

On the use of the Prandtl mixing length model in the cutting torch modeling

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Abstract. The Prandtl mixing length model has been used to take into account the turbulent effects in a 30 A high-energy density cutting torch model. In particular, the model requires the introduction of only one adjustable coefficient c corresponding to the length of action of the turbulence. It is shown that the c value has little effect on the plasma temperature profiles outside the nozzle (the differences being less than 10 %), but severely affects the plasma velocity distribution, with differences reaching about 100 % at the middle of the nozzle-anode gap. Within the experimental uncertainties it was also found that the value $c = 0.08$ allows to reproduce both, the experimental data of velocity and temperature

1. Introduction

Plasma hydrodynamic modeling by numerical simulation in cutting torches is a common tool to predict the values of the fundamental physical quantities, namely the plasma temperature, the particles concentration and the fluid velocity. These numerical codes are employed to understand the relevant physical processes ruling the plasma behavior in order to interpret the experimental results of several plasma diagnostics, and ultimately to obtain optimized designs of such devices.

In particular, the practical use of turbulent hydrodynamic codes requires the introduction of some numerical coefficient [1] whose value has to be obtained from a comparison between the model predictions and the experiment. Unfortunately, since most of the available experimental data on cutting arcs are related to temperatures and species concentrations in the external plasma region [2], the experimental validation of the existing cutting torch models [3-6] has been restricted to the temperature distribution in the nozzle-anode gap. Only recently [7], a measurement of the plasma flow velocity was reported in cutting torches.

The purpose of this work is to validate a turbulent code based in the Prandtl mixing length model employing not only temperature but also velocity values as the experimental data to be confronted with. In order to do this, a 2-D local thermodynamic equilibrium (LTE) plasma model was developed and applied to the same 30 A oxygen cutting torch that was used in a previous velocity measurement experiment [7].

2. Mathematical model

2.1 Computational domain

The schematic of the modeled domain for the simulation is presented in figure 1. The edge of the domain EF is located at a radius of $10 R_N$ [6]. FG represents the anode. A mass flow rate of 0.71 g s^{-1} with a vortex injection that leads to a ratio of the azimuthal to the axial inlet velocity of $\tan(13^\circ) \approx 0.23$ [7] was used at the torch inlet CD. More details on the torch characteristics can be found elsewhere [7].

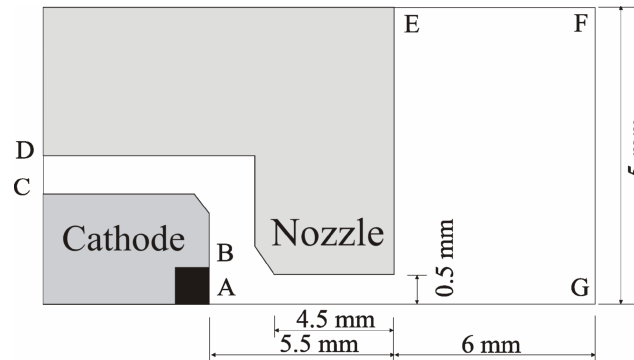


Figure 1. Cutting torch computational domain.

2.2 Model assumptions

- The plasma is in LTE, the fluid being characterized by a single temperature T .
- The plasma flow is two-dimensional and axisymmetric.
- The plasma is considered as a Newtonian fluid following the Navier-Stokes equation.
- The plasma gas is assumed to be pure Oxygen with thermodynamic and transports coefficients as calculated by Murphy [8].
- Hall currents and gravitational effects are considered negligible.
- In the energy equation the viscous dissipation term is considered negligible.
- The anode was considered as a porous free boundary characterized by its electrostatic potential.

2.3 Governing equations

The set of conservation equations describing such a flow can be expressed as follows.

Total mass conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0. \quad (1)$$

Momentum conservation

$$\frac{\partial (\rho \bar{u})}{\partial t} + \nabla \cdot (\rho \bar{u} \otimes \bar{u} + p \bar{\delta} - \bar{\tau}) - \bar{J} \times \bar{B} = 0. \quad (2)$$

Internal energy conservation

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho \bar{u} e + \bar{q}) - \bar{J} \cdot \bar{E} + p \nabla \cdot \bar{u} + 4\pi \epsilon_N \frac{p}{P_{ATM}} = 0, \quad (3)$$

where ρ represents the total mass density, \bar{u} the fluid velocity (having, axial $-u_x$, radial $-u_y$ and azimuthal $-u_z$ components), p the pressure, $\bar{\delta}$ the identity tensor, $\bar{\tau}$ the stress tensor, \bar{J} the current density, \bar{B} the magnetic field (only the azimuthal component was considered), e the

internal energy, \bar{q} the total heat flux, \bar{E} the electric field and ε_N the plasma radiation net emission coefficient (NEC).

Two further equations are required to describe the electromagnetic part of the plasma model. The first is the current continuity equation

$$\nabla \cdot \bar{J} = 0, \quad (4)$$

where

$$\bar{J} = -\sigma \nabla \phi, \quad (5)$$

and the second is one of Maxwell's equations

$$\nabla \times \bar{B} = \mu_0 \bar{J}, \quad (6)$$

where σ is the electric conductivity, ϕ is the electrostatic potential and μ_0 the magnetic permeability of free space.

The total heat flux in (3) describes the heat transported by conduction and the enthalpy transport by mass diffusion, and is defined as

$$\bar{q} \equiv -\kappa_e \nabla T + \bar{\Gamma}_e h_e, \quad (7)$$

where κ_e is the effective thermal conductivity and $\bar{\Gamma}_e$ is the electron mass diffusion that can be approximated by

$$\bar{\Gamma}_e \approx -\frac{m}{e'} \bar{J}, \quad (8)$$

where e' is the elementary electric charge and m is the electron mass. Equation (8) neglects the charge transported by ions. In (7) $h_e = 5k_B T / (2m)$ represents the specific electron enthalpy (k_B is the Boltzmann's constant).

The effective viscosity is

$$\mu_e = \mu_l + \mu_t, \quad (9)$$

and the effective thermal conductivity is

$$\kappa_e = \kappa + \frac{\mu_t C_p}{P_r}, \quad (10)$$

where C_p , μ_l and κ are the plasma specific heat at constant pressure, viscosity and thermal conductivity, respectively. The turbulent Prandtl number P_r and the turbulent viscosity μ_t are given in the next subsection.

The source terms in (3) account for the Joule effect, the compression work, and the radiation losses $4\pi\varepsilon_N$, where ε_N was taken for a plasma radius of 0.5 mm [3]. The NEC of pure Oxygen for one atmosphere [9] has been multiplied by the factor p/p_{ATM} for other pressures [5].

2.4 Turbulent modeling

The closure of the system equations requires extra relationships to calculate the turbulent enhanced viscosity and thermal conductivity. The simple Prandtl mixing length model was chosen. Such length is given as:

$$l_m \equiv c \lambda, \quad (11)$$

where c is an adjustable parameter and λ is a local thermal radius defined as the radial distance from the axis to the point at 2000 K [10]. It has been found that for transferred arcs the turbulent Prandtl number can be approximated by unity ($P_r \equiv 1$) thus only the parameter c in (11) needs to be adjusted by comparing the numerical results with the experiment. The turbulent viscosity for isotropic turbulence was calculated taking into account the effect of the vortex injection [11]

$$\mu_t = \rho l_m^2 \left((\partial u_x / \partial y)^2 + (y \partial (u_z / y) / \partial y)^2 \right)^{1/2}. \quad (12)$$

2.5 Boundary conditions

Table 1 summarizes the prescribed values of the physical quantities (or of their spatial derivatives) on the boundaries shown in figure 1. In addition, the voltage drop between the cathode AC and the anode FG was adjusted in order that the integrated value of the axial current density on a given section corresponds to the value of the electric current of the torch. An external source term to increase the temperature [1] was applied at the axis of the torch AG to initiate the current. A current value of 30 A was used in this study [7]. Also, at the hafnium insert AB the maximum value of the axial current density on the axis of the geometry was limited to $\leq 170 \text{ A mm}^{-2}$ [1]. Besides, the electrostatic potential value of the nozzle DE was calculated so as to preserve the zero current balance at its surface (i.e., the nozzle is electrically floating).

Table 1. Boundary conditions.

	p	u	T	ϕ
AB	–	0	3500 K	–
BC	–	0	500 K	–
CD	–	mass flow 0.71 g s^{-1}	300 K	$\frac{\partial \phi}{\partial x} = 0$
DE	–	0	500 K	–
EF	1 atm	$\frac{\partial u_x}{\partial y} = \frac{\partial u_y}{\partial y} = \frac{\partial u_z}{\partial y} = 0$	300 K	$\frac{\partial \phi}{\partial y} = 0$
FG	–	$\frac{\partial u_x}{\partial x} = \frac{\partial u_y}{\partial x} = \frac{\partial u_z}{\partial x} = 0$	$\frac{\partial T}{\partial x} = 0$	$\phi = 0$
GA	–	$\frac{\partial u_x}{\partial y} = 0, u_y = u_z = 0$	$\frac{\partial T}{\partial y} = 0$	$\frac{\partial \phi}{\partial y} = 0$

2.6 Numerical aspects

The unsteady form of the model equations was solved using a time-marching method [12]. The specific values used for the initial guesses did not impact on the final converged results. The set of governing equation was discretized in time using a Taylor series first-order accurate, in space using the finite volume method, and solved with the given boundary conditions on a 81×15 non uniform internal grid-points and 39×47 non uniform external grid-points, by using the predictor-corrector algorithm [12]. The time-step used in the time-marching algorithm was chosen so that the CFL criterion was fulfilled [12]. The calculation was stopped when the relative variation of the plasma variables between two consecutive time iterations was $< 10^{-3}$. The accuracy of the calculations was tested by repeating them with a 38×15 internal grid-points and 19×47 external grid-points. The change in the plasma temperature was everywhere less than 15 %, while the changes in the axial velocity were less than 20 %. The finer grid was then used for generating the results to be presented in the following Section.

3. Validation of the model

This section is devoted to the torch model validation for temperature and velocity by comparing the model results with experimental data. For the temperature, its radial profile at 3.5 mm from the nozzle exit was used. This profile was derived from electrostatic probe [13,14] and schlieren [15] measurements. For the axial velocity, the axial distribution derived from a time-of-flight technique, corresponds to light emitted from the arc central core [7].

Figure 2a) presents the radial profiles of the calculated temperature at 3.5 mm from the nozzle exit for $c = 0$ (i.e., laminar flow), $c = 0.08$ and $c = 0.20$. As shown, all the temperature profiles are

similar (their differences being smaller than the experimental temperature uncertainty $\approx 10\%$). Figure 2b) shows the comparison among the theoretical profile corresponding to $c = 0.08$ and the experimentally derived temperature profiles. It can be seen from this figure that the model results are in good agreement with the experimental data [13-15] for $c = 0.08$.

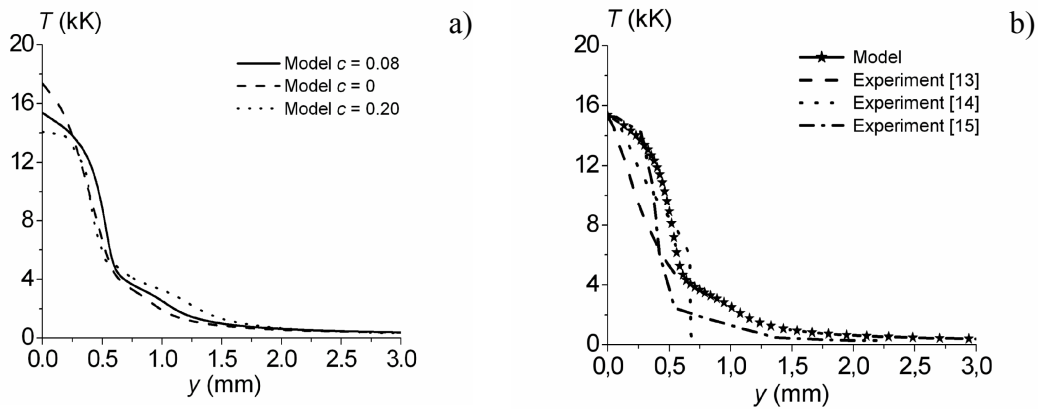


Figure 2. a) Radial profile of the calculated plasma temperature for $c = 0$, $c = 0.08$ and $c = 0.20$. b) Radial profile of the plasma temperature for $c = 0.08$ together with the experimental data.

The theoretical distributions of the axial velocity on the axis for the same c values presented in figure 2a) are shown in figure 3. For comparison purposes, the measured values of the axial velocity corresponding to light emitted from the arc central core [7] are also included in figure 3. It can be seen that the theoretical profiles are close among them at the vicinities of the nozzle exit (reflecting the well known fact of the little importance of the turbulence inside the nozzle [1]) but soon after the scatter in the u_x values is larger than those found for the temperature, reaching about 100 % at the middle of the gap. Hence, it can be concluded that the fluid velocity strongly depends on the particular value of the turbulent parameter c . On the other hand, the theoretical profile presenting the best matching with the experimental data is that corresponding to $c = 0.08$.

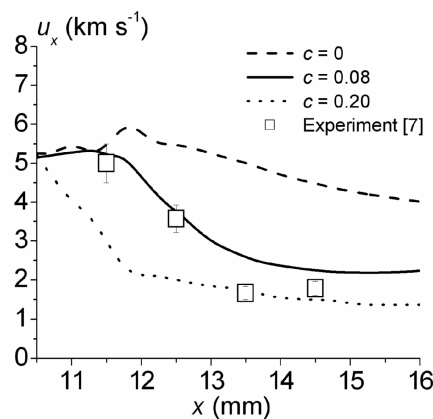


Figure 3. Comparison between calculated plasma velocity values at the axis for $c = 0$, $c = 0.08$ and $c = 0.20$, and the measured values of the axial velocity corresponding to light emitted from the arc central core.

4. Conclusions

Up to now, turbulent models for the plasma generated in cutting torches published during the last ten years have been validated using temperature data derived from spectroscopic measurements in

the nozzle-anode gap. It has been shown in this work that the plasma temperature is not the most appropriate quantity to validate numerical codes since it is not quite sensitive to changes in the model numerical parameters. Instead, it has been shown that the plasma velocity appears to be a more adequate quantity to perform such validation. In order to realize this validation to such a sensitive variable as the plasma velocity, a 2-D model similar to those proposed in the literature was developed and applied to the same 30 A high-energy density cutting torch that was used in the velocity measurements recently published by some of the authors [7]. Within the experimental uncertainties, it was found that a Prandtl mixing length turbulent parameter $c = 0.08$ allows to reproduce both the experimental data of velocity and temperature.

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