Eur. Phys. J. A **31**, 687–690 (2007)
DOI 10.1140/epja/i2006-10229-y

THE EUROPEAN PHYSICAL JOURNAL A

Special Article – QNP 2006

Excited baryons and heavy pentaquarks in large-N_c QCD

D. Pirjol[1] and C. Schat[2,3,a]

[1] Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[2] CONICET and Departamento de Física, FCEyN, Universidad de Buenos Aires, Ciudad Universitaria, Pab. 1, (1428) Buenos Aires, Argentina
[3] Departamento de Física, Universidad de Murcia, E30071 Murcia, Spain

Received: 8 November 2006
Published online: 8 March 2007 –

Abstract. We briefly discuss the large-N_c picture for excited baryons, present a new method for the calculation of matrix elements and illustrate it by computing the strong decays of heavy exotic states.

PACS. 11.15.Pg Expansions for large numbers of components (*e.g.*, $1/N_c$ expansions) – 14.20.-c Baryons (including antiparticles)

1 Introduction

The $1/N_c$ expansion of QCD has turned out to be a fruitful approach to its non-perturbative regime, as is shown by many examples [1]. The successful applications to the study of ground-state baryons make the excited baryons and exotic states especially interesting because they provide a wider testing ground for the $1/N_c$ expansion.

It is useful to recall a few general facts that make the large number of colors limit interesting and useful:

– The $1/N_c$ expansion is the only candidate for a perturbative expansion of QCD at all energies.
– In the $N_c \to \infty$ limit baryons fall into irreducible representations of the *contracted* spin-flavor algebra $SU(2n_f)_c$, also known as $\mathcal{K}$-symmetry, that relates properties of states in different multiplets of flavor symmetry.
– The breaking of spin-flavor symmetry can be studied order by order in $1/N_c$ as an operator expansion.

It is important to stress that already at leading order in the large-N_c limit it is possible to obtain significant insights into the structure of excited baryons, among which we would like to highlight the following:

– The three towers [2–4] predicted by $\mathcal{K}$-symmetry for the $L = 1$ negative-parity N^* baryons, labeled by $\mathcal{K} = 0, 1, 2$ with $\mathcal{K}$ related to the isospin I and spin J of the N^*'s by $\mathcal{K} \geq |I - J|$.
– The vanishing of the strong-decay width $\Gamma(N^*_{\frac{1}{2}} \to [N\pi]_S)$ for $N^*_{\frac{1}{2}}$ in the $\mathcal{K} = 0$ tower, which provides a natural explanation for the relative suppresion of pion decays for the $N^*(1535)$ [2,4,5].
– The order $\mathcal{O}(N_c^0)$ mass splitting of the $SU(3)$ singlets $\Lambda(1405)$-$\Lambda(1520)$ in the $[\mathbf{70}, 1^-]$ multiplet [6].

The general framework is based on the observation that at the fundamental level of QCD diagrams can be classified according to their scaling with N_c. Planar diagrams are the leading order, non-planar diagrams and quark loops are subleading in $1/N_c$. In order to obtain finite amplitudes the quark-gluon coupling constant must scale as $g \propto N_c^{-1/2}$. An m-body operator requires at least the exchange of $m - 1$ gluons which gives a suppression factor of N_c^{1-m}. However, the matrix elements of an operator can eventually be enhanced by coherence effects, as is the case of G^{ia} defined below[1]. In an explicit quark operator representation different hadronic operators like the masses, magnetic moments, axial currents, etc., can be expanded [7] in $1/N_c$. For example, for the mass operator we have schematically

$$\hat{M} = \sum_{k=0}^{N_c} \frac{1}{N_c^{k-1}} C_k \mathcal{O}_k \qquad (1)$$

with $\mathcal{O}_k$ a k-body operator. Both the coefficients C_k (which correspond to reduced matrix elements of QCD operators) and the matrix elements of the quark operators on baryon states $\langle \mathcal{O}_k \rangle$ have power expansions in $1/N_c$ with coefficients determined by nonperturbative dynamics. The basic building blocks to construct the $\mathcal{O}_k$ are the

[a] e-mail: schat@df.uba.ar

[1] $\langle G^{ia} \rangle \propto N_c$ when restricted to the subspace of states with spin and isospin of order N_c^0, which are the ones that will correspond to the $N_c = 3$ physical states.

generators of $SU(2n_f)$, where n_f is the number of flavors

$$S^i = \sum_{\alpha=1}^{N_c} s^i_{(\alpha)}, \quad T^a = \sum_{\alpha=1}^{N_c} t^a_{(\alpha)}, \quad G^{ia} = \sum_{\alpha=1}^{N_c} s^i_{(\alpha)} t^a_{(\alpha)} . \quad (2)$$

In the large-N_c limit we can define $X^0_{ia} \equiv \lim_{N_c\to\infty} \frac{G_{ia}}{N_c}$, because the matrix elements of G_{ia} scale like N_c for the states of interest, which is the coherence effect mentioned before. In this way we obtain for $n_f = 2$ the contracted algebra $SU(4)_c$

$$\begin{aligned} &[S_i, S_j] = i\epsilon_{ijk} S_k , && [S_i, X^0_{ja}] = i\epsilon_{ijk} X^0_{ka}, \\ &[T_a, T_b] = i\epsilon_{abc} T_c , && [T_a, X^0_{ib}] = i\epsilon_{abc} X^0_{ic} , \\ &[X^0_{ia}, X^0_{jb}] = 0. && \end{aligned} \quad (3)$$

The last commutation relations can also be obtained in a purely hadronic language. They are known as consistency relations [7] and are necessary to obtain finite amplitudes for pion-nucleon scattering. Consider the direct and crossed diagrams that contribute at tree level. The pion-nucleon coupling scales like $\sqrt{N_c}$, which makes each diagram separately to scale like N_c. To obtain a finite amplitude for the physical process we need a cancellation to happen. This requires $[X^0_{ia}, X^0_{jb}] = \mathcal{O}(1/N_c)$, which in the large-$N_c$ limit gives eq. (3). This symmetry structure gives rise to model-independent predictions like the three towers for excited baryons that was mentioned above. In an explicit quark operator representation this is manifest by the presence of two $\mathcal{O}(N_c^0)$ operators (that also involve the generator of $O(3)$ [8]) and has been checked by an explicit calculation [3,4].

2 Occupation number formalism

In this section we give an outline of the occupation number formalism [9] that we use to compute matrix elements for arbitrary N_c. In broken $SU(3)$, the $SU(6)$ spin-flavor generators can be decomposed into generators of the subgroup

$$\begin{aligned} &SU(6)_{SF} \supset SU(4)_{SI} \otimes SU(2)_{J_s} \otimes U(1)_{n_s} \\ &J^i, \qquad I^a = T^a, \qquad G^{ia} = G^{ia} \qquad (i, a = 1, \ldots, 3) , \\ &J^i_s = s^\dagger \frac{\sigma^i}{2} s, \qquad N_s = s^\dagger s \end{aligned}$$

plus operators mediating transitions between sectors of different n_s

$$\begin{aligned} &\tilde{t}^\alpha = q^{\dagger\alpha} s, \qquad t_\alpha = s^\dagger q_\alpha \qquad (\alpha = \pm 1/2) , \\ &\tilde{Y}^{i\alpha} = q^{\dagger\alpha} \frac{\sigma^i}{2} s, \qquad Y^i_\alpha = s^\dagger \frac{\sigma^i}{2} q_\alpha . \end{aligned}$$

We introduce the "$6n$-symbol" defined as ($N = \sum_{i=1}^6 n_i$)

$$\{n_1, n_2, n_3, n_4, n_5, n_6\} = \sqrt{\frac{n_1! n_2! n_3! n_4! n_5! n_6!}{N!}} \times (u_\uparrow^{n_1} u_\downarrow^{n_2} d_\uparrow^{n_3} d_\downarrow^{n_4} s_\uparrow^{n_5} s_\downarrow^{n_6} + \text{perms}) .$$

The nonstrange states in a $\mathcal{K} = 0$ tower have spin and isopin satisfying $I = J$. Their spin-flavor symmetric wave functions can be given in closed form as

$$|I I_3 J_3; N_{ud}\rangle = \sum_i \left(\begin{matrix} \frac{N_u}{2} & \frac{N_d}{2} \\ i & J_3 - i \end{matrix} \middle| \begin{matrix} I \\ J_3 \end{matrix} \right) \times \left\{ \frac{N_u}{2} + i, \frac{N_u}{2} - i, \frac{N_d}{2} + J_3 - i, \frac{N_d}{2} - J_3 + i \right\} ,$$

where $N_{u,d}$ are the number of up and down quarks: $N_u = \frac{N_{ud}}{2} + I_3, N_d = \frac{N_{ud}}{2} - I_3$ with $N_{ud} = N_c - n_s$.

A few representative nonstrange $J_3 = +\frac{1}{2}$ states are

$$p_\uparrow = \sqrt{\frac{2}{3}} \{2, 0, 0, 1\} - \frac{1}{\sqrt{3}} \{1, 1, 1, 0\}, \quad \Delta^{++}_\uparrow = \{2, 1, 0, 0\} .$$

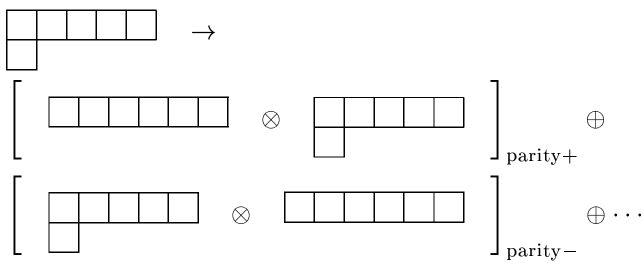
Acting with

$$\begin{aligned} &q_i \{\cdots, n_i, \cdots\} = \sqrt{n_i} \{\cdots, n_i - 1, \cdots\} , \\ &q_i^\dagger \{\cdots, n_i, \cdots\} = \sqrt{n_i + 1} \{\cdots, n_i + 1, \cdots\} \end{aligned} \quad (4)$$

we obtain the matrix elements of any operator for arbitrary N_c.

3 Pentaquark towers

For the exotic $q^{N_c+1}\bar{q}$ states with $N_c + 1$ quarks in a "$\bar{\mathbf{3}}$" of color, Fermi statistics implies the $SU(6) \otimes O(3)$ decomposition



The negative-parity states were studied in [10]. Here we reconsider the positive-parity states [11], which are all members of the two towers

$$\begin{aligned} &\mathcal{K} = 1/2 : \quad \overline{\mathbf{10}}_{\frac{1}{2}} , \quad \mathbf{27}_{\frac{1}{2},\frac{3}{2}} , \quad \mathbf{35}_{\frac{3}{2},\frac{5}{2}} , \cdots \\ &\mathcal{K} = 3/2 : \quad \overline{\mathbf{10}}_{\frac{3}{2}} , \quad \mathbf{27}_{\frac{1}{2},\frac{3}{2},\frac{5}{2}} , \quad \mathbf{35}_{\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}} , \cdots . \end{aligned}$$

In [11] only states in the first tower were considered. In the heavy-quark limit $m_Q \to \infty$ these two towers become degenerate and the tower label for the light degrees of freedom becomes a good quantum number:

$$\mathcal{K}_{light} = 1 : \ \bar{\mathbf{6}}_1 , \quad \mathbf{15}_{0,1,2} , \quad \mathbf{15'}_{1,2,3} , \quad \cdots . \quad (5)$$

On the other hand, the heavy pentaquarks considered in [11] belong to the tower

$$\mathcal{K}_{light} = 0 : \ \bar{\mathbf{6}}_0 , \quad \mathbf{15}_1 , \quad \mathbf{15'}_2 , \quad \cdots , \quad (6)$$

Table 1. Reduced matrix elements Y and large-N_c width for the $\mathcal{K}=1/2$ pentaquark $\Theta_{\bar{Q}J_\ell} \to NK, \Delta K$ decays.

Decay	$(I'J', IJ)$	$Y(I'J'\mathcal{K}', IJ\mathcal{K})$	$\frac{1}{p^3}\Gamma^{p\text{-wave}}_{N_c\to\infty}$
$\Theta_0(\frac{1}{2}) \to NK$	$(\frac{1}{2}\frac{1}{2}, 0\frac{1}{2})$	$-\frac{\sqrt{3}}{2}\sqrt{N_c+1}$	1
$\Theta_1(\frac{1}{2}) \to NK$	$(\frac{1}{2}\frac{1}{2}, 1\frac{1}{2})$	$\frac{1}{2}\sqrt{N_c+5}$	$\frac{1}{9}$
$\to \Delta K$	$(\frac{3}{2}\frac{3}{2}, 1\frac{1}{2})$	$\frac{1}{\sqrt{2}}\sqrt{N_c-1}$	$\frac{8}{9}$
$\Theta_1(\frac{3}{2}) \to NK$	$(\frac{1}{2}\frac{1}{2}, 1\frac{3}{2})$	$-\sqrt{2}\sqrt{N_c+5}$	$\frac{4}{9}$
$\to \Delta K$	$(\frac{3}{2}\frac{3}{2}, 1\frac{3}{2})$	$-\frac{1}{2}\sqrt{\frac{5}{2}}\sqrt{N_c-1}$	$\frac{5}{9}$

which arises naturally in the Skyrme model.

As an example we compute the strong decays of the $\mathcal{K}=1/2$ states in [11]. The reduced matrix elements of the transition operator are defined by

$$\langle I'I_3', J'J_3'; n_s-1|Y^{i\alpha}|II_3, JJ_3; n_s\rangle = \left(\begin{array}{cc|c} I & \frac{1}{2} & I' \\ I_3 & \alpha & I_3' \end{array}\right)\left(\begin{array}{cc|c} J & 1 & J' \\ J_3 & i & J_3' \end{array}\right) Y(I'J'\mathcal{K}', IJ\mathcal{K}). \quad (7)$$

In the large-N_c limit we find [12]

$$Y_0(I'J'\mathcal{K}', IJ\mathcal{K}) \propto \sqrt{[I][J]}\left\{\begin{array}{ccc} \frac{1}{2} & 1 & \frac{1}{2} \\ I & J & \mathcal{K} \\ I' & J' & \mathcal{K}' \end{array}\right\}. \quad (8)$$

The expressions for arbitrary N_c are found in table 1. Averaging over initial states and summing over final states the p-wave widths are obtained as

$$\Gamma(I'J'\mathcal{K}', IJ\mathcal{K}) \propto \frac{[I'][J']}{[I][J]}|Y(I'J'\mathcal{K}', IJ\mathcal{K})|^2 .$$

In the large-N_c limit all pentaquark states in the same tower have the same total width. This leads to sum rules like

$$\Gamma\left(\Theta_0\left(\frac{1}{2}\right) \to NK\right) = \Gamma\left(\Theta_1\left(\frac{1}{2}\right) \to NK\right) + \Gamma\left(\Theta_1\left(\frac{1}{2}\right) \to \Delta K\right) = \Gamma\left(\Theta_1\left(\frac{3}{2}\right) \to NK\right) + \Gamma\left(\Theta_1\left(\frac{3}{2}\right) \to \Delta K\right)$$

as can be verified from table 1. The results for $N_c = 3$ in [11] can also be verified from table 1.

4 Large-N$_c$ and heavy-quark limit predictions

Heavy-quark symmetry predicts the amplitudes in terms of a few reduced matrix elements f_i (see table 2). The decay amplitude for $\Theta_{\bar{Q}}(IJJ_\ell) \to [NH_{\bar{Q}}^{(*)}(J'J_\ell')]_{J_N}$, where $\mathbf{J}_N = \mathbf{S}_N + \mathbf{L}$ is the angular momentum carried by the final baryon, is given by [13]

$$A_i = \sqrt{(2J_\ell+1)(2J'+1)}\left\{\begin{array}{ccc} J_\ell & J_\ell' & J_N \\ J' & J & \frac{1}{2} \end{array}\right\} f_i . \quad (9)$$

Table 2. Heavy-quark symmetry predictions for the decay amplitudes $\Theta_{\bar{Q}J_\ell} \to [NH_{\bar{Q}}^{(*)}]_{p\text{-wave}}$.

Decay	$J_N = 1/2$	$J_N = 3/2$
$\Theta_{\bar{Q}0}(\frac{1}{2}) \to NH_{\bar{Q}}$	$-\frac{1}{2}f_0$	–
$\Theta_{\bar{Q}1}(\frac{1}{2}) \to NH_{\bar{Q}}$	$\frac{\sqrt{3}}{2}f_1$	–
$\Theta_{\bar{Q}1}(\frac{3}{2}) \to NH_{\bar{Q}}$	–	$-\frac{1}{2}\sqrt{\frac{3}{2}}f_2$
$\Theta_{\bar{Q}0}(\frac{1}{2}) \to NH^*_{\bar{Q}}$	$\frac{\sqrt{3}}{2}f_0$	–
$\Theta_{\bar{Q}1}(\frac{1}{2}) \to NH^*_{\bar{Q}}$	$\frac{1}{2}f_1$	$-f_2$
$\Theta_{\bar{Q}1}(\frac{3}{2}) \to NH^*_{\bar{Q}}$	$-f_1$	$\frac{1}{2}\sqrt{\frac{5}{2}}f_2$

Table 3. Ratios of strong-decay widths for heavy pentaquarks $R^I(J) = \Theta^I_{\bar{Q}}(J) \to (NH_{\bar{Q}}) : (NH^*_{\bar{Q}})$.

$I=1$	$R^I(J=\frac{1}{2})$	$R^I(J=\frac{3}{2})$
$\mathcal{K}_{light}=1$	$1:3 \quad (J_\ell=0)$	$\frac{1}{2}:\frac{7}{2} \quad (J_\ell=1)$
	$2:2 \quad (J_\ell=1)$	$\frac{5}{2}:\frac{3}{2} \quad (J_\ell=2)$
$\mathcal{K}_{light}=0$	$1:11 \quad (J_\ell=1)$	$4:8 \quad (J_\ell=1)$

Combining the heavy-quark symmetry predictions with the large-N_c amplitudes we can fix the reduced amplitudes f_i and obtain predictions for the ratios of decays widths, as summarized for the $I=1$ states in table 3. More details will be given elsewhere [12].

5 Conclusions

The large-N_c limit reveals a structure of mass degeneracies and sum rules for decay widths that is not apparent at $N_c = 3$. This picture can be corrected systematically in $1/N_c$. We presented a new method for computing matrix elements for arbitrary N_c which is useful for this purpose. As an illustration, we showed how the combined large-N_c and heavy-quark limit allows to compute decay width ratios that discriminate between different heavy-pentaquark states. In the heavy-quark limit the spin of the light degrees of freedom is a conserved quantum number. When this is combined with the large-N_c limit we can label the states by the new quantum number $\mathcal{K}_{light}$. The states considered in [11] have $\mathcal{K}_{light}=0$, while the states considered in this work have $\mathcal{K}_{light}=1$. The predictions for their strong decays differ, as can be seen in table 3.

The work of C.S. was supported in part by Fundación Antorchas, Argentina and Fundación Séneca, Murcia, Spain. C.S. thanks for the hospitality of the Departamento de Física, Universidad de Murcia during the completion of this work.

References

1. J.L. Goity, R.F. Lebed, A. Pich, C.L. Schat, N.N. Scoccola (Editors), *Proceedings of the Workshop Large N(C) QCD 2004, Trento, Italy* (World Scientific, Singapore, 2005).

2. D. Pirjol, T.M. Yan, Phys. Rev. D **57**, 5434; 1449 (1998).
3. D. Pirjol, C. Schat, Phys. Rev. D **67**, 096009 (2003).
4. T.D. Cohen, R.F. Lebed, Phys. Rev. Lett. **91**, 012001 (2003).
5. D. Pirjol, C. Schat, AIP Conf. Proc. **698**, 548 (2004).
6. C.L. Schat, J.L. Goity, N.N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002).
7. R.F. Dashen, E. Jenkins, A.V. Manohar, Phys. Rev. D **49**, 4713 (1994) [**51**, 2489 (1995)(E)]; **51**, 3697 (1995).
8. J.L. Goity, Phys. Lett. B **414**, 140 (1997); C.E. Carlson, C.D. Carone, J.L. Goity, R.F. Lebed, Phys. Rev. D **59**, 114008 (1999).
9. D. Pirjol, C. Schat, Phys. Lett. B **638**, 340 (2006).
10. D. Pirjol, C. Schat, Phys. Rev. D **71**, 036004 (2005); M.E. Wessling, Phys. Lett. B **603**, 152 (2004).
11. E. Jenkins, A.V. Manohar, JHEP **0406**, 039 (2004).
12. D. Pirjol, C. Schat, hep-ph/0612314, to be published in Phys. Rev. D.
13. N. Isgur, M.B. Wise, Phys. Rev. Lett. **66**, 1130 (1991).