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# Topologically massive gravity at the chiral point is not unitary

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ABSTRACT: A recent paper [1] claims that topologically massive gravity contains only chiral boundary excitations at a particular value of the Chern-Simons coupling. On the other hand, propagating bulk degrees of freedom with negative norm were found even at the chiral point in [2]. The two references use very different methods, making comparison of their respective claims difficult. In this letter, we use the method of [1] to construct a tower of physical propagating bulk states satisfying standard AdS boundary conditions. Our states have finite norm, with sign opposite to that of right-moving boundary excitations. Our results thus agree with [2] and disagree with [1].

KEYWORDS: AdS-CFT Correspondence, Models of Quantum Gravity.

Recently, interest in pure AdS gravity has been revived following Witten’s work [3], which seemed to offer a chance of finding an exact solution to *a* quantum gravity (albeit a particularly simple one). Shortly afterward [1, 4] appeared, which argued for the existence of a theory even simpler than pure gravity. The claim was that in topologically massive gravity (TMG) [5, 6] at a special value of the Chern-Simons coupling the only boundary degrees of freedom are chiral. If true, the theory could solve some of the problems with Witten’s original proposal [7, 8].

However in [9] a propagating mode was found even at the special, “chiral” point  $\mu l = 1$  (defined in eq. (13) below). This is not by itself in contradiction with [1], since the mode does not respect standard Brown-Henneaux boundary conditions [10]: near the boundary it diverges linearly in the AdS radius. On the other hand, propagating modes obeying Brown-Henneaux boundary conditions were found in [2, 11].<sup>1</sup> Those papers work in the Poincaré patch of AdS, which only covers a part of the space; ref. [1] instead uses global coordinates. This difference between coordinate systems makes direct comparison of the Poincaré patch modes with the global-coordinate ones difficult. Among other things, the Poincaré patch energy does not coincide with energy in global coordinates. One is an element of the Lorentz subgroup of AdS<sub>3</sub> isometries:  $SO(1, 2) \subset SO(2, 2)$ ; the other is the (cover of) one  $SO(2)$  in the subgroup  $SO(2) \times SO(2) \subset SO(2, 2)$ . In the Poincaré patch the global energy appears as the generator of dilatations. Finally, the Poincaré patch energy has a continuous spectrum while the global energy spectrum is discrete.

In this letter we will work in global coordinates and analyze the spectrum of topologically massive gravity at the chiral point using the same method as ref. [1]. By applying appropriate generators of the AdS<sub>3</sub> isometry group  $SO(2, 2) \sim SL(2, R) \times \overline{SL(2, R)}$  to the linearly-divergent mode of ref. [9], we find modes that obey standard Brown-Henneaux boundary conditions. They are all descendant of a field that is not quite primary: it transforms into a locally pure gauge mode when hit by the  $L_{+1}, \bar{L}_{+1}$  generators of  $SL(2, R) \times \overline{SL(2, R)}$ . This gauge mode has zero norm [1] and it is pure gauge under diffeomorphism that act on the boundary as left-moving conformal transformations. After using this larger diffeomorphisms group to factor out the zero norm state, the new modes fall into a standard discrete representation of  $SL(2, R) \times \overline{SL(2, R)}$ . The representation is spanned by  $SL(2, R) \times \overline{SL(2, R)}$  descendants of a primary of weights  $h = 2, \bar{h} = 1$ , and thus carries an intriguing — though as yet mysterious — kinship with the topologically massive spin-one field that TMG reduces to at the chiral point [2].

By factoring out all diffeomorphisms that act on the boundary as left moving conformal transformations one can define a theory where only the right moving boundary Virasoro algebra acts nontrivially on physical states [14]. However this theory cannot be unitary, since the bulk state is still not pure gauge and its norm is negative.

The action of topological massive gravity is

$$S = +\frac{1}{2\pi} \int d^3x \sqrt{-g} \left( R + \frac{2}{l^2} \right) + \frac{1}{4\pi\mu} \int d^3x \varepsilon^{\alpha\mu\nu} \left( \Gamma_{\alpha\sigma}^\beta \partial_\mu \Gamma_{\nu\beta}^\sigma + \frac{2}{3} \Gamma_{\alpha\sigma}^\beta \Gamma_{\mu\gamma}^\sigma \Gamma_{\nu\beta}^\gamma \right), \quad (1)$$

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<sup>1</sup>See also [12, 13] for a canonical analysis of TMG, which also shows a propagating degree of freedom at the chiral point.

where  $\Lambda = -l^{-2}$  and  $\mu$  is the coupling constant of the Chern-Simons term. We choose a positive sign for the Einstein-Hilbert term. With this choice, BTZ [15] black holes have positive energy for  $\mu l > 1$ , while massive gravitons have negative energy.

The equations of motion are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{l^2}g_{\mu\nu} + \frac{1}{\mu}C_{\mu\nu} = 0. \quad (2)$$

The Cotton tensor  $C_{\mu\nu}$  is defined as:

$$C_{\nu}^{\mu} = \frac{1}{2}\varepsilon^{\mu\alpha\beta}\nabla_{\alpha}R_{\beta\nu} + \frac{1}{2}\varepsilon_{\nu}^{\alpha\beta}\nabla_{\alpha}R_{\beta}^{\mu}. \quad (3)$$

The Cotton tensor is the three-dimensional analog of the Weyl tensor in the sense that  $C_{\mu\nu} = 0$  if and only if the metric is conformally flat. The Cotton tensor vanishes for any solution to Einstein gravity, so all GR solutions are also solutions of TMG.

In this note we want to determine whether TMG possesses degrees of freedom propagating on an AdS background. In other words, we are interested in the perturbative spectrum, i.e. linearized fluctuations around empty AdS space. Thus we expand  $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2)$ , with  $\bar{g}_{\mu\nu}$  the AdS<sub>3</sub> metric.

The perturbation must leave the metric asymptotically AdS<sub>3</sub>. This fixes the asymptotics to be [10]:

$$\begin{aligned} g_{tt} &= -r^2/l^2 + \mathcal{O}(1), & g_{rr} &= l^2/r^2 + \mathcal{O}(r^{-4}), & g_{\phi\phi} &= r^2 + \mathcal{O}(1) \\ g_{r\phi} &= \mathcal{O}(r^{-3}), & g_{rt} &= \mathcal{O}(r^{-3}), & g_{t\phi} &= \mathcal{O}(1). \end{aligned} \quad (4)$$

Here we have used a global coordinate system in which the AdS<sub>3</sub> metric is

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = -(1 + r^2/l^2)dt^2 + \frac{dr^2}{(1 + r^2/l^2)} + r^2d\phi^2. \quad (5)$$

$\phi$  is the angular direction, and the radial direction is  $r \geq 0$ . The boundary is located at  $r = \infty$ .

It is convenient to write the AdS<sub>3</sub> metric in the following form:

$$ds^2 = l^2 (-\cosh^2 \rho dt^2 + \sinh^2 \rho d\phi^2 + d\rho^2). \quad (6)$$

where we defined  $r = l \sinh \rho$  and rescaled  $t \rightarrow lt$ . This coordinate system also covers the whole space, with the boundary at  $\rho = \infty$ . In these coordinates the asymptotics (4) become

$$\begin{aligned} h_{\rho\rho} &\simeq \mathcal{O}(e^{-2\rho}), & h_{\rho t} &\simeq \mathcal{O}(e^{-2\rho}), & h_{\rho\phi} &\simeq \mathcal{O}(e^{-2\rho}), \\ h_{tt} &\simeq \mathcal{O}(1), & h_{\phi\phi} &\simeq \mathcal{O}(1), & h_{t\phi} &\simeq \mathcal{O}(1). \end{aligned} \quad (7)$$

The isometry group of AdS<sub>3</sub> space is  $SL(2, R) \times \overline{SL(2, R)}$ , and its generators are realized on scalar fields by

$$L_0 = i\partial_u, \quad L_{\pm 1} = ie^{\pm iu} \left( \frac{\cosh 2\rho}{\sinh 2\rho} \partial_u - \frac{1}{\sinh 2\rho} \partial_v \mp \frac{i}{2} \partial_{\rho} \right), \quad (8)$$

$$\bar{L}_0 = i\partial_v, \quad \bar{L}_{\pm 1} = ie^{\pm iv} \left( \frac{\cosh 2\rho}{\sinh 2\rho} \partial_v - \frac{1}{\sinh 2\rho} \partial_u \mp \frac{i}{2} \partial_{\rho} \right). \quad (9)$$

The two light-like variables introduced here are defined by  $u = t + \phi$  and  $v = t - \phi$ .

In the linear approximation about the solution  $\bar{g}_{\mu\nu}$ , and in the harmonic gauge  $\bar{\nabla}_\mu h^{\mu\nu} = 0$  and  $h^\mu_\mu = 0$ , the graviton equations of motion take the form

$$(\mathcal{D}^+ \mathcal{D}^- \mathcal{D}^M h)_{\mu\nu} = 0 \tag{10}$$

where the metric is  $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2)$ , and where

$$(\mathcal{D}^\pm)^\nu_\mu = \delta^\nu_\mu \mp l \varepsilon^{\alpha\nu} \bar{\nabla}_\alpha, \quad (\mathcal{D}^M)^\nu_\mu = \delta^\nu_\mu + \frac{1}{\mu} \varepsilon^{\alpha\nu} \bar{\nabla}_\alpha. \tag{11}$$

The covariant derivative  $\bar{\nabla}_\alpha$  is defined using the background metric  $\bar{g}_{\mu\nu}$ .

Since  $\mathcal{D}^-$ ,  $\mathcal{D}^+$  and  $\mathcal{D}^M$  commute with each other, one can obtain all linearized solutions in terms of three functions  $h^\pm_{\mu\nu}$  and  $h^M_{\mu\nu}$  which obey<sup>2</sup>

$$(\mathcal{D}^+ h^+)_{\mu\nu} = (\mathcal{D}^- h^-)_{\mu\nu} = (\mathcal{D}^M h^M)_{\mu\nu} = 0 \tag{12}$$

We want to analyze the theory at the “chiral” point

$$\mu l = 1, \tag{13}$$

which could yield a chiral gravity, i.e. a theory in which only boundary modes of definite chirality and black holes exist. According to ref. [1], the theory at the chiral point is consistent because all negative energy modes disappear. A first observation is that, at the point  $\mu l = 1$ ,  $\mathcal{D}^M = \mathcal{D}^-$ .

In [9] an additional solution — not considered in [1]—was found. It is proportional to  $\partial_\mu h_{\mu\nu}|_{\mu=1/l}$ , which manifestly solves eq. (10). Its explicit form is

$$h_{\mu\nu}^{(new)} = \text{Re } \psi_{\mu\nu}^{(new)}, \quad \text{with} \quad \psi_{\mu\nu}^{(new)} = y(t, \rho) \psi_{\mu\nu}^- = e^{-2iu} y(t, \rho) H_{\mu\nu}(\rho). \tag{14}$$

The function  $y(t, \rho)$  is

$$y(t, \rho) = -\frac{i}{2}(u + v) - \log(\cosh \rho), \tag{15}$$

while the  $H_{\mu\nu}(\rho)$  are the components of the tensor (in the  $t, \phi, \rho$  basis)

$$H(\rho) = \begin{pmatrix} \tanh^2 \rho & \tanh^2 \rho & i \frac{\sinh \rho}{\cosh^3 \rho} \\ \tanh^2 \rho & \tanh^2 \rho & i \frac{\sinh \rho}{\cosh^3 \rho} \\ i \frac{\sinh \rho}{\cosh^3 \rho} & i \frac{\sinh \rho}{\cosh^3 \rho} & -\frac{1}{\cosh^4 \rho} \end{pmatrix}. \tag{16}$$

Now the crucial observation is that, while  $\psi_{\mu\nu}^{(new)}$  and its descendants, of the form  $\mathcal{L}_{L-1}^n \psi_{\mu\nu}^{(new)}$  diverge linearly in  $y(t, \rho)$  near the boundary,<sup>3</sup> there also exist descendants that satisfy the standard Brown-Henneaux asymptotics.<sup>4</sup> Here we adopted the standard

<sup>2</sup>We defined  $\psi_{\mu\nu}^R = \psi_{\mu\nu}^+$  and  $\psi_{\mu\nu}^L = \psi_{\mu\nu}^-$ .

<sup>3</sup>That is, linearly in  $\rho$  and also linearly in time.

<sup>4</sup>For quasinormal modes in a BTZ black hole background a similar fact has been independently noticed in [16].

notation  $\mathcal{L}_v$  for the Lie derivative along the vector field  $v$ . From now on, to simplify notations, we shall denote by  $L_n, \bar{L}_n$  both the vector field and the Lie derivative along it, whenever unambiguous. Also, we must recall that metric fluctuations are not uniquely defined: two fluctuations that can be mapped into each other by diffeomorphisms that vanish at infinity as

$$\zeta^\rho = \mathcal{O}(e^{-2\rho}), \quad \zeta^t = \mathcal{O}(e^{-4\rho}), \quad \zeta^\phi = \mathcal{O}(e^{-4\rho}), \quad (17)$$

represent the same physical state. We shall denote equality up to these trivial diffeomorphisms by  $\stackrel{bh}{=}$ .

An example of one such state is obtained as follows: define first of all the tensor perturbation

$$Y_{\mu\nu} \equiv \bar{L}_{-1}\psi_{\mu\nu}^{(new)} = \frac{1}{2}e^{-iv} \tanh \rho \psi_{\mu\nu}^- + h_{\mu\nu} = \frac{1}{2}e^{-i(v+2u)} \tanh \rho H_{\mu\nu}(\rho) + h_{\mu\nu}. \quad (18)$$

A simple calculation shows that  $h_{\rho\phi} = \mathcal{O}[y(t, \rho)e^{-2\rho}]$ ,  $h_{\rho t} = \mathcal{O}[y(t, \rho)e^{-2\rho}]$ ,  $h_{\rho\rho} = \mathcal{O}[y(t, \rho)e^{-4\rho}]$ ; therefore,  $h_{\rho\phi}, h_{\rho t}$  do not obey the asymptotics (7), while  $h_{\rho\rho}$  does. Inspection of the AdS metric eq. (6) shows immediately that  $h_{\rho\phi}$  and  $h_{\rho t}$  can be canceled up to terms with proper asymptotics by an infinitesimal diffeomorphism

$$t \rightarrow t + \zeta^t, \quad \phi \rightarrow \phi + \zeta^\phi, \quad \zeta^t, \zeta^\phi = \text{constant } e^{-i(v+2u)-4\rho} [y(t, \rho) + \mathcal{O}(1)], \quad (19)$$

which does not spoil the good asymptotics of any other component of the metric.

So, a bulk mode with proper boundary conditions is

$$X_{\mu\nu} = Y_{\mu\nu} + \nabla_\mu \zeta_\nu + \nabla_\nu \zeta_\mu = Y_{\mu\nu} + \mathcal{L}_\zeta \bar{g}_{\mu\nu}. \quad (20)$$

The field  $X_{\mu\nu}$  defined above and its  $\text{SL}(2, R) \times \overline{\text{SL}(2, R)}$  descendants generate a tower of states with the standard Brown-Henneaux asymptotic behavior.

It is worth mentioning that, while the (2,0)-primary states corresponding to solutions to  $(\mathcal{D}^- \psi^-)_{\mu\nu} = 0$  satisfy

$$(L_0 + \bar{L}_0) \psi^- = 2\psi^-, \quad (L_0 - \bar{L}_0) \psi^- = 2\psi^-, \quad (21)$$

the state  $\psi_{\mu\nu}^{(new)}$  is not primary, but it obeys instead the following equation

$$(L_0 + \bar{L}_0) \psi^{(new)} = 2\psi^{(new)} + \psi^-, \quad (L_0 - \bar{L}_0) \psi^{(new)} = 2\psi^{(new)}. \quad (22)$$

Since  $L_0 - \bar{L}_0 = i\partial_\phi$ , is the angular momentum, this state carries one unit of angular momentum: if it were a primary, it would generate a spin-one representation of  $\text{SL}(2, R) \times \overline{\text{SL}(2, R)}$ .

$X_{\mu\nu}$  too fails to be a conventional primary field because  $\text{SL}(2, R) \times \overline{\text{SL}(2, R)}$  descent operators do not annihilate it. Instead:

$$\begin{aligned} L_{+1} X_{\mu\nu} &= \mathcal{L}_{[L_{+1}, \zeta]} \bar{g}_{\mu\nu}, & \bar{L}_{+1} X_{\mu\nu} &= \psi_{\mu\nu}^- + \mathcal{L}_{[\bar{L}_{+1}, \zeta]} \bar{g}_{\mu\nu}, \\ L_0 X_{\mu\nu} &= 2X_{\mu\nu} + \frac{1}{2} \bar{L}_{-1} \psi_{\mu\nu}^- + \mathcal{L}_{[L_0, \zeta]} \bar{g}_{\mu\nu}, & \bar{L}_0 X_{\mu\nu} &= X_{\mu\nu} + \frac{1}{2} \bar{L}_{-1} \psi_{\mu\nu}^- + \mathcal{L}_{[\bar{L}_0, \zeta]} \bar{g}_{\mu\nu}. \end{aligned} \quad (23)$$

In writing these equations we have used standard properties of the Lie derivative, the commutation relations of  $\text{SL}(2, R) \times \overline{\text{SL}(2, R)}$  and the equations  $L_0 y = \bar{L}_0 y = 1/2$ ,  $L_{+1} y = \bar{L}_{+1} y = 0$ . We also exploited the fact that  $\psi^-$  is a  $(2, 0)$ -primary and that  $L_{+1}, L_0, \bar{L}_{+1}, \bar{L}_0$  are Killing vectors of the background AdS metric  $\bar{g}_{\mu\nu}$ .

The asymptotic form of the vector field  $\zeta$  defined in eq. (19) is such that  $[L_{+1}, \zeta]$ ,  $[\bar{L}_{+1}, \zeta]$ ,  $[L_0, \zeta]$ ,  $[\bar{L}_0, \zeta]$  actually obey Brown-Henneaux boundary conditions and vanish at the AdS boundary; moreover,  $\bar{L}_{-1} \psi^- = \bar{L}_{-1} L_{-2} \bar{g}_{\mu\nu} \stackrel{bh}{=} L_{-2} \bar{L}_{-1} \bar{g}_{\mu\nu} = 0$ .<sup>5</sup> The first equality follows from the definition of  $\psi^-$  [1], the second from Virasoro commutation relations, the third from  $\bar{L}_{-1}$  being an isometry of the background metric. So eq. (23) can also be written as

$$L_{+1} X_{\mu\nu} \stackrel{bh}{=} 0, \quad \bar{L}_{+1} X_{\mu\nu} \stackrel{bh}{=} \psi_{\mu\nu}^-, \quad L_0 X_{\mu\nu} \stackrel{bh}{=} 2X_{\mu\nu}, \quad \bar{L}_0 X_{\mu\nu} \stackrel{bh}{=} X_{\mu\nu} \quad (24)$$

The second of these equations makes  $X$  non-primary. Notice however that  $\psi_{\mu\nu}^-$  is a pure gauge excitation. If we define physical states modulo locally pure-gauge states,  $X_{\mu\nu}$  would be a true primary.

We can easily compute the norm of  $X$  at the chiral point. Start at a generic value of  $\mu l$  and consider the mode  $\bar{L}_{-1} \psi_M$ , where  $\psi_M$  is the massive graviton defined e.g in [1]:

$$\bar{L}_{-1} \psi_{\mu\nu}^M \stackrel{bh}{=} \bar{L}_{-1} (\psi^M - \psi^-)_{\mu\nu} + \nabla_\mu (\zeta_\nu^{\bar{h}} - \zeta_\nu^0) + \nabla_\nu (\zeta_\mu^{\bar{h}} - \zeta_\mu^0) \equiv \bar{h} X_{\mu\nu}^{\bar{h}}, \quad (25)$$

where  $\bar{h} = \mu l / 2 - 1/2$  is the right-moving weight of the massive graviton and  $\zeta_\mu^{\bar{h}} \equiv e^{\bar{h}y} \zeta_\mu$  generates a trivial diffeomorphism. The utility of this expression is that it converges pointwise to  $X$  in the limit  $\mu l \rightarrow 1$ :

$$X(\rho, u, v) = \lim_{\mu l \rightarrow 1} X^{\bar{h}}(\rho, u, v). \quad (26)$$

We can now easily compute the norm of  $X$ :

$$\langle X | X \rangle = \lim_{\mu l \rightarrow 1} \langle X^{\bar{h}} | X^{\bar{h}} \rangle = \bar{h}^{-2} \langle \psi^M | \bar{L}_{-1}^\dagger \bar{L}_{-1} | \psi^M \rangle = 2\bar{h}^{-1} \langle \psi^M | \psi^M \rangle, \quad (27)$$

where the second equality is true assuming the norm is invariant under Brown-Henneaux-trivial diffeomorphisms, and the last follows from the Virasoro algebra. With the choice of sign which gives BTZ black holes positive energy,  $\langle \psi^M | \psi^M \rangle = \bar{h} C$  with  $C$  negative [1]. Hence the norm of  $X$  is finite and negative.

Metric fluctuations obeying Brown-Henneaux boundary conditions are completely determined by their asymptotically non-vanishing components. In the case of  $X_{\mu\nu}$ ,  $\psi_{\mu\nu}^-$  and their descendants, the only non-vanishing component is  $h_{uu}$ . Therefore identification modulo the diffeomorphisms (17) tells us that a generic physical state takes the form

$$\begin{aligned} h_{uu}|_{\rho=\infty} &= \sum_{n,m \geq 0} x_{m,n} e^{-(2+n)iu - (1+m)iv} + \sum_{n \geq 0} \psi_n^- e^{-(2+n)iu} + c.c., \\ h_{vv}|_{\rho=\infty} &= \sum_{n \geq 0} \psi_n^+ e^{-(2+n)iv} + c.c. \end{aligned} \quad (28)$$

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<sup>5</sup>We thank Alex Maloney for pointing this out to us.

Each of the Fourier coefficients  $\psi_n^\pm, x_{n,m}$  is physical, so  $X_{\mu\nu}$  is not a standard primary. It is worth noting here that the asymptotics of this state at large  $m, n$  are identical to the short wavelength asymptotics of the states found in [2].

At the chiral point  $\mu l = 1$ , we can also define a *different* theory, where states are identified modulo the larger group [14]

$$\zeta^u = \epsilon(u) + \mathcal{O}(e^{-4\rho}), \quad \zeta^v = \frac{1}{2}e^{-2\rho}\partial_u^2\epsilon(u) + \mathcal{O}(e^{-4\rho}), \quad \zeta^\rho = -\frac{1}{2}\partial_u\epsilon(u) + \mathcal{O}(e^{-2\rho}). \quad (29)$$

This theory is chiral by construction. Physical states are defined by identifying those in (28) modulo the Virasoro algebra generated on asymptotic states by eq. (29). A possible gauge choice is to set all  $\psi_n^- = 0$ . This leaves yet unfixed the  $SL(2, R)$  generated by  $L_{\pm 1}, L_0$ . We can use it to fix three real coefficients in  $x_{m,n}$ . *All other  $x_{m,n}$  coefficients define distinct physical states*; by construction, they are chiral primaries of the right-moving Virasoro algebra surviving factorization by (29).

In conclusion, physical states obeying the *standard* Brown-Henneaux  $AdS_3$  asymptotic exist at the “chiral” point  $\mu l = 1$ . They are descendants of an “improper” primary,  $\psi_{\mu\nu}^{(new)}$ , which does not have the right asymptotics. The lowest weight state obeying the Brown-Henneaux asymptotics is  $X_{\mu\nu}$ , given explicitly in eq. (20). It can be promoted to a true primary by defining physical states modulo the Virasoro algebra (29). Irrespective of the gauge group used to define physical states, the theory is non-unitary, because states with negative norm exist. There is a strong case against topologically massive gravity being chiral *and unitary* at the chiral point, which we summarize here:

- There is an extra mode  $X$  at the chiral point which obeys the Brown-Henneaux boundary conditions and which is not pure gauge (even with the prescription of [14]).
- Modulo trivial diffeomorphisms  $X$  can be obtained as a smooth limit of the  $\bar{L}_{-1}$  descendent of the massive graviton in the limit  $\mu l \rightarrow 1$ .
- The asymptotic wavefunction for  $X$  matches the Poincare-patch results of [2] at short wavelength.
- Including  $X$ , the counting of states matches the canonical analyses of [12, 13, 17].
- The norm of  $|X\rangle$  is negative.

A still unresolved and intriguing question is how the (2,1) representation we have found relates to the fact that TMG at the chiral point can be thought of as a topologically massive spin one field. A better understanding the chiral spectrum of [2] may shed light on this connection.

A possible way out is if the theory at the chiral point possesses another yet to be discovered gauge symmetry beyond diffeomorphisms (perhaps akin to the Weyl invariance of pure Chern-Simons gravity in flat space), which changes the definition of the energy and renders  $X$  pure gauge. Another possibility is that our mode becomes non-normalizable at higher than leading order.<sup>6</sup> However, the absence of local bulk degrees of freedom appears to be in conflict also with the non-perturbative canonical analyses of [12, 13, 17].

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<sup>6</sup>We thank A. Strominger for suggesting this possibility to us.



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## References

- [1] W. Li, W. Song and A. Strominger, *Chiral gravity in three dimensions*, *JHEP* **04** (2008) 082 [[arXiv:0801.4566](#)].
- [2] S. Carlip, S. Deser, A. Waldron and D.K. Wise, *Cosmological topologically massive gravitons and photons*, [arXiv:0803.3998](#).
- [3] E. Witten, *Three-dimensional gravity revisited*, [arXiv:0706.3359](#).
- [4] W. Li, W. Song and A. Strominger, *Comment on 'cosmological topological massive gravitons and photons'*, [arXiv:0805.3101](#).
- [5] S. Deser, R. Jackiw and S. Templeton, *Three-dimensional massive gauge theories*, *Phys. Rev. Lett.* **48** (1982) 975.
- [6] S. Deser, R. Jackiw and S. Templeton, *Topologically massive gauge theories*, *Ann. Phys. (NY)* **140** (1982) 372 [*Erratum ibid.* **185** (1988) 406] [*Ann. Phys. (NY)* **281** (2000) 409].
- [7] A. Maloney and E. Witten, *Quantum gravity partition functions in three dimensions*, [arXiv:0712.0155](#).
- [8] J. Manschot, *AdS<sub>3</sub> partition functions reconstructed*, *JHEP* **10** (2007) 103 [[arXiv:0707.1159](#)].
- [9] D. Grumiller and N. Johansson, *Instability in cosmological topologically massive gravity at the chiral point*, *JHEP* **07** (2008) 134 [[arXiv:0805.2610](#)].
- [10] J.D. Brown and M. Henneaux, *Central charges in the canonical realization of asymptotic symmetries: an example from three-dimensional gravity*, *Commun. Math. Phys.* **104** (1986) 207.
- [11] S. Carlip, S. Deser, A. Waldron and D.K. Wise, *Topologically massive AdS gravity*, *Phys. Lett.* **B 666** (2008) 272 [[arXiv:0807.0486](#)].
- [12] D. Grumiller, R. Jackiw and N. Johansson, *Canonical analysis of cosmological topologically massive gravity at the chiral point*, [arXiv:0806.4185](#).
- [13] S. Carlip, *The constraint algebra of topologically massive AdS gravity*, [arXiv:0807.4152](#).
- [14] A. Strominger, *A simple proof of the chiral gravity conjecture*, [arXiv:0808.0506](#).
- [15] M. Bañados, C. Teitelboim and J. Zanelli, *The black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [[hep-th/9204099](#)].
- [16] I. Sachs, *Quasi-normal modes for logarithmic conformal field theory*, *JHEP* **09** (2008) 073 [[arXiv:0807.1844](#)].
- [17] M.-I. Park, *Constraint dynamics and gravitons in three dimensions*, *JHEP* **09** (2008) 084 [[arXiv:0805.4328](#)].