Zero permeability and zero permittivity band gaps in 1D metamaterial photonic crystals

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Abstract

We consider layered heterostructures combining ordinary positive index materials and dispersive metamaterials. We show that these structures can exhibit a new type of photonic gap around frequencies where either the magnetic permeability μ or the electric permittivity ϵ of the metamaterial is zero. Although the interface of a semi-infinite medium with zero refractive index (a condition attained either when $\mu = 0$ or when $\epsilon = 0$) is known to give full reflectivity for all incident polarizations, here we show that a gap corresponding to $\mu = 0$ occurs only for TE polarized waves, whereas a gap corresponding to $\epsilon = 0$ occurs only for TM polarized waves. These band gaps are scale-length invariant and very robust against disorder, although they may disappear for the particular case of propagation along the stratification direction.
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1. Introduction

Photonic band gap (PBG) materials allow electromagnetic field propagation in certain frequency bands, but not in others, their essential feature being the periodic arrangement of high-contrast electromagnetic properties. The simplest version of a PBG material is the 1D planar layered stack, which has been commonly used in optics in the form of filters or Bragg reflectors [1].

Almost every available natural material has been used to construct PBG structures. Possibilities have been widened with the recent advent of metamaterials (MMs), artificially constructed composites exhibiting electromagnetic properties that are difficult or impossible to achieve with conventional, naturally occurring materials [2]. Key representatives of this new class of materials are MMs with negative index of refraction, a property that arises in media with a negative electric permittivity together with a negative magnetic permeability in the same frequency range [3]. Multilayered PBG structures containing MMs have attracted much attention in recent years [4–7]. Two new kinds of PBG, fundamentally different from the usual Bragg gaps originating from interference in the periodic structure, have been identified: the "zero averaged refractive index" gap [5] and the "zero effective phase" gap [6]. The first one appears in multilayers combining ordinary materials (positive refractive index) and MMs with negative refractive index, whereas the second one appears in multilayers containing two different single-negative (permittivity- or permeability-negative) MMs. In contrast to Bragg PBGs, both the "zero averaged refractive index" and the "zero effective phase" gaps

remain invariant to scale-length changes and are robust against disorder. PBGs with these characteristics can be useful as photonic barriers in quantum well applications, such as multiple channeled filtering [6].

Since Bragg PBGs result from interference between waves reflected at different layers, material dispersion does not play a crucial role in their appearance. In other words, Bragg PBGs in 1D multilayers always exist, even when the dispersion of the constituent materials is neglected. On the other hand, the existence of a "zero averaged refractive index" PBG highly relies on the dispersive characteristics of the material layers, since for nondispersive materials this PBG would cover all frequencies, except for singular points [5]. In this Letter we show that MM multilayers can also exhibit a new kind of PBG, where material dispersion—and not interference—plays the key role. These new PBGs correspond to frequency regions around the point where one of the constitutive parameters of the MM is zero, i.e., we consider that the refractive index of one of the materials of our 1D periodic multilayered structure changes its sign at a certain frequency. The MM multilayered structure under consideration is related to a layered electron gas, whose dispersion characteristics were analyzed in plasma physics (see, for instance, [8]). This situation differs fundamentally from that of a multilayer whose unit cell combines two materials in such a way that the average of the refractive index is zero. Since they are not based on the Bragg interference mechanism, they are also invariant to scaling and even more insensitive to disorder than the "zero averaged refractive index" and the "zero effective phase" gaps, as we discuss later.

2. Band structure of 1D metamaterial photonic crystals

If losses are neglected, the refractive index of a MM $n(\omega) = \sqrt{\mu(\omega)\epsilon(\omega)}$ is zero at the values of the frequency ω where the magnetic permeability $\mu(\omega)$ or the electric permittivity $\epsilon(\omega)$ change their sign. Therefore, at these frequency values one could expect to find gaps for both TE and TM polarization, since it is well known that the boundary of a semi-infinite zero refractive index material exhibits full reflectivity [9,10]. Contrary to expectation, we show that at frequencies where $\mu = 0$ a gap occurs for TE polarized waves, but not necessarily for TM polarized waves, whereas at frequencies where $\epsilon = 0$ a gap occurs for TM polarized waves, but not necessarily for TE polarized waves. Moreover, we show that these gaps can be completely absent for the particular case of propagation along the stratification direction, which could explain why its presence went unnoticed in previous studies (see, for example, Fig. 2 in [5]).

Although these new gaps can occur in aperiodic structures, we study a periodic stack in order to make evident their coexistence with previously reported PBGs. With this aim, we consider the 1D periodic structure created by alternated layers of two different materials: a conventional dielectric with permeability μ_1, permittivity ϵ_1 and thickness d_1, and a MM with permeability $\mu_2(\omega)$, permittivity $\epsilon_2(\omega)$ and thickness d_2. The stratification direction is the y axis and we consider wave propagation in the $x-y$ plane. Let the function $f(x, y)$ represent the z-directed component of the electric field for the TE-polarization case (electric field parallel to the layers), and the z-directed component of the magnetic field for the TM-polarization case (magnetic field parallel to the layers). The propagating waves in the periodic structure have the form of Bloch modes, for which the fields satisfy the condition $f(x, y + d) = f(0, y) \exp i(k_x x + Kd)$, where $d = d_1 + d_2$ is the period of the structure, k_x is the wave vector component along the layers and K is the Bloch wave number. For bi-layer periodic structures, the dispersion relation $K(\omega, k_x)$ can be found explicitly from [1]

$$\cos(Kd) = \xi$$
$$\equiv \cos(k_{1y}d_1)\cos(k_{2y}d_2)$$
$$- \frac{1}{2}\left[\frac{\sigma_2 k_{1y}}{\sigma_1 k_{2y}} + \frac{\sigma_1 k_{2y}}{\sigma_2 k_{1y}}\right]\sin(k_{1y}d_1)\sin(k_{2y}d_2), \quad (1)$$

where the index $j = 1, 2$ indicates the layer, $\sigma_j = \mu_j$ for TE polarization or $\sigma_j = \epsilon_j$ for TM polarization, $k_{jy}^2 = k_j^2 - k_x^2$, and $k_j = \omega n_j/c$ are wave numbers in each medium with refractive indexes n_j. The quantity ξ, half the trace of the matrix characterizing the unit cell translation operator [1], determines the band structure. It takes real values for lossless media and real k_x. Regimes where $|\xi| < 1$ correspond to real K and thus to propagating Bloch waves. In regimes where $|\xi| > 1$, K has an imaginary part, therefore the Bloch wave is evanescent, and this regime corresponds to forbidden bands (or gaps) of the periodic medium. The band edges are where $|\xi| = 1$.

Let us consider frequency regions where the refractive index $n_2(\omega)$ goes through zero, that is, where a constitutive parameter of the MM—either $\mu_2(\omega)$ or $\epsilon_2(\omega)$—changes its sign. Note that the term between square brackets in the right hand side of Eq. (1) becomes singular under these conditions. For propagation along the stratification direction ($k_x = 0$), $k_{2y} \to 0$ and the singularity is compensated. To be specific, and taking into account that μ_2 and ϵ_2 do not generally change their sign at the same frequency value, we assume that $\epsilon_2 \to 0$ but $\mu_2 \neq 0$. In this limit, $k_{2y} \to 0$ and the quantity ξ for both TE and TM polarizations adopts the form

$$\xi \approx \cos(k_1 d_1) - \frac{1}{2}\frac{\omega}{c}\sqrt{\frac{\epsilon_1}{\mu_1}}\mu_2 d_2 \sin(k_1 d_1), \quad (2)$$

which can take any real value, either outside or inside the interval $[-1, 1]$. A similar conclusion can be obtained when $\mu_2 \to 0$ but $\epsilon_2 \neq 0$. Thus, gaps near regions where a constitutive parameter changes its sign may not appear at normal incidence. It depends on the parameter values that determine the structure. For example, the multilayer considered to obtain Fig. 2 in Ref. [5], namely, with the following geometric and constitutive parameters, $d_1 = 12$ mm, $d_2 = 6$ mm, $\epsilon_1 = \mu_1 = 1$, and

$$\epsilon_2(f) = 1 + \frac{5^2}{0.9^2 - f^2} + \frac{10^2}{11.5^2 - f^2},$$
$$\mu_2(f) = 1 + \frac{3^2}{0.902^2 - f^2}, \quad (3)$$

where f is the frequency measured in GHz, exhibits Bragg gaps and a "zero averaged refractive index" gap. However there is no
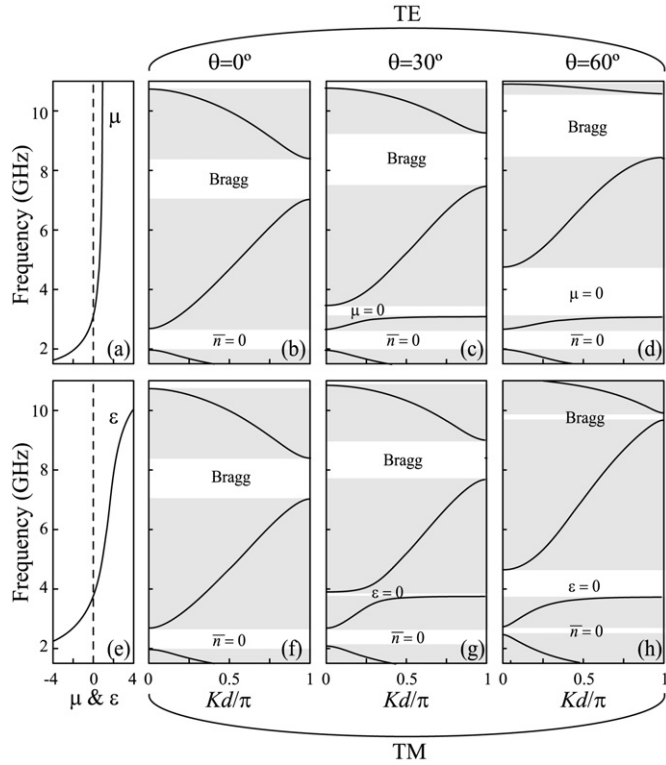
Fig. 1. Band structure for TE and TM polarizations and different angles of incidence corresponding to a periodic stack with alternating air layers ($\mu_1 = \epsilon_1 = 1$, $d_1 = 12$ mm) and MM layers (μ_2 and ϵ_2 given by Eq. (3), $d_2 = 6$ mm). The left column shows the frequency behavior of the constitutive parameters corresponding to layer 2.

gap around the regions where μ_2 and ϵ_2 change sign. This can be observed in the band diagrams shown in Figs. 1(b) and (f), obtained for $k_x = 0$.

For oblique incidence, on the other hand, $k_x \neq 0$, $k_{2y} \to \pm i k_x \neq 0$ and the second term in the right-hand side of Eq. (1) remains singular when $\mu_2 = 0$ (TE polarization) or when $\epsilon_2 = 0$ (TM polarization), thus originating new gaps, as shown in Fig. 1 for $\theta = 30°$ and $60°$ ($k_x = k_1 \sin\theta$). The nonsingular behavior of ξ at normal incidence and its singular behavior for oblique incidence is illustrated in Fig. 2 for both polarizations. We conclude that these new gaps always appear for nonzero propagation angles θ in layer 1: the $\mu_2 = 0$ gap appears for TE, but not for TM, polarization; whereas the $\epsilon_2 = 0$ gap appears for TM, but not for TE, polarization.

The projected band structure corresponding to this example is shown in Fig. 3. White regions indicate forbidden bands where there are no electromagnetic modes, regardless of K. The width of the "zero averaged refractive index" gap does not change appreciably for TE polarization, but it gets narrower when $\theta \to 90°$ for TM polarization. On the other hand, the new $\mu_2 = 0$ and $\epsilon_2 = 0$ gaps increase with the angle θ.

3. Transmission properties

Like the "zero averaged refractive index" gap [5] and the "zero effective phase" gap [6], the gap arising from $\mu_2 = 0$ or $\epsilon_2 = 0$ differs fundamentally from usual Bragg gaps. First, its
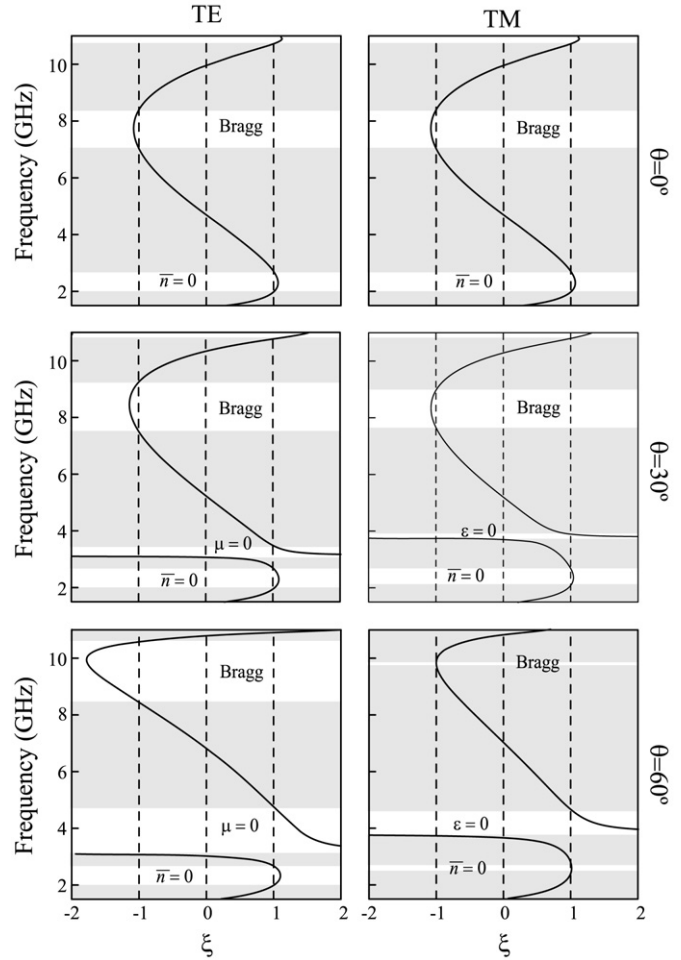

Fig. 2. Half of the trace of the unit cell translation matrix, ξ, as given by Eq. (1), for the structure considered in Fig. 1, and $\theta = 0°$, $30°$, and $60°$. The left column corresponds to TE polarization, whereas the right column corresponds to TM polarization.
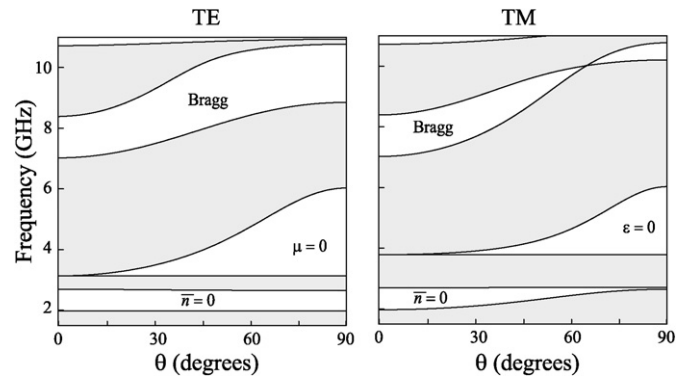

Fig. 3. Projected band structure for the MM multilayer considered in Fig. 1.

central frequency is independent of the lattice constant, while all Bragg gap frequencies must scale with the lattice constant. This fact is shown in Fig. 4, where we compare the transmittance through 16 unit cells corresponding to the periodic multilayer considered in Fig. 3, for $\theta = 45°$, with the transmittance obtained for similar structures with lattice constants scaled by factors of 2/3 and 4/3. While the Bragg gap shifts upward or
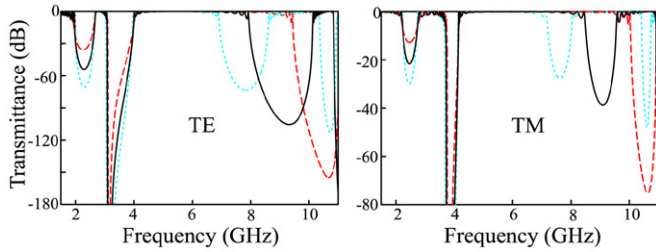
Fig. 4. TE and TM transmittance through 16 unit cells corresponding to the band structure in Fig. 3 for $\theta = 45°$ (solid line), the same structure but the lattice constant scaled by 2/3 (dashed line) and scaled by 4/3 (dotted line).
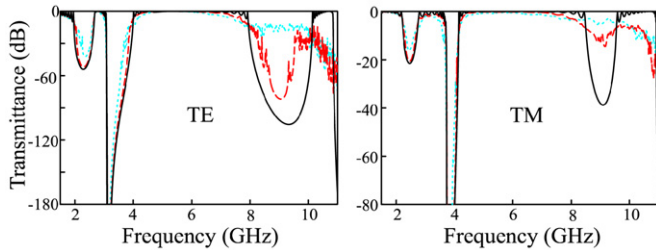

Fig. 5. TE and TM transmittance through 16 unit cells corresponding to the perfectly periodic multilayer considered in Fig. 3 for $\theta = 45°$ (solid line), to a similar structure with thickness fluctuation of ± 3 mm (dashed line), and thickness fluctuation of ± 6 mm (dotted line), averaged over 24 samples.

downward in frequency, the "zero averaged refractive index" gap—near 2.3 GHz (TE) and 2.55 GHz (TM)—and the new gaps—near 3.55 GHz (TE) and 3.9 GHz (TM)—remain unchanged. Second, the Bragg gap, relying on interference mechanisms, is destroyed by strong disorder, while the $\mu_2 = 0$ and the $\epsilon_2 = 0$ gaps, relying on constitutive properties of the MM, should be expected to be robust against disorder. In Fig. 5 we compare the transmittance through 16 unit cells corresponding to the periodic multilayer considered in Fig. 3 for $\theta = 45°$ and to similar structures with random thickness fluctuations of ± 3 mm and ± 6 mm, each ensemble averaged over 24 realizations. As expected, the Bragg gap is destroyed by disorder, but our "zero permeability" and "zero permittivity" gaps survive.

4. Conclusions

To summarize, we have shown that periodic multilayers containing MMs can exhibit a new type of photonic gap near frequencies where either the magnetic permeability μ or the electric permittivity ϵ of the MM changes sign. In contrast to Bragg PBGs, originating from interference mechanisms, it is

the material dispersion of the MM that mainly determines the appearance of these new gaps. Therefore, like the "zero averaged refractive index" and the "zero effective phase" gaps, the new gaps also remain invariant to scaling and insensitive to disorder. We point out that the physical origin of the "zero permeability" and "zero permittivity" PBGs is radically different from the rest of the above mentioned gaps and consequently their appearance and behavior are determined by different rules. In this sense, although electromagnetic waves would be completely reflected at the boundary of a semi-infinite medium with zero refractive index, our results show that multilayers with MM constituents do not automatically exhibit PBGs at frequencies where the refractive index is zero: for propagation along the stratification direction, zero refractive index PBGs may be completely absent, whereas for oblique propagation they can emerge for different polarizations, depending on which of the constitutive parameters $\mu(\omega)$ or $\epsilon(\omega)$ makes the refractive index zero. It is noteworthy that the above results are preserved even when we consider metamaterials exhibiting moderate losses. This situation will be discussed in detail elsewhere.

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