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Energy and reserve co-optimization within the Short Term Hydrothermal Scheduling under uncertainty: A proposed model and decomposition strategy

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ABSTRACT

Procurement of spare capacity is an essential mechanism in electric power systems. Through it the system is able to endure the disturbances induced by different sources of uncertainty, which also impact the system's operational cost. This paper introduces an optimization model for the Short Term Hydrothermal Scheduling that performs a co-optimization of energy and reserves for tertiary regulation, considering hydrology uncertainty and forced outages of generation units and transmission lines. The formulation presented is extended to consider intermittent stochastic generation from wind farms or hydro plants without storage capability. This work also proposes a decomposition scheme to deal with the resulting stochastic and large scale optimization problem. The proposed strategy relies on a coordinated application of two instances of the Bender's decomposition principle. By means of two study cases it will be demonstrated the effectiveness of the proposed model and the performance of the methodology.

1. Introduction

Short Term Hydrothermal Scheduling (STHS) is a fundamental task in modern power systems. As the restructuring of the electric power industry advances, and even when renewables as wind or photovoltaic power are gaining attention, hydroelectric power is still seen as a highly desirable form of energy from an economic, operational and reliability perspective. Hydroelectric plants remain preeminent suppliers within any power system due to the fact that a limited and low cost energy resource can be feasibly stored, and because of its flexible regulation capability (among others).

Hydrothermal scheduling aims to obtain the optimal operation policy that minimizes the total operation cost, including the production cost of thermal plants, the cost of reserve provision, and the expected costs for energy not supplied. However, because of the temporal linkage between the decision processes and the variability that characterizes the water inflows to reservoirs, this optimization problem is of higher complexity than in purely thermal systems. Independently of the above facts, when contingencies occur or the hydrology differs from its predicted values, the system must be restored to its energy balance and stability conditions while ensuring the minimal economic harm to users.

The security and reliability of the power system are faced through the procurement of sufficient operating reserves that provide three types of regulation. Reserves for primary and secondary regulation respond to deviations from the nominal frequency, and follow normal load variations through automatic generation control. This regulation must be provided by plants that are spinning and synchronized to the grid. Tertiary regulation may be supplied by units that are spinning or not spinning (supplemental reserves), which are not limited by the synchronized state of the generator. The deployment of supplemental reserves allows the repositioning of generating levels such that the faster spinning reserves become available to respond to further disturbances [1]. The emphasis of the present contribution is on spinning and supplemental reserves for tertiary regulation, considering the uncertainty affecting the hydrothermal system.

A singularity of hydrothermal systems is related to the different optimizations steps needed within a fixed horizon. For the context of the present work, a weekly scheduling is necessary because of the periodic but dissimilar load profile that characterizes the week, particularly accentuated between the work and the weekend days, and the water reserve that must be optimally allocated within this period. Decisions on the stored energy in reservoirs will considerably influence the total operation cost, especially when the penetration of hydroelectric power is significant.

The present work aims to show that the decisions about the water levels to store or release from reservoirs are different,

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Notation

Indexes/parameters

t	index of stages running from 1 to T
i	index of thermal plants running from 1 to $\mathcal{G}$
j	index of short-term regulation reservoirs, running from 1 to $\mathcal{J}^c$
y	index of seasonal regulation reservoirs, running from 1 to $\mathcal{J}^s$
n	index of buses running from 1 to $\mathcal{N}$
m	index of transmission lines running from 1 to $\mathcal{M}$
s	index of hydrology scenarios running from 1 to $\mathcal{S}$
k	index of contingencies running from 0 to $\mathcal{K}$
ξ	index of stochastic realization constituted by the pair of hydrology scenario s and contingency k, i.e., $\xi = \{s, k\}$ and running from 1 to Ξ

Sets

$\mathcal{G}_n$	set of thermal plants connected to bus n
$\mathcal{J}_n^c$	set of short-term hydroelectric plants connected to bus n
$\mathcal{J}_n^s$	set of seasonal hydroelectric plants connected to bus n

Parameters

Δt	duration of each stage [h]
T_i^{ON}	minimum ON time of thermal plant i [h]
T_i^{OFF}	minimum OFF time of thermal plant i [h]
SU_i	start-up costs of thermal plant i [$]
SD_i	shut-down costs of thermal plant i [$]
FC_i	fix costs of thermal plant i [$/h]
CE_i	cost of energy from thermal plant i [$/MWh]
WV_y	water value function (constant) of seasonal hydroelectric plant y [$/MWh]
$\mathrm{CR}_i^{\mathrm{U}}$	cost of spinning up reserve from thermal plant i [$/MWh]
$\mathrm{CR}_i^{\mathrm{D}}$	cost of spinning down reserve from thermal plant i [$/MWh]
$\mathrm{CR}_i^{\mathrm{NS}}$	cost of non-spinning reserve from thermal plant i [$/MWh]
$\mathrm{CR}_y^{\mathrm{U}}$	cost of up reserve from hydroelectric plant y [$/MWh]
$\mathrm{CR}_y^{\mathrm{D}}$	cost of down reserve from hydroelectric plant y [$/MWh]
VOLL	value of lost load [$/MWh]
$\mathcal{L}_{nt}$	demand at bus n during stage t [MW]
F_m^{MAX}	power flow limit of transmission line m [MW]
P_i^{MAX}	upper limit on power of thermal plant i [MW]
P_i^{MIN}	lower limit on power of thermal plant i [MW]
R_i^{Umax}	maximum up reserve of thermal plant i [MW]
R_i^{Dmax}	maximum down reserve of thermal plant i [MW]
$H_{y,j}^{\mathrm{MAX}}$	limit on power output of hydroelectric plant y or j [MW]
Q_j^{MAX}	maximum flow rate of short-term hydroelectric plant j [m^3/s]
V_j^{MAX}	maximum volume of reservoir associated to hydroelectric plant j [hm^3]
V_j^{MIN}	minimum volume of reservoir associated to hydroelectric plant j [hm^3]
$\boldsymbol{\kappa}_j$	power conversion factor of hydroelectric plant j [MW s/m^3]
C_0	constant term to convert m^3/s to Hm^3 ($C_0 = (3600/1e6) \cdot \Delta t$)
$\bar{\mathrm{EH}}_y$	available energy in seasonal reservoir y for the planning horizon under study [MWh]
I_{jt}^s	inflow volume to reservoir j for scenario s during stage t [m^3/s]
$\bar{\mathbf{I}}_{jt}$	forecast average inflow volume to reservoir j for most probable hydrology scenario during stage t [m^3/s]
U_{it}^k	status of thermal plant i for contingency k during stage t (1 = available, 0 = unavailable)
$\boldsymbol{\pi}_k$	probability of contingency k
$\boldsymbol{\sigma}_s$	probability of hydrology scenario s
$\boldsymbol{\psi}^k$	network sensitivity matrix for contingency k during stage t

Variables

u_{it}	commitment of thermal plant i for period t
a_{it}	start-up of thermal plant i at the beginning of period t
z_{it}	shut-down of thermal plant i at the beginning of period t
v_{it}	commitment of thermal plant i for non-spinning up reserve during period t
R_{it}^{U}	scheduled spinning up reserve of thermal plant i for stage t [MW]
R_{it}^{D}	scheduled spinning down reserve of thermal plant i for stage t [MW]
R_{it}^{NS}	scheduled non-spinning up reserve of thermal plant i for stage t [MW]
W_{yt}^{U}	scheduled spinning up reserve of hydroelectric plant y for stage t [MW]
W_{yt}^{D}	scheduled spinning down reserve of hydroelectric plant y for stage t [MW]
V_{jt}	volume of reservoir j at the end of stage t [hm^3]
F_{mt}^{sk}	power flow through line m for scenario s and contingency k during stage t [MW]
P_{it}^{sk}	output power of thermal plant i for scenario s and contingency k during stage t [MW]
H_{jt}^{sk}	output power of hydroelectric plant j in scenario s and contingency k during stage t [MW]
H_{yt}^{sk}	output power of hydroelectric plant y in scenario s and contingency k during stage t [MW]
Q_{jt}^{sk}	discharge volume of reservoir j in scenario s and contingency k during stage t [m^3/s]
S_{jt}^{sk}	spilled volume of reservoir j in scenario s and contingency k during stage t [m^3/s]
ENS_{nt}^{sk}	energy not supplied at bus n in scenario s and contingency k during stage t [MW]

depending on whether or not the inherent aspects of reserves for tertiary regulation are taken into consideration.

1.1. Literature review

The current practice in many systems is to solve the STHS problem using a deterministic approach. This results in misleading operation policies that could force the System Operator (SO) to purchase expensive energy from interconnections, or even worse, to schedule load curtailment actions. When this is the case, the STHS models have failed to provide a proper adjustment of the schedules derived from longer term planning models, especially with respect to the volume in reservoirs, whose primary resource must still be optimized within the short term model.

Authors of [2] proposed an all-encompassing deterministic model that is solved using a Genetic Algorithm (GA), solving the Unit Commitment (UC) problem, energy and spinning reserve scheduling, and an economic load dispatch within a single algorithm. Other approaches for a daily context perform a deterministic STHS with UC and exogenous co-optimization of spinning reserve, emphasizing active and reactive power flow constraints, and where the nonlinear multi-stage problem is addressed using General Benders Decomposition (GBD) [3,4]. Recent contributions propose a stochastic security constrained model considering load and hydrology uncertainty, and forced outages of plants. The problem is addressed by means of a two-stage stochastic programming model for which they propose a hybrid decomposition strategy [5]. A similar approach is offered in [6], except that stochastic production comes from wind farms instead of hydroelectric plants.

Contribution from [7] proposes a robust optimization model for the daily term (24 h) considering load uncertainty, and they handle the stochastic variable (load demand) employing information gap decision theory (IGDT). In this approach, the Independent System Operator (ISO) is able to assess the robustness of its decisions against high load using a risk-averse model. Based on the structure of the formulation, this can be extended to include exogenous reserves and a DC or AC load flow model, but due to the increase in size of the resulting MINLP problem, the model remain suitable for a daily scheduling scope. Authors of [8,9] propose a novel multi-objective optimization model to address the daily scheduling of hydrothermal systems, considering non smooth fuel cost functions of thermal plants (due to the valve-point effects) and nonlinear hydro productivity functions. The model is relevant where environmental restrictions must be measured. Two objective functions are considered, the thermal costs of thermal plants and an emissions amount function, expressed as a nonlinear function of the power output. They approach the problem applying lexicographic optimization and then using Normal Boundary Intersection (NBI) to determine the optimal Pareto solutions. When their model was tested, it achieved a reduced solution time. Their model remains in the deterministic optimization realm, as there are no stochastic variables involved.

With respect to energy & reserve co-optimization, the trend a few years ago was for simultaneous optimization of these services using exogenous rules, defining the reserve needs beforehand. In this line, some models addressed the problem including the transmission network, considering reliability data from equipment and a set of credible contingencies to assess compliancy with reliability indexes [10]. It has also been proposed a deterministic security constrained energy and reserve dispatch accounting for power flow limits and lines losses [11]. Nowadays, there is a widely recognized consensus that energy supply and reserve provision services not only must be scheduled jointly due to the strong coupling between them, but also that the optimal levels of reserve to procure should be a byproduct of the optimization process [12–14]. Under this perspective and due to the large scale, stochastic, nonlinear and non-convex characterization of the problem, evolutionary strategies have been also proposed [15,16].

The latter proposals represent a proper theoretical framework over which a balanced approach to the STHS problem may be shaped. The stand adopted in the present work is that the reserve procurement cost should match the value it provides to users, and that the criteria for optimal reserve allocation should not ignore the stochastic nature of the factors that call for balancing energy. Consequently, the levels of reserves to procure should be obtained considering the magnitude of the uncertainties affecting the power system and the impact this would have on the system's operational cost [17].

1.2. Motivation

This paper proposes an optimization model for STHS that jointly optimizes the reserves for tertiary regulation (spinning and supplementary). The reserves to allocate are obtained endogenously by the optimization process. The model incorporates the transmission network, uncertainty of water, and considers random failure of generating units and transmission lines. This paper also proposes a decomposition strategy to cope with the large scale, multi-stage, stochastic and non-convex problem that the model represents. The intention pursued is to offer a scheduling instrument that adequately anticipates the planning models of shorter duration (as the daily scheduling or the economic load dispatch), by optimally setting the volumes in reservoirs in the light of the inflow variations and the uncertainties that are likely to happen in the real operation.

The methodology relies on the General Benders' Decomposition principle. Because the resulting problem possess a decomposable structure, it is adequate to be split into three blocks: a master problem, a sub-problem, and a set of stochastic problems. The master problem deals with the UC and contains integer variables only; the sub-problem deals with the thermal energy, hydro energy and reserve dispatch; the set of stochastic problems deals with the expected operation cost for the realizations of hydrology scenarios and contingencies.

The following lines describe the main contributions from this work:

(a) An extensive stochastic STHS formulation that includes the UC problem and considers hydrology uncertainty and contingencies.
(b) A joint energy-reserve optimization for the STHS problem where the amounts of spinning and supplementary reserves for tertiary regulation are obtained endogenously.
(c) A three-level decomposition architecture that allows solving problems of moderate dimension and known structure within each level. This implies a significant advantage because a realistic set of credible contingencies and hydrology scenarios may be considered in the problem, without the computational limitations imposed by non-decomposable structures.

The structure of this paper is as follows: Section 2 presents the formulation of the stochastic STHS; Section 3 provides a background on the Benders' decomposition scheme, and develops the proposed methodology; Section 4 presents two study cases, oriented to assess the proposed model against existing reserve determination criteria, and to demonstrate the application and performance of the strategy; Section 5 extends the formulation to consider intermittent energy generation from wind farms and non-storable hydroelectric stations. The paper is ended in Section 6 with the conclusions.

2. Problem formulation

2.1. Model assumptions

The modelling assumptions considered in this work are exposed below.

1. The envisioned power system is one on which the System Operator or other regulation entity must perform a short term optimization model based on the producers' costs, in a centralized manner.
2. The demand is assumed to be price-inelastic.
3. Seasonal hydroelectric plants are assumed to have obtained water value or future cost functions, after running a medium or long term optimization.

4. The production costs of short-term reservoirs are assumed to be nil, and allocation of water is left to the entity responsible of the short term scheduling.
5. Supplementary reserves for tertiary regulation are available to be deployed within a time delay of 10 min or less, following a disturbance that have required the activation of regulation services.
6. Providers of supplementary reserves have submitted reserve provision costs in the light of their known probability of utilization. Consequently, they have embedded the start-up and shut-down costs in their offer cost ($\mathrm{CR}^{\mathrm{NS}}$).
7. In case of calling for generation supply after a disturbance, providers of supplementary reserves are able to handle by themselves the technical aspects of the minimum ON and OFF times of their plants, so that these constraints are not binding in the post-contingency analysis of the present formulation.
8. The transmission network is modelled using a DC power flow representation.
9. The productivity function of reservoirs is considered linear.
10. The sources of uncertainty considered are the inflows to reservoirs and forced outages of generating units and transmission lines. It is assumed that the stochastic processes that induce these phenomena are known in the form of continuous or discrete time functions.

2.2. Formulation

The objective function (OF) of the proposed model entails the minimization of the commitment costs, energy and reserve costs, expected operation and load shedding costs (balancing costs), and the future cost of seasonal reservoirs. It is given by:

$$\mathrm{OF} = \min \sum_{t=1}^{T} \left(C_t^{\mathrm{UC}} + (\boldsymbol{\sigma}_1 \boldsymbol{\pi}_0) \cdot C_t^{\mathrm{OPER}} + C_t^{\mathrm{BAL}} + C_t^{\mathrm{FUTURE}} \right) \tag{2.1}$$

where

$$C_t^{\mathrm{UC}} = \mathrm{FC}_i \cdot u_{it} + \mathrm{SU}_i \cdot a_{it} + \mathrm{SD}_i \cdot z_{it} \tag{2.2}$$

is the Unit Commitment (UC) costs, which entails the fixed costs, the start-up and shut-down costs of thermal plants. The second term is:

$$C_t^{\mathrm{OPER}} = \sum_{i=1}^{\mathcal{G}} [\mathrm{CE}_i P_{it} + \mathrm{CR}_i^{\mathrm{U}} R_{it}^{\mathrm{U}} + \mathrm{CR}_i^{\mathrm{D}} R_{it}^{\mathrm{D}} + \mathrm{CR}_i^{\mathrm{NS}} R_{it}^{\mathrm{NS}}] + \sum_{y=1}^{\mathcal{F}^S} [\mathrm{CR}_y^{\mathrm{U}} W_{yt}^{\mathrm{U}} + \mathrm{CR}_y^{\mathrm{D}} W_{yt}^{\mathrm{D}}] + \sum_{n=1}^{\mathcal{N}} \mathrm{VOLL} \cdot ENS_{nt} \tag{2.3}$$

The above term given by (2.3) contains the energy and reserve costs for the scheduling phase, that is, the dispatch cost corresponding to the non-contingency state ($k = 0$) and the most probable hydrology scenario ($s = 1$). Notice that reserves from seasonal hydroelectric plants are being considered in the OF; in this way the decision about deployment of hydroelectric power to face contingencies becomes an output of the optimization process that would depend mainly on the water value functions of these reservoirs for the period under study.

For $\xi = \{s, k\} \neq \{1, 0\}$ we have:

$$C_t^{\mathrm{BAL}} = \sum_{s=1}^{\mathcal{S}} \sum_{k=0}^{K} (\boldsymbol{\sigma}_s \cdot \boldsymbol{\pi}_k) \cdot \left[\sum_{i=1}^{\mathcal{G}} \mathrm{CE}_i P_{it}^{sk} + \sum_{n=1}^{\mathcal{N}} \mathrm{VOLL} \cdot ENS_{nt}^{sk} \right] \tag{2.4}$$

$$C_t^{\mathrm{FUTURE}} = \sum_{y=1}^{\mathcal{F}^S} \left[\mathrm{WV}_y \cdot H_{yt} + \sum_{s=1}^{S} \sum_{k=0}^{K} (\boldsymbol{\sigma}_s \cdot \boldsymbol{\pi}_k) \cdot \mathrm{WV}_y \cdot H_{yt}^{sk} \right] \tag{2.5}$$

The third term of the OF, detailed in (2.4), contains the expected operation and load curtailment costs for all the scenarios and contingencies, except $\xi = \{1, 0\}$. The last term given by Eq. (2.5) is the future cost of seasonal reservoirs. This cost is computed using their water values, assumed to be constant for the short term horizon under study.

The optimization problem is subject to the following constraints:

- Start-up, shut-down and commitment of thermal plants:

$$u_{it} - u_{i(t-1)} = a_{it} - z_{it} \tag{2.6}$$

$$a_{it} + z_{it} \leq 1 \tag{2.7}$$

$$u_{it} + v_{it} \leq 1 \tag{2.8}$$

$$u_{it}, v_{it}, a_{it}, z_{it} \in \{0, 1\} \tag{2.9}$$

- Minimum ON and OFF times of thermal plants:

$$\sum_{l=0}^{T_i^{\mathrm{ON}}-1} u_{i(t+l)} \geq T_i^{\mathrm{ON}} \cdot (u_{it} - u_{i(t-1)}) \tag{2.10}$$

$$\sum_{l=0}^{T_i^{\mathrm{OFF}}-1} (1 - u_{i(t+l)}) \geq T_i^{\mathrm{OFF}} \cdot (u_{i(t-1)} - u_{it}) \tag{2.11}$$

- Power balance equations for a DC flow representation:

$$\mathbf{F}^{sk} = \boldsymbol{\psi}^k \cdot (\mathbf{G}^{sk} - \mathbf{L}) \tag{2.12}$$

$$\sum_{i=1}^{\mathcal{G}} P_{it}^{sk} + \sum_{y=1}^{\mathcal{F}^S} H_{yt}^{sk} \sum_{j=1}^{\mathcal{F}^C} H_{jt}^{sk} = \sum_{n=1}^{\mathcal{N}} (\mathcal{L}_{nt} - ENS_{nt}^{sk}) \tag{2.13}$$

$$H_{jt}^{sk}, H_{yt}^{sk}, ENS_{nt}^{sk} \geq 0 \tag{2.14}$$

$$ENS_{nt}^{sk} \leq \mathcal{L}_{nt} \tag{2.15}$$

The symbols $\mathbf{F}^{sk}$, $\mathbf{G}^{sk}$ and $\mathbf{L}$ in bold, are vector forms of the power flows, net nodal power injections and nodal demands respectively. From now on, when no super index is assigned to variables P_{it}, H_{yt}, H_{jt}, and ENS_{nt}, it denotes variables at the scheduling phase, that is, for $\{s, k\} = \{1, 0\}$.

- Capacity limits of transmission lines:

$$-F_m^{\mathrm{MAX}} \cdot U_{mt}^k \leq F_{mt}^{sk} \leq F_m^{\mathrm{MAX}} \cdot U_{mt}^k \tag{2.16}$$

- Maximum and minimum capacity of thermal plants for energy supply and provision of spinning reserve:

$$P_{it} \leq P_i^{\mathrm{MAX}} \cdot u_{it} \quad : \bar{\lambda}_{it} \tag{2.17}$$

$$P_{it} \geq P_i^{\mathrm{MIN}} \cdot u_{it} \quad : \underline{\lambda}_{it} \tag{2.18}$$

$$R_{it}^{\mathrm{U}} \leq R_i^{\mathrm{UMAX}} \cdot u_{it} \quad : \theta_{it}^{\mathrm{U}} \tag{2.19}$$

$$R_{it}^{\mathrm{D}} \leq R_i^{\mathrm{DMAX}} \cdot u_{it} \quad : \theta_{it}^{\mathrm{D}} \tag{2.20}$$

$$P_{it} + R_{it}^{\mathrm{U}} \leq P_i^{\mathrm{MAX}} \tag{2.21}$$

$$P_{it} - R_{it}^{\mathrm{D}} \geq 0 \tag{2.22}$$

$$R_{it}^{\mathrm{U}}, R_{it}^{\mathrm{D}} \geq 0 \tag{2.23}$$

- Maximum and minimum capacity of thermal plants for provision of non spinning reserve [18]:

$$R_{it}^{\text{NS}} \leq \text{NR}_i^{\text{MAX}} \cdot \nu_{it} \quad : \bar{\gamma}_{it} \tag{2.24}$$

$$R_{it}^{\text{NS}} \geq P_i^{\text{MIN}} \cdot \nu_{it} \quad : \underline{\gamma}_{it} \tag{2.25}$$

- Output power and capacity limits of hydroelectric plants (for seasonal and short-term regulation reservoirs):

$$H_{jt} = \boldsymbol{\kappa}_j \cdot Q_{jt} \tag{2.26}$$

$$Q_{jt} \leq Q_j^{\text{MAX}} \tag{2.27}$$

$$H_{yt} \leq H_y^{\text{MAX}} \tag{2.28}$$

$$H_{yt} + W_{yt}^{\text{U}} \leq H_y^{\text{MAX}} \tag{2.29}$$

$$H_{yt} - W_{yt}^{\text{D}} \geq 0 \tag{2.30}$$

$$Q_{jt}, H_{yt}, W_{yt}^{\text{U}}, W_{yt}^{\text{D}} \geq 0 \tag{2.31}$$

- Available energy from seasonal reservoirs for the planning horizon under study:

$$0 \leq \sum_{t=1}^{T} (H_{yt} + W_{yt}^{\text{U}}) \cdot \Delta_t \leq \bar{\text{EH}}_y \tag{2.32}$$

- Water balance equation and volume limits of reservoirs:

$$V_{jt} - V_{j(t-1)} = C_0 \cdot [\bar{\mathbf{I}}_{jt} - (Q_{jt} + S_{jt})] \tag{2.33}$$

$$V_j^{\text{MIN}} \leq V_{jt} \leq V_j^{\text{MAX}} \tag{2.34}$$

$$S_{jt} \geq 0 \tag{2.35}$$

- Limits on output power of thermal and seasonal hydroelectric plants at the reserve deployment phase:

$$P_{it}^{sk} \leq (P_{it} + R_{it}^{\text{U}} + R_{it}^{\text{NS}}) \cdot U_{it}^{k} \quad : \bar{\mu}_{it} \tag{2.36}$$

$$P_{it}^{sk} \geq (P_{it} - R_{it}^{\text{D}}) \cdot U_{it}^{k} \quad : \underline{\mu}_{it} \tag{2.37}$$

$$H_{yt}^{sk} \leq (H_{yt} + W_{yt}^{\text{U}}) \quad : \bar{\Phi}_{it} \tag{2.38}$$

$$H_{yt}^{sk} \geq (H_{yt} - W_{yt}^{\text{D}}) \quad : \underline{\Phi}_{it} \tag{2.39}$$

- Water balance equation and volume limits of reservoirs at the reserve deployment phase:

$$Q_{jt}^{sk} + S_{jt}^{sk} = I_{jt}^{s} + C_0^{-1}(V_{j(t-1)} - V_{jt}) \quad : \phi_{it} \tag{2.40}$$

$$H_{jt}^{sk} = (\kappa_j Q_{jt}^{sk}) \tag{2.41}$$

$$Q_{jt}^{sk} \leq Q_j^{\text{MAX}} \tag{2.42}$$

$$H_{jt}^{sk}, \quad Q_{jt}^{sk}, \quad S_{jt}^{sk} \geq 0 \tag{2.43}$$

The above formulation corresponds to a stochastic Mixed Integer Linear Programming (MILP) problem. Constraints (2.6)–(2.11) are classical functions in a UC problem, except for (2.8) which is normally left aside, since supplemental reserve (non-spinning reserve for tertiary regulation) is not included in the STHS problem. The integer variable $\nu_{it} \in \{0, 1\}$ has been included to define the commitment of plant *i* for provision of non-spinning reserve. Constraint (2.8) assures that if a thermal plant has been committed for generation and spinning reserve, it cannot provide non-spinning reserve.

Eqs. (2.21) and (2.22) establish that provision of energy and up reserve, or allocation of down reserve, cannot surpass the plant capacity. Constraints (2.24) and (2.25) set the upper and lower limits on the non-spinning reserve levels that a unit is able to provide, assuring that if the plant is called up for generation, it must work within its allowed levels.

Because the present model does not consider hydraulic variables and parameters from seasonal reservoirs, the variables W_{yt}^{U} and W_{yt}^{D} (up and down reserve from hydroelectric plants respectively) have been defined, so that the contribution to reliability from these plants may be quantified. Eqs. (2.29) and (2.30) assure that provision of energy and up reserve, and allocation of down reserve to seasonal hydro plants do not surpass their technical limits.

Constraint (2.32) establishes limits on the total hydroelectric energy from seasonal reservoirs. Since it has been assumed that the future cost function of seasonal hydro plants are available, and that they are linear functions for the short-term study period, the present formulation only models electrical parameters from these plants. Thus, the constraints on their volume levels are treated according to the energy equivalent (in MWH), and its usage is penalized in the OF by the inclusion of (2.5).

Eqs. (2.33) and (2.34) are the water balance equation and the technical limits on the volumes of short-term regulation reservoirs.

Constraints (2.36)–(2.39) relate the scheduling and the operation phase by limiting the total power that can be deployed in a particular hydrology/contingency realization. Constraint (2.40) provides the linkage between the scheduling and the reserve deployment phase in relation to the storable energy and the stochastic nature of inflows. Notice that V_t and V_{t-1} are those from the scheduling phase, which means that once optimal reservoir levels have been obtained, the operator decides based on those values and the updated hydrology information available. This is not a water balance equation in the sense of an inter-temporal constraint; because the model intention is to replicate the real hydrothermal operation, where the operator does not have information on future hydrology or the exact time and duration of forced outages.

In the interest of clarity and to keep this research work within a reasonable length, the above formulation does not include ramp rate constraints of thermal plants, forced outages of hydro units, hydrological coupling between reservoirs, nor spinning reserve requirement for the primary or secondary regulation interval. However, their inclusion follows easily from the formulation presented here, as it implies constraints belonging to the scheduling phase only. In the particular case of spinning reserve for primary and secondary regulation, because the time frame on which these reserves are deployed is much smaller than that on which tertiary regulation reserves are active, their employment is not assessed in the context of the present model.

3. Proposed methodology

3.1. Benders decomposition background

For a linear optimization problem where **x** and **y** are state vectors such that:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_T \end{bmatrix}^{\text{T}}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \dots & \mathbf{y}_T \end{bmatrix}^{\text{T}}$$

the optimization problem:

$$\text{Minimize}\,\{\mathbf{c} \cdot \mathbf{x} + \mathbf{e} \cdot \mathbf{y}\} \tag{3.1}$$

Subject to:

$$\mathbf{Bx} \leq \mathbf{d} \tag{3.2}$$

$$\mathbf{A}_1\mathbf{y}_1 - \mathbf{E}_1\mathbf{x}_1 \leq \mathbf{b}_1 \tag{3.3}$$

$$\vdots$$

$$\mathbf{A}_T\mathbf{y}_T - \mathbf{E}_T\mathbf{x}_T \leq \mathbf{b}_T \tag{3.4}$$

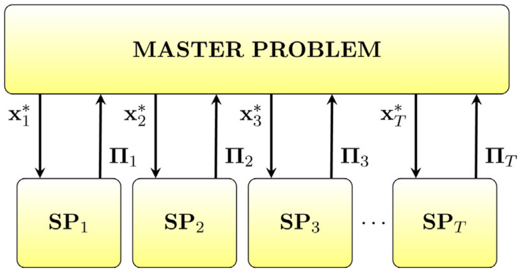

Fig. 1. Implicit structure of problem (3.1)–(3.4).

possess a separable structure that can be addressed using Benders decomposition. Notice that constraints (3.3) and (3.4) relates second phase vectors $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T$ with first phase vector $\mathbf{x}$ only, and that there are no constraints linking second phase vectors among them. Application of the Benders decomposition principle to this problem demands splitting the original formulation into a master and a sub-problem [23,24].

The Benders' cut to add to the master problem at each iteration of the algorithm is given by:

$$\boldsymbol{\beta} \geq \mathcal{Z}^* + \sum_{t=1}^{T} \boldsymbol{\Pi}_t^{\mathrm{T}} \mathbf{E}_t (\mathbf{x}_t - \mathbf{x}_t^*) \tag{3.5}$$

where $\boldsymbol{\beta}(\mathbf{x})$ is the approximation of the sub-problem solution's cost; $\boldsymbol{\Pi}_t$ is the vector of dual variables (Lagrange multipliers) corresponding to the set of constraints given by (3.3 and 3.4); and $\mathcal{Z}^*$ is the optimal cost of the sub-problem [21,22]. The overall implicit structure of problem (3.1)–(3.4) is shown in Fig. 1.

3.2. Definition of problems

The STHS problem formulated in the previous section is a multi-stage, stochastic, large scale, non-convex optimization problem. Nevertheless, we deal with a MILP problem to which the General Benders Decomposition (GBD) can be applied in two different instances. It possess a structure that is able to be divided into three problems of moderate size and complexity: a master problem that must be solved using a MIP tool, a multi-stage sub problem that can be solved using LP, and a set of linear stochastic problems that can also be addressed with a LP solver.

3.2.1. Master problem

The master problem is defined as the Unit Commitment part of the STHS problem, and its objective function entails de fixed costs, start-up costs and shut-down costs of thermal plants. This is the term given by (2.2), and the constraints are those given by (2.6)–(2.11). The Benders' cut to add at each iteration of the process is constructed using the Lagrange multipliers (shadow prices) of constraints (2.17)–(2.20) and (2.24) and (2.25). It can be shown that the cut in the Unit Commitment problem is given by:

$$\beta^{\mathrm{M}} - \sum_{t=1}^{T}\sum_{i=1}^{\mathcal{G}} \left(\left[\bar{\lambda}_{it}^{(r)} \cdot \mathrm{P}_i^{\mathrm{MAX}} - \underline{\lambda}_{it}^{(r)} \cdot \mathrm{P}_i^{\mathrm{MIN}} + \theta_{it}^{\mathrm{U}(r)} \cdot \mathrm{R}_i^{\mathrm{UMAX}} + \theta_{it}^{\mathrm{D}(r)} \cdot \mathrm{R}_i^{\mathrm{DMAX}} \right] \cdot u_{it} + \left[\bar{\gamma}_{it}^{(r)} \cdot \mathrm{NR}_i^{\mathrm{MAX}} - \underline{\gamma}_{it}^{(r)} \cdot \mathrm{P}_i^{\mathrm{MIN}} \right] \cdot v_{it} \right) \geq \Lambda^{(r)} \tag{3.6}$$

where r is the iterations counters; $\bar{\lambda}_{it}$ is the dual variable of constraint (2.17); $\underline{\lambda}_{it}$ is the dual variable of constraint (2.18); θ_{it}^{U} is the dual variable of constraint (2.19); θ_{it}^{D} is the dual variable of constraint (2.20); $\bar{\gamma}_{it}$ is the dual variable of constraint (2.24), and $\underline{\gamma}_{it}$ is the dual variable of constraint (2.25). The scalar term $\Lambda^{(r)}$ is defined as:

$$\Lambda^{(r)} = \mathcal{Z}^{*(r)} - \sum_{t=1}^{T}\sum_{i=1}^{\mathcal{G}} \left(\left[\bar{\lambda}_{it}^{(r)} \cdot \mathrm{P}_i^{\mathrm{MAX}} - \underline{\lambda}_{it}^{(r)} \cdot \mathrm{P}_i^{\mathrm{MIN}} + \theta_{it}^{\mathrm{U}(r)} \cdot \mathrm{R}_i^{\mathrm{UMAX}} + \theta_{it}^{\mathrm{D}(r)} \cdot \mathrm{R}_i^{\mathrm{DMAX}} \right] \cdot u_{it}^{*(r-1)} + \left[\bar{\gamma}_{it}^{(r)} \cdot \mathrm{NR}_i^{\mathrm{MAX}} - \underline{\gamma}_{it}^{(r)} \cdot \mathrm{P}_i^{\mathrm{MIN}} \right] \cdot v_{it}^{*(r-1)} \right) \tag{3.7}$$

where $\mathcal{Z}^{*(r)}$ is the optimal cost of sub-problem for iteration r.

Then, the master problem to solve at each iteration is:

$$\mathrm{OF}^{\mathrm{M}} = \min \left\{ \sum_{t=1}^{T}\sum_{i=1}^{\mathcal{G}} (\mathrm{FC}_i \cdot u_{it} + \mathrm{SU}_i \cdot a_{it} + \mathrm{SD}_i \cdot z_{it}) + \beta^{\mathrm{M}} \right\} \tag{3.8}$$

Subject to:

- Constraints (2.6)–(2.11)
- Benders' cut given by (3.6)

3.2.2. Sub-problem

In the proposed methodology, once candidate values for the commitment of plants are proposed by the master problem, the sub-problem solves de energy and reserve dispatch in a deterministic sense. The output will also provide the optimal volume levels and hydroelectric power of the short term regulation reservoirs. Because the integer variables have been already dealt with at the master problem, the sub-problem is now a multi-stage Linear Programming (LP) problem. The objective function is the minimization of the operation cost corresponding to the non-contingency state and most probable hydrology scenario, plus the opportunity cost of hydroelectric energy from seasonal reservoirs.

The constraints included in the sub-problem are the energy balance equations (DC flow), the water balance equation, the capacity limits on transmission lines and hydroelectric plants, and the capacity limits on energy supply and reserve provision. These last constraints are precisely the ones dependent on the binary variables of the UC problem, and from which dual information is later delivered to the master problem.

The objective function is then formed by (2.3) and the first term of (2.5). Notice that the second term in the future cost corresponds to the reserve deployment phase. It is done this way to keep the size of the problem moderate, and to maintain the homogeneity of the architecture, by dealing with stochastic aspects in a different set of problems. The Benders' cut of the sub-problem is constructed using dual information from the set of stochastic problems. This cut is defined as:

$$\beta^{\mathrm{S}} - \sum_{t=1}^{T}\sum_{s=1}^{\mathcal{S}}\sum_{k=0}^{\mathcal{K}} \left(\sum_{i=1}^{\mathcal{G}} [\bar{\mu}_{it}^{sk(p)} \cdot (P_{it} + R_{it}^{\mathrm{U}} + R_{it}^{\mathrm{NS}}) - \underline{\mu}_{it}^{sk(p)} \cdot (P_{it} - R_{it}^{\mathrm{D}})] \cdot \mathrm{U}_{it}^{k} + \sum_{y=1}^{\mathcal{J}^{\mathcal{S}}} [\bar{\Phi}_{yt}^{sk(p)} \cdot (H_{yt} + W_{yt}^{\mathrm{U}}) - \underline{\Phi}_{yt}^{sk(p)} \cdot (H_{yt} - W_{yt}^{\mathrm{D}})] - \sum_{j=1}^{\mathcal{J}^{\mathcal{C}}} \left(\frac{1}{\mathrm{C}_0} \right) \cdot \phi_{jt}^{sk(p)} \cdot (V_{jt} - V_{j(t-1)}) \right) \geq \Gamma^{(p)} \tag{3.9}$$

where p is the iteration counter of the sub-routine (different from r); $\bar{\mu}_{it}^{sk(p)}$ is the dual variable of constraint (2.36); $\underline{\mu}_{it}^{sk(p)}$ is the dual variable of constraint (2.37); $\bar{\Phi}_{yt}^{sk(p)}$ is the dual variable of constraint (2.38); $\underline{\Phi}_{yt}^{sk(p)}$ is the dual variable of constraint (2.39); and $\phi_{jt}^{sk(p)}$

is the dual variable of constraint (2.40). The scalar term $\Gamma^{(p)}$ is calculated as:

$$\Gamma^{(p)} = \sum_{t=1}^{T}\sum_{s=1}^{\mathcal{S}}\sum_{k=0}^{\mathcal{K}}\Bigg(\mathcal{Z}_t^{sk(p)} - \sum_{i=1}^{\mathcal{G}}\Big[\bar{\mu}_{it}^{sk(p)}\cdot\big(P_{it}^{*(p-1)}+R_{it}^{\mathrm{U}*(p-1)}+R_{it}^{\mathrm{NS}*(p-1)}\big) -\underline{\mu}_{it}^{sk(p)}\cdot\big(P_{it}^{*(p-1)}-R_{it}^{\mathrm{D}*(p-1)}\big)\Big]\cdot \mathrm{U}_{it}^{k} - \sum_{y=1}^{\mathcal{J}}\Big[\bar{\Phi}_{yt}^{sk(p)}\cdot\big(H_{yt}^{*(p-1)}+W_{yt}^{\mathrm{U}*(p-1)}\big) -\underline{\Phi}_{yt}^{sk(p)}\cdot\big(H_{yt}^{*(p-1)}-W_{yt}^{\mathrm{D}*(p-1)}\big)\Big] + \sum_{j=1}^{\mathcal{J}}\left(\frac{1}{\mathrm{C}_0}\right)\cdot\phi_{jt}^{sk(p)}\cdot\big(V_{jt}^{*(p-1)}-V_{j(t-1)}^{*(p-1)}\big)\Bigg) \tag{3.10}$$

where $\mathcal{Z}_t^{sk(p)}$ is the optimal cost of stochastic problem corresponding to hydrology scenario s and contingency k at stage t for iteration p. The definition of the sub-problem is summarized below:

$$\mathrm{OF}^{\mathrm{SUB}} = \min\left\{\sum_{t=1}^{T}\left[(\sigma_1\cdot\pi_0)\cdot\left(\sum_{i=1}^{\mathcal{G}}\left[\mathrm{CE}_iP_{it}+\mathrm{CR}_i^{\mathrm{U}}R_{it}^{\mathrm{U}}+\mathrm{CR}_i^{\mathrm{D}}R_{it}^{\mathrm{D}}+\mathrm{CR}_i^{\mathrm{NS}}R_{it}^{\mathrm{NS}}\right] + \sum_{y=1}^{\mathcal{J}}\left[\mathrm{CR}_y^{\mathrm{U}}W_{yt}^{\mathrm{U}}+\mathrm{CR}_y^{\mathrm{D}}W_{yt}^{\mathrm{D}}\right]+\sum_{n=1}^{\mathcal{N}}\mathrm{VOLL}\cdot ENS_{nt}\right)+\sum_{y=1}^{\mathcal{J}}\mathrm{WV}_y\cdot H_{yt}\right]+\beta^{\mathrm{S}}\right\} \tag{3.11}$$

Subject to:

- Constraints (2.12)–(2.35)
- Benders' cut given in (3.9)

3.2.3. Set of stochastic problems

Uncertainty in hydrology and availability of generation plants have a significant impact in the total operation cost of a power system. This third set of problems accounts for uncertainty. Although a set of credible contingencies and hydrology scenarios can cause the problem to grow to a very large size, the proposed decomposition scheme provides a framework to deal with these problems independently from the scheduling problem. The goal is to provide a schedule that can be followed even in the events of unforeseen inflows or forced outages of plants and transmission lines.

The objective function of the stochastic problem is defined by (2.4) and the second term in (2.5). The constraints to include are (2.12)–(2.16) and (2.36)–(2.43). Because of its intrinsic form, the stochastic problem is suitable to be addressed using parallel or distributed computing, since its constraints are not time-coupled. This is a unique and important feature of the present work; the problems where hydrology scenarios and contingencies are realized are only dependent on variables at the scheduling phase that correspond to the same period.

If the water balance equation (2.40) were modified as to link the reservoir levels at each stage for each hydrology and contingency realization (as in two-stage stochastic programming problems) this will certainly yield a lower value for the OF, but that will not be a realistic anticipation of the real operation, as the operator cannot predict the next day hydrology, or know with certainty the time and duration of outages. In that sense, the present model is not a two-stage stochastic programming model in its strict sense. The definition of the stochastic problems is as follows. For each $t = 1 \dots T$ solve:

$$C_t^{\mathrm{BAL}} = \sum_{s=1}^{\mathcal{S}}\sum_{k=0}^{K}(\sigma_s\cdot\pi_k)\cdot\left(\sum_{i=1}^{\mathcal{G}}\mathrm{CE}_iP_{it}^{sk}+\sum_{y=1}^{\mathcal{J}}\mathrm{WV}_yH_{yt}^{sk}+\sum_{n=1}^{\mathcal{N}}\mathrm{VOLL}\cdot ENS_{nt}^{sk}\right) \tag{3.12}$$

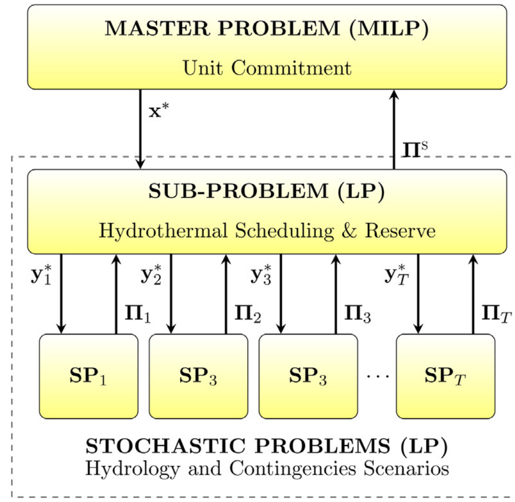

Fig. 2. Proposed partitioning process of the STHS problem (2.1)–(2.43).

Subject to:

- Constraints (2.12)–(2.16) and (2.36)–(2.43)

When solving the stochastic problems, dual information from constraints (2.36), (2.37), (2.38), (2.39) and (2.40) must be collected to construct the Benders' cut needed at the sub-problem.

The general structure of the proposed strategy is depicted in Fig. 2.

3.3. Lower and upper bounds

In order to solve the co-optimization problem in the STHS model, the proposed strategy splits the original problem into three blocks and applies the GBD scheme in two instances. The main routine is performed by solving iteratively the UC problem and the set of Sub Problem-Stochastic Problems (enclosed by dotted lines in Fig. 2). To test for convergence of the overall process, it is necessary to define lower and upper bounds for the algorithm. The lower bound for the main routine is obtained from the solution of (3.8), and hence is computed as:

$$\underline{Z}^{\mathrm{M}} = \sum_{t=1}^{T}\sum_{i=1}^{\mathcal{G}}(\mathrm{FC}_iu_{it}^{*}+\mathrm{SU}_ia_{it}^{*}+\mathrm{SD}_iz_{it}^{*})+\beta^{\mathrm{M}^*} \tag{3.13}$$

Once candidate values from the UC problems are proposed, a subroutine must be executed until convergence is achieved. This subroutine is controlled by the counter p and not r, which controls the execution of the main loop. If in the sub-routine $\underline{Z}^{\mathrm{S}}$ and $\bar{Z}^{\mathrm{S}}$ are the lower and upper bounds respectively, and ζ is the parameter for convergence test, the lower bound for the sub-routine is obtained from the solution to (3.11), and is given by:

$$\underline{Z}^{\mathrm{S}} = \mathrm{OF}^{\mathrm{SUB}^*} \tag{3.14}$$

while the upper bound is given by the sum of the optimal cost of all stochastic problems (3.12) plus the total operation cost of the scheduling phase; that is:

$$\bar{Z}^{\mathrm{S}} = \sum_{t=1}^{T}C_t^{\mathrm{BAL}*}+\left(\underline{Z}^{\mathrm{S}}-\beta^{\mathrm{S}*}\right) \tag{3.15}$$

Once the subroutine process has converged ($\bar{Z}^{\mathrm{S}}-\underline{Z}^{\mathrm{S}}\le\zeta$, and $\mathcal{Z}^{*(r)}\approx\bar{Z}^{\mathrm{S}}\approx\underline{Z}^{\mathrm{S}}$), the convergence of the main loop must be tested. To perform this test the upper bound of the overall process must be obtained. This bound is obtained by adding the total UC costs to the upper or lower bound (as they have already converged) of the previous process. Thus, the upper bound of the overall process is given by:

$$\begin{aligned} \bar{Z}^{M} &= (\underline{Z}^{M} - \beta^{M^*}) + \bar{Z}^{s} \\ &= (\underline{Z}^{M} - \beta^{M^*}) + \underline{Z}^{s} \end{aligned} \quad (3.16)$$

since $\bar{Z}^s \approx \underline{Z}^s$ when the sub-routine process is ended. The proposed strategy is outlined in Fig. 3.

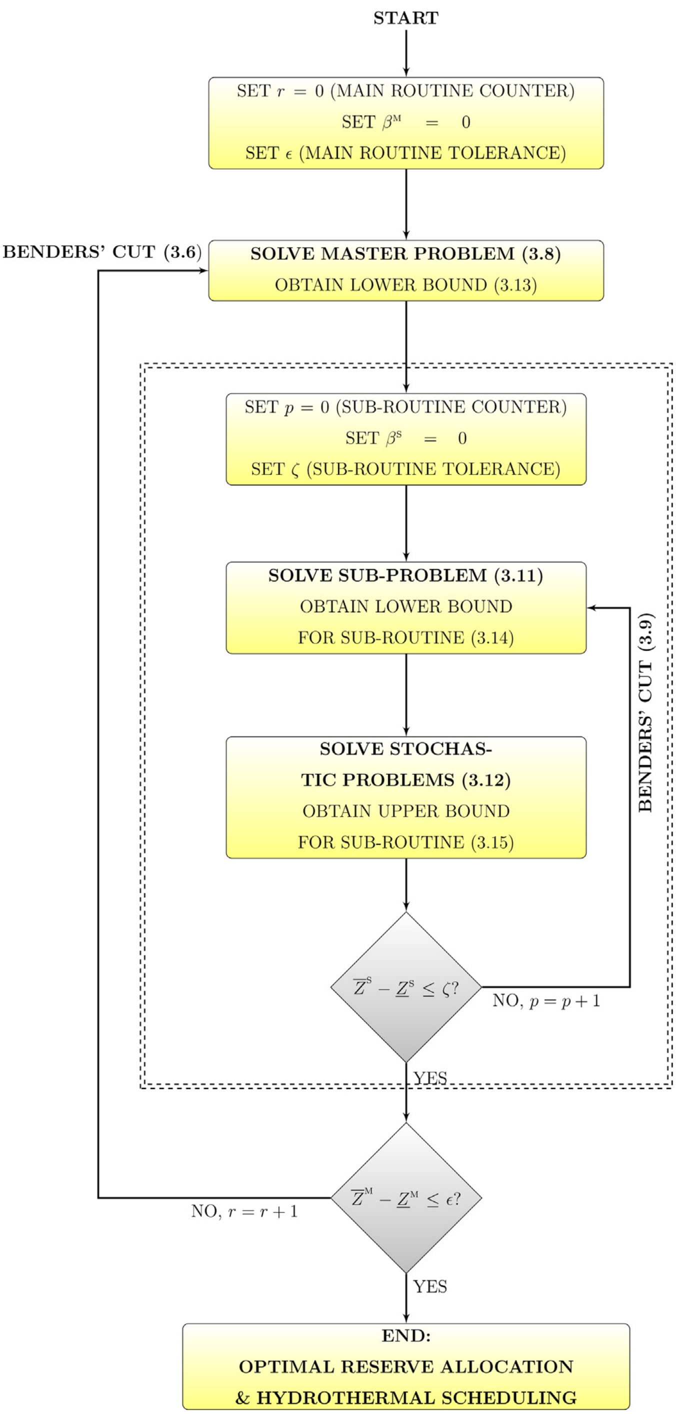

Fig. 3. Proposed solution strategy.

3.4. Convergence and sub-problems infeasibility

In the present application, the convergence of the algorithm is guaranteed as long as the envelope of the functions β^M and β^s is convex. A convexity proof for these functions is presented in [25]. The GBD has been applied successfully to other kind of problems in power systems [26,27] and some of these utilize a distinction between feasibility and optimality cuts. The infeasibility of the sub-problem and stochastic problems can be treated by means of artificial variables and objective function penalties. This is done implicitly in the present formulation by the inclusion of artificial generators ENS_{nt} at every bus, which plays a fundamental role in the model. Through this variable, it can be measured the economic loss that the user experiences due to load interruptions. Since the inclusion of this variable does modify the optimal cost of the problem, there is no need to define feasibility cuts [25].

On the other hand, in order for constraint (2.40) to hold for every stochastic problem, it is necessary to assure that the reservoir levels are reachable even for the lowest possible hydrology realization. This is done by including the additional constraint

$$V_{jt} \leq \min\left\{\left(V_{j(t-1)} + C_0 \cdot \min\left\{I_{jt}^1, I_{jt}^2, \ldots, I_{jt}^S\right\}\right), V_j^{MAX}\right\} \quad (3.17)$$

in the sub-problem, or by replacing the "=" in constraint (2.40) by a "≤" symbol. It must be stated though, that this last change is only necessary because of the decomposition scheme applied to the problem.

4. Study cases

To illustrate the effectiveness of the proposed model and the application of the proposed strategy, a 3-bus interconnected test system and a small single bus system are considered. The simulations presented have been carried using different solvers (properly indicated for each case) within AMPL [29].

4.1. Case 1

The purpose of this case is to evaluate the proposed model and compare its output data with that of other existing approaches that apply exogenous reserve determination criteria. Therefore in order to simplify the analysis of results, the UC problem is not included in this example. The problem is defined by a three 3-area system with 5 generating units, as shown in Fig. 4. At buses 1 and 2, there are 1 thermal and 1 hydroelectric plant at each bus, while at bus 3 there is only 1 thermal plant. The data for thermal plants is presented in Table 1.

Each line's reactance is 0.13 PU on a 230 kV and 100 MVA basis. The data of reservoirs and hydroelectric plants is shown in Table 2,

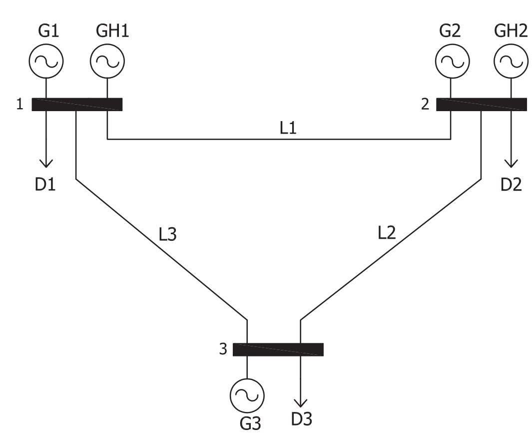

Fig. 4. Hydrothermal system for case 1.

Table 1
Data for thermal plants and transmission lines of case 1.

	P^{MIN}	P^{MAX}	CE_i [$/MWh]	CR_i^U [$ h/MW]	MTTF [h]	MTTR [h]
G_1	0	60	50.00	8.50	200	17
G_2	0	75	52.00	10.00	200	17
G_3	0	60	45.00	12.00	200	17
L_1	0	80	0	0	600	24
L_2	0	80	0	0	600	24
L_3	0	80	0	0	600	24

Table 2
Data for hydroelectric plants in case 1.

	Volume [hm^3]				Q^{MAX} [m^3/s]	κ_j [MW s/m^3]
	V^{MIN}	V^{MAX}	V_0	V^{END}		
GH_1	25.00	140.00	50	50	190	0.4
GH_2	10.00	96.4	15	≥13	100	0.5

and the demands and water inflows to reservoirs for each period are shown in Table 3. The planning horizon is 1 day, having each stage a duration of 4 h. For each reservoir, three possible scenarios are considered: the *average* scenario is the one having the highest probability (60%). Scenarios 'LOW' and 'HIGH' have been assigned probabilities of 20% each.

The reliability data of thermal plants and transmission lines that are included in Table 1 (MTTF and MTTR) have been used to obtain a credible set contingencies that are valid for the study horizon. This set of contingencies may include failures at any stage (or stages) of the study horizon, and each disturbance can have a duration of $t_{FAILURE} \leq 24$ h. A Capacity Outage Probability Table (COPT) for all stages was obtained with a total of 36 possible states, including the pre-contingency state with a probability of 65%.

The intention is to examine the combined impact of component failure and hydrology uncertainty in the scheduling, considering the allocation of contingency reserves. In order to simplify the analysis, the hydroelectric plants remain in the pre-contingency state for the entire horizon. Under the assumption that hydrology realizations and component failures are independent events, the probability of each scenario is computed as $\boldsymbol{\sigma}_s \times \boldsymbol{\pi}_k$, so the case has a total of $3 \times 36 = 108$ scenarios to consider.

4.1.1. Hydrothermal scheduling

This case has been addressed using the proposed formulation. It was employed the XPRESS 27.01 solver within the AMPL modelling environment. Table 4 compares the costs of energy and reserve obtained with the proposed model, against the costs obtained when using a deterministic reserve determination criteria, being the required reserve a 25% of the total demand for each stage. Election of 25% is explained below when a simulation of the daily operation is done. Table 5 summarizes the optimal schedules of energy and reserve for both of the methodologies presented in Table 4. Results show that with the proposed model more reserve is procured, and that the variations in the amounts of spare capacity by stage, between one model and the other, are accentuated in the early and final stages.

Table 4
Costs from proposed and one existing model.

Method	Energy cost [$]	Reserve cost [$]	Total cost [$]
Exogenous	164,788	12,400	177,188
Endogenous	165,188	14,270	179,458

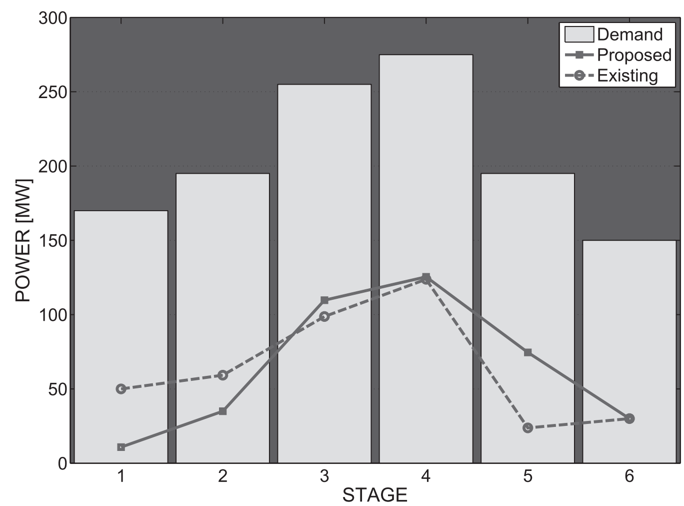

Fig. 5. Hydropower profiles for proposed and one existing model.

Fig. 5 shows the total hydropower (dotted and continuous lines) for each stage, in relation to the total demand of the period. The dotted line corresponds to the deterministic case (25% of total demand criteria), while the continuous line represents the proposed model.

Observe that with the proposed model, the decisions on the hydro energy are more conservative in the stages preceding the peak load hours (stages 3 and 4), where it will tend to make an intensive use of hydro energy. We may interpret this as the model storing an additional amount of water to face disturbances. In other words the directive in the allocation of water is not only to replace expensive thermal energy in future stages, but also to optimally accommodate water in the light of the expected availability of the plants providing this energy and the hydrology deviations from their forecast values. In this way the consideration of the magnitude of the contingencies that call for reserve deployment, helps to ensure optimal levels of reserves in reservoirs.

Results from Table 4 clearly indicate that if using a deterministic model a lower cost can be achieved for the scheduling phase. A further test though, will provide insight on the model outcome when contingencies and hydrology variations are assessed.

4.1.2. Simulation of daily operation

To evaluate the proposed STHS model, we performed 100 simulations of the daily operation using the results of the proposed formulation and other models based on exogenous rules for reserve

Table 3
Demand at each bus and reservoir's inflow for each stage in case 1.

Period	Demand [MW]			I_1^s [m^3/s]			I_2^s [m^3/s]		
	n_1	n_2	n_1	Low	Med	High	Low	Med	High
t_1	10	50	110	57.49	75	91.91	32.46	40	49.57
t_2	15	60	120	64.37	76	88.43	34.23	41	50.79
t_3	25	80	150	58.35	76	88.29	34.62	41	51.22
t_4	35	90	150	63.69	80	92.76	37.33	44	52.29
t_5	20	50	125	67.43	82	96.53	36.97	44	52.11
t_6	10	40	100	68.73	86	99.33	31.63	42	51.27

Table 5
Optimal schedules with proposed and one existing model.

Period	Power [MW]					Reserve [MW]		
	P_1	P_2	P_3	H_1	H_2	R_1^{U}	R_2^{U}	R_3^{U}
Energy & endogenous reserve schedule								
t_1	10.00	89.23	60.00	7.00	3.77	0.00	10.78	0.00
t_2	60.00	40.00	60.00	4.65	30.35	0.00	60.00	0.00
t_3	60.00	16.46	60.00	68.54	50.00	0.00	73.54	0.00
t_4	60.00	29.00	60.00	76.00	50.00	0.00	71.00	0.00
t_5	60.00	9.87	60.00	15.13	50.00	0.00	69.34	0.00
t_6	60.00	0.00	60.00	18.67	11.32	0.00	72.09	0.00
Energy & exogenous reserve schedule (25% of demand)								
t_1	60.00	0.00	60.00	0.00	50.00	0.00	42.50	0.00
t_2	60.00	15.81	60.00	14.25	44.94	0.00	48.75	0.00
t_3	60.00	36.25	60.00	76.00	22.75	0.00	63.75	0.00
t_4	60.00	31.25	60.00	76.00	47.75	0.00	68.75	0.00
t_5	60.00	51.25	60.00	23.75	0.00	0.00	48.75	0.00
t_6	60.00	0.00	60.00	0.00	30.00	0.00	37.50	0.00

determination. The following steps were followed before running the simulations:

(i) A new credible set of contingencies was obtained, using reliability data given in Table 1. Details on the computation of Time to Failure (TTF) and TTR (Time to Repair) can be found in [28].
(ii) New inflow scenarios were obtained, using the average values of Table 3, their probabilities, and a random number generator from a normal probability distribution.
(iii) It was obtained a new vector of bus demands for each stage, using as reference the values given in Table 3, but allowing variations between −10 and +10%, obtained with a random number generator from a uniform probability distribution.

The obtained results are summarized in Table 6. The lowest cost obtained with a deterministic model (joint or sequential) corresponded to a reserve criteria of 25% of the total demand, and this is the reason why this percentage was used before. In this particular example the system is not able to provide more than this percentage, unless some load is shed at the scheduling phase, a not permitted situation. However, and even when the system is performing at its very limits of system adequacy, it can be seen that with the proposed formulation, a lower expected value of the total cost was reached. In terms of performance under uncertainty the proposed model surpassed all of the deterministic models. If this test system represented a real one, using the proposed model would yield savings of about $ 2.2 million/year, compared to the costs from the best deterministic model.

Table 6
Simulation of daily operation using schedules from different methodologies.

Model	Reserve criteria	Costs [$]		
	% of demand	Reserve	$\mathbb{E}$(Operation cost)	Total
Joint	5	1,930	261,990	263, 920
	10	4,960	230,479	235, 439
	15	7,450	215,245	222, 696
	20	9,920	202,346	212, 284
	25	12,400	192,573	204, 973
Sequential	5	2,480	264,965	267, 445
	10	4,960	233,068	238, 038
	15	7,451	215,932	223, 383
	20	9,920	202,311	212, 231
	25	12,400	192,313	204, 713
Proposed	–	14,270	178,518	192, 788

Table 7
Thermal plants data for case 2.

	P^{MIN}	P^{MAX}	CE_i [$/MWh]	$\mathrm{CR}_i^{\mathrm{U}}$ [$ h/MW]
G_1	10	50	50	15
G_2	10	50	55	10

	$\mathrm{CR}_i^{\mathrm{NS}}$ [$ h/MW]	FC_i [$/h]	SU_i [$]	SD_i [$]
G_1	15	10	300	50
G_2	10	0	200	0

Table 8
Reservoir data for case 2.

	Volume [hm³]				Q^{MAX} [m³/s]	κ_j [MW s/m³]
	V^{MIN}	V^{MAX}	V^{START}	V^{END}		
GH	0	10	1	1	200	0.4

4.2. Case 2

The objective with this example is to test the implementation of the full proposed strategy, including the UC problem, and to assess its performance in terms of computational cost. The system under analysis is single bus power system consisting of 2 thermal plants and 1 hydroelectric plant with water reservoir. The scheduling horizon has been set in 16 h using $\Delta_t = 4$ h. Data for the thermal plants, including fix, start up and shut down costs are shown in Table 7; the hydro plant-reservoir data is shown in Table 8; and the demand at each stage along with water inflow to the reservoir for 3 scenarios are shown in Table 9. Table 10 shows the set of contingencies considered with their respective probabilities. The units are assumed to be OFF at the beginning of the scheduling horizon, and it has been assumed that all plants' capacity is available as spinning down reserve at no additional cost.

Table 9
Demand and inflow scenarios for each period in case 2.

Period	$\mathcal{L}$ [MW]	$\bar{I}$ [m³/s]	I^s [m³/s]		
			Low	Med	High
t_1	65	80	70	80	100
t_2	100	80	70	80	100
t_3	120	80	70	80	100
t_4	50	80	70	80	100

Table 10
Set of credible contingencies for case 2.

Contingency	Plant	Plant status				Probability
		t_1	t_2	t_3	t_4	
1	G_1 G_2	1 1	1 1	1 1	1 1	0.84
2	G_1 G_2	0 1	1 1	1 1	1 1	0.04
3	G_1 G_2	1 1	1 0	1 0	1 1	0.04
4	G_1 G_2	1 0	1 1	1 1	0 1	0.04
5	G_1 G_2	1 1	1 0	0 1	1 1	0.04

Table 11
Unit commitment of thermal plants for case 2.

Commitment for energy &reserve				
Period	Energy &spinning reserve		Supplemental reserve	
	u_1	u_2	v_1	v_2
t_1	1	0	0	1
t_2	1	1	0	0
t_3	1	0	0	1
t_4	1	0	0	1

4.2.1. Results

Results for the commitment of thermal plants for energy supply and reserve provision, and the optimal energy and reserve schedules are summarized in Tables 11 and 12 respectively. It can be seen that only thermal unit 2 was committed for supplementary reserve provision at stages 1, 2 and 4. This unit is then scheduled with power and spinning reserve at stage 3.

Although the purpose of the example is to test the proposed decomposition methodology, an interesting point to notice in the schedules presented in Table 12, is the reserve allocation for the last stage. It can be seen that all capacity of plant 2 is committed to reserve provision (non-spinning). This can be understood by looking at the credible set of contingencies in Table 10. Observe that only in contingency 4 a forced outage at stage 4 has been simulated, and that outage corresponds to unit 1. It is because of this contingency and its expected impact on the total operation cost that non-spinning reserve is allocated to a more expensive plant. This fact underlines the importance of a stochastic optimization model for the short term planning that considers the uncertainty in the availability of generation plants.

Using a tolerance of $ 0.1 for both routines, the algorithm converged in 28 iterations, and the resulting optimal cost was $ 47,400. The convergence process is depicted in Fig. 6. Although this study case does not represent a real size system, the intention is to show that the proposed methodology indeed reaches the global optima,

Table 12
Optimal schedule for problem of case 2 using the proposed strategy.

Energy &endogenous reserve schedule							
Period	Scheduled power [MW]			Scheduled reserve [MW]			
	P_1	P_2	H	R_1^{U}	R_2^{U}	R_1^{NS}	R_2^{NS}
t_1	50	0	15	0	0	0	50
t_2	50	11	39	0	4	0	0
t_3	50	0	70	0	0	0	50
t_4	46	0	4	0	0	0	50

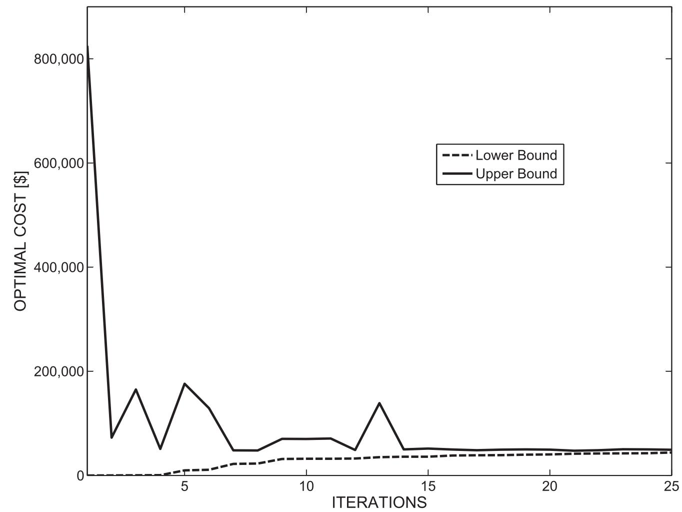

Fig. 6. Convergence of lower and upper bounds for case 2.

Table 13
Assessment of computational effort of proposed strategy in case 2.

Solver	Optimal cost [$]	Computation time [s]
Cplex 12.6.1.0	47,400	0.17
Xpress 27.01	47,400	0.34
Minlp	47,400	0.16
Knitro 9.1.0	47,400	7.35
Couenne 0.5.3	47,400	20.01
Bonmin 1.8.0	47,400	41.65
Proposed Strategy	47,400	113.00

and that the methodology can naturally be extended to the modelling of real size systems.

In order to have a quantitative assessment of the computational cost involved in the proposed strategy, this case has been also implemented using the AMPL modelling language and tested with different MILP solvers available. Results are summarized in Tables 13 and 14 outline the size of the resulting sub-problems when applying decomposition. The drawback of decomposition in terms of computational cost is evident; a natural and expected outcome, that nevertheless remains within reasonable margins.

However, the main objective pursued with the strategy is to show that since the problem possess an inherent decomposable structure, it can be split into three blocks of problems of reduced size and complexity, as depicted in Fig. 2. This seems suitable to deal with real size large scale problems: allowing their separation, exploiting the benefits of powerful commercial MILP and LP solvers available today, and finally taking advantage of parallel computing.

A second phase of the current proposal and a relevant topic for future research, is the testing and implementation of the present methodology in real size power systems with significant penetration of hydroelectric power from short term controllable reservoirs. The interested reader is referred to [20], where 3 decomposition outlines have been tested in a real medium-size system, including 11 hydroelectric plants with 6 controllable reservoirs, and at both

Table 14
Assessment of the computational size of problem.

	Direct	Decomposition		
		Master	Sub-problem	Stochastic problems
# of integer variables	32	32	–	–
# of continuous variables	340	–	40	300
# of constraints	1005	48	657	300

24-h and 1-week scheduling terms. Although that model follows a deterministic approach with an emphasis on the nonlinear character of the problem, the results provide a good reference of the benefit of applying decomposition. The numbers obtained there significantly surpassed those when employing existing MINLP solvers.

5. Consideration of stochastic intermittent production

The *downward* reserve has been considered in the formulation presented in Section 2. Unless forced outages of transmission lines are modelled, this type of reserve is not normally scheduled in systems with limited fluctuating generation, since reserves for tertiary regulation is usually required to cope with forced plant outages. However, in systems with high penetration of wind power, it may be required to perform a sudden reduction of conventional generation, and hence to deploy a negative reserve to cope with increased wind generation [31]. A similar argument applies when the power system contains a considerable penetration of hydro power from non-storable facilities, except that the stochastic processes that model the natural inflows to these plants have a lower variance profile than that of the wind farms.

This section presents an extension of the formulation to include energy provided by wind farms and hydroelectric plants without water reservoirs, referred to here as pass hydroelectric plants. In order to do it, the following notation is added:

f	Index of wind farms from $1, \ldots, \mathcal{F}$
g	Index of pass hydroelectric plants from $1\ldots\mathcal{J}^P$
w	Index of wind scenario from $1, \ldots, \mathcal{W}$
$\mathcal{F}_n$	Set of wind farms connected to bus n
$\mathcal{J}_n^P$	Set of pass hydroelectric plants connected to bus n
E_f^{MAX}	Maximum generation capacity of wind farm f [MW]
H_g^{MAX}	Maximum generation capacity of hydroelectric plant g [MW]
$\bar{E}_{ft}$	Forecast production of wind farm f for most probable wind scenario during period t [MW]
$\bar{I}_{gt}$	Forecast average water flow through hydroelectric plant g for most probable hydrology scenario during stage t[m^3/s]
E_{ft}^w	Forecast production of wind farm f for wind scenario w during stage t [MW]
I_{gt}^s	Forecast water flow through hydroelectric plant g for hydrology scenario s during stage t [m^3/s]
$\boldsymbol{\rho}_w$	Probability of wind scenario w
E_{ft}^S	Scheduled wind power production for wind farm f at stage t [MW]
H_{gt}	Scheduled output power of hydroelectric plant g at stage t [MW]
ES_{ft}^{wsk}	Spilled wind power from wind farm f for wind scenario w, hydrology scenario s and contingency k during stage t [MW]
H_{gt}^{wsk}	Output power of hydroelectric plant g for wind scenario w, hydrology scenario s and contingency k during stage t [MW]

From the formulation given in Section 2, the following constraints are modified to account for the inclusion of fluctuating generation.

The objective function (2.1) is only altered by adding an operator to the terms given by (2.4) and (2.5), which now become: for all $\xi = \{w, s, k\} \neq \{1, 1, 0\}$:

$$C_t^{\text{BAL}} = \sum_{w=1}^{\mathcal{W}}\sum_{s=1}^{\mathcal{S}}\sum_{k=0}^{K}(\boldsymbol{\rho}_w \cdot \boldsymbol{\sigma}_s \cdot \boldsymbol{\pi}_k) \cdot \left[\sum_{i=1}^{\mathcal{G}} \text{CE}_i P_{it}^{wsk} + \sum_{n=1}^{\mathcal{N}} \text{VOLL} \cdot ENS_{nt}^{wsk}\right] \tag{5.1}$$

$$C_t^{\text{FUTURE}} = \sum_{y=1}^{\mathcal{J}^S}\left[\text{WV}_y \cdot H_{yt} + \sum_{w=1}^{\mathcal{W}}\sum_{s=1}^{\mathcal{S}}\sum_{k=0}^{K}(\boldsymbol{\rho}_w \cdot \boldsymbol{\sigma}_s \cdot \boldsymbol{\pi}_k) \cdot \text{WV}_y \cdot H_{yt}^{wsk}\right] \tag{5.2}$$

The constraint (2.12) is now written as:

$$\sum_{i=1}^{\mathcal{G}} P_{it} + \sum_{y=1}^{\mathcal{J}^S} H_{yt} + \sum_{j=1}^{\mathcal{J}^C} H_{jt} + \sum_{g=1}^{\mathcal{J}^P} H_{gt} + \sum_{f=1}^{\mathcal{F}} E_{ft}^S = \sum_{n=1}^{\mathcal{N}} (\mathcal{L}_{nt} - ENS_{nt}) \tag{5.3}$$

where the nth element of $\mathbf{G}$ is given by:

$$\mathbf{G}(n) = \sum_{i \in \mathcal{G}_n} P_{it} + \sum_{y \in \mathcal{J}_n^S} H_{yt} + \sum_{j \in \mathcal{J}_n^C} H_{jt} + \sum_{g \in \mathcal{J}_n^P} H_{gt} + \sum_{f \in \mathcal{F}_n} E_{ft}^S + ENS_{nt} \tag{5.4}$$

The constraints for energy production of pass hydroelectric plants and wind farms at the scheduling phase are:

$$H_{gt} \leq \boldsymbol{\kappa}_g \cdot \bar{I}_{gt} \tag{5.5}$$

$$H_{gt} \leq H_g^{\text{MAX}} \tag{5.6}$$

$$E_{ft}^S \leq E_f^{\text{MAX}} \tag{5.7}$$

$$E_{ft}^S \leq \bar{E}_{ft} \tag{5.8}$$

At the balancing phase, the power balance equation becomes:

$$\sum_{i=1}^{\mathcal{G}} P_{it}^{wsk} + \sum_{y=1}^{\mathcal{J}^S} H_{yt}^{wsk} + \sum_{j=1}^{\mathcal{J}^C} H_{jt}^{wsk} + \sum_{g=1}^{\mathcal{J}^P} H_{gt}^{wsk} + \sum_{f=1}^{\mathcal{F}} (E_{ft}^w - ES_{ft}^{wsk}) = \sum_{n=1}^{\mathcal{N}} (\mathcal{L}_{nt} - ENS_{nt}^{wsk}) \tag{5.9}$$

where the nth element of $\mathbf{G}^{wsk}$ is now:

$$\mathbf{G}^{wsk}(n) = \sum_{i \in \mathcal{G}_n} P_{it}^{wsk} + \sum_{y \in \mathcal{J}_n^S} H_{yt}^{wsk} + \sum_{j \in \mathcal{J}_n^C} H_{jt}^{wsk} + \sum_{g \in \mathcal{J}_n^P} H_{gt}^{wsk} + \sum_{f \in \mathcal{F}_n} (E_{ft}^w - ES_{ft}^{wsk}) + ENS_{nt}^{wsk} \tag{5.10}$$

and where

$$H_{gt}^{wsk} = \boldsymbol{\kappa}_g \cdot I_{gt}^s \tag{5.11}$$

The solution algorithm presented in Section 3 can be extended to incorporate the changes presented above.

6. Conclusions

This work introduced a Short Term Hydrothermal Scheduling model with co-optimization of energy and reserve in the presence of hydrology uncertainty and equipment failures. The proposed model is suitable for systems with a significant penetration of hydroelectric power from controllable reservoirs, over which a weekly optimization must be run. Although the three types of reserves can be included in the formulation, the focus of this paper has been on reserves for tertiary regulation (spinning and supplementary) which are obtained endogenously to the optimization process. It has been shown that the proposed model exceeds the existing deterministic approaches that are based either on sequential or simultaneous directives, and that decisions on the water levels in reservoirs are different when the inherent aspects of contingency reserves are included in the model.

It has also been proposed a decomposition strategy to handle the stochastic, large scale, non-convex, and multi-stage optimization problem derived from the formulation. At the expense of computational cost, the proposed scheme allows to divide the problem into three problems of known structure and moderate dimension and complexity. The proposed strategy can be naturally extended to consider real size problems, an AC Power Flow representation, and fluctuating generation as that coming from wind farms and pass hydroelectric plants.

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