Electric currents induced by twisted light in Quantum Rings

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Abstract: We theoretically investigate the generation of electric currents in quantum rings resulting from the optical excitation with twisted light. Our model describes the kinetics of electrons in a two-band model of a semiconductor-based mesoscopic quantum ring coupled to light having orbital angular momentum (twisted light). We find the analytical solution, which exhibits a "circular" photon-drag effect and an induced magnetization, suggesting that this system is the circular analog of that of a bulk semiconductor excited by plane waves. For realistic values of the electric field and material parameters, the computed electric current can be as large as μA; from an applied perspective, this opens new possibilities to the optical control of the magnetization in semiconductors.

OCIS codes: (000.0000) General.

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1. Introduction

Intense research efforts are currently devoted to the development and application of appropriate optical means capable of tracing the quantum dynamics down to the atto-second time scale and with a spatial resolution down to the atomic scale [1–3]. The key element in these studies is the coupling of matter to the electric field of the light source. For the study of the spin-dependent dynamics [4], one relies then on intrinsic mechanisms of the sample that couple the spin to the charge. Another route to access the spin dynamics is to utilize the magnetic field generated by a charge current [5–8] which can be triggered in the sample by light irradiation ([9] and references therein). To achieve temporal and local control of the photo-generated current (and hence the magnetic field) one may exploit the conduction intra-band dynamics initiated by asymmetric electromagnetic pulses in semiconductor-based nanorings that are positioned on a scanning tip [10–13]. Such rings are with the appropriate parameters ready available [14–17]. Photo-induced currents associated with inter-band excitations may be driven by quantum interference of one- and two-photon carrier excitations [18]. In this contribution we point

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out the possibility of triggering directly, i.e. via one-photon processes, current-carrying inter-band excitations by using twisted light (TL), i.e. light carrying orbital angular momentum (OAM). Since the pioneering work of Allen *et al* [20] in 1992 on TL, the application of TL has expanded substantially ([21] and references therein) to cover diverse areas such as the interaction with mesoscopic particles (optical tweezers) [22–24], the interaction with atoms and molecules [25, 26], and the interaction with Bose-Einstein condensates [27]. Research on the coupling between inhomogeneous electromagnetic fields and semiconductor has been however scarcely pursued. On the other hand, the interaction of homogeneous fields with inhomogeneous system (including synthesized structures such as quantum dots) is being extensively investigated due to important potential applications.
Recent research on the interaction between TL and semiconductors were devoted to the absorption of TL by bulk semiconductors [29], and to the study of the electronic transitions in quantum dots [28]. Here we show that TL, i.e. an inhomogeneous electromagnetic field carrying orbital angular momentum (OAM), tuned to inter-band frequencies generates electrical currents in quantum rings (QRing) and hence a light-controlled magnetic field localized around the ring. We propose this method as a new mean for the light-controlled spin dynamics.

The paper is organized as follows: The physical system is introduced in Sect. 2, while Sect. 3 presents the Heisenberg equations of motion applied to the study of the kinetics of electrons in the QRing. The OAM transferred from the light to the electrons is derived in Sect. 4. Section 5 describes our calculations of the electric current together with possible applications. The article ends in Sect. 6 with our conclusions.

2. Model

We consider a phase-coherent semiconductor-based quantum ring illuminated by twisted light. Such rings are readily feasible experimentally [14–17]. The ring we consider is such that its width d and height h are much smaller than its radius r_0. In a cylindrical coordinate system where the z axis is perpendicular to the ring plane and intersects its center, we write the carrier wave function as (r is the radial distance from the z axis and ϕ is the azimuthal angle)

$$\psi_{bm}(\mathbf{r}) = [\Phi_m(\phi)\,R(r)\,Z(z)]\,u_b(\mathbf{r})\,\chi\,. \tag{1}$$

Here m is the angular quantum number (OAM of the envelope function), $u_b(\mathbf{r})$ is the microscopic and periodic wave function for the band b. The spin part and the envelope functions are χ, and $[\Phi_m(\phi)\,R(r)\,Z(z)]$. The system parameters are such that the carrier wave length is longer than d and h and hence only the lowest radial subbands are populated. Explicitly, within the ring the envelope wave functions are

$$\Phi_m(\phi) = \frac{1}{\sqrt{2\pi}}\,e^{im\phi}, \tag{2}$$

$$R(r) = \sqrt{\frac{2}{r_0\,d}}\,\sin\left[\frac{\pi}{d}(r-r_0+d/2)\right], \tag{3}$$

$$Z(z) = \sqrt{\frac{2}{h}}\cos\left[\frac{\pi}{h}(z-z_0)\right], \tag{4}$$

with $R(r)=0$ and $Z(z)=0$ outside the ring. The energy spectrum reads

$$\varepsilon_{vm} = \frac{\hbar^2\,m^2}{2\,m_v^*\,r_0^2}$$

$$\varepsilon_{cm} = E_g+\frac{\hbar^2\,m^2}{2\,m_c^*\,r_0^2}$$

#116406 - $15.00 USD Received 28 Aug 2009; revised 23 Sep 2009; accepted 24 Sep 2009; published 23 Oct 2009

with the effective masses m_b^*, and the band gap energy E_g and with the light frequency tuned to inter-band transitions. Due to the aforementioned conditions, we can restrict to a two-band model, one valence and one conduction bands. *

The ring is subjected to TL pulse propagating along the ring z axis. The pulse duration is much longer than the time scale of the carrier dynamics (as specified below). A twisted light field, carrying OAM $\hbar l$, is characterized by the angular dependence $\exp(i\,l\,\phi)$, and a radial dependence either of the Laguerre-Gaussian or the Bessel mode type. For the purpose of our study, the radial dependence of the field is irrelevant, since we have assumed that $d \ll r_0$. The field inhomogeneity is preserved in the angular dependence.

The main contribution to the TL-QRing interaction originates from the transverse part of the vector potential, as readily seen for the case of a TL Bessel beam [21, 29]

$$\mathbf{A}_{\pm}(\mathbf{r},t) = \mathbf{e}_{\pm}\, F(r) e^{i(q_z z - \omega t)} e^{il\phi} + c.c.. \tag{5}$$

The polarization vector $\mathbf{e}_{\pm} = \hat{x} \pm i\hat{y}$ is in the ring plane. $F(r)$ is the amplitude of the vector potential, q_z is the wave number, ω is the light frequency, and l quantifies the amount of OAM carried by the photons.

3. Equation of motion

The Heisenberg equations of motion describe the evolution of the populations and the inter- and intra- band coherences of the carriers under the action of the TL-electromagnetic field. To setup these equations we start from the total Halmiltonian

$$H = \sum_{bm} \varepsilon_{bm}\, a^{\dagger}_{bm} a_{bm} + \sum_{bm,b'm'} \langle b'm'|\, h_I\, |bm\rangle\, a^{\dagger}_{b'm'} a_{bm}\,. \tag{6}$$

$a^{\dagger}_{bm}\,(a_{bm})$ creates (annihilates) single particles in the orbitals $\psi_{bm}(\mathbf{r}) = \langle \mathbf{r}|\, bm\rangle$. For moderate photon intensities, the light-matter interaction is described by the lowest order in the vector potential $h_I = (-q/m)\mathbf{p}\cdot\mathbf{A}_{\sigma}(\mathbf{r},t)$ (q is the carrier charge).

Expressed in the basis $\{\psi_{bm}\}$ (eq.1), the Heisenberg equations of motion read

$$i\hbar \frac{d}{dt} a^{\dagger}_{b'm'} a_{bm} \quad = \quad [a^{\dagger}_{b'm'} a_{bm}, H]. \tag{7}$$

As usual the quantities of interest are the expectation values $\rho_{vm',cm} = \langle \hat{a}^{\dagger}_{vm'} \hat{a}_{cm}\rangle$, $\rho_{v,m'm} = \langle \hat{a}^{\dagger}_{vm'} \hat{a}_{vm}\rangle$, and $\rho_{c,m'm} = \langle \hat{a}^{\dagger}_{cm} \hat{a}_{cm}\rangle$ where the average $\langle \ldots \rangle$ is taken over the initial state of the semiconductor. These expectation values represent populations when they have repeated indices (diagonal elements of the density matrix) and quantum coherences when they are off-diagonal elements. To calculate the matrix elements of the interaction $\langle b'm'|\, h_I\, |bm\rangle$ [Eq. (6)] in the basis of Eq. (1) we proceed as in Quinteiro *et al* [28] and obtain for the photoabsorption

$$\langle cm'|\, h_I^{(+)}\, |v\, m\rangle \quad = \quad \xi\, \delta_{m-m',-l}\,,$$

and for the photoemission

$$\langle vm'|\, h_I^{(-)}\, |c\, m\rangle \quad = \quad \xi^*\, \delta_{m-m',l}\,,$$

where $\xi = -\frac{q}{m}\,(\mathbf{e}_{\sigma}\cdot\mathbf{p}_{cm'\,vm}) F(r_0)\, e^{iq_z z_0}$, and $\mathbf{p}_{cm'\,vm}$ the transition matrix element of the momentum operator between the indexed states ($\sigma \equiv \pm$). In what follows ξ does not play any

*Two arguments justify this assumption. First, the confinement causes a splitting of the heavy- and the light-hole valence bands. Tuning the frequency of the light beam one or the other valence band can be selected. Second, when no fine frequency tuning is desirable or possible, both the heavy- and light-hole bands are excited and evolve independently.

#116406 - $15.00 USD Received 28 Aug 2009; revised 23 Sep 2009; accepted 24 Sep 2009; published 23 Oct 2009

dynamical role, as it is simply as a complex constant. The spin indices has been already taken into account (they determine the selection rule for the transitions between specific valence and conduction bands). The use of these matrix elements in Eq. (7) yields the final equations of motion

$$i\hbar\frac{d}{dt}\rho_{v,m'm} = (\varepsilon_{vm}-\varepsilon_{vm'})\,\rho_{v,m'm} + \xi^*\,\widetilde{\rho}_{vm',cm+l} - \xi\,\widetilde{\rho}_{cm'+l,vm}, \tag{8}$$

$$i\hbar\frac{d}{dt}\rho_{c,m'm} = (\varepsilon_{cm}-\varepsilon_{cm'})\,\rho_{c,m'm} + \xi\,\widetilde{\rho}_{cm',vm-l} - \xi^*\,\widetilde{\rho}_{vm'-l,cm}, \tag{9}$$

$$i\hbar\frac{d}{dt}\widetilde{\rho}_{vm',cm} = (\varepsilon_{cm}-\varepsilon_{vm'}-\hbar\omega)\,\widetilde{\rho}_{vm',cm} + \xi\,\left(\rho_{v,m'm-l}-\rho_{c,m'+lm}\right), \tag{10}$$

with $\hat{\rho}_{vm',cm} = \widetilde{\rho}_{vm',cm}e^{-i\omega t}$.

3.1. Exact solution

We obtained an exact solution of equations Eqs. (8)-(10). This is done by first inspecting the population of the valence band, i.e. by specializing Eq. (8) to $m = m'$. It is readily seen that this equation couples to the intra-band coherence $\widetilde{\rho}_{vm,cm+l}$. In turn, the differential equation for the latter couples to the conduction $\rho_{c,m+l\,m+l}$ and valence $\rho_{v,m\,m}$ band populations. Thus, a closed system of equations is found:

$$\begin{aligned}
\hbar\frac{d}{dt}\rho_{v,mm} &= 2\Im\left[\xi^*\,\widetilde{\rho}_{vm,cm+l}\right] \\
\hbar\frac{d}{dt}\rho_{c,m+l\,m+l} &= -2\Im\left[\xi^*\,\widetilde{\rho}_{vm,cm+l}\right] \\
i\hbar\frac{d}{dt}\widetilde{\rho}_{vm,cm+l} &= \Delta_{cm+l,vm}\,\widetilde{\rho}_{vm,cm+l} + \xi\,\left(\rho_{v,mm}-\rho_{c,m+l\,m+l}\right),
\end{aligned}$$

with $\Im[\ldots]$ the imaginary part, and $\Delta_{cm+l,vm} = \varepsilon_{cm+l} - \varepsilon_{vm} - \hbar\omega$. Due to their resemblance, we rename the functions to the terms used in conventional Optical Bloch Equations [30], i.e. the population difference $W = \rho_{c,m+l\,m+l} - \rho_{v,m\,m}$, and coherences † $U = \widetilde{\rho}_{vm,cm+l} + \widetilde{\rho}^*_{vm,cm+l}$, and $V = i\,\widetilde{\rho}_{vm,cm+l} - i\,\widetilde{\rho}^*_{vm,cm+l}$. Then,

$$\begin{aligned}
\dot{W} &= \frac{2}{\hbar}\left[\Re(\xi)V - i\Im(\xi)U\right] \\
\dot{U} &= -\frac{\Delta}{\hbar}V - \frac{2}{\hbar}\Im(\xi)W \\
\dot{V} &= \frac{\Delta}{\hbar}U - \frac{2}{\hbar}\Re(\xi)W,
\end{aligned}$$

with $\Re[\ldots]$ the real part. These are the Bloch equations in the absence of decay for the Rabi frequencies $\mathscr{R}^2 = \mathscr{R}_0^2 + (\Delta_{cm+l,vm}/\hbar)^2$, $\mathscr{R}_0 = -\frac{2}{\hbar}\Re(\xi)$, and $\Im(\xi) = 0$ (for brevity we replace $\Delta_{cm+l,vm}$ by Δ, unless otherwise stated). For the initial conditions of an unexcited QRing $\{\rho_{vc}(0) = 0; \rho_v(0) = 1; \rho_c(0) = 0\}$ the solution reads

$$\begin{aligned}
W(t) &= \left(\frac{\mathscr{R}_0}{\mathscr{R}}\right)^2\left[1-\cos(\mathscr{R}t)\right] - 1 \\
U(t) &= \frac{\mathscr{R}_0\,\Delta}{\hbar\mathscr{R}^2}\left[1-\cos(\mathscr{R}t)\right] \\
V(t) &= -\frac{\mathscr{R}_0}{\mathscr{R}}\sin(\mathscr{R}t).
\end{aligned}$$

† In the standard Optical Bloch Eqs. problem U and V represent the dipole moment.

#116406 - $15.00 USD Received 28 Aug 2009; revised 23 Sep 2009; accepted 24 Sep 2009; published 23 Oct 2009

Transforming back we find

$$\begin{aligned}
\rho_{v,m\,m}(t) &= 1+\frac{1}{2}\left(\frac{\mathscr{R}_0}{\mathscr{R}}\right)^2[\cos(\mathscr{R}t)-1] \\
\rho_{c,m+l\,m+l}(t) &= -\frac{1}{2}\left(\frac{\mathscr{R}_0}{\mathscr{R}}\right)^2[\cos(\mathscr{R}t)-1] \\
\rho_{vm,cm+l}(t) &= \frac{\mathscr{R}_0}{2\mathscr{R}}\,e^{-i\omega t}\left\{\frac{\Delta}{\hbar\mathscr{R}}[1-\cos(\mathscr{R}t)]+i\sin(\mathscr{R}t)\right\}. \qquad (11)
\end{aligned}$$

For a better understanding of our results, we briefly digress on two particular issues of the interaction of plane-waves with bulk semiconductors. The photon-drag effect [32, 33] accounts for the transfer of linear momentum of the photons to the electrons; it is explained in terms of transitions between states differing in their linear quasi-momentum by the small amount q_z carried by the photon. It has been studied both theoretically as well as demonstrated in experiments [34], and it is shown to induce an electric voltage between the sides of the material (which may be viewed as a macroscopic polarization) or equivalently an electric current; this effect has technology applications, such as light detection [35]. On the other hand, when q_z is neglected (the usual approximation of "vertical" transitions) the global polarization of the semiconductor is directly related to the induced inter-band coherence [31]. This polarization has a microscopic origin, since it mathematically arises from the integral $\int_{a^3} u^*_{b'}(\mathbf{r})\,e\mathbf{r}\,u_b(\mathbf{r})d\mathbf{r}$ over the semiconductor's unit cell of the linear size a. Both sources of the polarization are distinct, and in principle may be simultaneously present.

Equations (11) show that the action of the TL field is to induce "tilted" transition between the valence and conduction bands, connecting states differing in their OAM $\hbar m$ by the amount $\hbar l$ given/taken by the TL field. Here, we appreciate the connection to the photon-drag effect of a bulk semiconductor. The inter-band coherence will be shown later to produce a magnetization; therefore, we think this is the equivalent to the inter-band polarization of the linear system. Summarizing, we assert that our problem is the circular analog of the well-known problem of bulk interacting with plane-waves. We further support our view by noting that the QRing is a rotationally invariant system, and the TL transfers to it OAM; this is in contrast to plane waves transferring linear momentum to a translationally invariant system. As usual [31], our inter-band coherence has a frequency component at light frequency ω, whereas the populations oscillate with the Rabi frequency $\mathscr{R} \ll \omega$ (typically $\mathscr{R}/\omega < 0.1$).

It is of interest to analyze two particular cases of the general solution Eqs. (11). The first we address is the "low excitation" regime (or perturbation theory solution) which, from a physical point of view, is realized when the semiconductor is illuminated by a weak or out of resonance electromagnetic field. In mathematical terms we seek a solution in the limit $|\xi/\Delta| \ll 1$ in which case $\mathscr{R} = \Delta/\hbar$ and the equations simplify to

$$\begin{aligned}
\rho_{v,m\,m}(t) &\simeq 1+2\left(\frac{\xi}{\Delta}\right)^2\left[\cos\left(\frac{\Delta}{\hbar}t\right)-1\right] \\
\rho_{c,m+l\,m+l}(t) &\simeq -2\left(\frac{\xi}{\Delta}\right)^2\left[\cos\left(\frac{\Delta}{\hbar}t\right)-1\right] \\
\rho_{vm,cm+l}(t) &\simeq \left(\frac{\xi}{\Delta}\right)\left(1-e^{-i\Delta t/\hbar}\right)e^{-i\omega t}, \qquad (12)
\end{aligned}$$

valid for all times. We note that the lowest order is the inter-band coherence. Our second case is that of short times and any value of $|\xi/\Delta|$; hereafter this regime is defined as that for which t satisfies the condition $2\pi/\omega < t < \hbar/\Delta$ (typically 1 fs$< t <$ 1 ps). By a Taylor expansion around

#116406 - $15.00 USD Received 28 Aug 2009; revised 23 Sep 2009; accepted 24 Sep 2009; published 23 Oct 2009

$t = 0$ we find

$$\begin{aligned} \rho_{v,mm}(t) &\simeq 1 - \frac{1}{4}\mathscr{R}_0^2 t^2 \\ \rho_{c,m+l\,m+l}(t) &\simeq \frac{1}{4}\mathscr{R}_0^2 t^2 \\ \rho_{vm,cm+l}(t) &\simeq \frac{i}{2}\mathscr{R}_0\, t\, e^{-i\omega t}\,. \end{aligned}$$

We note that the inter-band coherence evolves in time linearly while the populations does it quadratically. The magnitude of the coherence and populations depend on the Rabi frequency at zero detuning $\mathscr{R}_0$. Both approximate solutions are in full agreement with the Optical Bloch solutions for the atomic case. In addition, we have seen that the inter-band coherence is the largest effect for either short times or weak excitation. This may be exploited to simplify the study of additional effects, such as the electron-electron interaction; to this end one would need to solve only for the differential equation of the inter-band coherence (including the additional terms to account for the new effects), assuming constant populations.

4. Orbital angular momentum

The OAM of electrons is of special interest as it is related to the photo-induced electric current and the magnetization. We have derived a second quantization formula for the operator $L_z = -i\hbar\partial_\phi$, and we refer the interested reader to App. A for details.

The z-component of the OAM of electrons can be split into two contributions, namely the "coherence-" and the "population-" based OAM

$$\begin{aligned} L_z^{(coh)} &= \sum_{mm'} 2\Re\left[\mathscr{L}^{(p)}_{vm'cm}\,\rho_{vm',cm}\right] \\ L_z^{(pop)} &= \sum_m \hbar m\,\rho_{c,mm} + \sum_m (l_z + \hbar m)\,\rho_{v,mm}\,, \end{aligned}$$

with $\mathscr{L}^{(p)}_{vm'cm} \neq 0$ only for $m = m' \pm 1$. In what follows we will not study the l_z term. It is a consequence of the difference in "band" OAM between valence and conduction bands, and even though interesting, it is independent of the OAM of the electromagnetic field, being also present when a plane-wave light is used. The terms $L_z^{(coh)}$ and $L_z^{(pop)}$ may be interpreted as microscopic and macroscopic contributions to the OAM, respectively.

Figure 1 shows the temporal evolution of $L_z^{(coh)}$ and $L_z^{(pop)}$ for different values of the OAM of the TL pulsed beam, and same number M of excited electrons (or excited states in the Brillouin Zone). We assign a value to M based on the physical argument that a light pulse having a spectral width $\Delta\omega$ will resonantly excite M levels, according to the relation $\hbar\Delta\omega = \hbar^2 M^2/(2m_c^* r_0^2)$. We have verified that $L_z = 0$ when $l = 0$. We stress that, due to $\mathscr{L}^{(p)}_{vm'cm}$, the coherence-based contribution only exists for $l = \pm 1$. Both contributions to the OAM exhibit distinctive temporal behaviors, that should easily be resolved in an experiment (e.g. Faraday rotation).

Clear expressions for both OAM contributions are obtained for short times in the regime of weak excitation. We start with the coherence-based OAM

$$L_z^{(coh)} \simeq \pm\frac{1}{2}\,\delta_{l,\pm 1}\, r_0 (2M+1)\mathscr{R}_0\ \left[p_{x,vc}\cos(\omega t) \pm p_{y,vc}\sin(\omega t)\right] t\,, \tag{13}$$

where $p_{\pm,vc} = p_{x,vc} \pm i\, p_{y,vc}$. This can be compared to the contribution $L_z^{(pop)}$

$$L_z^{(pop)} \simeq \frac{1}{4}\hbar l\,(2M+1)\,\mathscr{R}_0{}^2\, t^2\,. \tag{14}$$

#116406 - $15.00 USD Received 28 Aug 2009; revised 23 Sep 2009; accepted 24 Sep 2009; published 23 Oct 2009

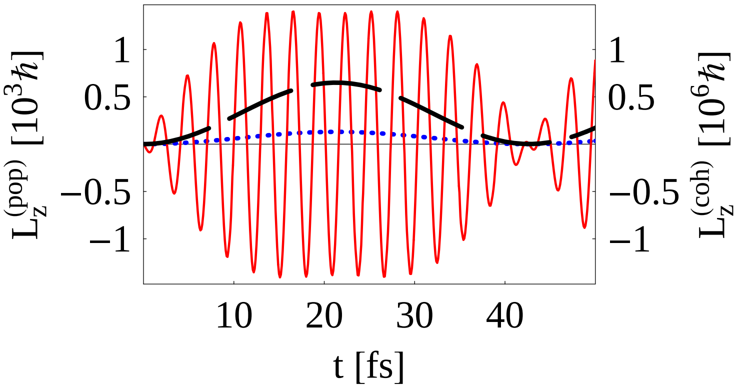

Fig. 1. (color online) Orbital angular momentum: population-based contribution for $l = 1$ in dotted (blue) and for $l = 5$ in dashed (black) lines, and coherence-based contribution for $l = 1$ in full (red) line. The ordinate is given in values of $\hbar$, the OAM of a single electron orbiting the QRing. The parameters for the ring are: GaAs semiconductor QRing, number of electrons excited $M = 100$, $r_0 = 1\mu$m; for the TL-beam pico-seconds pulsed beam: wavelength $\lambda = 300$nm, $\omega = 2\,10^{15}\text{s}^{-1}$, average electric field per pulse $5\,10^{7}$V/m, detuning 0.06eV. The Rabi frequency is $\mathscr{R} \simeq 10^{14}\text{s}^{-1}$.

To this level of approximation, the non-verticality of the induced transition (or asymmetry of the population distribution in the conduction band) is responsible for the net transfer of OAM, by the factor $2M + 1$; this shows clearly the “circular” photon-drag effect. For more precise approximate solutions (as well as for the exact solution) the Rabi frequency $\mathscr{R}$ shows up, and new phenomena arise. For instance, at long times interference and/or beating of the OAM signal is observed, due to the superposition of many different frequency contributions from each value of m.

As anticipated, the OAM may be used to derive other quantities; a simple analysis – based on classical electrodynamics – yields the electric current $I = qL_z/(2\pi r_0^2 m_i)$ and magnetization $\mathscr{M} = \pi r_0^2 I$ or $\mathscr{M} = qL_z/(2m_i)$ with m_i the mass. More will be said about the electric current in Sect. 5.

The use of the picture of a current loop to relate the current with the magnetization is well justified here because the rings are quite large. Equations (12) indicates that the inter-band coherence is the source for the linear response of the system; then, within linear response the magnetization is fully determined by the inter-band coherence. We here have found a significant difference with the model of a plane wave coupled to a bulk semiconductor, where the quantity that relates to the inter-band coherence is actually the polarization of the system. On the other hand, a linear susceptibility may be defined to link magnetization and vector potential. Even though the vector potential gives rise to both electric and magnetic fields, the semiconductor response to optical fields is mainly due to the electric component. Thus, our analysis predicts a cross term linking the magnetization to the electric field, which is still in line with symmetry consideration as the light electric field carries OAM.

5. Electric current

Viewing the excited ring as a classical current loop we write $I = qL_z/(2\pi r_0^2 m_i)$, where the mass m_i is the effective valence or conduction band mass for the case of $J_z^{(pop)}$, or the electron

#116406 - $15.00 USD Received 28 Aug 2009; revised 23 Sep 2009; accepted 24 Sep 2009; published 23 Oct 2009

mass for the case of $J_z^{(coh)}$, and L_z is either $L_z^{(coh)}$ or $L_z^{(pop)}$; a plot is displayed in Figure 2. As clearly seen, the fast oscillation of $J_z^{(coh)}$ is at the frequency of the light. We remind that the coherence-based contribution only exists for $l = \pm 1$.

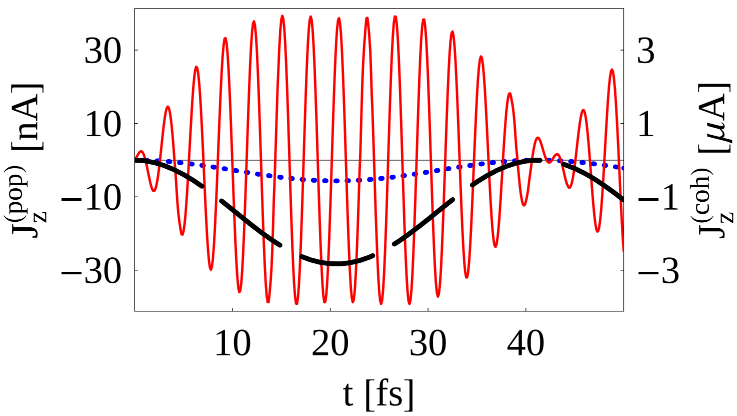

Fig. 2. (color online) Electric current: population-based contribution for $l = 1$ in dotted (blue) and for $l = 5$ in dashed (black) lines, and coherence-based contribution for $l = 1$ in full (red) line. The parameters for the ring are: GaAs semiconductor QRing, number of electrons excited $M = 100$, $r_0 = 1\mu$m; for the TL-beam pico-seconds pulsed beam: wavelength $\lambda = 300$nm, $\omega = 2\,10^{15}\mathrm{s}^{-1}$, average electric field per pulse $5\,10^7$V/m, detuning 0.06eV. The Rabi frequency is $\mathscr{R} \simeq 10^{14}\mathrm{s}^{-1}$.

The distinct origin of $J_z^{(coh)}$ and $J_z^{(pop)}$ suggests possible ways to utilize the electric current. A simple model of a two-level system coupled to light tells that, after the light has been turned off, the populations become frozen, while the coherence develops oscillations at the frequency of the level splitting. After long times – compared to the period of coherence oscillations – the population will decay due to processes not accounted for here. In addition, when a multitude of non-interacting two-level systems, having slightly different splitting (e.g. the QRing bands), is analyzed a new element arises: any quantity containing the sum of coherences (e.g. the OAM) exhibits destructive interference for times comparable to the inverse of the frequency spread of the whole set of excited levels. In view of these considerations, a steady electric current can be established in the system.

The easiest way to set a constant electric current is to use a TL field with $|l| > 1$ that causes $J_z^{(coh)} = 0$. By choosing the duration of the TL pulse, one can freeze the electric current after the pulse is turned off to the desired value. For instance, a TL source generating 30fs top-hat pulses with an average electric field per pulse of $3\,10^7$V/m and $l = 5$ exciting a $r_0 = 1\mu$m GaAs QRing (detuning 0.06eV at band edge, and estimated $M = 470$) yields a steady electric current of $J_z^{(pop)}(t) \simeq 0.2\mu$A for $t > 30$fs.

For a TL field having $l = \pm 1$ we envisage two cases, namely that of short or long pulses, where a steady electric current may be obtained. For a short pulse of TL, we expect a large amount of states to be excited, leading to a rapid destructive interference in the coherence-based OAM, leaving only the frozen population-based contribution. Let us consider a specific situation: a GaAs QRing of $r_0 = 1\mu$m is excited by a TL source generating 30fs top-hat pulses with an average electric field per pulse of $3\,10^7$V/m and $l = 1$, with an estimated $M = 470$ and detuning 0.06eV. After the TL pulse is turned off, the free evolution of the coherence-

#116406 - $15.00 USD Received 28 Aug 2009; revised 23 Sep 2009; accepted 24 Sep 2009; published 23 Oct 2009

based current will exhibit destructive interfere, roughly after 30fs; as a result, the effective value of the current will be that of the population-based one $J_z^{(pop)}(t) \simeq 40$nA for $t > 30$fs. For long pulses, the amount of excited levels is smaller, and the destructive interference of the coherence-based current may occur at times comparable to the decay time of the population. However, when the purpose is the control of other degrees of freedom, having slow dynamics, it becomes admissible to take a time-average of the electric current($< \ldots >= (1/T)\int_t^{t+T} \ldots$ in a period $T = 2\pi/\omega$); then, the average current will again be that of the frozen population-based contribution.

In this work we did not address the relaxation and decoherence pathways of the excited system. In a previous study [11, 36] we investigated these issues for the intra-band dynamics triggered by asymmetric pulses with the following results: In the low excitation regime, which also applies to the present case, relaxation of the excited intraband population is caused by scattering from longitudinal acoustic phonons; optical phonons play no role but are expected to be important for interband relaxation processes. The intraband decay time scale can be as large as nanoseconds [11,36]; effects due to radiative damping and emission of coherent phonons act on a longer or a comparable time scale [36]. As we are interested here in the sub-picosecond time scale (cf. Fig.2, Fig.1) we expect that relaxation and decoherence, as treated in Refs. [11, 36], will not qualitatively modify our conclusions.

6. Conclusions

The generation of electric currents in semiconductor-based quantum rings induced by twisted light has been investigated theoretically, using the formalism of Heisenberg equations of motion of the population and coherence of electrons.

A full analytical solution for the kinetics of electrons without dissipation has been found, describing the transition from states in the valence band to states in the conduction band differing in their orbital angular momentum by the amount $\hbar l$ conveyed by the twisted light field. We have compared these findings with the well-known phenomena exhibit by the interaction of plane-waves with bulk semiconductors, namely the photon-drag effect and the polarization of the system. We conclude that our problem is the circular analog of the problem of a plane wave interacting with a bulk semiconductor.

As a result of the transfer of orbital angular momentum from the light to the electrons, a net electric current appears in the system. Our model indicates that, for realistic values of the parameters, this current could be as large as μA. Moreover, if the field is turned off when there is a finite current, this will persist for times comparable to the typical decay time. The photo-induced magnetic field is exploitable for the study and for the temporal and local control of magnetization, e.g. of magnetic nanoparticles positioned within the ring.

A. Orbital angular momentum

We seek to derive a second quantization formula for the z component of the OAM $\hat{L}_z = \sum_{mm'b'b} \langle b'm'|(-i\hbar\partial_\phi)|bm\rangle a^\dagger_{b'm'} a_{bm}$ with respect to the center of the laser profile. The matrix element is given by

$$\begin{aligned}\langle b'm'|(-i\hbar\partial_\phi)|bm\rangle &= \int_{L^3} d^3r\,|R(r)|^2\,|Z(z)|^2 \times \\ \left\{\Phi_{m'}(\phi)^* u^*_{b'}(\mathbf{r})\Phi_m(\phi)[-i\hbar\partial_\phi u_b(\mathbf{r})] \right.&+ \left.\Phi_{m'}(\phi)^* u^*_{b'}(\mathbf{r})[\hbar m\Phi_m(\phi)]u_b(\mathbf{r})\right\}.\end{aligned}$$

Our next step is to resolve the integration over the whole system into a sum over all unit cells and an integral inside a unit cell; we notice that the required change of variables leads to $l_z \mapsto l_z + \mathbf{r}\times\mathbf{p}|_z$. In addition, we argue that there is no need to consider large values of $\{m, m'\}$, since

#116406 - $15.00 USD Received 28 Aug 2009; revised 23 Sep 2009; accepted 24 Sep 2009; published 23 Oct 2009

the equations of motion will only be studied close to the band edge. This procedure yields three terms $\langle b'm'|\hat{L}_z|bm\rangle = l_{z,b'b}\,\delta_{m'm} + \delta_{b'b}\,\delta_{m'm}\,\hbar m + \mathscr{L}^{(p)}_{b'm'bm}$, where

$$\begin{aligned} l_{z,b'b} &= \frac{1}{a^3}\int_{a^3} d^3r u^*_{b'}(\mathbf{r})\,(-i\hbar\partial_\phi)\,u_b(\mathbf{r}) \\ \mathscr{L}^{(p)}_{b'm'bm} &= \int dr r|R(r)|^2 \int d\phi\,\Phi_{m'}(\phi)^*\Phi_m(\phi)\,(\mathbf{r}\times\mathbf{p}_{b'b}|_z)\,. \end{aligned}$$

The $u_b(\mathbf{r})$ has a given symmetry according to what band represents; we choose the quantization axis in z. For the case of the heavy hole band, this is an eigenfunction of $\hat{l}_z$ with eigenvalues $m_z = \pm 1$. For the case of the conduction band $m_z = 0$. Then, $l_{z,b'b} = \langle m b'|\,l_z\,|m b\rangle = l_z\langle m b'|m b\rangle = l_z\delta_{b'b}$ which does not depend on m, and for each combination of bands it is: $l_{z,cv} = l_{z,cc} = 0$ and $l_{z,vv} = l_z \neq 0$. After some algebra $\mathscr{L}^{(p)}_{b'm'bm}$ is simplified to

$$\mathscr{L}^{(p)}_{b'm'bm} = \frac{r_0}{2}\left\{(p_{y,b'b} - i\,p_{x,b'b})\,\delta_{m-m',1} + (p_{y,b'b} + i\,p_{x,b'b})\,\delta_{m-m',-1}\right\},$$

where we used that $\int dr r^2|R(r)|^2 \simeq r_0$.

The final result is

$$\begin{aligned} \hat{L}_z &= \sum_{mm'}\left(\mathscr{L}^{(p)}_{cm'vm}\,\rho_{cm',vm} + \mathscr{L}^{(p)}_{vm'cm}\,\rho_{vm',cm}\right) + \\ &\quad \sum_m \hbar m\,\rho_{c,mm} + \sum_m [l_z + \hbar m]\,\rho_{v,mm}\,. \end{aligned}$$

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