PHYSICAL REVIEW D **71,** 034016 (2005)

Decays of nonstrange negative parity baryons in the $1/N_c$ expansion

J. L. Goity,[1,2,*] C. Schat,[3,4,†] and N. N. Scoccola[3,4,5,‡]

[1]*Department of Physics, Hampton University, Hampton, Virginia 23668, USA*
[2]*Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*
[3]*Physics Department, Comisión Nacional de Energía Atómica, (1429) Buenos Aires, Argentina*
[4]*CONICET, Rivadavia 1917, (1033) Buenos Aires, Argentina*
[5]*Universidad Favaloro, Solís 453, (1078) Buenos Aires, Argentina*
(Received 5 November 2004; published 16 February 2005)

The decays of nonstrange negative parity baryons via the emission of single π and η mesons are analyzed in the framework of the $1/N_c$ expansion. A basis of spin-flavor operators for the partial wave amplitudes is established to order $1/N_c$ and the unknown effective coefficients are determined by fitting to the S- and D-wave partial widths as provided by the Particle Data Group. A set of relations between widths that result at the leading order, i.e., order N_c^0, is given and tested with the available data. The rather large errors of the input partial widths, that result from the often discrepant results for the resonance parameters from different analyses of the data, lead to a rather good fit at the leading order N_c^0. The next to leading order fit fails for the same reason to pin down with satisfactory accuracy the subleading effective coefficients. The hierarchy expected from the $1/N_c$ expansion is however reflected in the results.

DOI: 10.1103/PhysRevD.71.034016 PACS numbers: 14.20.Gk, 11.15.Pg, 12.39.Jh

I. INTRODUCTION

The $1/N_c$ expansion has proven to be a very useful tool for analyzing the baryon sector. This success is mostly a consequence of the emergent contracted spin-flavor symmetry in the large N_c limit [1,2]. In the sector of ground state baryons (identified for $N_c = 3$ with the spin $1/2$ octet and spin $3/2$ decuplet in the case of three flavors), that symmetry gives rise to several important relations that hold at different orders in the $1/N_c$ expansion [3–5]. The domain of excited baryons (baryon resonances) has also been explored in the framework of the $1/N_c$ expansion [6–14] with very promising results. The analyses carried out so far have been constrained to states that belong to a definite spin-flavor and orbital multiplet, i.e., the possibility of mixing of different such multiplets (so called configuration mixing) has been disregarded. One good reason for this approximation is that the resonance data are not sufficient for a full fledged $1/N_c$ analysis that includes those effects. It is also very likely that such effects are small for dynamical reasons. Indeed, it has been shown that the only configuration mixings that are not suppressed by $1/N_c$ factors involve couplings to the orbital degrees of freedom [15]. The $1/N_c$ analyses of excited baryon masses have shown that orbital angular momentum couplings turn out to be very small [8,9], which is in agreement with older results in the quark model [16]. This strongly suggests that a similar suppression, which is not a consequence of the $1/N_c$ expansion but rather of QCD dynamics, also takes place in configuration mixings. Thus, disregarding configuration mixing is likely to be a good approximation for the purpose of phenomenology. Within such a framework, a few analyses of excited baryon strong decays have been carried out, namely, the decays of the negative parity $SU(6)$ 70-plet [10] and of the Roper 56-plet [11]. In the case of interest in the present work, namely, the 70-plet, the analysis in Ref. [10] used an incomplete basis of operators at subleading order in $1/N_c$. One of the motivations of the present work is to provide a complete analysis to $\mathcal{O}(1/N_c)$ for the decays of the nonstrange members of the 70-plet [i.e., the mixed symmetry 20-plet of $SU(4)$] into ground state baryons plus a pion or an eta meson. In particular, a complete basis of effective operators that provide the various S- and D-wave amplitudes is furnished. It should be advanced that the lack of a complete basis of operators in [10] does not affect in a significant way the conclusions of that work. As discussed later in this work, the fact that the input partial widths have large errors implies a rather uncertain determination of subleading effects.

The negative parity 70-plet is the experimentally best established and known excited baryon multiplet. In particular the S- and D-wave partial decay widths have been determined from different data analyses [17] with varying degrees of certainty. In all, these available widths provide sufficient input for the analysis at $\mathcal{O}(1/N_c)$ pursued in this work. To make the analysis more conclusive, however, higher precision in the inputs would be required.

This paper is organized as follows: Sec. II contains the framework for calculating the decays, Sec. III provides the basis of effective operators, Sec. IV presents the results, and finally the conclusions are given in Sec. V.

II. FRAMEWORK FOR DECAYS

In the application of the $1/N_c$ expansion to excited baryons the assumption is made that there is an approxi-

*Email address: goity@jlab.org
†Email address: schat@tandar.cnea.gov.ar
‡Email address: scoccola@tandar.cnea.gov.ar

1550-7998/2005/71(3)/034016(11)$23.00

mate spin-flavor symmetry. This assumption is rigorous in the large N_c limit for the ground state baryons. For excited states, however, the symmetry is broken at order N_c^0. This has been found to be the case for the masses of the negative parity baryons, where it was shown [6,8] that effective mass operators of spin-orbit type produce such a breaking. In the large N_c limit a different scheme becomes rigorously valid [13,14]. In the real world with $N_c = 3$ there is, however, plenty of evidence that the dominating spin-flavor breaking in masses are the hyperfine effects that are order $1/N_c$, while the spin-orbit effects are substantially smaller [8,9]. Thus, a scheme based on an approximate spin-flavor symmetry is very convenient for phenomenological purposes.

The excited baryons are therefore classified in multiplets of the $O(3) \times SU(2N_f)$ group. $O(3)$ corresponds to spatial rotations and $SU(2N_f)$ is the spin-flavor group where N_f is the number of flavors being considered, equal to two in the present work. The ground state baryons, namely, the N and Δ states, belong to the $(\mathbf{1}, \mathbf{20_S})$ representation, where the $\mathbf{20_S}$ is the totally symmetric representation of $SU(4)$. The negative parity baryons considered here belong instead to the $(\mathbf{3}, \mathbf{20_{MS}})$ representation, where $\mathbf{20_{MS}}$ is the mixed-symmetric representation of $SU(4)$. For general N_c the spin-flavor representations involve, in the Young tableaux language, N_c boxes and are identified with the totally symmetric and the mixed-symmetric representations of type $(N_c - 1, 1)$ for ground and excited negative parity states, respectively. Since in the mixed-symmetric spin-flavor representation one box of the Young tableaux is distinguished, such a box is associated with the "excited quark" in the baryon. In a similar fashion, and without any loss of generality it is possible to distinguish one box in the ground state multiplet as well. This is a very convenient procedure that has been used repeatedly in previous works. The spin and isospin quantum numbers of the distinguished box will be denoted with lower cases, and the corresponding quantum numbers of the rest of the $N_c - 1$ boxes [which are in a totally symmetric representation of $SU(4)$ and form the so-called "core" of the large N_c baryon] after they are coupled to eigenstates of spin and isospin will be denoted by S_c and I_c respectively. Notice that $S_c = I_c$ for totally symmetric representations of $SU(4)$. For a given core state, the coupling of the excited quark gives eigenstates of spin and isospin:

$$
\begin{aligned}
|S, S_3; I, I_3; S_c\rangle = \sum_{s_3, i_3} &\left\langle S_c, S_3 - s_3; \frac{1}{2}, s_3 \,\middle|\, S, S_3 \right\rangle \\
&\left\langle I_c = S_c, I_3 - i_3; \frac{1}{2}, i_3 \,\middle|\, I, I_3 \right\rangle \\
|S_c, S_3 - s_3; I_c = S_c, I_3 - i_3\rangle &\left|\frac{1}{2}, s_3; \frac{1}{2}, i_3\right\rangle. \qquad (1)
\end{aligned}
$$

These states are not in an irreducible representation of $SU(4)$, as they are not in an irreducible representation of the permutation group. The totally symmetric states are given by

$$
|S, S_3; I, I_3\rangle_{\mathbf{S}} = \sum_{\eta = \pm 1/2} C_{\mathbf{S}}(S, \eta) |S, S_3; I, I_3; S_c = S + \eta\rangle, \qquad (2)
$$

where

$$
C_{\mathbf{S}}\left(S, \pm\frac{1}{2}\right) = \sqrt{\frac{(2S + 1 \pm 1)[N_c + 1 \mp (2S + 1)]}{2N_c(2S + 1)}}, \qquad (3)
$$

while the mixed-symmetric (**MS**) states $(N_c - 1, 1)$ are given by

$$
\begin{aligned}
|S, S_3, I, I_3\rangle_{\mathbf{MS}} = \sum_{\eta = \pm 1/2} &C_{\mathbf{MS}}(I, S, \eta) |S, S_3; I, I_3; \\
&S_c = S + \eta\rangle, \qquad (4)
\end{aligned}
$$

where

$$
C_{\mathbf{MS}}\left(I, S, \pm\frac{1}{2}\right) = \begin{cases} 1, & \text{if } I = S \pm 1, \\ 0, & \text{if } I = S \mp 1, \\ \pm\sqrt{\frac{(2S+1\mp 1)[N_c+1\pm(2S+1)]}{2N_c(2S+1)}}, & \text{if } I = S. \end{cases} \qquad (5)
$$

Finally, upon coupling the orbital degrees of freedom, the excited baryons in the $(\mathbf{3}, \mathbf{MS})$ representation are given by

$$
\begin{aligned}
|J, J_3; I, I_3; S\rangle_{\mathbf{MS}} = \sum_m &\langle 1, m; S, J_3 - m \\
&| J, J_3\rangle |1, m\rangle |S, J_3 - m; I, I_3\rangle_{\mathbf{MS}}. \qquad (6)
\end{aligned}
$$

For $N_c = 3$ the states are displayed in Table I along with their quantum numbers, masses, decay widths and branching ratios.

Note that there are two sets of N^* states each consisting of two states with the same spin and isospin. The physical states are admixtures of such states, and are given by

$$
\begin{pmatrix} N_J^* \\ N_J^{*\prime} \end{pmatrix} = \begin{pmatrix} \cos\theta_{2J} & \sin\theta_{2J} \\ -\sin\theta_{2J} & \cos\theta_{2J} \end{pmatrix} \begin{pmatrix} {}^2N_J^* \\ {}^4N_J^* \end{pmatrix}, \qquad (7)
$$

where $J = \frac{1}{2}$ and $\frac{3}{2}$, $N_J^{*(\prime)}$ are mass eigenstates, and the two mixing angles can be constrained to be in the interval $[0, \pi)$. Here the notation ${}^{2S+1}N_J^*$ has been used.

The possible strong decays with emission of a single π or η meson (including those in the η channel that turn out to be kinematically forbidden) are shown in Table I. Only S- and D-wave decays are considered [18]. The effective amplitudes for the emission of a pseudoscalar meson have the most general form:

$$
\begin{aligned}
M^{[\ell_P, I_P]}(\vec{k}_P) = (-1)^{\ell_P} &\sqrt{2M_{B^*}} Y^*_{\ell_P, m_P}(\hat{k}_P) \\
&\times \sum_{\mu, \alpha} \langle \ell_P, m_P; I_P, I_{P3} | P^{[\ell_P, I_P]}_{[\mu, \alpha]} | 0 \rangle \\
&\times {}_{\mathbf{S}}\langle S, S_3; I, I_3 | B^{[\ell_P, I_P]}_{[-\mu, -\alpha]} | J^*, J_3^*; I^*, I_3^*; S^* \rangle_{\mathbf{MS}}, \qquad (8)
\end{aligned}
$$

where ℓ_P and I_P are the orbital angular momentum and

TABLE I. Negative parity nonstrange baryons and their decay widths and branching ratios from the Particle Data Group. Channels not explicitly indicated are forbidden.

State	Notation	Mass (MeV)	Total width (MeV)	Branching ratios (%) S wave	D wave
$N(1535)$	$N^*_{1/2}$	1538 ± 18	150 ± 50	πN: 45 ± 10 ηN: 42.5 ± 12.5	$\pi\Delta$: 0.5 ± 0.5
$N(1520)$	$N^*_{3/2}$	1523 ± 8	122 ± 13	$\pi\Delta$: 8.5 ± 3.5	πN: 55 ± 5 $\pi\Delta$: 12 ± 2 ηN: unknown
$N(1650)$	$N^{*\prime}_{1/2}$	1660 ± 20	167 ± 23	πN: 72.5 ± 17.5 ηN: 6.5 ± 3.5	$\pi\Delta$: 4 ± 3
$N(1700)$	$N^{*\prime}_{3/2}$	1700 ± 50	100 ± 50	$\pi\Delta$: unknown	πN: 10 ± 5 $\pi\Delta$: unknown ηN: unknown
$N(1675)$	$N^*_{5/2}$	1678 ± 8	160 ± 20		πN: 45 ± 5 $\pi\Delta$: unknown ηN: unknown
$\Delta(1620)$	$\Delta^*_{1/2}$	1645 ± 30	150 ± 30	πN: 25 ± 5	$\pi\Delta$: 45 ± 15 $\eta\Delta$: kinematically forbidden
$\Delta(1700)$	$\Delta^*_{3/2}$	1720 ± 50	300 ± 100	$\pi\Delta$: 37.5 ± 12.5 $\eta\Delta$: kinematically forbidden	πN: 15 ± 5 $\pi\Delta$: 4 ± 3 $\eta\Delta$: kinematically forbidden

isospin of the pseudoscalar meson, and μ and α the corresponding projections. $P^{[\ell_P,I_P]}_{[\mu,\alpha]}$ is a mesonic operator that creates the final state pseudoscalar and $B^{[\ell_P,I_P]}_{[-\mu,-\alpha]}$ is a baryonic operator that transforms the initial excited baryon into a ground state baryon. Here, the partial wave is projected onto a meson momentum eigenstate with momentum $\vec{k}_P$. The factor $\sqrt{2M_{B*}}$, where M_{B*} is the mass of the excited baryon, has been added for convenience. Since as shown later, the dynamics of the decay can be encoded in effective coefficients, the mesonic operator matrix elements can be chosen to be

$$\langle \ell_P, m_P; I_P, I_{P3} | P^{[\ell_P,I_P]}_{[\mu,\alpha]} | 0\rangle = \sqrt{(2\ell_P + 1)(2I_P + 1)}\,\delta_{\mu m_P}\delta_{\alpha I_{P3}}. \quad (9)$$

The baryonic operator admits an expansion in $1/N_c$ and has the general form:

$$B^{[\ell_P,I_P]}_{[\mu,\alpha]} = \left(\frac{k_P}{\Lambda}\right)^{\ell_P} \sum_q C^{[\ell_P,I_P]}_q(k_P)(B^{[\ell_P,I_P]}_{[\mu,\alpha]})_q, \quad (10)$$

where

$$(B^{[\ell_P,I_P]}_{[\mu,\alpha]})_q = \sum_m \langle 1, m; j, j_z \mid \ell_P, \mu\rangle \xi^1_m (\mathcal{G}^{[j,I_P]}_{[j_z,\alpha]})_q, \quad (11)$$

and the factor $(k_P/\Lambda)^{\ell_P}$ is included to take into account the chief meson momentum dependence of the partial wave. The scale Λ is chosen in what follows to be 200 MeV. Here, ξ^1_m is an operator that produces a transition from the triplet to the singlet $O(3)$ state, and $(\mathcal{G}^{[j,I_P]}_{[j_z,\alpha]})_q$ is a spin-flavor operator that produces the transition from the mixed-symmetric to the symmetric $SU(4)$ representation. The label j denotes the spin of the spin-flavor operator, and as it is clear, its isospin coincides with the isospin of the emitted meson. As it was already mentioned, the dynamics of the decay is encoded in the effective dimensionless coefficients $C^{[\ell_P,I_P]}_n(k_P)$.

The terms in the right-hand side of Eq. (10) are ordered in powers of $1/N_c$. As it has been explained in earlier publications [3], the order in $1/N_c$ is determined by the spin-flavor operator. For an n-body operator, this order is given by

$$\nu = n - 1 - \kappa, \quad (12)$$

where κ is equal to zero for incoherent operators and can be equal to 1 or even larger for coherent operators. More details can be found in the following section where a basis of operators $\mathcal{G}$ is explicitly built.

With the definition of effective operators used in this work, all coefficients $C^{[\ell_P,I_P]}_q(k_P)$ in Eq. (10) are of zeroth order in N_c. The leading order (LO) of the decay amplitude is in fact N_c^0 [15]. At this point it is important to comment on the momentum dependence of the coefficients. The spin-flavor breakings in the masses, of both excited and ground state baryons, give rise to different values of the momenta k_P. In the expansion where the spin-flavor breaking is treated as small, $k_P = \overline{k}_P + \delta k_P$, where $\overline{k}_P$ is the spin-flavor symmetry limit value, and δk_P is taken as small and expanded upon. Only the momentum dependence of the coefficients associated with leading order operators should be included if one takes δk_P to be of the same order as $1/N_c$ corrections. The goal would be to treat these corrections in a model independent fashion. In an operator analysis, they require the consideration of reducible effective operators that are the product of a mass operator that gives the spin-flavor breaking mass shifts (e.g. of the ground state baryons) times the leading order operator to which the coefficient is associated with. These corrections should be considered to be of the same order as the subleading in $1/N_c$ corrections given by irreducible operators that are analyzed below in full detail. In principle, this could be achieved with no difficulties in a world where N_c is large. The problem is that for $N_c = 3$ the number of independent amplitudes for the decays here under consideration is essentially the same as the number of irreducible operators that appear in the analysis to order $1/N_c$. There is, therefore, no room to separate the momentum dependence effects in the coefficients from the $1/N_c$ effects due to irreducible operators. Thus, one can adopt the scheme followed in this paper where the only momentum dependence assigned to the coefficients is the explicitly shown factor $(k_P/\Lambda)^{\ell_P}$ that takes into account the chief momentum dependence of the corresponding partial wave, and the rest of the dependence is then encoded in the coefficients of the subleading operators. The other possibility is to model the momentum dependence, as it was done in [11], with a profile function motivated for instance by a quark model, in such a way that less of the momentum dependence is absorbed by subleading operators. Both ways of proceeding are equally valid.

Using the standard definition for the decay width and averaging over the initial- and summing over the corresponding final-baryon spins and isospins, the decay width for each $[\ell_P, I_P]$ channel is given by

$$\Gamma^{[\ell_P,I_P]} = f_{ps} \frac{|\sum_q C_q^{[\ell_P,I_P]} \mathcal{B}_q(\ell_P, I_P, S, I, J^*, I^*, S^*)|^2}{\sqrt{(2J^*+1)(2I^*+1)}}, \quad (13)$$

where the phase space factor f_{ps} is

$$f_{ps} = \frac{k_P^{1+2\ell_P}}{8\pi^2 \Lambda^{2\ell_P}} \frac{M_{B^*}}{M_B} \quad (14)$$

and $\mathcal{B}_q(\ell_P, I_P, S, I, J^*, I^*, S^*)$ are reduced matrix elements defined via the Wigner-Eckart theorem as follows:

$$\begin{aligned} &{}_{\mathbf{S}}\langle S, S_3; I, I_3 | (B^{[\ell_P, I_P]}_{[m_P, I_{P_3}]})_q | J^*, J_3^*; I^*, I_3^*; S^* \rangle_{\mathbf{MS}} \\ &= \frac{(-1)^{\ell_P - J^* + S + I_P - I^* + I}}{\sqrt{(2S+1)(2I+1)}} \langle \ell_P, m_P; J^*, J_3^* | S, S_3 \rangle \\ &\quad \times \langle I_P, I_{P3}; I^*, I_3^* | I = S, I_3 \rangle \mathcal{B}_q(\ell_P, I_P, S, I, J^*, I^*, S^*). \end{aligned} \quad (15)$$

These reduced matrix elements can be easily calculated in terms of the reduced matrix elements of the spin-flavor operators, namely

$$\mathcal{B}_q(\ell_P, I_P, S, I, J^*, I^*, S^*) = (-1)^{j+J^*+\ell_P+J+1} \sqrt{(2J^*+1)(2\ell_P+1)} \begin{Bmatrix} J^* & S^* & 1 \\ j & \ell_P & S \end{Bmatrix} {}_{\mathbf{S}}\langle S; I \| (\mathcal{G}^{[j,I_P]})_q \| S^*; I^* \rangle_{\mathbf{MS}}, \quad (16)$$

where, due to the fact that the unknown dynamics can be included in the effective coefficients, the operators ξ^1_m can be chosen such that their matrix elements are simply given by

$$\langle 0 | \xi^1_{m'} | 1m \rangle = -\sqrt{3}\delta_{mm'}. \quad (17)$$

Note that in the present case, where ℓ_P can be 0 or 2 only, Eq. (16) implies that the spin-flavor operators can carry spin j that can be 1, 2 or 3.

III. BASIS OF OPERATORS

The construction of a basis of spin-flavor operators follows similar lines as in previous works on baryon masses. The spin-flavor operators considered in the present paper must connect a mixed-symmetric with a symmetric representation. Generators of the spin-flavor group acting on the states obviously do not produce such connection. However, generators restricted to act on the excited quark or on the core of $N_c - 1$ quarks can do this. The spin-flavor operators can, therefore, be represented by products of generators of the spin-flavor group restricted to act either on the excited or on the core states. In the following the generators acting on the core are denoted by S_c, G_c, T_c, and the ones acting on the excited quark by s, g, t. The generators G_c are known to be coherent operators, while all the rest are incoherent. In order to build a basis of operators for the present problem one has to consider products of such generators with the appropriate couplings of spins and isospins. The n-bodyness (nB) of an operator is given by the number of such factors, and the level of coherence of the operator is determined by how many factors G_c appear in the product. It should be noticed that in the physical case where $N_c = 3$, only operators of at most 3B have to be considered. Still, to order $1/N_c$ there is a rather long list of operators of given spin j and isospin I_P. This list can be drastically shortened by applying several reduction rules. The first rule is that the product of two or more generators acting on the excited quark can always be reduced to the identity operator or to a linear combination of such generators. The second set of rules can be easily derived for products of operators whose matrix elements are taken between a mixed-symmetric and a symmetric representation. These reduction rules are as follows for 1B and 2B operators (here λ represents generators acting on the excited quark and Λ_c represent generators acting on the core):

$$\begin{aligned} \lambda &= -\Lambda_c, \\ (\Lambda_c)_1(\Lambda_c)_2 &= -\lambda_1(\Lambda_c)_2 - \lambda_2(\Lambda_c)_1 + \text{1B operators}. \end{aligned} \tag{18}$$

Therefore, only the following types of operators should be considered

$$\begin{array}{ll} \text{1B} & \lambda \\ \text{2B} & \frac{1}{N_c}\lambda_1(\Lambda_c)_2 \\ \text{3B} & \frac{1}{N_c^2}\lambda_1(\Lambda_c)_2(\Lambda_c)_3. \end{array} \tag{19}$$

It is convenient to make explicit the transformation properties of each basic operator under spin j and isospin t. In what follows we use the notation $O^{[j,t]}$ to indicate that the operator O has spin j and isospin t. It is easy to see that

$$\begin{aligned} \lambda^{[j,t]} &= s^{[1,0]}, t^{[0,1]}, g^{[1,1]}, \\ (\Lambda_c)^{[j,t]} &= (S_c)^{[1,0]}, (T_c)^{[0,1]}, (G_c)^{[1,1]}. \end{aligned} \tag{20}$$

For decays in the η channels the spin-flavor operators transform as $[j, 0]$ while for decays in the pion channels they should transform as $[j, 1]$, where in both cases $j = 1, 2, 3$. From the transformation properties of each basic operator given in Eq. (20) it is easy to construct products of the forms given in Eq. (19) with the desired spin and isospin. An example of a 2B operator that transforms as $[2, 1]$ is

$$(gG_c)^{[2,1]}_{[\mu,\alpha]} = \langle 1, m; 1, m' \mid 2, \mu\rangle \times \langle 1, a; 1, a' \mid 1, \alpha\rangle g_{ma}(G_c)_{m'a'}, \tag{21}$$

where the spin and isospin projections (e.g. m, m', a, etc.) are considered to be spherical.

Thus, the 1B operators that contribute to a given $[j, t]$ are just those $\lambda^{[j,t]}$ given in Eq. (20) which have the proper spin and isospin quantum numbers. Similarly, the possible 2B operators are given by the products $\lambda_1^{[j_1,t_1]}(\Lambda_c)_2^{[j_2,t_2]}$ coupled to the required $[j, t]$ by means of the conventional Clebsch-Gordan (CG) coefficients.

In order to construct all possible 3B operators, it is convenient to note that there are additional reduction rules for the products $(\Lambda_c)_2(\Lambda_c)_3$. These rules apply to matrix elements of products of generators between states in the symmetric representation [2]. The starting point is to consider all possible products of two core operators. Since one is interested in keeping contributions of at most order $1/N_c$, and because these products will appear only in 3B operators, at least one of the operators must be a G_c. Thus, the list of 2B core operators to be considered is

$$\begin{aligned} &(T_cG_c)^{[1,t]}, \quad (G_cT_c)^{[1,t]}, \quad (S_cG_c)^{[j,1]}, \\ &(G_cS_c)^{[j,1]}, \quad (G_cG_c)^{[j,t]}, \end{aligned} \tag{22}$$

where $j, t = 0, 1, 2$. Using transformation properties of the CG coefficients and dropping all the combinations leading to commutators (because the basic operators are generators of the algebra, the commutators are proportional to another basic operator), only the following products remain:

$$(\{T_c, G_c\})^{[1,0]}, \quad ([T_c, G_c])^{[1,1]}, \quad (\{T_c, G_c\})^{[1,2]}, \tag{23}$$

$$(\{S_c, G_c\})^{[0,1]}, \quad ([S_c, G_c])^{[1,1]}, \quad (\{S_c, G_c\})^{[2,1]}, \tag{24}$$

$$\begin{aligned} &(G_cG_c)^{[0,0]}, \quad (G_cG_c)^{[0,2]}, \quad (G_cG_c)^{[1,1]}, \\ &(G_cG_c)^{[2,0]}, \quad (G_cG_c)^{[2,2]}, \end{aligned} \tag{25}$$

where $([T_c, G_c])^{[1,1]}$ denotes $(T_cG_c)^{[1,1]} - (G_cT_c)^{[1,1]} = \langle 1a1b \mid 1d\rangle(T_c^aG_c^{mb} - G_c^{ma}T_c^b) = \langle 1a1b \mid 1d\rangle\{G_c^{ma}, T_c^b\}$, etc. Using now the reduction relations [2], it can be shown that several of these 11 operators can be eliminated. The final list of independent products of two core operators turns out to be

$$\begin{aligned} &(\{T_c, G_c\})^{[1,2]}, \quad ([S_c, G_c])^{[1,1]}, \\ &(\{S_c, G_c\})^{[2,1]}, \quad (G_cG_c)^{[2,2]}. \end{aligned} \tag{26}$$

By coupling any of these operators with one of the excited core operators $\lambda_1^{[j_1,t_1]}$ [see Eq. (20)] to the required $[j, t]$ all the possible 3B operators are obtained.

Using this scheme to couple products of generators it is straightforward to construct lists of operators with spin 1, 2 and 3 and isospin 0 and 1. Further reductions result from the fact that not all the resulting operators are linearly independent up to order $1/N_c$. The determination of the final set of independent operators for each particular decay channel is more laborious. This is achieved by coupling the resulting spin-flavor operators with the $\xi^{[1,0]}$ orbital transition operator to the corresponding total spin and isospin and by explicitly calculating all the relevant matrix elements. In this way operators that are linearly dependent at the corresponding order in the $1/N_c$ expansion can be eliminated. The resulting basis of independent operators $(O^{[\ell_P,I_P]}_{[m_P,I_{P_3}]})_n$ is shown in Table II, where for simplicity the corresponding spin and isospin projections have been omitted. Their reduced matrix elements are given in Tables III, IV, V, and VI. In the bottom rows of these tables normalization coefficients $\alpha_n^{[\ell,I_P]}$ are displayed. These coefficients are used to define normalized basis operators such that, for $N_c = 3$, their largest reduced matrix element is equal to 1 for order N_c^0 operators and equal to 1/3 for order $1/N_c$ operators. Thus,

$$(B^{[\ell_P,I_P]}_{[m_P,I_{P_3}]})_n = \alpha_n^{[\ell,I_P]}(O^{[\ell_P,I_P]}_{[m_P,I_{P_3}]})_n \tag{27}$$

furnishes the list of basis operators normalized according to the $1/N_c$ power counting.

TABLE II. Basis operators.

	n bodyness	Name	Operator	Order in $1/N_c$
	1B	$O_1^{[0,1]}$	$(\xi g)^{[0,1]}$	0
Pion		$O_2^{[0,1]}$	$\frac{1}{N_c}(\xi(sT_c)^{[1,1]})^{[0,1]}_{[0,a]}$	1
S wave	2B	$O_3^{[0,1]}$	$\frac{1}{N_c}(\xi(tS_c)^{[1,1]})^{[0,1]}_{[0,a]}$	1
		$O_4^{[0,1]}$	$\frac{1}{N_c}(\xi(gS_c)^{[1,1]})^{[0,1]}_{[0,a]}$	1
	1B	$O_1^{[2,1]}$	$(\xi g)^{[2,1]}_{[i,a]}$	0
		$O_2^{[2,1]}$	$\frac{1}{N_c}(\xi(sT_c)^{[1,1]})^{[2,1]}_{[i,a]}$	1
Pion		$O_3^{[2,1]}$	$\frac{1}{N_c}(\xi(tS_c)^{[1,1]})^{[2,1]}_{[i,a]}$	1
D wave	2B	$O_4^{[2,1]}$	$\frac{1}{N_c}(\xi(gS_c)^{[1,1]})^{[2,1]}_{[i,a]}$	1
		$O_5^{[2,1]}$	$\frac{1}{N_c}(\xi(gS_c)^{[2,1]})^{[2,1]}_{[i,a]}$	1
		$O_6^{[2,1]}$	$\frac{1}{N_c}(\xi(sG_c)^{[2,1]})^{[2,1]}_{[i,a]}$	0
	3B	$O_7^{[2,1]}$	$\frac{1}{N_c^2}(\xi(s(\{S_c, G_c\})^{[2,1]})^{[2,1]})^{[2,1]}_{[i,a]}$	1
		$O_8^{[2,1]}$	$\frac{1}{N_c^2}(\xi(s(\{S_c, G_c\})^{[2,1]})^{[3,1]})^{[2,1]}_{[i,a]}$	1
Eta	1B	$O_1^{[0,0]}$	$(\xi s)^{[0,0]}_{[0,0]}$	0
S wave	2B	$O_2^{[0,0]}$	$\frac{1}{N_c}(\xi(sS_c)^{[1,0]})^{[0,0]}_{[0,0]}$	1
Eta	1B	$O_1^{[2,0]}$	$(\xi s)^{[2,0]}_{[i,0]}$	0
D wave	2B	$O_2^{[2,0]}$	$\frac{1}{N_c}(\xi(sS_c)^{[1,0]})^{[2,0]}_{[i,0]}$	1
		$O_3^{[2,0]}$	$\frac{1}{N_c}(\xi(sS_c)^{[2,0]})^{[2,0]}_{[i,0]}$	1

IV. RESULTS

The different S- and D-wave partial widths used in the analysis are the ones provided by the Particle Data Group [17]. The values for the widths and branching ratios are taken as the ones indicated there as ‘‘our estimate,’’ while the errors are determined from the corresponding ranges. The total widths and branching fractions are given in Table I, while in Table VIII the partial widths calculated from those values are explicitly displayed. The entries in Table I indicated as unknown reflect channels for which no width is provided by the Particle Data Group or where the authors consider that the input is unreliable, such as in the $\pi\Delta$ decay modes of the $N(1700)$ and $N(1675)$ and the D-wave η decay modes. At this point it is important to stress the marginal precision of the data for the purposes of this work. This work performs an analysis at order $1/N_c$, which means that the theoretical error is order $1/N_c^2$. This implies that amplitudes are affected by a theoretical uncertainty at the level of 10%. Thus, in order to pin down the coefficients of the subleading operators, the widths provided by the data should not be affected by errors larger than about 20%. As shown in Tables I and VIII, the experimental errors are in most entries 30% or larger. In consequence, the determination of the subleading effective coefficients is affected by large errors as the results below show.

TABLE III. Reduced matrix elements of pion S-wave operators.

Pion S waves	$O_1^{[0,1]}$	$O_2^{[0,1]}$	$O_3^{[0,1]}$	$O_4^{[0,1]}$	Overall factor
$2N^*_{1/2} \to N$	-1	$-\frac{1}{2N_c}$	$-\frac{1}{2N_c}$	0	$\frac{\sqrt{(N_c+3)(N_c-1)}}{3N_c}$
$2N^*_{3/2} \to \Delta$	$\frac{1}{3}$	$-\frac{1}{3N_c}$	$-\frac{1}{3N_c}$	$\frac{1}{2\sqrt{2}N_c}$	$-\frac{\sqrt{3(N_c+5)(N_c+3)}}{\sqrt{6}N_c}$
$4N^*_{1/2} \to N$	$-\frac{1}{6}$	$\frac{2}{3N_c}$	$\frac{1}{6N_c}$	$\frac{1}{4\sqrt{2}N_c}$	$-\sqrt{2\frac{N_c-1}{N_c}}$
$4N^*_{3/2} \to \Delta$	$-\frac{1}{6}$	$\frac{1}{6N_c}$	$-\frac{1}{3N_c}$	0	$\sqrt{10\frac{N_c+5}{N_c}}$
$\Delta^*_{1/2} \to N$	$\frac{1}{6}$	$-\frac{1}{6N_c}$	$-\frac{2}{3N_c}$	0	$-\sqrt{2\frac{N_c-1}{N_c}}$
$\Delta^*_{3/2} \to \Delta$	$-\frac{1}{6}$	$-\frac{1}{3N_c}$	$\frac{1}{6N_c}$	$-\frac{1}{4\sqrt{2}N_c}$	$-\sqrt{10\frac{N_c+5}{N_c}}$
$\alpha^{[0,1]}$	$\frac{3\sqrt{3}}{2\sqrt{5}}$	$\frac{3\sqrt{3}}{4\sqrt{5}}$	$-\frac{3\sqrt{3}}{4\sqrt{5}}$	$\sqrt{\frac{6}{5}}$	

Before presenting the results of the fits, it is convenient to derive some parameter independent relations that can be obtained to leading order. These relations serve as a test of the leading order approximation. Since at this order there are only four coefficients and two angles to be fitted, and there are a total of 20 partial widths (excluding all D-wave η channels but including kinematically forbidden η-channel decays), there are 14 independent parameter free relations that can be derived. These relations are more conveniently written in terms of reduced widths $\tilde{\Gamma}$, where the phase space factor f_{sp} [see Eq. (14)] has been removed.

TABLE IV. Reduced matrix elements of pion D-wave operators.

Pion D waves	$O_1^{[2,1]}$	$O_2^{[2,1]}$	$O_3^{[2,1]}$	$O_4^{[2,1]}$	$O_5^{[2,1]}$	$O_6^{[2,1]}$	$O_7^{[2,1]}$	$O_8^{[2,1]}$	Overall factor
$2N^*_{1/2} \to \Delta$	$-\frac{1}{3\sqrt{5}}$	$\frac{1}{3\sqrt{5}N_c}$	$\frac{1}{3\sqrt{5}N_c}$	$-\frac{1}{2\sqrt{10}N_c}$	$-\frac{1}{2\sqrt{30}N_c}$	$\frac{N_c-1}{2\sqrt{30}N_c}$	$\frac{1}{4\sqrt{5}N_c^2}$	0	$5\frac{\sqrt{(N_c+3)(N_c+5)}}{2N_c}$
$2N^*_{3/2} \to N$	$-\frac{1}{\sqrt{5}}$	$-\frac{1}{2\sqrt{5}N_c}$	$-\frac{1}{2\sqrt{5}N_c}$	0	0	0	0	0	$-5\frac{\sqrt{(N_c+3)(N_c-1)}}{3N_c}$
$2N^*_{3/2} \to \Delta$	$-\frac{1}{3\sqrt{5}}$	$\frac{1}{3\sqrt{5}N_c}$	$\frac{1}{3\sqrt{5}N_c}$	$-\frac{1}{2\sqrt{10}N_c}$	$\frac{1}{2\sqrt{30}N_c}$	$-\frac{N_c-1}{2\sqrt{30}N_c}$	$-\frac{1}{4\sqrt{5}N_c^2}$	0	$5\frac{\sqrt{(N_c+3)(N_c+5)}}{2N_c}$
$4N^*_{1/2} \to \Delta$	$-\frac{1}{6\sqrt{5}}$	$\frac{1}{6\sqrt{5}N_c}$	$-\frac{1}{3\sqrt{5}N_c}$	0	$-\frac{1}{\sqrt{30}N_c}$	$\frac{N_c-1}{4\sqrt{30}N_c}$	$-\frac{N_c-3}{4\sqrt{5}N_c^2}$	$\frac{\sqrt{7}}{20}\frac{N_c-1}{N_c^2}$	$-5\sqrt{\frac{N_c+5}{2N_c}}$
$4N^*_{3/2} \to N$	$-\frac{1}{4\sqrt{5}}$	$\frac{1}{\sqrt{5}N_c}$	$\frac{1}{4\sqrt{5}N_c}$	$\frac{3}{8\sqrt{10}N_c}$	$-\frac{3\sqrt{3}}{8\sqrt{10}N_c}$	$\frac{3\sqrt{3}(N_c+2)}{8\sqrt{10}N_c}$	$-9\frac{N_c+1}{16\sqrt{5}N_c^2}$	0	$-\frac{2}{3}\sqrt{5\frac{N_c-1}{N_c}}$
$4N^*_{3/2} \to \Delta$	$-\frac{1}{3\sqrt{5}}$	$\frac{1}{3\sqrt{5}N_c}$	$-\frac{2}{3\sqrt{5}N_c}$	0	$-\frac{1}{\sqrt{30}N_c}$	$\frac{N_c-1}{4\sqrt{30}N_c}$	$-\frac{N_c-3}{4\sqrt{5}N_c^2}$	$-\frac{\sqrt{7}}{40}\frac{N_c-1}{N_c^2}$	$-2\sqrt{5\frac{N_c+5}{N_c}}$
$N^*_{5/2} \to N$	$-\frac{1}{4\sqrt{5}}$	$\frac{1}{\sqrt{5}N_c}$	$\frac{1}{4\sqrt{5}N_c}$	$\frac{3}{8\sqrt{10}N_c}$	$\frac{1}{8\sqrt{30}N_c}$	$-\frac{N_c+2}{8\sqrt{30}N_c}$	$\frac{N_c+1}{16\sqrt{5}N_c^2}$	0	$-2\sqrt{5\frac{N_c-1}{N_c}}$
$N^*_{5/2} \to \Delta$	$\frac{1}{2\sqrt{5}}$	$-\frac{1}{2\sqrt{5}N_c}$	$\frac{1}{\sqrt{5}N_c}$	0	$-\frac{1}{\sqrt{30}N_c}$	$\frac{N_c-1}{4\sqrt{30}N_c}$	$-\frac{N_c-3}{4\sqrt{5}N_c^2}$	$-\frac{N_c-1}{20\sqrt{7}N_c^2}$	$5\sqrt{\frac{7}{10}\frac{N_c+5}{N_c}}$
$\Delta^*_{1/2} \to \Delta$	$\frac{1}{6\sqrt{5}}$	$\frac{1}{3\sqrt{5}N_c}$	$-\frac{1}{6\sqrt{5}N_c}$	$\frac{1}{4\sqrt{10}N_c}$	$\frac{1}{4\sqrt{30}N_c}$	$\frac{N_c+2}{20\sqrt{30}N_c}$	$\frac{3N_c+1}{40\sqrt{5}N_c^2}$	0	$5\sqrt{5\frac{N_c+5}{N_c}}$
$\Delta^*_{3/2} \to N$	$-\frac{1}{6\sqrt{5}}$	$\frac{1}{6\sqrt{5}N_c}$	$\frac{2}{3\sqrt{5}N_c}$	0	0	0	0	0	$-5\sqrt{2\frac{N_c-1}{N_c}}$
$\Delta^*_{3/2} \to \Delta$	$\frac{1}{6\sqrt{5}}$	$\frac{1}{3\sqrt{5}N_c}$	$-\frac{1}{6\sqrt{5}N_c}$	$\frac{1}{4\sqrt{10}N_c}$	$-\frac{1}{4\sqrt{30}N_c}$	$-\frac{N_c+2}{20\sqrt{30}N_c}$	$-\frac{3N_c+1}{40\sqrt{5}N_c^2}$	0	$5\sqrt{5\frac{N_c+5}{N_c}}$
$\alpha^{[2,1]}$	$\sqrt{\frac{3}{7}}$	$\frac{3\sqrt{3}}{10\sqrt{2}}$	$\frac{\sqrt{3}}{2\sqrt{7}}$	$\frac{2\sqrt{3}}{5}$	$\frac{3}{4}$	$-\frac{12}{5}$	$-\frac{3\sqrt{6}}{5}$	$-\frac{3\sqrt{3}}{\sqrt{7}}$	

Considering the S-wave decays in the π mode, there are six decays and three parameters in the fit. Thus, three parameter free relations must follow. These relations and the corresponding comparison with experimental values read

$$\begin{array}{lccccccc} & \begin{array}{c}\tilde{\Gamma}_{N(1535)\to\pi N}\\ +\tilde{\Gamma}_{N(1650)\to\pi N}\end{array} & : & \begin{array}{c}\tilde{\Gamma}_{N(1520)\to\pi\Delta}\\ +\tilde{\Gamma}_{N(1700)\to\pi\Delta}\end{array} & : & \tilde{\Gamma}_{\Delta(1620)\to\pi N} & : & \tilde{\Gamma}_{\Delta(1700)\to\pi\Delta} \\ \text{Th.} & 1 & : & 1 & : & 0.17 & : & 0.42 \\ \text{Exp.} & 1 & : & \text{unknown} & : & 0.19 \pm 0.07 & : & 0.62 \pm 0.33. \end{array} \tag{28}$$

Within the experimental errors the relations are satisfied. They can be used to give a leading order prediction for the unknown S-wave width: $\Gamma_{N(1700)\to\pi\Delta} = 160 \pm 40$ MeV. The following expression for the θ_1 mixing angle also follows:

$$\frac{\tilde{\Gamma}_{N(1535)\to\pi N} - \tilde{\Gamma}_{N(1650)\to\pi N}}{\tilde{\Gamma}_{N(1535)\to\pi N} + \tilde{\Gamma}_{N(1650)\to\pi N}} = \frac{1}{3}[\cos(2\theta_1) - \sqrt{8}\sin(2\theta_1)]. \tag{29}$$

Using the empirical values the angle that results from this relation is $\theta_1 = 1.62 \pm 0.12$ or 0.29 ± 0.11. One of these angles, namely, the latter, turns out to be close to the angle obtained in the next to leading order (NLO) fit.

In a similar fashion, relations involving D-wave decays in the π mode can be obtained. There are in this case four parameters to fit and 11 partial widths. Thus, seven parameter free relations can be obtained. In general these relations are quadratic and/or involve some of the unknown decay widths. However, the following testable three linear relations can be obtained:

TABLE V. Reduced matrix elements of eta S-wave operators.

Eta S waves	$O_1^{[0,0]}$	$O_2^{[0,0]}$	Overall factor
$2N^*_{1/2} \to N$	1	0	$-\frac{\sqrt{(N_c+3)(N_c-1)}}{\sqrt{3}N_c}$
$4N^*_{1/2} \to N$	1	$-\frac{3}{2\sqrt{2}N_c}$	$-\sqrt{2\frac{N_c-1}{3N_c}}$
$\Delta^*_{1/2} \to \Delta$	1	$\frac{3}{2\sqrt{2}N_c}$	$\sqrt{2\frac{N_c+5}{3N_c}}$
$\alpha^{[0,0]}$	$\frac{3}{4}$	$\frac{1}{\sqrt{2}}$	

TABLE VI. Reduced matrix elements of eta D-wave operators.

Eta D waves	$O_1^{[2,0]}$	$O_2^{[2,0]}$	$O_3^{[2,0]}$	Overall factor
$2N^*_{3/2} \to N$	1	0	0	$5\frac{\sqrt{(N_c+3)(N_c-1)}}{\sqrt{15}N_c}$
$4N^*_{3/2} \to N$	1	$-\frac{3}{2\sqrt{2}N_c}$	$\frac{3\sqrt{3}}{2\sqrt{2}N_c}$	$-\sqrt{\frac{N_c-1}{3N_c}}$
$N^*_{5/2} \to N$	1	$-\frac{3}{2\sqrt{2}N_c}$	$-\frac{1}{2\sqrt{6}N_c}$	$-\sqrt{3\frac{N_c-1}{N_c}}$
$\Delta^*_{1/2} \to \Delta$	1	$\frac{3}{2\sqrt{2}N_c}$	$\frac{\sqrt{3}}{2\sqrt{2}N_c}$	$5\sqrt{\frac{N_c+5}{15N_c}}$
$\Delta^*_{3/2} \to \Delta$	1	$\frac{3}{2\sqrt{2}N_c}$	$-\frac{\sqrt{3}}{2\sqrt{2}N_c}$	$5\sqrt{\frac{N_c+5}{15N_c}}$
$\alpha^{[2,0]}$	$\frac{3}{2\sqrt{10}}$	$\frac{1}{\sqrt{5}}$	$-\sqrt{\frac{3}{5}}$	

$$\begin{array}{lccc} & 2\tilde{\Gamma}_{\Delta(1620)\to\pi\Delta} + \tilde{\Gamma}_{\Delta(1700)\to\pi\Delta} & = & 8\tilde{\Gamma}_{\Delta(1700)\to\pi N} + \frac{15}{4}\tilde{\Gamma}_{N(1675)\to\pi N}, \\ \text{Exp.} & 5.24 \pm 1.95 & = & 2.19 \pm 0.62, \end{array} \tag{30}$$

$$\begin{array}{lccc} & \frac{2}{9}(\tilde{\Gamma}_{\Delta(1535)\to\pi\Delta} + \tilde{\Gamma}_{N(1650)\to\pi\Delta}) + \frac{20}{3}\tilde{\Gamma}_{\Delta(1620)\to\pi\Delta} & = & 16\tilde{\Gamma}_{\Delta(1700)\to\pi N} + 15\tilde{\Gamma}_{N(1675)\to\pi N}, \\ \text{Exp.} & 16.87 \pm 6.46 & = & 6.49 \pm 1.65, \end{array} \tag{31}$$

$$\begin{array}{lccc} & \frac{1}{36}(\tilde{\Gamma}_{\Delta(1520)\to\pi N} + \tilde{\Gamma}_{N(1700)\to\pi N}) + \frac{5}{12}\tilde{\Gamma}_{\Delta(1620)\to\pi\Delta} & = & \tilde{\Gamma}_{\Delta(1700)\to\pi N} + \tilde{\Gamma}_{N(1675)\to\pi N}, \\ \text{Exp.} & 1.07 \pm 0.40 & = & 0.42 \pm 0.11. \end{array} \tag{32}$$

These relations are not well satisfied by the empirical data. Thus, one can anticipate a poor leading order fit to the D-wave pion decays. In the case of S-wave η mode decays, there are two parameters to fit (since there is no dependence on the mixing angle θ_3), and three possible decays. However, one of the decays is kinematically forbidden, namely, the $\Delta(1700) \to \eta\Delta$. It is also possible to express the mixing angle θ_1 in terms of the reduced S-wave widths:

$$\frac{\tilde{\Gamma}_{N(1535)\to\eta N} - \tilde{\Gamma}_{N(1650)\to\eta N}}{\tilde{\Gamma}_{N(1535)\to\eta N} + \tilde{\Gamma}_{N(1650)\to\eta N}} = -\frac{1}{3}[\cos(2\theta_1) - \sqrt{8}\sin(2\theta_1)]. \tag{33}$$

TABLE VII. Fit parameters. Fit No. 1: Pion S waves LO. In this case there is fourfold ambiguity for the angles $\{\theta_1, \theta_3\}$ given by the two values shown for each angle. Fit No. 2: Pion S and D waves, eta S waves, LO. In this case there is twofold ambiguity for the angle θ_1. For the angle θ_3 there is an *almost* twofold ambiguity given by the two values indicated in parenthesis and which only differ in the two slightly different values of $C_6^{[2,1]}$. Fit No. 3: Pion S and D waves, eta S waves, NLO, no 3-body operators. No degeneracy in θ_1 but *almost* twofold ambiguity in θ_3 given by the two values indicated in parenthesis. Values of coefficients which differ in the corresponding fits are indicated in parentheses.

Coefficient	No. 1 LO	No. 2 LO	No. 3 NLO
$C_1^{[0,1]}$	31 ± 3	31 ± 3	23 ± 3
$C_2^{[0,1]}$	...	...	$(7.4, 32.5) \pm (27, 41)$
$C_3^{[0,1]}$	...	...	$(20.7, 26.8) \pm (12, 14)$
$C_4^{[0,1]}$	...	...	$(-26.3, -66.8) \pm (39, 65)$
$C_1^{[2,1]}$	...	4.6 ± 0.5	3.4 ± 0.3
$C_2^{[2,1]}$	...	...	-4.5 ± 2.4
$C_3^{[2,1]}$	...	...	$(-0.01, 0.08) \pm 2$
$C_4^{[2,1]}$	...	...	5.7 ± 4.0
$C_5^{[2,1]}$	...	...	3.0 ± 2.2
$C_6^{[2,1]}$	...	$(-1.86, -2.25) \pm 0.4$	-1.73 ± 0.26
$C_7^{[2,1]}$	...	...	...
$C_8^{[2,1]}$	...	...	...
$C_1^{[0,0]}$	...	11 ± 4	17 ± 4
$C_2^{[0,0]}$	...	...	...
θ_1	1.62 ± 0.12 0.29 ± 0.11	1.56 ± 0.15 0.35 ± 0.14	0.39 ± 0.11
θ_3	3.01 ± 0.07 2.44 ± 0.06	$(3.00, 2.44) \pm 0.07$	$(2.82, 2.38) \pm 0.11$
$\chi^2_{\rm d.o.f.}$	0.25	1.5	0.9
d.o.f.	2	10	3

TABLE VIII. Partial widths resulting from the different fits in Table I. Values indicated in parentheses correspond to the cases in which *almost* degenerate values of θ_3 lead to different partial widths.

Decay		Empirical width (MeV)	No. 1 LO (MeV)	No. 2 LO (MeV)	No. 3 NLO (MeV)
	$N(1535) \to \pi N$	68 ± 27	74	62	(58,68)
	$N(1520) \to \pi\Delta$	10 ± 4	10	9.7	9.5
π *S*	$N(1650) \to \pi N$	121 ± 40	132	144	122
wave	$N(1700) \to \pi\Delta$	unknown	175	175	(259,156)
	$\Delta(1620) \to \pi N$	38 ± 13	35	35	38
	$\Delta(1700) \to \pi\Delta$	112 ± 53	81	81	(135,112)
	$N(1535) \to \pi\Delta$	1 ± 1		0.01	0.5
	$N(1520) \to \pi N$	67 ± 9		70	65
	$N(1520) \to \pi\Delta$	15 ± 3		2.8	13
	$N(1650) \to \pi\Delta$	7 ± 5		0.12	8
π *D*	$N(1700) \to \pi N$	10 ± 7		10	(11,9)
wave	$N(1700) \to \pi\Delta$	unknown		4	(4,9)
	$N(1675) \to \pi N$	72 ± 12		85	76
	$N(1675) \to \pi\Delta$	unknown		45	79
	$\Delta(1620) \to \pi\Delta$	68 ± 26		30	87
	$\Delta(1700) \to \pi N$	45 ± 21		49	32
	$\Delta(1700) \to \pi\Delta$	12 ± 10		15	18
η *S*	$N(1535) \to \eta N$	64 ± 28		17	(57,61)
wave	$N(1650) \to \eta N$	11 ± 6		14	12

Using the empirical values the angle that results from this relation is $\theta_1 = 1.26 \pm 0.14$ or 0.65 ± 0.14. Although these results seem not to be far from the results derived from Eq. (29), the leading order fit discussed below indicates a poor description of the ratio of the $N(1650) \to \eta N$ to $N(1535) \to \eta N$ widths. This is due to the high sensitivity of this ratio to the mixing angle θ_1. Indeed, it is necessary to include some $1/N_c$ effects as discussed below to arrive at a good description of the η modes together with the other modes, and in this case the resulting angle is 0.39 ± 0.11.

In Table VII the results of several fits are displayed. In these fits the decay amplitudes are expanded keeping only the terms that correspond to the order in $1/N_c$ of the fit. Similarly, when performing the fits the errors have been taken to be equal or larger than the expected accuracy of the fit (30% to the LO fits and about 10% for the NLO ones).

The first LO fit only considers the *S*-wave π modes. As expected, the values for the mixing angle θ_1 resulting from this fit turn out to be equal to the ones obtained through the relation given by Eq. (29). Notice that θ_3 also has a twofold ambiguity at this order. The second leading order fit includes the *D* waves and η modes. The angles remain within errors equal to the ones from the first fit. Table VIII shows that the $N(1535) \to \eta N$ width results to be a factor 4 smaller than the empirical one (this having, however, a rather generous error). In the *D* waves, several widths involving decays with a Δ in the final state are also too small. The *S*-wave π modes are well fitted and there is no real need for NLO improvement. In the *D*-wave decays in the π channel there are two leading order operators that contribute. The fit shows that the 1B operator has a coefficient whose magnitude is a factor 2 to 3 larger than that of the coefficient of the 2B operator. The 1B *D*-wave operator as well as the 1B *S*-wave operator $O_1^{[0,1]}$ stem from the 1B coupling of the pion via the axial current. Such a coupling naturally occurs as the dominant coupling in the chiral quark model [19]. Thus, the result of the *D*-wave fit indicates that such a mechanism is dominant over other mechanisms that give rise to the 2B operator $O_6^{[2,1]}$.

The NLO fit involves a rather large number of effective constants. In addition to the four effective constants and the two mixing angles that appear at LO, there are ten new effective constants, three of them in the *S*-wave pion channels, six in the *D*-wave pion channels, and one in the *S*-wave eta channels. Since there are only 16 data available, some operators must be discarded for the purpose of the fit. It is reasonable to choose to neglect the 3B operators and the subleading *S*-wave operator for η emission. The NLO fit has been carried out by demanding that the LO coefficients are not vastly different from their values obtained in the LO fit. This demand is reasonable if the assumption is made that the $1/N_c$ expansion makes sense. The fact that the NLO coefficients do not have unnaturally large values with respect to the scale set by the LO fit indicates the consistency of the assumption. This clearly is no proof, however, that the $1/N_c$ expansion is working. As mentioned earlier, the chief limitation here is due to the magnitude of the errors in the inputs. This leads to results for the NLO coefficients being affected with rather large uncertainties. Indeed, no clear NLO effects can be pinned down, as most NLO coefficients are no more than 1 standard deviation from zero. Because the number of coefficients is approximately equal to the number of inputs, there are important correlations between them. For instance, the S-wave NLO coefficients are very correlated with each other and with the angle θ_3. For the S waves the LO fit is already excellent, and therefore nothing significantly new is obtained by including the NLO corrections. Correlations are smaller for the D-wave coefficients. As mentioned before, here the LO fit has room for improvement, and thus the NLO results are more significant than in the case of the S waves. Still, no clear pattern concerning the NLO corrections is observed. One interesting point, however, is that without any significant change in the value of θ_1 the η modes are now well described. The reason for this is that in the LO fit the matrix elements of the operator $O_1^{[0,0]}$ were taken to zeroth order in $1/N_c$, while in the NLO fit the $1/N_c$ terms are included. These subleading corrections enhance the amplitude for the ${}^2N^*_{1/2}$ and suppress the amplitude for the ${}^4N^*_{1/2}$. This along with an increment in the coefficient brings the fit in line with the empirical widths. One important point is that at NLO the twofold ambiguity in θ_1 that results at LO is eliminated. The smaller mixing angle turns out to be selected. The angle θ_3 remains ambiguous and close to the values obtained in the LO fits. It should be noticed that the present values of both mixing angles are somewhat different from the values $\theta_1 = 0.61$, $\theta_3 = 3.04$ obtained in other analyses [10,16,20]. Finally, a clear manifestation of the lack of precision in the data is the ratio between the errors affecting the coefficients of NLO versus those of LO operators. In a situation where the data would have precision of the order of NNLO corrections, that ratio should be $\simeq N_c$. In most cases the ratios turn out to be much larger than that, as Table VII shows.

V. CONCLUSIONS

In this work the method of analysis of excited baryon decays in the $1/N_c$ expansion has been presented. It is limited to the situation where configuration mixings are neglected, and therefore the baryon states are taken to belong to a single multiplet of $O(3) \times SU(4)$. The application to the decays of the negative parity baryons illustrates the method. A basis of effective operators, in which the S- and D-wave amplitudes are expanded, was constructed to order $1/N_c$. All dynamical effects are then encoded in the effective coefficients that enter in that expansion.

The application to the decays addressed here shows that a consistent description within the $1/N_c$ expansion is possible. Indeed, up to the relatively poor determination of $1/N_c$ corrections that results from the magnitude of the errors in the input widths, these corrections are of natural size. A few clear cut observations can be made. The most important one is that the S-wave π and η channels are well described by the leading order operators (one for each channel) provided one includes the contributions subleading in $1/N_c$ in the matrix elements for the η decays. The mixing angle θ_1 is then determined by these channels up to a twofold ambiguity, which is lifted when all channels are analyzed at NLO. The angle θ_3 is also determined up to a twofold ambiguity at LO. The ambiguity remains when the NLO is considered. The LO results also indicate the dominance of the 1B effective operators that have the structure that would result from a chiral quark model [19]. This is explicitly seen in the D-wave channels where the LO 2B operator turns out to be suppressed with respect to the 1B one. The subleading operators are shown to be relevant to fine-tune the S-wave decays and improve the D-wave decays. Because of the rather large error bars in and significant correlations between the resulting effective coefficients, no clear conclusions about the physics driving the $1/N_c$ corrections can be made. The mixing angles $\theta_1 = 0.39 \pm 0.11$ and $\theta_3 = (2.82, 2.38) \pm 0.11$ that result at NLO are similar to the ones determined at LO. They are, however, somewhat different from the angles $\theta_1 = 0.61$ and $\theta_3 = 3.04$ obtained in other analyses [10,16,20].

ACKNOWLEDGMENTS

This work was supported by DOE Contract No. DE-AC05-84ER40150 under which SURA operates the Thomas Jefferson National Accelerator Facility, by the National Science Foundation (USA) through Grants No. PHY-9733343 and No. PHY-0300185 (J. L. G.), by the Department of Energy Grant No. DE-AC05-84ER40150 (C. S.), by CONICET (Argentina) Grant No. PIP 02368 and by ANPCyT (Argentina) Grant No. PICT 00-03-08580 (C. S. and N. N. S.), and by the Fundación Antorchas (Argentina) (C. S.).

[1] J. L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984); Phys. Rev. D **30**, 1795 (1984).
[2] R. Dashen and A. V. Manohar, Phys. Lett. B **315**, 425 (1993); **315**, 438 (1993); E. Jenkins, Phys. Lett. B **315**, 441 (1993).
[3] R. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D **49**, 4713 (1994); **51**, 3697 (1995).
[4] C. D. Carone, H. Georgi, and S. Osofsky, Phys. Lett. B **322**, 227 (1994); M. A. Luty and J. March-Russell, Nucl. Phys. **B426**, 71 (1994); M. A. Luty, J. March-Russell, and M. White, Phys. Rev. D **51**, 2332 (1995).
[5] E. Jenkins and R. F. Lebed, Phys. Rev. D **52**, 282 (1995); J. Dai, R. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D **53**, 273 (1996); A. Buchmann and R. F. Lebed, Phys. Rev. D **62**, 096005 (2000); A. Buchmann, J. A. Hester, and R. F. Lebed, Phys. Rev. D **66**, 056002 (2002); E. Jenkins, X. d. Ji, and A. V. Manohar, Phys. Rev. Lett. **89**, 242001 (2002); T. D. Cohen, Phys. Lett. B **554**, 28 (2003).
[6] J. L. Goity, Phys. Lett. B **414**, 140 (1997).
[7] D. Pirjol and T. M. Yan, Phys. Rev. D **57**, 1449 (1998); **57**, 5434 (1998).
[8] C. E. Carlson, C. D. Carone, J. L. Goity, and R. F. Lebed, Phys. Lett. B **438**, 327 (1998); Phys. Rev. D **59**, 114008 (1999).
[9] C. L. Schat, J. L. Goity, and N. N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002); J. L. Goity, C. L. Schat, and N. N. Scoccola, Phys. Rev. D **66**, 114014 (2002); Phys. Lett. B **564**, 83 (2003).
[10] C. D. Carone, H. Georgi, L. Kaplan, and D. Morin, Phys. Rev. D **50**, 5793 (1994).
[11] C. E. Carlson and C. D. Carone, Phys. Lett. B **484**, 260 (2000).
[12] N. Matagne and F. Stancu, Phys. Rev. D **71**, 014010 (2005).
[13] T. D. Cohen and R. F. Lebed, Phys. Rev. Lett. **91**, 012001 (2003); Phys. Rev. D **67**, 096008 (2003); T. D. Cohen, D. C. Dakin, A. Nellore, and R. F. Lebed, Phys. Rev. D **69**, 056001 (2004).
[14] D. Pirjol and C. Schat, Phys. Rev. D **67**, 096009 (2003); AIP Conf. Proc. **698**, 548 (2004).
[15] J. L. Goity, hep-ph/0405304 [Yad. Fiz. (to be published)].
[16] S. Capstick and W. Roberts, Prog. Part. Nucl. Phys. **45**, S241 (2000), and references therein.
[17] Particle Data Group, S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
[18] There is, in addition, the pion G-wave decay $N^*_{5/2} \rightarrow \pi\Delta_{3/2}$ which we are not going to consider here.
[19] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); L. Y. Glozman and D. O. Riska, Phys. Rep. **268**, 263 (1996).
[20] D. Faiman and D. E. Plane, Nucl. Phys. **B50**, 379 (1972); A. J. G. Hey, P. J. Litchfield, and R. J. Cashmore, Nucl. Phys. **B95**, 516 (1975).