Symmetry entropy and isoscaling

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We suggest, on general principles, that the isotopic distributions and thus isoscaling are affected by an entropic symmetry term, which is present even when the symmetry energy term is absent.

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The isoscaling property has been widely observed in the analysis of nuclear multifragmentation experiments [1] and it has been traced to the symmetry energy term in the nuclear mass formula. Let us consider two fragmentation reactions at a given energy E/A, whose fragmenting systems have mass, charge, (A_1, Z_1), (A_2, Z_2), respectively. It was found empirically that the following relation holds

$$R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} \propto \exp(\alpha N + \beta Z), \tag{1}$$

where $Y_i(N, Z)$ represents the number of fragments characterized by N neutrons and Z protons produced in the nucleus-nucleus collisions from experiment i and α and β are fitting parameters. The two experiments can be ordered, say, with $A_2 > A_1$. Usually the reactions are chosen so that the Z of the two systems is the same ($Z_1 = Z_2$). The observed scaling produced by this ratio has been called isoscaling [1].

This scaling has attracted the attention of nuclear physicists because of the possibility that such a quantity might allow one to infer the density dependence of the symmetry energy term.

The connection between isoscaling and the symmetry energy term has been proposed in different works [2] by analyzing the fragmentation process in the framework of the grand canonical ensemble or in the Emitting Expanding Source (EES) model [3].

The standard result is

$$\alpha \sim \frac{4C_{\rm sym}}{T}\left[\left(\frac{Z_1}{A_1}\right)^2 - \left(\frac{Z_2}{A_2}\right)^2\right], \tag{2}$$

in which the fitting parameter α is directly related to the symmetry energy term

$$E_{\rm sym} = C_{\rm sym}\frac{(N-Z)^2}{A} = C_{\rm sym}\frac{(A-2Z)^2}{A}. \tag{3}$$

It is commonly believed that the isoscaling phenomenon is due to the existence of such a term in the binding energy of a nucleus, for which the corresponding yield distribution is of the type

$$P = \exp[-E_{\rm sym}/T] = \exp\left[-\frac{C_{\rm sym}}{T}\frac{(N-Z)^2}{A}\right]. \tag{4}$$

This can be seen by calculating the energy required to create a fragment with a given N and Z (with $A = N + Z$) from a parent nucleus with N_1 and Z_1 (and $A_1 = N_1 + Z_1$). Considering only the symmetry energy terms, the energy ΔE_1 needed to go from $(Z_1, A_1) \to (Z_1 - Z, A_1 - A) + (Z, A)$ is

$$\Delta E_1 = \left[\frac{(A_1 - A - 2(Z_1 - Z))^2}{A_1 - A} + \frac{(A-2Z)^2}{A} - \frac{(A_1 - 2Z_1)^2}{A_1}\right] C_{\rm sym}. \tag{5}$$

A similar expression exists for emission from a nucleus of (Z_2, A_2). The ratio defined in Eq. (1) is given by

$$R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} \propto e^{(-\Delta E_2 + \Delta E_1)/T}. \tag{6}$$

It can be shown that

$$\Delta E_1 - \Delta E_2 = 4C_{\rm sym}\left[\frac{(Z_1 - Z)^2}{A_1 - A} - \frac{(Z_2 - Z)^2}{A_2 - A} + \frac{Z_2^2}{A_2} - \frac{Z_1^2}{A_1}\right]. \tag{7}$$

In the limit that $Z \ll Z_1$ and Z_2 and also that $A \ll A_1$ and A_2, the Taylor expansion of $\Delta E_1 - \Delta E_2$ gives

$$\Delta E_1 - \Delta E_2 = 4C_{\rm sym}\left[\left(\frac{Z_1}{A_1}\right)^2 - \left(\frac{Z_2}{A_2}\right)^2\right] N + 4C_{\rm sym}\left[\left(\frac{N_1}{A_1}\right)^2 - \left(\frac{N_2}{A_2}\right)^2\right] Z, \tag{8}$$

which has the form also observed in Eq. (1) with α and β given by

$$\alpha = \frac{4C_{\rm sym}}{T}\left[\left(\frac{Z_1}{A_1}\right)^2 - \left(\frac{Z_2}{A_2}\right)^2\right] \tag{9}$$

$$\beta = \frac{4C_{\rm sym}}{T}\left[\left(\frac{N_1}{A_1}\right)^2 - \left(\frac{N_2}{A_2}\right)^2\right]. \tag{10}$$

An example of the difference in symmetry energies, $\Delta E_1 - \Delta E_2$, for carbon isotopes emitted from ^{124}Sn and ^{112}Sn is given in Fig. 1. A comparison of the exact form calculated with Eq. (7) (solid symbols) and the approximation given by Eq. (8) (line) shows that the approximation is good to about 10% for the mass range shown in the figure.

In the approximate form of Eq. (8) the isoscaling correlation depends upon the parameter $C_{\rm sym}/T$. An analysis of this parameter for a variety of reactions generating different temperature/density conditions is expected to yield the symmetry energy parameter $C_{\rm sym}$ in terms of the same variables (temperature, density, etc.). Such analyses have been undertaken

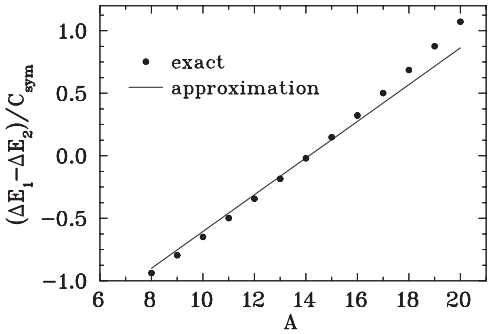

FIG. 1. The difference in symmetry calculated for emission from two systems $[(Z1, A1)$ and $(Z2, A2)]$ scaled the symmetry energy term $C_{\rm sym}$ calculated for carbon nuclei with masses indicated by the abscissa. The solid symbols are from an exact expression given by Eq. (7) and the line is for the approximation given by Eq. (8).

despite the difficulties of determining temperature, density, and pressure at any stage of the reaction; the relative weight for the values of the evolving parameters in the course of the reaction; and the effect of the sequential decay on the primary distribution [4]. Finally, the level of approximation in the extraction of $C_{\rm sym}$ must also be addressed before one can hope to explore the density and temperature dependence of the symmetry energy based on an isoscaling analysis.

Among the many simulations that have been performed, lattice simulations hold a particular interest because of their inherent simplicity and interpretability. In these lattice calculations, particles, randomly assigned the colors red and blue, are distributed in a lattice allowing them to interact according to the matrix element J_{ij}, for instance, $J_{ii} = J_{jj} < J_{ij}$. A temperature T assigned to the lattice leads to the formation of clusters that are characterized by their sizes and color composition. This procedure leads to a strong isoscaling correlation that is dependent, as expected, upon T and J_{ij} [5].

It came as a surprise [6] that a good isoscaling signature appeared even when there was no interaction at all and fragments were defined via a simple bond percolation approach. Such a calculation was performed considering a simple cubic D-dimensional lattice with nodes occupied by "particles" of N colors. The total number of nodes is A and $A = N + Z + R + \ldots$ with $N, Z, R, \ldots,$ denoting the number of nodes of given colors. Each color is assigned with a given probability p_Z, p_N, p_R, etc., with $1 = p_Z + p_N + p_R + \cdots$. These probabilities are taken to be independent and homogeneous.

Why does the killing of the difference in the interaction allow isoscaling to survive?

To answer this question, let us consider an arbitrary cluster resulting from a lattice calculation or, in fact, from any kind of calculation that produces clusters of arbitrary size A. There is no need to get into the details of the mechanism leading to the formation of the cluster itself.

We can ask what is the probability that a cluster of size A is constituted by N blue particles and Z red particles? Because of the random nature of the color assignation and in the absence of any other difference in the colored particles, the distribution is to be binomial

$$P_A(N, Z) = \binom{A}{Z} p_Z^Z p_N^N = \frac{A!}{Z!N!} p_Z^Z (1 - p_Z)^N. \quad (11)$$

In this expression p_Z and $p_N = (1 - p_Z)$ are the complementary probabilities for the two colors, which can be related to the corresponding chemical potential of the environment. Assuming for the sake of simplicity that $p_N = p_Z = \frac{1}{2}$, then

$$P(A, Z) = \frac{A!}{Z!(Z - A)!} \left(\frac{1}{4}\right)^A. \quad (12)$$

This distribution is sharply peaked around $Z = A/2$ and has a variance $\sigma_Z^2 = A/4$ or

$$P(A, Z) \approx \left(\frac{1}{2\pi A/4}\right)^{1/2} \exp\left[-\frac{(Z - A/2)^2}{2A/4}\right] \quad (13)$$

or

$$P(N - Z) = \frac{2}{\sqrt{2\pi A}} \exp\left[-\frac{1}{2}\frac{(N - Z)^2}{2A}\right], \quad (14)$$

with $\sigma_{N-Z}^2 = A$.

This distribution is more generally known to arise from the entropy of mixing. Its role under a variety of conditions is well studied in the thermodynamics of solutions. The simple form of the entropy from our simple example should capture the essence of the problem. In reality, the problem of symmetry energy, entropy, and other thermodynamic quantities at low and high temperature has been explored in some detail [7–10], although a clear description in terms of entropy of mixing has been missing.

Comparing Eq. (4) (describing the energy dependence of the yield distributions) with Eq. (14) (describing the entropic dependence of the same yields) we first notice that both distributions have the same shape. Aside from multiplicative constants, the terms to be compared are $C_{\rm sym}/T$ and $1/2$. Replacing $C_{\rm sym}$ with the standard liquid drop value, we are to compare $1/2$ with ≈ 25 MeV$/T$ and see that under standard density conditions and temperature (up to 6 MeV) the mixing entropy contribution does not exceed 10% of the symmetry energy contribution.

It is then clear that two different effects contribute to the isoscaling property. The one, related to the the symmetry energy, is temperature dependent and the other, of entropic origin, is temperature independent. Both terms should be considered together in the symmetry free energy $\Delta F = \Delta E\text{-}T\Delta S$.

To have a clear view of the coexistence of these effects we have performed numerical simulations using the Illinois potential (for details, see Ref. [11] and references therein). In this case the Pauli principle is roughly taken into account by allowing the p-p and n-n interactions to be purely repulsive while the n-p interaction displays an attractive tail. We have performed molecular dynamics simulations of systems composed of $A = 80$ particles with 40 "protons" and 40 "neutrons" and a second system of $A = 96$ particles with $N = 56$ and $Z = 40$ inside spherical containers, thus allowing the systems to equilibrate. The density has been fixed and we have performed simulations in a rather ample

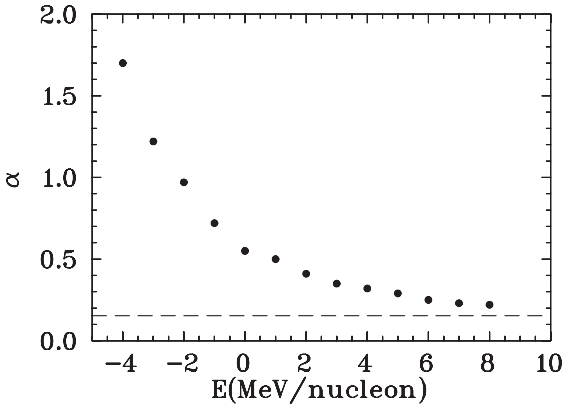

FIG. 2. The isoscaling parameter α resulting from the analysis of confined systems is plotted as a function of the total energy of the system. In this case the number density was fixed to 0.007. It can be seen that as the energy is increased the value of α tends toward the entropic limit of 0.154 (dashed line).

energy range. The results of such a calculation are displayed in Fig. 2. It is clear that as the energy is increased (and as a consequence the temperature increases), the value of the parameter α approaches the entropic limit [6] given by $\ln[\frac{p_2}{p_1}] = \ln[\frac{56/96}{40/80}] \simeq 0.154$.

The effect described above is purely classical, and, while it explains fully the percolation calculations mentioned above, its role in the nuclear systems dominated by quantum effects is not straightforward. It is safe, however, to guess that its role would be best observed in nuclear systems at high temperature and low density.

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