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Decoherence of domains and defects at phase transitions

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Abstract

In this further Letter on the onset of classical behaviour in field theory due to a phase transition, we show that it can be phrased easily in terms of the decoherence functional, without having to use the master equation. To demonstrate this, we consider the decohering effects due to the displacement of domain boundaries, with implications for the displacement of defects, in general. We see that decoherence arises so quickly in this event, that it is negligible in comparison to decoherence due to field fluctuations in the way defined in our previous papers.

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The standard big bang cosmological model of the early universe, with its period of rapid cooling, gives a strong likelihood of phase transitions, with concomitant symmetry breaking. This Letter is a further Letter in a sequence by ourselves and collaborators [1–3] in which we explore the way in which such phase transitions naturally take us from a quantum to classical description of the universe.

That (continuous) transitions can lead rapidly to classical behaviour is not surprising. Classical behaviour has two attributes. (i) *Classical correlations*: By this is meant that the Wigner function(al) $W[\pi,\phi]$ peaks on classical phase-space trajectories. (ii) *Diagonalisation* of the decoherence functional, whose role is to describe consistent histories.

Continuous transitions supply both ingredients. Firstly, the field ordering after such a transition is due to the growth in amplitude of unstable long-wavelength modes, which arise automatically from unstable maxima in the potential. From the papers of Guth and Pi [4] onwards, it has been appreciated that unstable modes lead to correlations through squeezing, and we shall not consider it further. Secondly, we understand diagonalisation to be an almost inevitable consequence of coarse-graining. The stable short-wavelength modes of the field [9], together with the other fields with which it interacts, form an environment whose coarse-graining enforces diagonalisation and makes the long-wavelength modes decohere. It is how this is implemented that is the basis of this Letter.

We stress that there is more than one way to formulate diagonalisation. In our earlier papers [1–3] we required the density matrix $\rho(t)$ itself to become diagonal, rather than the decoherence functional. Whichever approach we adopt, there is an issue as to which field basis we attempt to enforce diagonalisation, which can only be approximately achieved. For an infinite degree of freedom system field, ideally we should think of diagonalisation functionally. In practice this is impossible to achieve, and we are forced to adopt a piecemeal approach in which we make 'mini-superspace' approximations in which a finite number of degrees of freedom are isolated as the most significant. On comparing these, the relevant ones are those which decohere last. Since we are looking for the onset of classical behaviour, we take adjacent classical solutions with which to require no quantum interference. With these caveats we shall see later that

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the two approaches lead to the same results. In each case a probabilistic description is obtained, but the approach given here permits easier calculation.

Since phase transitions take place in a finite time, causality guarantees that correlation lengths remain finite. In our previous work, and this, we use the formation of domains after a transition to characterise the onset of classicality. To recapitulate, we showed [1–3], by using the master equation for the reduced density matrix, that the environment renders the long-wavelength modes of the order parameter field classical at early times, by or before the transition is complete. In particular, those modes on the scale of the domain size will have decohered, even though the modes on the scale of domain boundary thickness do not. In this Letter we recreate that result rather more simply and show that a parallel result holds due to small displacements of domain boundaries, in a coarse-graining that is insensitive to their positions.

This latter result has another consequence. If the symmetry breaking permits non-trivial homotopy groups the frustration of the order parameter fields is resolved by the creation of topological defects to mediate between the different ground states [5,6]. Since defects are, in principle, observable, they provide an excellent experimental tool for determining how phase transitions occur. For the simple theory that we shall consider here, that of a real scalar field with double-well potential, the domain boundaries *are* the defects (domain walls) and we can view our results as decoherence as induced by small displacements of defects. The generalisation to a complex field ϕ is straightforward, and has been considered elsewhere [7]. This gives more substance to our preliminary attempts to show that vortex defects are also classical by the time of their production [7].

As in our earlier work, we restrict ourselves to flat space–time. The extension to non-trivial metrics is straightforward in principle [8].

We now consider the case of a real quantum field ϕ with double-well potential in detail. As we have said, the field ordering after the transition begins is due to the growth of long-wavelength modes. For these modes the environment consists of the short-wavelength modes of the field, together with all the other fields with which ϕ inevitably interacts in the absence of selection rules [9,10]. The inclusion of explicit environment fields is both a reflection of the fact that a scalar field in isolation is physically unrealistic, as well as providing us with a systematic approximation scheme [1]. To be specific, the simplest classical action with scalar and environmental fields $\chi_{\rm a}$ is

$$S[\phi,\chi] = S_{\rm syst}[\phi] + S_{\rm env}[\chi] + S_{\rm qu}[\phi,\chi], \tag{1}$$

where (with $\mu^2, m^2 > 0$)

$$S_{\rm syst}[\phi] = \int d^4x \left\{ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}\phi^4 \right\},$$

$$S_{\rm env}[\chi] = \sum_{\rm a=1}^{N} \int d^4x \left\{ \frac{1}{2}\partial_\mu\chi_{\rm a}\partial^\mu\chi_{\rm a} - \frac{1}{2}m_{\rm a}^2\chi_{\rm a}^2 \right\},$$

and the most relevant interactions between system and environment are of the biquadratic form

$$S_{\rm qu}[\phi,\chi] = -\sum_{\rm a=1}^{N} \frac{g_{\rm a}}{8} \int d^4x\, \phi^2(x)\chi_{\rm a}^2(x). \tag{2}$$

Even if there were no external χ fields with a quadratic interaction of kind of Eq. (2), the interaction between short- and long-wavelength modes of the ϕ-field can be recast, in part, in this form, showing that such a term is obligatory.

Although the system field ϕ can never avoid the decohering environment of its own short-wavelength modes [9], to demonstrate the effect of an environment we first consider the case in which the environment is taken to be composed only of the fields $\chi_{\rm a}$. Since environments have a cumulative effect on the onset of classical behaviour, the inclusion of a further component of the environment *reduces* the time it takes for the system to behave classically. Thus it makes sense to include the environment one part after another, since we can derive an *upper* bound on that time at each step.

We have shown elsewhere [1–3] that, in order to make our calculations as robust as possible, we need a significant part of the environment to have a strong impact upon the system-field, but not vice versa. The simplest way to implement this is to take a large number $N \gg 1$ of scalar $\chi_{\rm a}$ fields with comparable masses $m_{\rm a} \simeq \mu$ weakly coupled to the ϕ, with λ, $g_{\rm a} \ll 1$ (for details see Ref. [1]). Thus, at any step, there are N weakly coupled environmental fields influencing the system field, but only one weakly self-coupled system field to back-react upon the explicit environment.

For one-loop consistency it is sufficient, at order of magnitude level, to take identical $g_{\rm a} = g/\sqrt{N}$. Further, at the same order of magnitude level, we take $g \simeq \lambda$. This is very different from the more usual large-N $O(N+1)$-invariant theory with one ϕ-field and N $\chi_{\rm a}$ fields, dominated by the $O(1/N)$ $(\chi^2)^2$ interactions, that has been the standard way to proceed for a *closed* system [11]. With our choice there are no direct χ^4 interactions, and the indirect ones, mediated by ϕ loops, are depressed by a factor $g/\sqrt{N}$. In this way the effect of the external environment qualitatively acts as a proxy for the effect of the internal environment provided by the short-wavelength modes of the ϕ-field, but in a more calculable way.

We shall assume that the initial states of the system and environment are both thermal, at a high temperature $T_0 > T_{\rm c}$. We then imagine a change in the global environment (e.g. expansion in the early universe) that can be characterised by a change in temperature from T_0 to $T_{\rm f} < T_{\rm c}$. That is, we do not attribute the transition to the effects of the environment-fields.

Given our thermal initial conditions it is not the case that the full density matrix has ϕ and χ fields uncorrelated initially, since it is the interactions between them that leads to the restoration of symmetry at high temperatures. Rather, on incorporating the hard thermal loop 'tadpole' diagrams of the χ (and ϕ) fields in the ϕ mass term leads to the effective action for ϕ quasiparticles,

$$S^{\rm eff}_{\rm syst}[\phi] = \int d^4x \left\{ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2(T_0)\phi^2 - \frac{\lambda}{4}\phi^4 \right\}, \tag{3}$$

where, in a mean-field approximation, $m_\phi^2(T_0) = -\mu^2(1 - T_0^2/T_c^2)$ for $T \approx T_c$. As a result, we can take an initial factorised density matrix at temperature T_0 of the form $\hat{\rho}[T_0] = \hat{\rho}_\phi[T_0] \times \hat{\rho}_\chi[T_0]$, where $\hat{\rho}_\phi[T_0]$ is determined by the quadratic part of $S_{\text{syst}}^{\text{eff}}[\phi]$ and $\hat{\rho}_\chi[T_0]$ by $S_{\text{env}}[\chi_a]$. That is, the many χ_a fields have a large effect on ϕ, but the ϕ-field has negligible effect on the χ_a.

Provided the change in temperature is not too slow the exponential instabilities of the ϕ-field grow so fast that the field has populated the degenerate vacua well before the temperature has dropped significantly below T_c [12]. Since the temperature T_c has no particular significance for the environment fields, for these early times we can keep the temperature of the environment fixed at $T_\chi \approx T_c$ (our calculations are only at the level of orders of magnitude). Meanwhile, for simplicity the χ_a masses are fixed at the common value $m \simeq \mu$.

It is sufficient for our purposes here to take an instantaneous quench. Slower quenches make the analytic calculations very much more difficult, without changing the qualitative nature of the results [1].

The notion of consistent histories provides an alternative approach to classicality to trying to solve the master equation, as we have done previously. Quantum evolution can be considered as a coherent superposition of fine-grained histories. Since we need to be able to distinguish different classical system-field configurations evolving after the transition, we work in the field-configuration basis. If one defines the c-number field $\phi(x)$ as specifying a fine-grained history, the quantum amplitude for that history is $\Psi[\phi] \sim e^{iS[\phi]}$ (we work in units in which $\hbar = 1$).

In the quantum open system approach that we have adopted here, we are concerned with coarse-grained histories

$$\Psi[\alpha] = \int \mathcal{D}\phi e^{iS[\phi]} \alpha[\phi], \tag{4}$$

where $\alpha[\phi]$ is the filter function that defines the coarse-graining. In the first instance this filtering corresponds to tracing over the χ_a degrees of freedom.

From this we define the decoherence functional for two coarse-grained histories as

$$\mathcal{D}\big[\alpha^+, \alpha^-\big] = \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{i(S[\phi^+] - S[\phi^-])} \alpha^+\big[\phi^+\big] \alpha^-\big[\phi^-\big]. \tag{5}$$

$\mathcal{D}[\alpha^+, \alpha^-]$ does not factorise because the histories $\phi^\pm$ are not independent; they must assume identical values on a spacelike surface in the far future. Decoherence means physically that the different coarse-graining histories making up the full quantum evolution acquire individual reality, and may therefore be assigned definite probabilities in the classical sense.

A necessary and sufficient condition for the validity of the sum rules of probability theory (i.e. no quantum interference terms) is [13]

$$\operatorname{Re} \mathcal{D}\big[\alpha^+, \alpha^-\big] \approx 0, \tag{6}$$

when $\alpha^+ \neq \alpha^-$ (although in most cases the stronger condition $\mathcal{D}[\alpha^+, \alpha^-] \approx 0$ holds [14]). Such histories are consistent [15].

For our particular application, we wish to consider as a single coarse-grained history all those fine-grained ones where the full field ϕ remains close to a prescribed classical field configuration ϕ_{cl}. The filter function takes the form

$$\alpha_{\text{cl}}[\phi] = \int \mathcal{D}J e^{i \int J(\phi - \phi_{\text{cl}})} \alpha_{\text{cl}}[J]. \tag{7}$$

In the general case, $\alpha[\phi]$ is a smooth function (we exclude the case $\alpha[\phi] = \text{const}$, where there is no coarse-graining at all). Using

$$J\phi \equiv \int d^4x\, J(x)\phi(x), \tag{8}$$

we may write the decoherence functional between two classical histories as

$$\mathcal{D}\big[\alpha^+, \alpha^-\big] = \int \mathcal{D}J^+ \mathcal{D}J^- e^{iW[J^+, J^-] - (J^+\phi_{\text{cl}}^+ - J^-\phi_{\text{cl}}^-)} \times \alpha^+\big[J^+\big] \alpha^{-*}\big[J^-\big], \tag{9}$$

where

$$e^{iW[J^+, J^-]} = \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{i(S[\phi^+] - S[\phi^-] + J^+\phi^+ - J^-\phi^-)}, \tag{10}$$

is the closed-path-time generating functional [16].

In principle, we can examine adjacent general classical solutions for their consistency but, in practice, it is simplest to restrict ourselves to particular solutions $\phi_{\text{cl}}^\pm$, according to the nature of the decoherence that we are studying. Initially, as we said earlier, we have made a de facto separation into the order parameter field ϕ and its explicit environmental fields χ_a whereby, in a saddle-point approximation over J,

$$\mathcal{D}\big[\phi_{\text{cl}}^+, \phi_{\text{cl}}^-\big] \sim F\big[\phi_{\text{cl}}^+, \phi_{\text{cl}}^-\big]. \tag{11}$$

$F[\phi^+, \phi^-]$ is the Feynman–Vernon [17] influence functional (IF) (see Refs. [1,9] for details). The influence functional is written in terms of the influence action $A[\phi_{\text{cl}}^+, \phi_{\text{cl}}^-]$ as

$$F\big[\phi_{\text{cl}}^+, \phi_{\text{cl}}^-\big] = \exp\big\{iA\big[\phi_{\text{cl}}^+, \phi_{\text{cl}}^-\big]\big\}. \tag{12}$$

As a result,

$$\big|\mathcal{D}\big[\phi_{\text{cl}}^+, \phi_{\text{cl}}^-\big]\big| \sim \exp\big\{-\operatorname{Im} \delta A\big[\phi_{\text{cl}}^+, \phi_{\text{cl}}^-\big]\big\}, \tag{13}$$

where δA is the contribution to the action due to the environment.

From this viewpoint, once we have chosen the classical solutions of interest, adjacent histories become consistent at the time t_D, for which

$$1 \approx \operatorname{Im} \delta \mathrm{A}|_{t=t_D}. \tag{14}$$

As we are considering weak coupling with the environment fields, we may expand the influence functional $F[\phi^+, \phi^-]$ up to second non-trivial order in coupling strengths for large N. Higher terms are depressed by powers of N. The general form of the influence action is then [9,18]

$$\begin{aligned}\delta A\big[\phi^+, \phi^-\big] = {} & \big\{\big\langle S_{\text{int}}\big[\phi^+, \chi_a^+\big]\big\rangle_0 - \big\langle S_{\text{int}}\big[\phi^-, \chi_a^-\big]\big\rangle_0\big\} \\ & + \frac{i}{2}\big\{\big\langle S_{\text{int}}^2\big[\phi^+, \chi_a^+\big]\big\rangle_0 - \big[\big\langle S_{\text{int}}\big[\phi^+, \chi_a^+\big]\big\rangle_0\big]^2\big\} \\ & - i\big\{\big\langle S_{\text{int}}\big[\phi^+, \chi_a^+\big] S_{\text{int}}\big[\phi^-, \chi_a^-\big]\big\rangle_0\end{aligned}$$

$$- \langle S_{\rm int}[\phi^+, \chi_{\rm a}^+]\rangle_0 \langle S_{\rm int}[\phi^-, \chi_{\rm a}^-]\rangle_0 \}$$
$$+ \frac{i}{2}\{\langle S_{\rm int}^2[\phi^-, \chi_{\rm a}^-]\rangle_0 - [\langle S_{\rm int}[\phi^-, \chi_{\rm a}^-]\rangle_0]^2\}. \tag{15}$$

As a further step we make a separation of the ϕ-field itself into its long wavelength modes with $|k| < \mu$, which determine the domains, and the short wavelength modes ($|k| > \mu$) which act as their implicit environment. However, in calculating t_D this latter step only serves to further reduce decoherence times which, as we shall see, are already short enough.

In reality, even if we ignore this step for the purpose of bounding t_D there is not a unique bound, since it depends on the class of classical solutions considered. However, in practice all sensible choices seem, qualitatively, to give the same upper bound on t_D. The reasons for this are the following. Firstly, for the long wavelength system modes the spatial profile of the classical solution is not particularly relevant, since it is its exponential growth in time that sets the scale for the onset of classical behaviour. Thus, not only is t_D insensitive to wavelength for small-k modes, but it only depends logarithmically on the parameters of the theory. Further, with the common mass-scale μ that we assume here, the Compton wavelength μ^{-1}, both determines the rate of exponential growth and provides the natural distance scale over which we do not wish to discriminate between classical solutions. Different choices lead to differences in μt_D of order unity, significantly smaller than μt_D itself, which we ignore. For weak couplings it is relatively easy to compute the upper bound on this time due to the interactions $S_{\rm int}[\phi, \chi]$ of (2). We shall now see that, in our particular model, it is in general shorter than the time t^* at which the transition is complete, defined in terms of the system field as the time for which

$$\langle \phi^2 \rangle_{t^*} \sim \eta^2 = 6\mu^2/\lambda, \tag{16}$$

where the average is taken over the system field. That is, in (16) we ignore the contributions from the dominant environment given by the short wavelength ($|k| > \mu$) modes of ϕ. This renders $\langle\phi^2\rangle_{t^*}$ finite. (More explicitly, in the language of our earlier papers based on the master equation, e.g., [1], $\langle\phi^2\rangle_{t^*} = {\rm Tr}\{\rho_{\rm r}\phi^2\}$, where $\rho_{\rm r}$ is the *reduced* density matrix obtained on tracing out the environment.) Any ambiguities are again of order unity in μt_D, the timelag for non-linear effects to occur, in comparison to the exponential runaway of the free field [19].

Since the effect of the environment is to induce damping, the classical behaviour of the field is expressed through the classical Langevin stochastic equations that it satisfies [20,21]. We are assuming that coupling is sufficiently strong for the system not to recohere after t^* [22].

For the biquadratic coupling of Eq. (2), the IF is given by

$$\mathrm{Re}\,\delta A = \frac{g^2}{8}\int d^4x \int d^4y\, \Delta(x) K(x-y) \Sigma(y),$$
$$\mathrm{Im}\,\delta A = -\frac{g^2}{16}\int d^4x \int d^4y\, \Delta(x) N(x,y) \Delta(y),$$

where $K(x-y) = \mathrm{Im}\, G_{++}^2(x,y)\theta(y^0 - x^0)$ is the dissipation kernel and $N(x-y) = \mathrm{Re}\, G_{++}^2(x,y)$ is the noise (diffusion) kernel. G_{++} is the relevant closed-time-path correlator of the χ-field at temperature T_0 [1]. We have defined $\Delta = \frac{1}{2}(\phi^{+2} - \phi^{-2})$ and $\Sigma = \frac{1}{2}(\phi^{+2} + \phi^{-2})$.

We look for classical solutions of the form

$$\phi_{\rm cl}(\vec{x}, s) = f(s,t)\Phi(\vec{x}),$$

where, in principle, $f(s,t)$ satisfies $f(0,t) = \phi_{\rm i}$ and $f(t,t) = \phi_{\rm f}$ and $\Phi(\vec{x})$ gives the space-field configuration.

We begin by showing that, using (14), we recreate the results obtained previously in Refs. [1,2] on using the more complicated master equation, in which the field is spread through space, and decoherence is due to different field amplitudes. In anticipation that $t_D < t^*$, it is sufficient to restrict ourselves to the initial Gaussian free field evolution with negative (mass)^2. In fact, for idealised Langevin equations, this can be a good approximation for domain formation into the non-linear regime [12]. We look for classical fields that, after a sudden quench, have the form [2]

$$\phi_{\rm cl}^{\pm}(s, \vec{x}) = e^{\mu s}\phi_{\rm f}^{\pm}\cos(k_0 x)\cos(k_0 y)\cos(k_0 z), \tag{17}$$

where $\phi_{\rm f}^{\pm}$ is the final field configuration. This is a single mode approximation to a regular chequer-board domain structure. Shorter wavelengths can be introduced without altering the result significantly. The reader is referred to [1] for more details. For an instantaneous quench, we will use the late time behaviour of the longest wavelengths ($k_0 = 0$), $\phi_{\rm cl}^{\pm}(s, \vec{x}) \sim e^{\mu s}\phi_{\rm f}^{\pm}$. The exponential factor, as always, arises from the growth of the unstable long-wavelength modes.

Thus, $\mathrm{Im}\,\delta A[\phi_{\rm cl}^+, \phi_{\rm cl}^-]$ takes the form

$$\mathrm{Im}\,\delta A = \frac{g^2 V T_c^2 \pi}{64}\Delta_{\rm f}^2 \int_0^\infty \frac{dk}{(k^2+\mu^2)^2} \times \frac{1 + e^{4\mu t} - e^{2\mu t}\cos(2\sqrt{k^2+\mu^2})}{(k^2 + 2\mu^2)}, \tag{18}$$

where $\Delta_{\rm f} = 1/2(\phi_{\rm f}^{+2} - \phi_{\rm f}^{-2})$, and T_c is the critical environmental-temperature. As we noted in previous publications, the volume V is due to the fact we are considering field configurations spread over all space. V is interpreted as the minimal volume inside which there are no coherent superpositions of macroscopically distinguishable states for the field. Later, we shall consider localised configurations where this factor does not appear.

After assuming $\mu t \gg 1$, the integral in momenta can be done analytically obtaining,

$$\mathrm{Im}\,\delta A \sim \frac{g^2 V T_c^2 \pi^2}{256}\frac{(3 - 2\sqrt{2})}{\mu^3}\Delta_{\rm f}^2 e^{4\mu t}. \tag{19}$$

With this expression at hand, we are able to evaluate the decoherence time t_D for amplitude variation as

$$\mu t_D \sim \frac{1}{2}\ln\left\{\frac{16\mu^{3/2}}{g T_c \Delta_{\rm f} V^{1/2}\pi\sqrt{(3-2\sqrt{2})}}\right\}. \tag{20}$$

Using a conservative value for the volume, $V = \mathcal{O}(\mu^{-3})$, we get

$$\mu t_D \sim \frac{1}{2}\ln\left\{\frac{16\mu^3}{gT_c\Delta_f}\right\}. \tag{21}$$

We re-write last expression in terms of $\Delta_f = \bar{\phi}\bar{\Delta}/2$, with $\bar{\phi} = \phi_f^+ + \phi_f^-$ and $\bar{\Delta} = \phi_f^+ - \phi_f^-$. At the completion of the transition $\bar{\phi}^2 \simeq \eta^2 \sim \lambda^{-1}$, and we will adopt, at time t_D, $\bar{\phi}^2 \sim \mathcal{O}(\mu^2\alpha/\lambda)$. $\lambda < \alpha < 1$ is to be determined self-consistently from the condition that, at time t_D, $\langle\phi^2\rangle_t \sim \alpha\eta^2$. We have shown in Ref. [2] that the value of α is determined as $\alpha \approx \sqrt{\mu/T_c}$. We also set $\bar{\Delta} \sim 2\mu$ (i.e. we do not discriminate between field amplitudes which differ by $\mathcal{O}(\mu)$), where μ^{-1} characterises the thickness of domain boundaries (walls) as the field settles into its ground-state values. Additionally, for simplicity we take the couplings $g \sim \lambda$. Therefore, we obtain an upper bound on t_D,

$$\mu t_D \sim \frac{1}{2}\ln\left\{\frac{\eta}{T_c\sqrt{\alpha}}\right\}, \tag{22}$$

which exactly coincides with the result found in Ref. [2] from the master equation for a sudden quench but, in the present case, using the decoherence functional approach.

If we now trace over short-wavelength modes, as in [9], but for unstable modes, we would get a further term in $\mathrm{Im}\,\delta A$ qualitatively similar to (19), which will serve to preserve $t_D < t^*$ by making t_D even smaller. We shall not consider this implicit environment in subsequent analysis.

For comparison, we find t^*, for which $\langle\phi^2\rangle_t \sim \eta^2$, given by [1,2]

$$\mu t^* \sim \frac{1}{2}\ln\left\{\frac{\eta}{\sqrt{\mu T_c}}\right\}, \tag{23}$$

whereby $\mu^{-1} < t_D < t^*$, with

$$\mu t^* - \mu t_D \simeq \frac{1}{4}\ln\left\{\frac{T_c}{\mu}\right\} > 1, \tag{24}$$

for weak enough coupling, or high enough initial temperatures.

Having seen that the decoherence functional approach gives identical conclusions to the solution of master equations, the new work in this Letter is to evaluate the IF for different field configurations which, from the point of view of the Master equation, would be much more taxing analytically. Instead of the classical solutions used before [1,3], in which the field is spread in an infinite chequer-board through space, here we are concerned with a different field configuration; a localised domain wall.

The orientation of such a wall is irrelevant, as was the orientation of the chequer-board solution used earlier. Most simply, we consider classical domain wall solutions (for the $k_0 = 0$ mode) of the form

$$\phi^{\pm}(s,\vec{x}) = \phi_f^{\pm} e^{\mu s}\tanh(\mu x), \tag{25}$$

which link adjacent domains.

Our main new result is to determine the decoherence induced by a small displacement in the domain wall, by evaluating the influence action for the classical field configurations $\phi^{\pm}(x,s) = \eta f(s,t)\Phi^{\pm}(x)$, where $\Phi^{\pm}(x) = \Phi(x \pm \delta/2)$. We will consider δ as a small displacement in the position of the wall, and consequently we expand the classical solution (or, equivalently, $\Delta(x)$) in powers of $\mu\delta$, up to linear order

$$\Delta(s,\vec{x}) \approx \mu\delta\eta^2 e^{2\mu s}\tanh(\mu x)\mathrm{sech}^2(\mu x).$$

Doing this, the imaginary part of the influence action can be written as (after integrating over time and assuming $\mu t \gg 1$),

$$\mathrm{Im}\,\delta A \approx \frac{g^2T_c^2\eta^4\delta^2\mu^2}{64(2\pi)^6}e^{4\mu t}\int d^3x\int d^3y \times \int d^3k\int d^3p\,\frac{e^{-i(\vec{p}+\vec{k})(\vec{x}-\vec{y})}}{(k^2+\mu^2)(p^2+\mu^2)} \times \frac{\tanh(\mu x)\mathrm{sech}^2(\mu x)\tanh(\mu y)\mathrm{sech}^2(\mu y)}{[(\sqrt{k^2+\mu^2}+\sqrt{p^2+\mu^2})^2+4\mu^2]}.$$

These integrations can be exactly evaluated (in part analytically and in part numerically), to give

$$\mathrm{Im}\,\delta A \approx 0.2\frac{g^2T_c^2\eta^4L^2\delta^2}{1024}\frac{e^{4\mu t}}{\mu^2}, \tag{26}$$

where L^2 is a surface term, analogous to the volume V in the chequer-board analysis. This coefficient comes from the fact that we are considering a one-dimensional kink solution embedded in a three-dimensional space. The L^2 coefficient comes from the “free surface” or wall in two directions of the three-dimensional kink. Considering very conservative values (such as to obtain upper bounds for t_D), we can set $L = \mathcal{O}(\mu^{-1})$ as the minimum length scale in which there could be not coherent superpositions of macroscopic states of the field.

If the decoherence time due to displacements is $\bar{t}_D$, let us suppose that $\mu\bar{t}_D > 1$. Then $\bar{t}_D$ can be evaluated from the last equation, and its order of magnitude is given by

$$\mu\bar{t}_D \sim \frac{1}{2}\ln\left\{\frac{74\mu}{gT_c\eta^2L\delta}\right\}. \tag{27}$$

Superficially, this result looks similar to previous results, where the difference between the field amplitudes has been replaced by the scale of displacement suffered by the domain boundary wall, δ. The main difference is in the power of η inside the log. If one takes $\delta = \gamma\mu^{-1}$, we may bound $\bar{t}_D$ as

$$\mu\bar{t}_D \sim \frac{1}{2}\ln\left\{\frac{12\mu}{\gamma T_c}\right\}. \tag{28}$$

Since $T_c \gg \mu$ (in fact one can show $T_c^2/\mu^2 \sim 24/\lambda$ [1]), this result puts a bound on the possible value of γ in order to have $\mu\bar{t}_D > 1$, i.e. $\gamma \leqslant \mu/T_c \ll 1$, and hence $\delta \ll \mu^{-1}$. If γ is larger, at very early times in our model $t \sim \mu^{-1}$, we get from Eq. (26) $\mathrm{Im}\,\delta A > 1$ immediately. Thus, the system behaves classically from this earliest permissible time, $\mu\bar{t}_D = \mathcal{O}(1)$.

Since the size of the core of the domain wall is $\mathcal{O}(\mu^{-1})$, we take $\delta \sim \mathcal{O}(\mu^{-1})$ (i.e. $\gamma = \mathcal{O}(1)$) as the minimum length scale in which there could be not coherent superpositions of macroscopic states of the field. With decoherence occurring at the earliest possible time it follows automatically that

$$\mu t^* - \mu\bar{t}_D > 0. \tag{29}$$

Even if we demand decoherence at scales a fraction of the domain wall core thickness (i.e. $\gamma \ll 1$) we can ensure that

$$\mu t^* - \mu \bar{t}_D \sim \frac{1}{4} \ln\left\{\frac{\gamma^2 T_c}{\lambda\mu}\right\} > 0. \qquad (30)$$

This result implies that decoherence due to the displacement of the boundary is a very early time event $\bar{t}_D < t_D$.

The outcome of this analysis is that configurations of field domains with displaced boundaries are less important for decoherence than fixed domains in which there are field fluctuations, which decohere later.

What is more appealing is, if one perform all the calculation in $1+1$ dimensions, the coefficient L appears only in our first example (i.e. the plane-wave); not for the kink-like solutions.

In fact, rather than consider a whole chequer-board of domains as in [1–3], we can restrict ourselves to two adjacent domains with boundary given by (25). Consider the simple case where the difference between walls is given by $\phi_{\rm f}^+ = \phi_{\rm f}^- - \epsilon$, with ϵ a small fluctuation around the final value of the field configuration. We assume that the final field configuration is $\phi_{\rm f}^+ = \sqrt{\alpha}\eta$, with α the self-consistent coefficient shown earlier and $\epsilon = \mathcal{O}(\mu)$. Perhaps surprisingly, we recreate the result of (24) exactly. Yet again, we have decoherent behaviour before the transition is completed.

In addition, the field possesses classical correlations at early times by virtue of the quasi-Gaussian nature of the regime [7, 23] to give a fully classical picture by time t^*.

Finally, we return to the other view of domain boundaries mentioned in the introduction, as topological defects. What does this analysis have to say about the classical behaviour of defects like vortices, whose separation also measures the size of domains in simple circumstances [24]? Treating vortices as having their cores as line zeroes (in the same way that the cores of our domain walls are sheet zeroes) we have shown elsewhere that the mechanism for the production of classical vortices has several parts [7]. Yet again, the environment renders the long-wavelength modes of the order parameter field classical at early times, by or before the transition is complete. In particular, those on the scale of the separation of the line-zeroes that will characterise the classical domains will have decohered by the time the transition is complete, even though the field modes on the scale of classical vortex thickness do not decohere ever. For line-zeroes to mature into vortex cores the field needs to have an energy profile commensurate with the vortex solutions to the ordinary classical Euler–Lagrange equations. This requires that the field fluctuations are peaked around long-wavelengths, to avoid fluctuations causing wiggles in the cores and creating small vortex loops, a related condition satisfied in our models. The resultant density of line-zeroes can already be inferred in the linear regime, whose topological charges are well defined even though close inspection of their interior structure is not permitted classically. What the analysis of this Letter shows is that we expect decoherence due to vortex displacement to be irrelevant in comparison to decoherence due to field fluctuations.

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