.85 ± 0.02	0.92 ± 0.02
Sphericity, Ψ	0.42 ± 0.02	0.71 ± 0.02	0.90 ± 0.02
Bulk density, ρ_a (g/cm^3)	0.80 ± 0.01	0.80 ± 0.01	0.74 ± 0.01
Specific surface area, SSA(1/mm)	1.68 ± 0.02	0.87 ± 0.02	0.88 ± 0.02

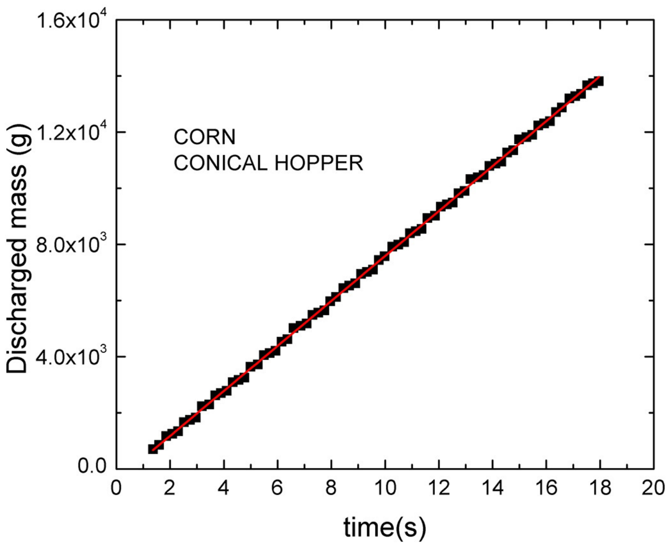

Fig. 2. Example of a typical plot of the mass discharge from the silo as a function of time. Corn seeds are used with a conical hopper with $D = 5.95$ cm. The linear fit is indicated.

a conical hopper with outlet diameter 5.95 cm. The line indicates the linear fit from which flow rate is calculated.

Although the average behavior in all the discharges is perfectly linear, fluctuations are observed in the slope of the line, i.e., flow rate oscillations are typical in these experiments [10]. During each experimental run, one can obtain the mass discharge rate by differentiating the cumulative mass with time. An example is presented in Fig. 3, where a plot of the fluctuations is displayed for the discharge of soy in a flat-bottomed hopper with outlet diameter 4.39 cm. The horizontal line is the average flow rate, approximately 292 g/s for this case. These fluctuations have been reported in former works [9–11], both experimentally and by simulations. They are associated with the formation and collapse of arches [11] and their amplitude is expected to be higher for irregular particles than for rounded particles, since the likelihood of arching is higher because of friction and irregular shape [10]. This feature is also present in our experiments, where fluctuations for wheat are a little stronger than those for soy or even corn.

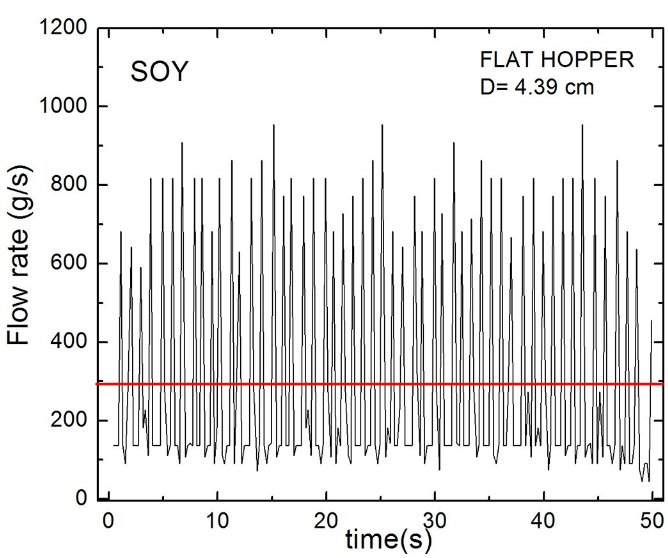

Fig. 3. The mass fluctuations during the course of a typical discharge. The horizontal line indicates the mean flow rate, approximately 292 g/s in this case.

We analyze the Fourier transform of the time series of the fluctuating mass rate to find whether a periodic component is present. The Fourier analysis of the fluctuation patterns allows obtaining the typical frequency related to the formation and destruction of arches [10,11]. An extensive explanation of the procedure and a physical interpretation of this behavior are aimed in [10].

In Figs. 4 and 5 we present the resulting frequencies from the Fourier analysis over all our present data, for the two hopper geometries, and as a function of the relative outlet diameter. This relative diameter is calculated by dividing the hopper outlet size, D, by the corresponding d_v values in order to plot all the results together. For flat-bottomed hoppers, it is clear from Fig. 4 that the frequency grows for larger outlets up to a certain relative size after which saturation is found. This effect is due to the decrease in the probability for the formation of arches that try to stop the flow.

In the case of conical hoppers, Fig. 5 demonstrates that saturation is attained before, especially for corn and soy particles. Observation of silo discharges in our experiments proofs that a mass regime is attained in most conical hoppers. Thus, low and constant arch formation likelihood at the outlet is expected [11] and, consequently, no changes are found in the fluctuation frequencies [10,11]. On the other hand, for flat-bottomed hoppers, a funnel flow regime is observed up to the bigger outlets. Thus, the structure and succeeding of arch formation change, showing a varying frequency pattern as a function of size, with lower frequencies compared to the cases of conical hoppers (see the different scales in both figures). Comparing the behavior among different seed types, wheat presents a larger relative size for saturation. This is related to the larger probability for the survival of the arches given to the geometry of the grains. This greater ease to form arches in the case of particles whose geometry is close to that of an ellipsoid has also been described by other authors [29].

4.2. Calculation of C and k

By using Eq. (1), it is straightforward to obtain parameters C and k through a linear fitting of $W^{2/5}$ vs. D. Indeed, it has been proved that appropriate values for those parameters make the Beverloo's equation still a good and simpler tool for silo designing [7,18,10,30]. Besides, to take into account the hopper angle, one has to multiplying Eq. (1) by a

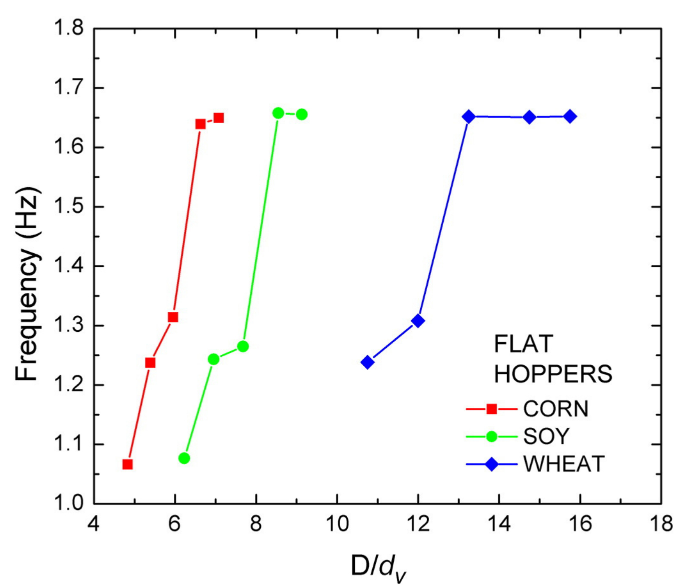

Fig. 4. Frequencies coming from the Fourier analysis over the mass fluctuations corresponding to flat-bottomed hopper discharges. The different frequencies are plotted against the ratio between outlet diameter, D, and equivalent particle diameter, d_v.

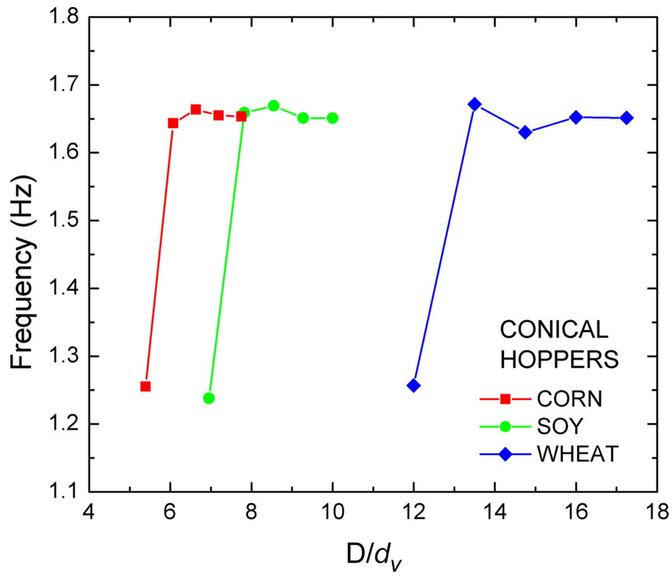

Fig. 5. Frequencies coming from the Fourier analysis over the mass fluctuations corresponding to conical hopper discharges. The different frequencies are plotted against the ratio between outlet diameter, D, and equivalent particle diameter, d_v.

function of that angle. The later analysis of the flow data would also give as a result the values for C and k that best fit the whole results but, this procedure ends up by being the same one than before, not contributing nothing relevant to our subsequent correlation with the shape of the grains.

Figs. 6 and 7 show the experimental values for the flow rates as a function of the outlet D for flat and conical hoppers, respectively. The linear fits indicated on the plots give rise to the parameter values displayed in Table 3. It should be noted that in order to calculate k from the linear fits, the equivalent diameter d_v is considered.

The values obtained for all the seeds and hoppers are inside the expected ranges found in the literature. Parameter C is quite constant for both, the type of hopper geometry and the type of seed. We should remember that a constant value of 0.58 is frequently used in most applications of the Beverloo's eq. [3,7,22]. Our present value is a little bit higher but almost constant inside the error limits. It is important to remark that

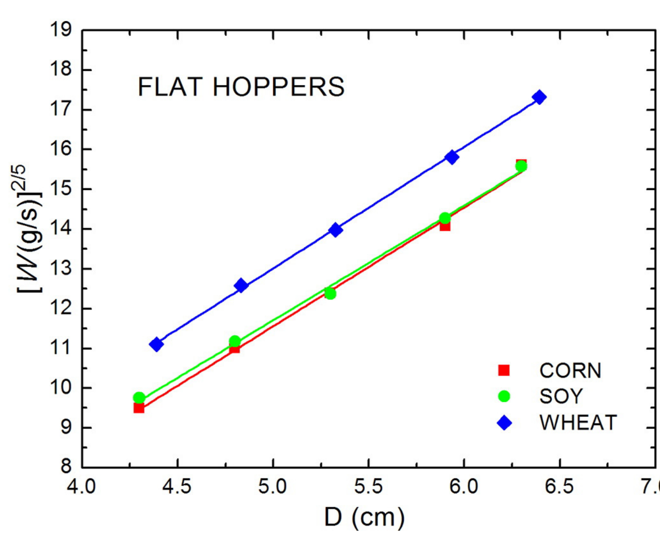

Fig. 6. Plot of the mean flow rates as a function of the outlet diameter for flat-bottomed hoppers.

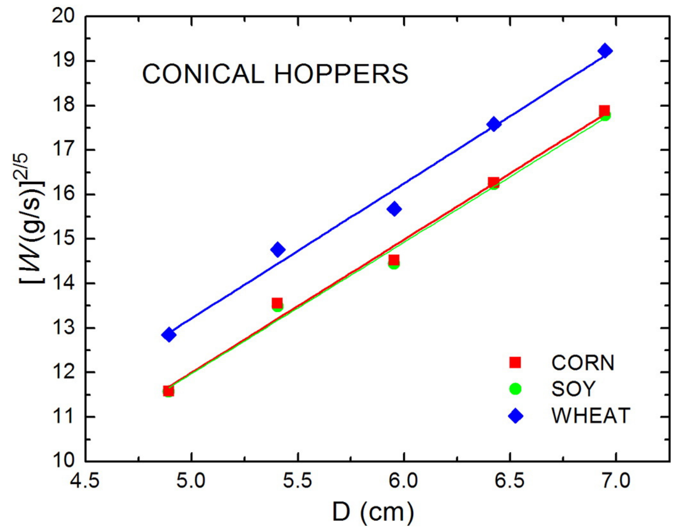

Fig. 7. Plot of the mean flow rates as a function of the outlet diameter for conical hoppers.

two features of the discharge rate are included in the parameter C: eventual angle dependence and internal friction of particles. On the one hand, in our present experiments, most of the discharging regime is mass flow or near mass flow, thus, a dependence on the inclination of the hopper walls is not expected [11]. On the other hand, the internal friction coefficients for the type of seeds used here could be related to the differences from the typical reported value of 0.58.

It is important to comment that the values obtained for k are inside the range reported in the literature [7–11,15–17,23]. It shows to be greater for flat hoppers and for the case of wheat grains. We get back to the discussion of this behavior further below.

4.3. Correlations between flow and particle shape

In what follows, we plot the values of the different shape factors and the values for k to see possible correlations between them. For a more conclusive discussion, we add to the present results previous ones corresponding to quartz particles studied in a former work [10]. Those particles belong to two different size ranges whose typical d_v values are 8.04 and 6.09 mm.

The correlation between k and d_v is presented in Fig. 8. The "Q" letters indicate the points corresponding to large and small quartz particles. The lines are fits drawn to guide the eye. A linear trend is observed where the parameter k decreases as the particle size increases. This behavior is in agreement with the fact that *SSA* also decreases, as shown in the inset of the figure. In general, the specific surface area of a grain increases due to two main reasons: the actual decrease of its volume (particle size decreases) and the increase of its angularity, where *CI* and Ψ play an important role as control parameters. In the case of soy, although its size is in between the quartz grains, it presents a quite

Table 3
Values for parameters C, k and for the elongation.

Seed-hopper	C	k	L/W_{th}
Corn-flat	0.62 (±0.01)	1.32 (±0.01)	1.3 (±0.05)
Soy-flat	0.62 (±0.01)	1.48 (±0.01)	1.1 (±0.05)
Wheat-flat	0.65 (±0.01)	1.90 (±0.01)	2.3 (±0.05)
Corn-conical	0.62 (±0.01)	1.13 (±0.01)	1.3 (±0.05)
Soy-conical	0.64 (±0.01)	1.38 (±0.01)	1.1 (±0.05)
Wheat-conical	0.64 (±0.01)	1.61 (±0.01)	2.3 (±0.05)

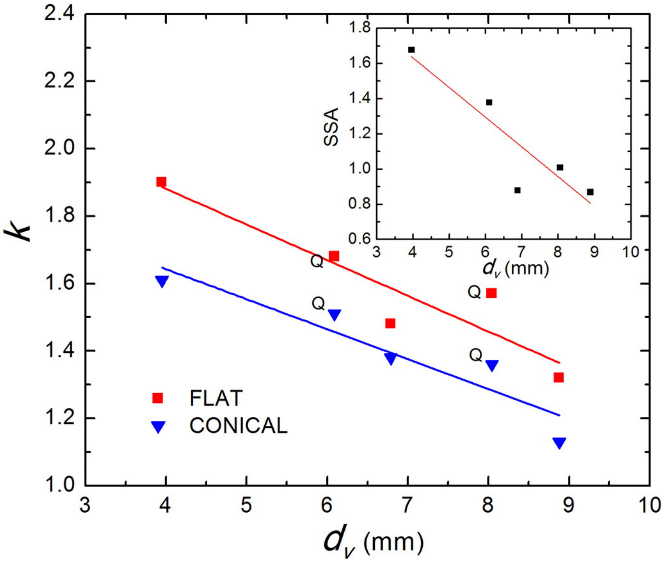

Fig. 8. Correlation between the values of parameter k and the equivalent volume diameter, d_v. Inset: specific surface area dependence on d_v. Lines are fits to see the trends. "Q" letters correspond to values for quartz particles.

lower *SSA* due to its high circularity and sphericity. Data in Fig. 9 help to see this point. Both parts of this figure show the decaying behavior of *SSA* as the particles are closer to spherical objects. The lines indicate exponential fits performed to guide the eye. Circularity shows to be closer to an exponential decay. In this way, the apparent singular behavior of soy in Fig. 8 can be understood.

It is interesting to comment that the values obtained for k are greater for elongated particles (see Table 3). On the other hand, when scaling the outlet size D by the grain diameter d_v in Figs. 6 and 7, the values of the flow rate for wheat, for example, move to the right of the figure, given that d_v for these grains is the smallest. Thus, one can infer that the mass flow rate, in terms of the relative outlet diameter D/d_v, for the most elongated particles has the lowest value for the same D/d_v. Consequently, a greater value of k corresponds to a less mass flow rate. This is in correspondence with the trend reported by Mellmann et al. where the Froude number against elongation is studied [13].

Closely connected to the above explanation for Fig. 9, the behavior observed in Fig. 10 is a consequence of the fact that an increment in the specific surface is expected when the diameter of the grains decreases and, as a result (Fig. 8), an increment in k occurs. A contribution to the physical interpretation of these results is provided by Fig. 11, where the correlation between k and the circularity is presented in part (a), while in part (b) the corresponding correlation between k and the sphericity is shown. In both cases, a parabolic fit is drawn to guide the eye. As soon as a particle increases its circularity or its sphericity, the so called "empty annulus" represented by k is smaller. In other words, the effective outlet aperture approaches to the difference between the outlet size and the particle diameter. Besides, it is important to remember that k decreases when the particle size is smaller (Fig. 8) [18]. Nevertheless, the fact that soy has a greater size but higher circularity and sphericity, makes that k presents a non-monotonous trend with both shape factors. In other words, the behavior of k is the result of the competition of two effects: its growth with *SSA* and its decrease with the size of the grain. As soon as both effects are present in a silo discharge, the correlation of k with the circularity and/or the sphericity is not straightforward. This explains the presence of the minimum values in Fig. 11. As in Fig. 9, we observe that the values for Ψ are more scattered. This could be due to the necessary assumptions made on the 3D shape of the seeds and the quartz particles, which affect the calculation of sphericity more than that of circularity.

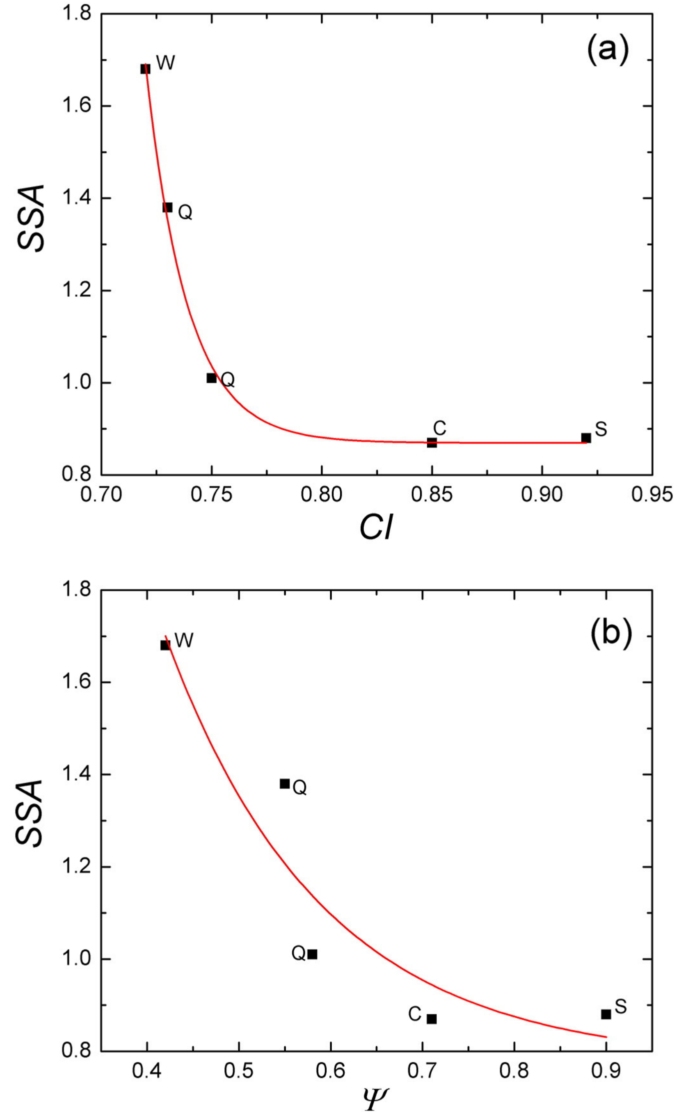

Fig. 9. Correlation between specific surface area and: (a) the circularity of the particles; (b) the sphericity of the particles. The lines are to guide the eye. The letters help distinguishing each type of grain.

5. Conclusions

The experiments performed here for silo discharging with different type of seeds allow correlating Beverloo's equation parameters with those related to the geometry of the grains. From the results and discussion presented so far, we can draw some interesting conclusions that are important for a silo design.

The fact that parameter C in Beverloo's equation is not too sensitive to the hopper geometry or to the type of seed is a practical and useful feature for designing silos in the agro food industry, where many production steps use the same type of silo for different type of seeds. In this way, when mass flow is involved, a constant value for C can be assumed and the geometrical aspects given by the specific surface area and shape factors can be included in the parameter k, whenever appreciable differences do not exist in the seeds used. This is an advantage because the determination of C with the help of a few experimental runs turns the Beverloo's equation into a one parameter equation, as proposed by other authors [11].

All along the geometrical analysis of the particles, we note that the shape assumed for the seeds (ellipsoids, spheres, parallelepipeds) is crucial when setting the correlation with k. A too simplified shape

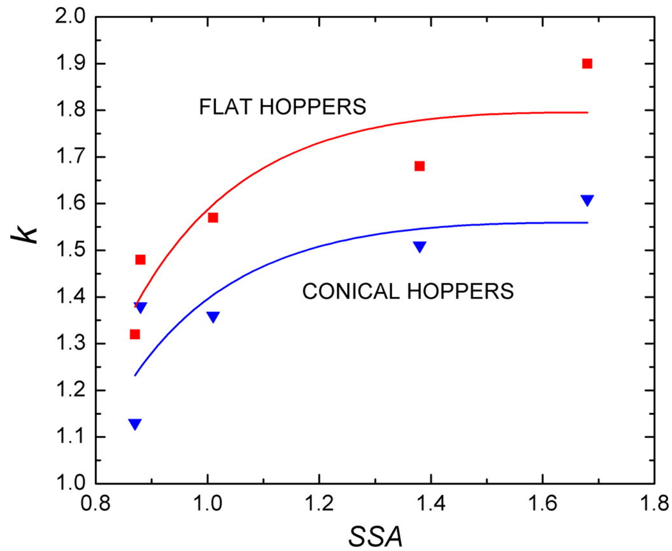

Fig. 10. Behavior of k for both types of hoppers as the specific surface area changes. The lines are to see the trends.

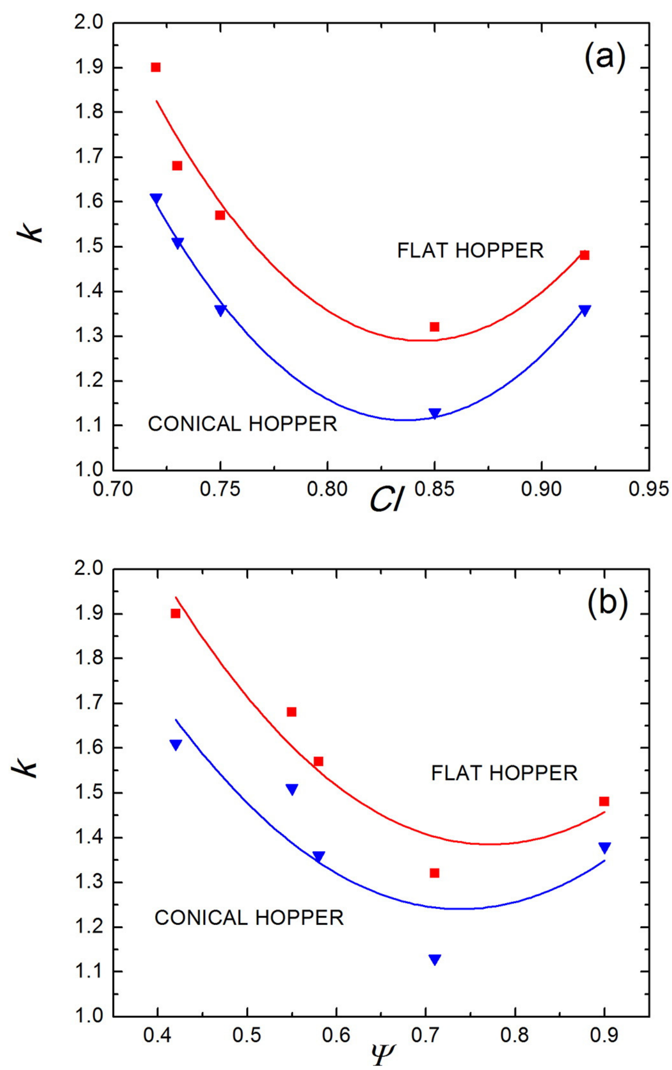

Fig. 11. Correlation between k and (a) the circularity, (b) the sphericity, for both types of hoppers. The lines represent parabolic fits.

assumption could drive to a not so obvious correlation between the parameters involved. Noting this, our results demonstrate that k can be successfully correlated with the size, circularity and specific area of the grains. When the size of the grains increases, k decreases. On the other hand, as the specific surface area increases, k also increases. This gives rise to a particular non-monotonous behavior when k is plotted against circularity. Thus, size and surface area have to be taken into account when predictions for the behavior of silo discharge are handled through the "empty annulus" concept introduced in Beverloo's equation through the parameter k. As a consequence, the circularity (or sphericity) of the grains, alone, is not enough to predict the value of k in a grain discharging process.

Nomenclature

A_p	projected area (*mm*^{2})
C	empirical constant in Beverloo's equation (*dimensionless*)
CI	circularity (*dimensionless*)
d	particle diameter (*mm*)
d_v	equivalent volume diameter (*mm*)
D	outlet diameter of the hopper (*cm*)
D_S	silo's diameter (*cm*)
g	gravity acceleration (*cm/s*^{2})
h	hopper's height (*cm*)
H_S	silo's height (*cm*)
k	Beverloo's constant (*dimensionless*)
L	particle's length (*mm*)
m	particle's mass (*g*)
M	sample mass (*g*)
P	perimeter (*mm*)
s	particle surface area (*mm*^{2})
S_p	particle's area (*mm*^{2})
S_c	circumscribed circle area (*mm*^{2})
SSA	specific surface area (1/*mm*)
T	particle's thickness (*mm*)
V	particle's volume (*mm*^{3})
V_T	sample's volume (*mm*^{3})
W	discharge rate (*g/s*)
W_{flat}	discharge rate in a flat hopper (*g/s*)
W_{th}	particle's width (*mm*)

Greek

α	hopper's angle from the horizontal (°)
ε	equivalent volume sphere surface area (*mm*^{3})
ρ	density (*g/mm*^{3})
ρ_a	bulk density (*g/mm*^{3})
ξ	shape factor (*dimensionless*)
Ψ	sphericity (*dimensionless*)

Acknowledgements

Authors want to acknowledge financial support from Universidad Nacional de San Luis through PROICO 310114 and from CONICET through PIP 353. Especial thanks are due to the professional support of Engineer V. Schmidt in the characterization of the grains.

References

[1] N. Job, A. Dardenne, J.-P. Pirard, Silo flow-pattern diagnosis using the tracer method, J. Food Eng. 91 (2009) 118–125.
[2] J.J. Fitzpatrick, T. Iqbal, C. Delaney, T. Twomey, M.K. Keogh, Effect of powder properties and storage conditions on the flowability of milk powders with different fat contents, J. Food Eng. 64 (2004) 435–444.
[3] J.R. Johanson, Method of calculating rate of discharge from hoppers and bins, Trans. Soc. Min. Eng. AIME 232 (1965) 69–80.
[4] R.M. Nedderman, C. Laohakul, The thickness of the shear zone of flowing granular media, Powder Technol. 25 (1980) 91–100.

[5] G.H. Ristow, Outflow rate and wall stress for two-dimensional hoppers, Phys. A 235 (1997) 319–326.
[6] C. Mankoc, A. Janda, R. Arévalo, J.M. Pastor, I. Zuriguel, A. Garcimartín, D. Maza, The flow rate of granular materials through an orifice, Granul. Matter 9 (2007) 407–414.
[7] A. Anand, J.S. Curtis, C.R. Wassgren, B.C. Hancock, W.R. Ketterhagen, Predicting discharge dynamics from a rectangular hopper using the discrete element method (DEM), Chem. Eng. Sci. 63 (2008) 5821–5830.
[8] J. Wu, J. Binbo, J. Chen, Y. Yang, Multi-scale study of particle flow in silos, Adv. Powder Technol. 20 (2009) 62–73.
[9] C. González-Montellano, A. Ramírez, E. Gallego, F. Ayuga, Validation and experimental calibration of 3D discrete element models for the simulation of the discharge flow in silos, Chem. Eng. Sci. 66 (2011) 5116–5126.
[10] R.O. Uñac, O.A. Benegas, A.M. Vidales, I. Ippolito, Experimental study of discharge rate fluctuations in a silo with different hopper geometries, Powder Technol. 225 (2012) 214–220.
[11] I. Oldal, I. Keppler, B. Csizmadia, L. Fenyvesi, Outflow properties of silos: the effect of arching, Adv. Powder Technol. 23 (2012) 290–297.
[12] P. Parafiniuk, M. Molenda, J. Horabik, Discharge of rapeseeds from a model silo: physical testing and discrete element method simulations, Comput. Electron. Agric. 97 (2013) 40–46.
[13] J. Mellmann, T. Hoffmann, C. Fürll, Mass flow during unloading of agricultural bulk materials from silos depending on particle form, flow properties and geometry of the discharge opening, Powder Technol. 253 (2014) 46–52.
[14] S.D. Liu, Z.Y. Zhou, R.P. Zou, D. Pinson, A.B. Yu, Flow characteristics and discharge rate of ellipsoidal particles in a flat bottom hopper, Powder Technol. 253 (2014) 70–79.
[15] W.A. Beverloo, H.A. Leniger, J. van de Velde, The flow of granular solids through orifices, Chem. Eng. Sci 15 (1961) 260–269.
[16] B.J. Crewdson, A.L. Ormond, R.M. Nedderman, Air-impeded discharge of fine particles from a hopper, Powder Technol. 16 (1977) 197–207.
[17] P. Arteaga, U. Tüzün, Flow of binary mixtures of equal density gradients in hoppers, Chem. Eng. Sci. 45 (1990) 205–223.
[18] S. Humby, U. Tüzün, A.B. Yu, Prediction of hopper discharge rates of binary granular mixtures, Chem. Eng. Sci. 53 (1998) 483–494.
[19] A.P. Huntington, N.M. Rooney, Chemical Engineering Tripos Part 2, Research Project Report, University of Cambridge, 1971.
[20] R.L. Brown, J.C. Richards, Principles of Powder Mechanics, Pergamon Press, London, 1970.
[21] M.E. Myers, M. Sellers, Chemical Engineering Tripos Part 2, Research Project Report, University of Cambridge, 1971.
[22] P. Mort, J.N. Michaels, R.P. Behringer, C.S. Campbell, L. Kondic, M. Kheiripour Langroudi, M. Shattuck, J. Tang, G.I. Tardos, C. Wassgrenk, Dense granular flow — a collaborative study, Powder Technol. 284 (2015) 571–584.
[23] P. Toson, J.G. Khinast, Impulse-based dynamics for studying quasi-static granular flows: application to hopper emptying of non-spherical particles, Powder Technol. 313 (2017) 353–360.
[24] R.M. Nedderman, U. Tüzün, S.B. Savage, G.T. Houlsby, The flow of granular materials – I. Discharge rates from hoppers, Chem. Eng. Sci. 37 (1982) 1597–1609.
[25] D.A. Robinson, S.P. Friedman, Observations of the effects of particle shape and particle size distribution on avalanching of granular media, Phys. A 311 (2002) 97–110 (and references there in).
[26] F. Podczeck, A shape factor to assess the shape of particles using image analysis, Powder Technol. 93 (1997) 47–53.
[27] P.A. Arias García, R.O. Uñac, A.M. Vidales, A. Lizcano, Critical parameters for measuring angles of stability in natural granular materials, Phys. A 390 (2011) 4095–4104.
[28] M. Hunger, H.J.H. Brouwers, Flow analysis of water–powder mixtures: application to specific surface area and shape factor, Cem. Concr. Compos. 31 (2009) 39–59.
[29] P.W. Cleary, M.L. Sawley, DEM modelling of industrial granular flows: 3D case studies and the effect of particle shape on hopper discharge, Appl. Math. Model. 26 (2002) 89–111.
[30] T.M. Verghese, R.M. Nedderman, The discharge of fine sands from conical hoppers, Chem. Eng. Sci. 50 (1995) 3143–3153.