

# Multifractal behavior of commodity markets: fuel vs. non-fuel products

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## Abstract

We investigate multifractal properties of commodity time series using multifractal detrended fluctuation analysis (MF-DFA). We find that agricultural and energy-related commodities exhibit very similar behavior, while the multifractal behavior of daily and monthly commodity series is rather different. Daily series demonstrate overall uncorrelated behavior, lower degree of multifractality and the dominance of small fluctuations. On the other hand monthly commodity series show overall persistent behavior, higher degree of multifractality and the dominance of large fluctuations. After shuffling the series, we find that the multifractality is due to broad probability density function for daily commodities series, while for monthly commodities series multifractality is caused by both probability density function and long term correlations.

**Keywords:** Commodities; Agricultural markets; Energy markets; Multifractal detrended fluctuation analysis

## 1 Introduction

A remarkable aspect of the world economy at the beginning of the XXI century and up to 2013/2014 was the steep increase of the price of commodities, that is, primary and essentially fungible goods like agricultural products, minerals and oil [1]. The main explanation for this is the increasing demand for such products by China and other fast growing Asian economies [2]. Also the incentives for the production of biofuels contributed to this trend [3]. Furthermore, the rapid gains of trading in commodities lead to the creation of financial instruments based on them, more sophisticated than the traditional futures contracts [4]. The trade in these secondary markets may have added impulse for the increment in the prices of commodities [5].

An obvious consequence of this appreciation of commodities was the induction of rapid growth processes of other emerging economies in Latin America and Africa, specialized in the production of such goods [6]. Another consequence involves the increase of prices for the low-income segments of the population around the world, whose consumption baskets are constituted by food and fuel [7]. This, in turn, is deemed as a major cause of political upheavals like the so-called “Arab Spring” [8], but also a contributing cause of the sub-prime crisis of 2007 which later triggered the larger crisis that affected the developed economies [9].

In any case, it is remarkable that, as shown in Figure 1, the composite price indexes of the main commodity types (agricultural and energy goods) aligned around 2003/2004. This is rather surprising, given the different market structures of those types of goods. It is well known that agricultural markets are close to be competitive (i.e. where no producer can fix prices) while energy markets, are highly concentrated [10]. In particular, the oil market has only a few big participants that are able to change the prices [11].<sup>1</sup>

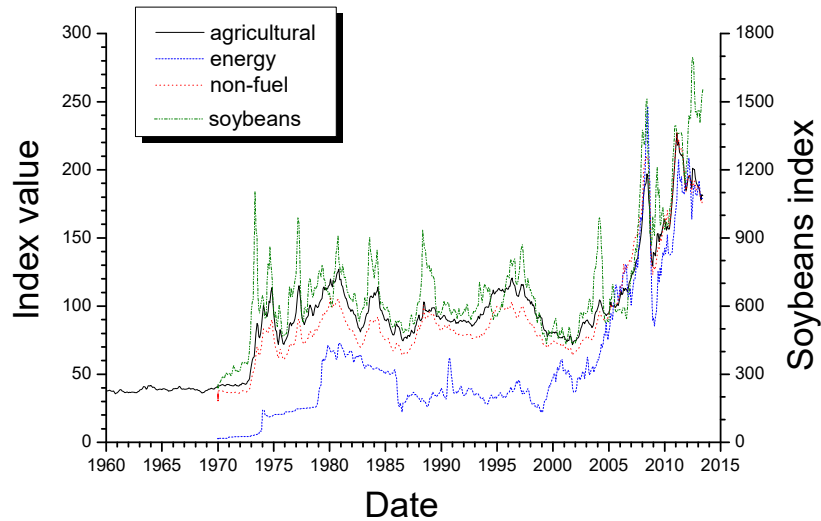


Figure 1: Evolution of commodity prices

In this paper we will show that the evidence exhibited in Figure 1 is backed by further analyses of the data. We examine the time series of the price indexes to look for their multifractal pattern. Multifractal properties of financial markets were extensively studied for different market variables such as market indices [13], stock prices [14,15] and foreign exchange rate [16,17]. Recently more attention was shifted toward commodities, as the economies of developing countries are often strongly affected by fluctuations of commodities prices. For countries that export commodities the increase (decrease) in price has positive (negative) effect on the balance of payments, for countries that import commodities the increase in price can result in an increase of overall inflation. Recent studies showed that prices of different types of commodities (agricultural, metals, nonmetals, crude oil) also display multifractal correlations [18,19] and cross correlations between commodities [20], as well as between commodities and other financial indices [21,22]. In order to contribute to a more complete understanding of the scaling behavior of commodity markets, we investigate multifractal properties of daily and monthly series of agricultural and energy related commodities. The changes of prices of these commodities over the last years have had a significant impact on global economy. By comparing the properties of the multifractal spectrum for different commodities at different temporal scales we discuss the characteristics of commodities price fluctuations and how they are related to the market efficiency.

<sup>1</sup> Consider the jump in prices induced by OPEC in 1973, or the more subtle manipulation of Saudi Arabia in 2014 that led to much lower prices than in the previous decade [12].

## 2 Methodology

While scaling behavior of monofractal temporal series can be described by a single scaling exponent, for multifractal processes a hierarchy of scaling exponents is required for a full description of different scaling behavior of subsets with small and large fluctuations [23]. Several methods have been proposed for non stationary time series [24,25,26]. In this work we use the Multifractal Detrended Fluctuation Analysis (MF-DFA) method [25], which has been successfully applied in analysis of physiological signals [27, 28], geophysical data [29, 30], weather data [31, 32], hydrological records [33, 34], forest fires records [35], traffic time series [36], and financial time series [17, 19, 37, 38]. The MF-DFA method is implemented through the following sequence of steps [25]:

- (i) First, the original temporal series  $x(i), i = 1, \dots, N$  is integrated to produce  $X(k) = \sum_{i=1}^k [x(i) - \langle x \rangle]$ , where  $\langle x \rangle$  is the mean value of  $x(i)$  and  $k = 1, \dots, N$ .
- (ii) Next, the integrated series  $X(k)$  is divided into  $N_n = \text{int}(N/n)$  non-overlapping segments of length  $n$  and in each segment  $v = 1, \dots, N_n$  the local trend  $X_{n,v}(k)$  (calculated from a  $m$ -th order polynomial regression) is subtracted from  $X(k)$ .
- (iii) The detrended variance is calculated for each segment (by subtracting the local trend) and averaged over all segments to obtain the  $q$ -th order fluctuation function:

$$F_q(n) = \left\{ \frac{1}{N_n} \sum_{v=1}^{N_n} \left[ \frac{1}{n} \sum_{k=(v-1)n+1}^{vn} [X(k) - X_{n,v}(k)]^2 \right]^{\frac{q}{2}} \right\}^{\frac{1}{q}}$$

where  $q$  can take any real value except zero.

- (iv) This calculation is repeated for a large number of choices of  $n$  (typically,  $4 < n < N/4$ ) and different values of  $q$  (typically  $q = -10, -9, \dots, 10$ ), and linear regression of  $\log F_q(n)$  versus  $\log(n)$  is performed for every chosen  $q$  value.

If long-term correlations are present,  $F_q(n)$  should increase with  $n$  as a power law  $F_q(n) \sim n^{h(q)}$ , where the scaling exponent  $h(q)$  (also called generalized Hurst exponent) is extracted as the slope of the linear regression of  $\log F_q(n)$  versus  $\log(n)$ . For monofractal series  $h(q)$  is constant, in which case value of  $h(q) = 0.5$  indicates absence of correlations (white noise, or uncorrelated random walk behavior, while  $h(q) > 0.5$  indicates persistent long-term correlations meaning that large (small) values are more likely to be followed by large (small) values, and  $h(q) < 0.5$  indicates anti-persistent correlations, meaning that large values are more likely to be followed by small values, and vice versa. For multifractal series  $h(q)$  is a decreasing function of  $q$ , which in itself represents a sort of "magnifying glass": negative  $q$  values enhance small fluctuations, while large positive  $q$  values enhance large fluctuations.

The generalized Hurst exponent is a constant for monofractal processes while for multifractal time series  $h(q)$  is a decreasing function of  $q$ . For positive values of  $q$ , the exponent  $h(q)$  describes the scaling of large fluctuations. For negative values of  $q$ ,  $h(q)$  describes the scaling of small fluctuations [25]. Multifractal series are also described by the singularity spectrum  $f(\alpha)$  through the Legendre transform

$$\alpha(q) = \frac{d\tau(q)}{dq} ,$$

$$f(\alpha(q)) = q\alpha(q) - \tau(q) ,$$

where  $\tau(q) = qh(q) - 1$  ,  $f(\alpha)$  is the fractal dimension of the series subset characterized by the singularity strength (or Holder exponent)  $\alpha$ . For monofractal series, the singularity spectrum is represented by a single point in the  $f(\alpha)$  plane, whereas multifractal processes exhibit a humped function [25].

Multifractality in a time series may be caused by: (i) different long-term correlations for small and large fluctuations and (ii) a broad probability density function for the values of the time series. The type of multifractality can be determined by analyzing the corresponding randomly shuffled series. The shuffled series from multifractals of type (i) exhibit simple random behavior with  $h(q) = 0.5$  and  $f(\alpha)$  being reduced to a single point, while for multifractals of type (ii) the original  $h(q)$  dependence (and the width of multifractal spectrum) is not changed. If both kinds of multifractality are present, the shuffled series demonstrates weaker multifractality (smaller width of  $f(\alpha)$  spectrum) than the original one [25]. In order to measure the complexity of the series, we fit the singularity spectra to a fourth degree polynomial [17,35]

$$f(\alpha) = A + B(\alpha - \alpha_0) + C(\alpha - \alpha_0)^2 + D(\alpha - \alpha_0)^3 + E(\alpha - \alpha_0)^4$$

and calculate the multifractal spectrum parameters: the position of the maximum  $\alpha_0$  (representing the overall Hurst exponent), the width of the spectrum,  $W = \alpha_{max} - \alpha_{min}$  obtained from equating the fitted curve to zero, and the skew parameter  $r = (\alpha_{max} - \alpha_0) / (\alpha_0 - \alpha_{min})$  , where  $r = 1$  for symmetric shapes,  $r > 1$  for right-skewed shapes, and  $r < 1$  for left-skewed shapes. The skew parameter  $r$  determines which fractal exponents are dominant: low fractal exponents that describe the scaling of subsets with large fluctuations (left-skewed spectrum), or high fractal exponents that describe the scaling of subsets with small fluctuations (right-skewed spectrum). These parameters can be used to describe the complexity of the series: a signal with a high value of  $\alpha_0$ , a wide range of fractal exponents  $W$  , and a right-skewed shape ( $r > 1$ ) may be considered more complex than one with opposite characteristics [39].

### 3 Data and analysis

The main sources for our analysis are the corresponding price indexes of commodities drawn from the World Bank ([data.worldbank.org](http://data.worldbank.org)): WB agricultural, WB soy, WB non-fuel, WB energy and from Dow Jones (<http://us.spindices.com/index-family/commodities/dj-commodities>): DJ energy, DJ Brent crude, DJ petroleum, the last ones obtained from DATASTREAM. The series taken from the WB have a monthly frequency from 1960 until 2013 for WB agricultural, WB non-fuel and WB energy, and from 1968 until 2013 for WB soy. The DJ series include daily data from January 1999 to January 2015 for DJ Brent crude and from January 2006 to January 2015 for DJ energy and DJ petroleum.

For each series we calculate the logarithmic returns  $R(t) = \ln P(t + \Delta t) - \ln P(t)$ , where  $P(t)$  is the closing price at time  $t$ ,  $\Delta t = 1$  (1 month for the WB series, 1 day for the DJ series). The original and return time series are shown in Fig. 2, and Fig. 3.

We apply the MF-DFA method on the logarithmic returns of commodities time series, and also perform a fourth order polynomial regression on the singularity spectra  $f(\alpha)$  to determine the position of maximum  $\alpha_0$  and the zeros of the polynomial  $\alpha_{max}$  and  $\alpha_{min}$  which are used to calculate the width of spectrum  $W$  and the asymmetry parameter  $r$ . The multifractal spectra of all commodities are shown in Fig.4.

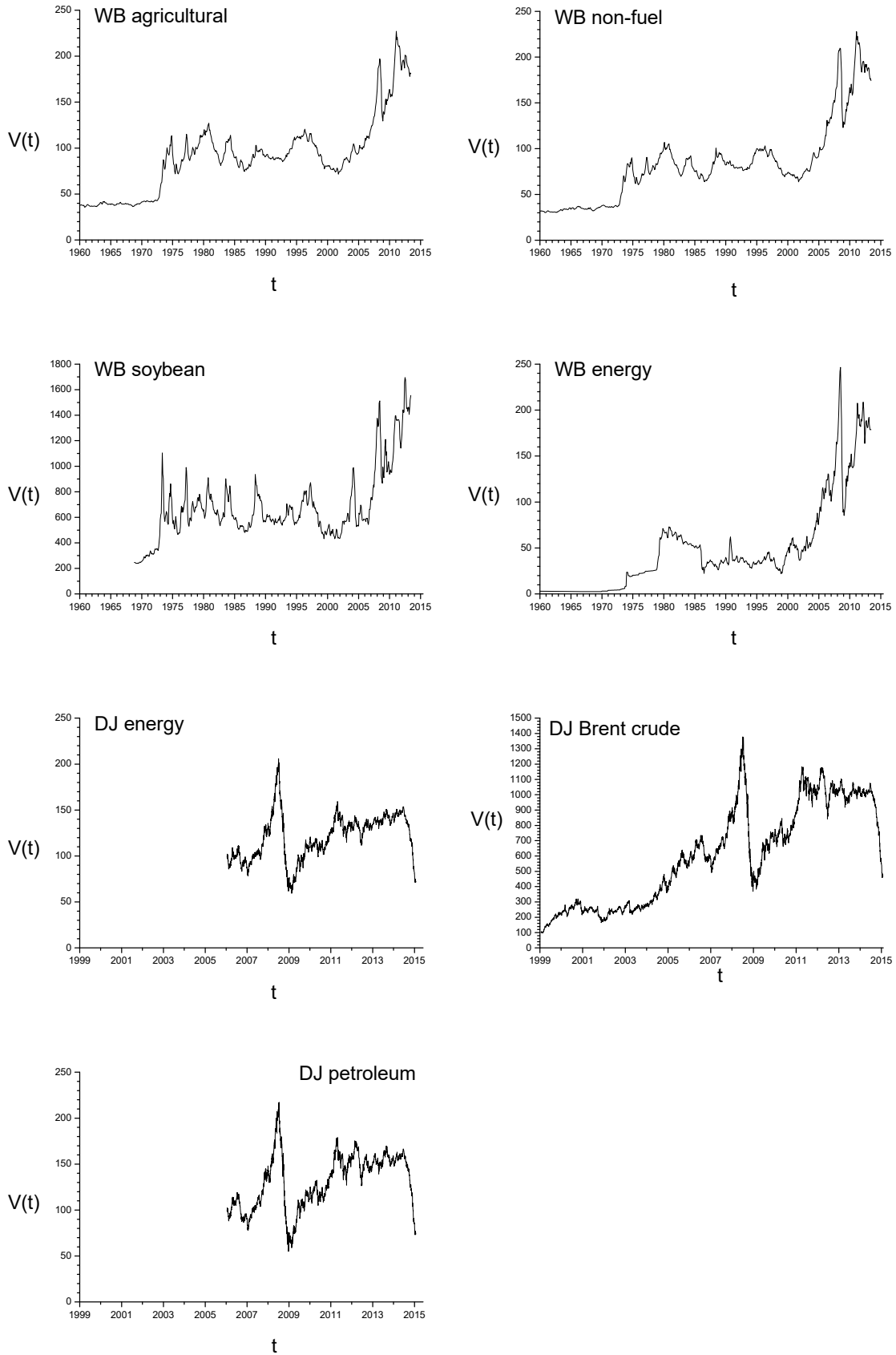


Fig. 2: Time series of commodity price  $P$ . Time is expressed in months (for the WB series) and in days (for the DJ series), starting from the beginning of the recording period.

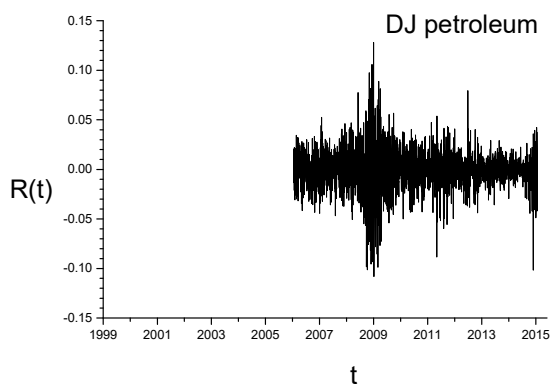
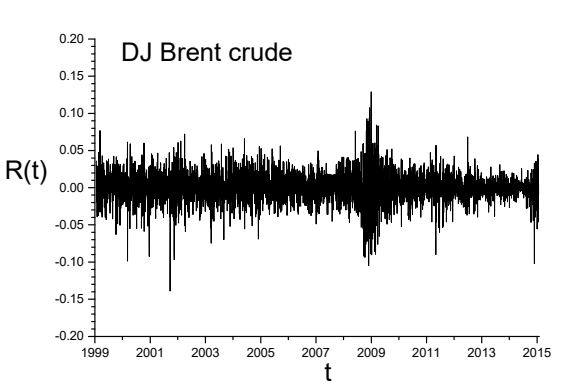
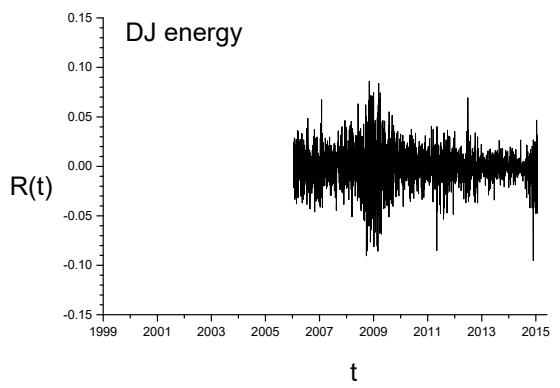
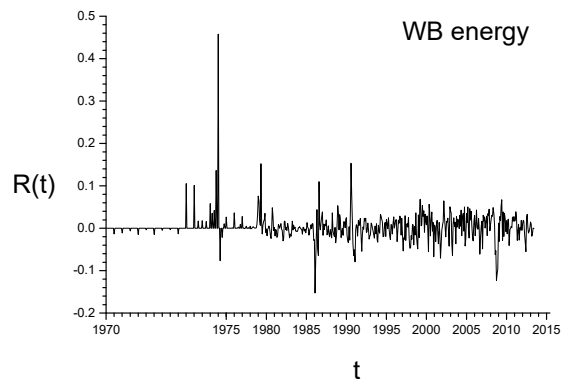
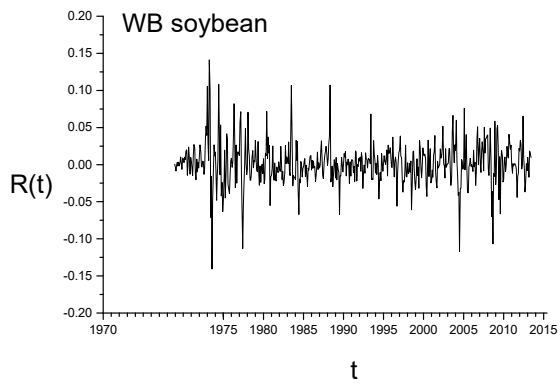
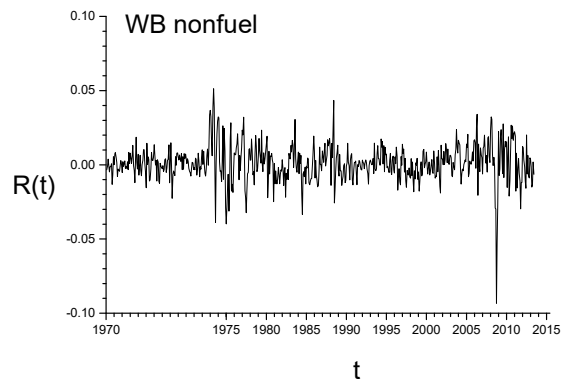
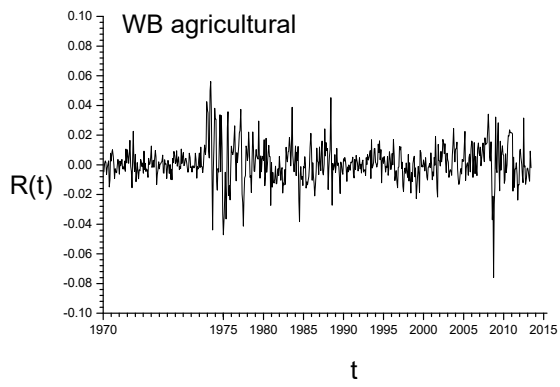


Fig. 3: Time series of commodity returns  $R$ . Time is expressed in months (for the WB series) and in days (for DJ series), starting from the beginning of the recording period.

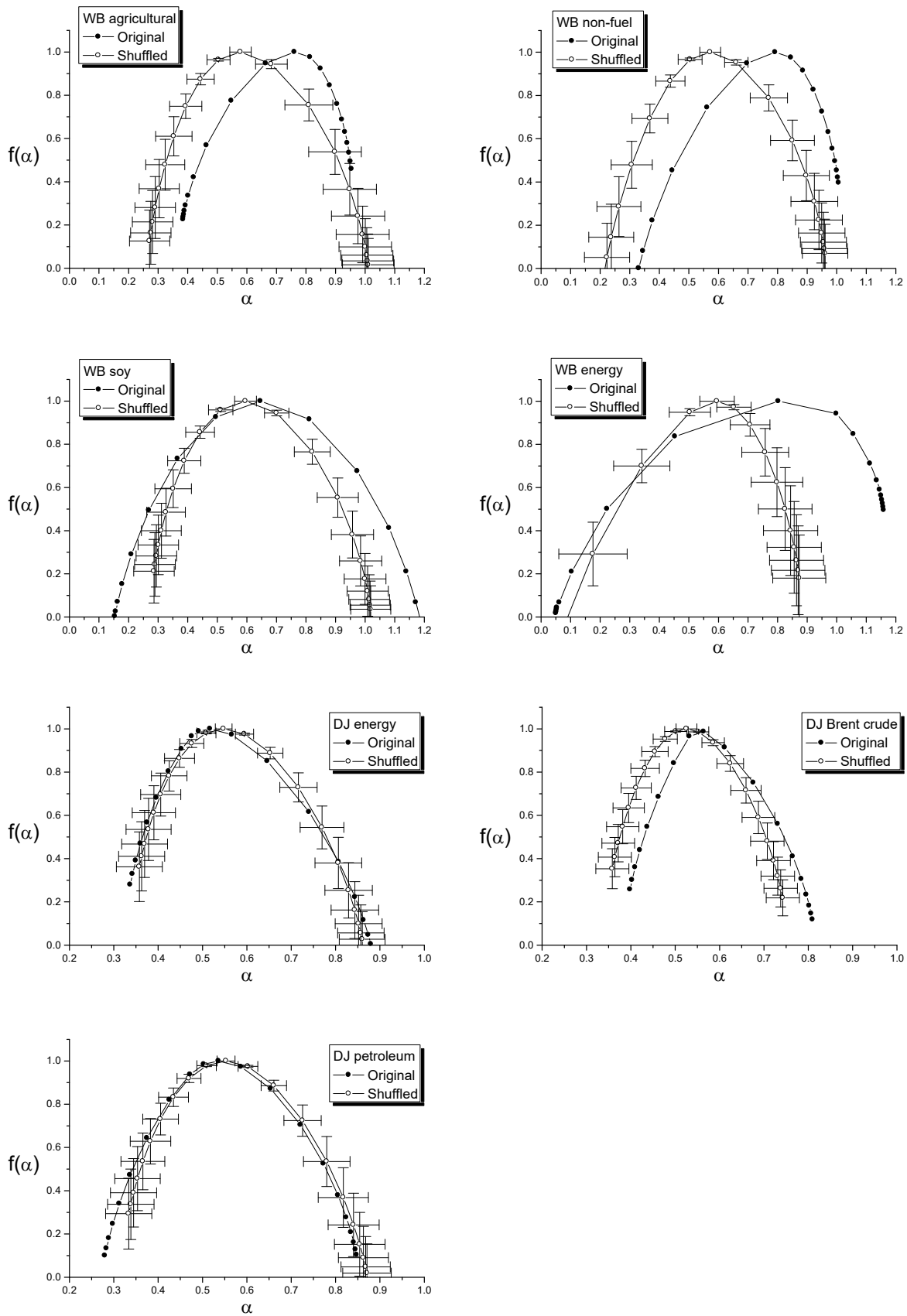


Fig. 4: Multifractal spectrum  $f(\alpha)$  for commodities returns  $R$  of the original and shuffled series (along with their error bars).



The complexity parameters  $(\alpha_0, W, r)$  are shown in Table 1. It is seen from Figure 4 and Table 1 that (i) The position of maximum of the  $f(\alpha)$  spectrum approaches an uncorrelated regime  $\alpha_0 \rightarrow 0.5$  for all the daily DJ series, indicating higher market efficiency than for the monthly WB series where  $\alpha_0 > 0.5$ . More precisely, the efficient market hypothesis (EMH) states that all information relevant to the asset prices is already incorporated into the present value and future returns are unpredictable [40]. However, financial markets can become inefficient and deviate from a random walk behavior [41-44], and the value of Hurst exponent different from 0.5. (ii) The width  $W$  of the multifractal spectra is smaller for the DJ series, which suggests lower heterogeneity for daily commodities time series (and higher efficiency and lower market risk [38]) than for the monthly WB series. (iii) The values of the asymmetry parameter  $r$  reveal that for the daily DJ the multifractality is more influenced by the scaling of small fluctuations ( $r > 1$  and right skewed spectrum) while for the monthly WB series large fluctuations contribute more to multifractality ( $r < 1$  and left skewed spectrum) with exception of WB soy where both large and small fluctuations have equal contribution to multifractality ( $r \approx 1$  and symmetric spectrum).

Table 1. Multifractal parameters  $\alpha_0$ ,  $W$  and  $r$  for commodities returns  $R$ , together with the corresponding values calculated after shuffling (standard deviation is shown in parentheses).

	$\alpha_0$	$W$	$r$	Shuffled data		
				$\alpha_0(\sigma)$	$W(\sigma)$	$R(\sigma)$
WB agricultural	0.768	0.657	0.567	0.579 (0.036)	0.739 (0.155)	1.406 (0.878)
WB soy	0.647	1.028	1.078	0.595 (0.037)	0.733 (0.136)	1.368 (0.805)
WB non-fuel	0.795	0.724	0.562	0.572 (0.035)	0.752 (0.152)	1.070 (0.633)
WB energy	0.848	1.211	0.509	0.595 (0.058)	0.817 (0.201)	0.517 (0.437)
DJ energy	0.528	0.570	1.617	0.548 (0.019)	0.502 (0.103)	1.643 (0.983)
DJ Brent crude	0.571	0.458	1.317	0.525 (0.023)	0.384 (0.075)	1.298 (0.835)
DJ petroleum	0.549	0.599	1.121	0.553 (0.020)	0.537 (0.107)	1.449 (0.819)

We also apply MF-DFA on shuffled series, where the shuffling procedure with  $1000 \times N$  transpositions on each series is repeated 100 times with different random number generator seeds. The multifractal spectra of original and shuffled series are shown in Table 1 and Figure 4. We find that for daily DJ series the multifractal spectrum remains unchanged indicating a broad probability density function as the source of multifractality. For monthly WB series the width of  $f(\alpha)$  spectrum decreases after shuffling for WB soy and WB energy, indicating that the multifractality stems from both a broad probability density function and long term correlations. For WB non-fuel and WB agricultural, only a broad probability density function contributes to multifractality, as the width of the  $f(\alpha)$  spectrum) remains essentially unchanged after shuffling.

#### 4. Conclusion

In this work we compared the multifractal behavior of commodity time series using MF-DFA method. We calculated the multifractal spectra and used a four degree polynomial fit to estimate the complexity parameters. We find distinct multifractal properties for the returns of commodity prices on daily and monthly scales: Daily series show overall scaling behavior approaching the uncorrelated regime ( $\alpha_0 \rightarrow 0.5$ ), a lower degree of multifractality, the dominance of high fractal exponents and a broad

probability density function as the source of multifractality. Monthly commodity series show overall persistent behavior ( $\alpha_0 > 0.5$ ), a higher degree of multifractality, the dominance of low fractal exponents and both i) long-term correlations for small and large fluctuations and ii) a broad probability density function as the source of multifractality. This indicates that commodity markets show higher efficiency when evaluated on daily scale than on monthly scale. Both agricultural and energy markets show similar behavior as indicated by properties of multifractal spectra of monthly WB time series.

From an economic point of view the meaning of this is that agricultural and energy-related commodities exhibit very similar behavior. This confirms the trend of the last decade and a half of parallel increases in both types of primary goods. While this runs against the received wisdom, i.e. that agricultural markets are competitive while energy ones are oligopolistic, this supports relatively recent claims pointing out the importance of the corporate management of international agribusiness [45,46]. The strength of this argument is reinforced by the evidence that short term behavior (i.e. daily transactions) of markets tends to be more efficient, responding to immediate variations, but showing more persistence in longer periods. This indicates that with more time these markets accommodate to new circumstances, a behavior more closely associated to oligopolistic supply [47].

### **Acknowledgments**

This work is supported by research grants from CNPq (Brazilian research agency) grant 306719/2012-6, and CONICET (Argentinean Council of Scientific and Technical Research), grant PIP 11220110100804.

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