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Highlights of the manuscript "On the efficiency of sovereign bond markets" by Luciano Zunino, Aurelio Fernández Bariviera, M. Belén Guercio, Lisana B. Martinez and Osvaldo A. Rosso

- Efficiency of sovereign bond markets is analyzed.
- The complexity-entropy causality plane is implemented to reach this goal.
- Correlations and hidden structures in the daily values of bond indices are unveiled.
- Consistency with qualifications assigned by major rating companies is obtained.
- A link between the entropy measure, economic growth and market size is also found.

On the efficiency of sovereign bond markets

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Abstract

The existence of memory in financial time series has been extensively studied for several stock markets around the world by means of different approaches. However, fixed income markets, i.e. those where corporate and sovereign bonds are traded, have been much less studied. We believe that, given the relevance of these markets, not only from the investors', but also from the issuers' point of view (government and firms), it is necessary to fill this gap in the literature. In this paper, we study the sovereign market efficiency of thirty bond indices of both developed and emerging countries, using an innovative statistical tool in the financial literature: the *complexity-entropy causality plane*. This representation space allows us to establish an efficiency ranking of different markets and distinguish different bond market dynamics. We conclude that the classification derived from the complexity-entropy causality plane is consistent with the qualifications assigned by major rating companies to the sovereign instruments. Additionally, we find a correlation between permutation entropy, economic development and market size that could be of interest for policy makers and investors.

Keywords: sovereign bond market efficiency, complexity-entropy causality plane, permutation entropy, permutation statistical complexity, Bandt and Pompe method, ordinal time series analysis

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05.45.Tp (Time series analysis), 89.70.Cf (Entropy and other measures of information)

1. Introduction

The study of the informational efficiency is maybe one of the most elusive issues in financial economics. In spite of the fact that the first model of an informational efficient market was based on the price changes of French government bonds [1], the literature focused its efforts on the study of stock markets rather than bond markets. The reason for this bias is probably twofold. On the one hand, stock markets trading figures are much larger than bond markets. On the other hand, sovereign bonds[1] began to be traded in exchange markets much more recently in time for many countries, specially for emerging ones. More details about the development of fixed income markets for emerging countries can be found in Refs. [3, 4]. Among the studies on the fixed income markets we can cite Ref. [5] in which January effect in returns of corporate bonds of the Dow Jones Composite Bond Average is found, Ref. [6] in which patterns of daily seasonality in high yield corporate bonds are observed, and Ref. [7] where it is shown the existence of daily seasonalities in the Spanish sovereign bonds for different maturities. Also the patterns of comovements in government bond market yields have been recently analyzed by implementing the minimum spanning tree approach [8, 9]. Useful conclusions are obtained by examining the dynamic evolution of market linkages.

The traditional definition of informational efficiency corresponds to a market where prices fully reflect all available information [10]. Therefore, the key element in assessing efficiency is to determine the information set against which prices should be tested. Informational

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[1]"A bond is an instrument in which the issuer (debtor/borrower) promises to repay to the lender/investor the amount borrowed plus interest over some specified period of time". Definition extracted from Ref. [2, p. 213]. "Bonds issued by autonomous nation states are included in sovereign debt". Definition extracted from Ref. [2, p. 223].

efficiency is classified into three categories, depending on this information set [11, 12]. The first category is the weak efficiency, where stock prices reflect all the information contained in the history of past prices. The second category is semi-strong efficiency, where the information set is all public known information. Finally, the third category is strong efficiency, where prices reflect all kind of information, public and private. Although it may seem at first sight a sign of irrationality, random changes in stock prices reflect the quest of rational investors to catch mispriced securities in the market. The Efficient Market Hypothesis (EMH) is a necessary condition for the existence of equilibrium in a competitive market, in which arbitrage opportunities cannot exist. Ross [13] indicates that this definition evokes the idea that prices are the result of decisions made by individual agents and, therefore, they depend on the underlying information. As a corollary, with the same information set it is not possible to obtain superior returns. It implies, also, that future returns depend to a great extent not only on historic information but also on the new information that arrives at the market. Therefore, an investor, whose information set is the same or inferior to the market information set, cannot beat the market. In addition, investors cannot control the flow of their informative endowment towards the market, since their own transactions (according to its direction and volume) act as signals to the market, tending, thus, to an equalization of the informative sets of the different participants in the market. This produces that, in average, participants cannot beat the market on a regular basis. In an attempt to relax such strict assumptions, Grossman and Stiglitz [14] expand the concept of efficiency, arguing that when information is costly, prices will reflect the information of informed individuals, but only partially, so that information gathering is rewarded.

The aim of this paper is to analyze the sovereign bond market efficiency. More precisely, we want to: (i) classify bond indices, giving a rationale for the bond qualifications of the main rating agencies such as Standard & Poor's (S&P) and Moody's and (ii) analyze the link between sovereign bond market efficiency, economic development and market size. The relationship between economic growth and financial system development has been extensively studied in the economic literature [15–21]. Nevertheless, these studies consider the financial system only composed by the banking sector and the stock market. There is a

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scarce literature that includes the bond market and their results are contradictory [22–25]. The present paper extends the coverage of the empirical literature, considering a potential relationship between economic growth and the development of sovereign bond markets, as an important part of the financial system.

In order to quantify the efficiency related to government bond market indices we use the *complexity-entropy causality plane*, i.e. the representation space with the permutation entropy of the system in the horizontal axis and an appropriate permutation statistical complexity measure in the vertical one. This novel information-theory-tool was recently shown to be a practical and robust way to discriminate the linear and nonlinear correlations present in stock and commodity markets [26, 27]. The location in the complexity-entropy causality plane allows to quantify the inefficiency of the system under analysis because the presence of temporal patterns derives in deviations from the ideal position associated to a totally random process. Consequently, the distance to this random ideal location can be used to define a ranking of efficiency. As will be shown in detail below, we have found that this permutation information-theory-tool is also useful for detecting and quantifying the presence of correlations and hidden structures in the temporal evolution of government bond markets.

This article contributes in several ways to the research field. First, to the best of our knowledge, this is the most comprehensive study of efficiency in the sovereign bond markets covering a total of thirty bond indices of both developed and emerging countries. Second, we detect a coherence of agencies' ratings with the time series efficiency endowment. Third, we find a statistically significant link between bond market randomness and economic development and market size. Fourth, we prove the practical utility of the complexity-entropy causality plane for quantifying efficiency in a financial context.

The remainder of the paper is organized as follows. In the next section, in order to keep our description as self-contained as possible, we introduce the complexity-entropy causality plane. In Sec. 3 we present the data and results. Finally, in Sec. 4, the main conclusions of this paper are summarized.

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2. Complexity-entropy causality plane

Black box time series, given by the discrete set $\{x_t, t = 1, \ldots, N\}$, recorded from observable quantities associated to a system are very often the starting point to study the underlying dynamical phenomenon. They should be carefully analyzed in order to extract relevant information for simulation and forecasting purposes. Information-theory-derived quantifiers can be good candidates for this task because they are able to characterize some properties of the probability distribution associated with the observable or measurable quantity. Shannon entropy is the most paradigmatic example. Its usefulness as a measure of the volatility phenomenon in the financial domain has been proved [28]. Given any arbitrary discrete probability distribution $P = \{p_i : i = 1, \ldots, M\}$, Shannon's logarithmic information measure is given by $S[P] = -\sum_{i=1}^{M} p_i \ln p_i$. It is equal to zero when we are able to predict with full certainty which of the possible outcomes i whose probabilities are given by p_i will actually take place. Our knowledge of the underlying process described by the probability distribution is maximal in this instance. In contrast, this knowledge is minimal for a uniform distribution. It is well known, however, that the degree of structure or patterns present in a process is not quantified by randomness measures and, consequently, measures of statistical or structural complexity are necessary for a better characterization [29]. This is why we have proposed to consider also the statistical complexity for the analysis of financial time series [26, 27]. The opposite extremes of perfect order and maximal randomness (a periodic sequence and a fair coin toss, for example) are very simple to describe because they do not have any structure. The former situation is fully predictable and the latter one has a very simple statistical description. The statistical complexity should be zero in both these cases. At a given distance from these extremes, a wide range of possible degrees of physical structure exists, that should be discriminated by the complexity measure. In this work we have considered the effective statistical complexity measure (SCM) introduced by Lamberti *et al.* [30] since it is able to detect essential details of the dynamics and discriminate different degrees of periodicity and chaos. This statistical complexity measure is defined, following

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the seminal and intuitive notion advanced by López-Ruiz *et al.* [31], through the product

$$\mathcal{C}_{JS}[P] = \mathcal{Q}_J[P, P_e]\,\mathcal{H}_S[P] \tag{1}$$

of the normalized Shannon entropy

$$\mathcal{H}_S[P] = S[P]/S_{\max} \tag{2}$$

79 with $S_{\max} = S[P_e] = \ln M$, $(0 \leq \mathcal{H}_S \leq 1)$ and $P_e = \{1/M, \ldots, 1/M\}$ the uniform dis-

80 tribution, and the disequilibrium $\mathcal{Q}_J$ defined in terms of the Jensen-Shannon divergence.

81 That is, $\mathcal{Q}_J[P, P_e] = \mathcal{Q}_0 \mathcal{J}[P, P_e]$ with $\mathcal{J}[P, P_e] = \{S[(P + P_e)/2] - S[P]/2 - S[P_e]/2\}$ the

82 above-mentioned Jensen-Shannon divergence and $\mathcal{Q}_0$ a normalization constant, equal to the

83 inverse of the maximum possible value of $\mathcal{J}[P, P_e]$. This value is obtained when one of the

84 components of P, say p_m, is equal to one and the remaining p_i are equal to zero. Note that

85 the above SCM depends on two different probability distributions, the one associated to the

86 system under analysis, P, and the uniform distribution, P_e. Furthermore, it was shown that

87 for a given value of $\mathcal{H}_S$, the range of possible $\mathcal{C}_{JS}$ values varies between a minimum $\mathcal{C}_{JS}^{\min}$ and

88 a maximum $\mathcal{C}_{JS}^{\max}$, restricting the possible values of the SCM in a given complexity-entropy

89 plane [32]. Thus, it is clear that important additional information related to the correla-

90 tional structure between the components of the physical system is provided by evaluating

91 the statistical complexity measure. Of course there exist many other complexity measures.

92 For a comparison among them see the paper by Wackerbauer *et al.* [33].

93   In order to calculate the two above-mentioned information-theory-derived quantifiers, a

94 probability distribution should be estimated from the time series associated to the measur-

95 able quantity of the system. The Bandt and Pompe permutation methodology was employed

96 in our analysis due to its simplicity and effectiveness [34]. This efficient symbolic technique,

97 based on the ordinal relation between the amplitude of neighboring values, arises naturally

98 from the time series and allows to avoid amplitude threshold sensitivity dependences. It is

99 clear that, with this way of symbolizing time series, some details of the original amplitude in-

100 formation and variability are lost. However, a meaningful reduction of the complex systems

101 to their basic intrinsic structure is provided. Furthermore, the ordinal pattern distribution

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is invariant with respect to nonlinear monotonous transformations. Thus, nonlinear drifts or scalings artificially introduced by a measurement device do not modify the quantifiers' estimations, a property highly desired for the analysis of experimental data. These are the main advantages with respect to more conventional methods based on range partitioning. The ordinal pattern probability distribution is obtained once we fix the embedding dimension D (pattern length) and the embedding delay time τ. The former parameter, D, refers to the number of symbols that forms the ordinal pattern. Its choice depends on the length N of the time series in such a way that the condition $N \gg D!$ must be satisfied in order to obtain a reliable statistics. It is worth remarking that there are $D!$ possible permutations, and accessible states, for a D-dimensional vector. For practical purposes, Bandt and Pompe recommend $3 \leq D \leq 7$ [34]. The embedding delay, τ, is the time separation between symbols, which is directly related to the sampling time of the time series. By changing the embedding delays of the symbolic reconstruction, different time scales are taken into account. Hereafter, we have fixed $\tau = 1$, focusing the analysis on the highest frequency (daily values) contained within the time series. Please see Refs. [26, 27] for further details about the Bandt and Pompe permutation methodology. A very related approach, based on computing the number of forbidden ordinal patterns present in time series, has been successfully used to find evidence of determinism in noisy time series [35]. By employing this methodology, Zanin [36] has found a clear deterministic behavior for the ten years U.S. bond interest rates. In the present work the normalized Shannon entropy, $\mathcal{H}_S$ (Eq. (2)), and the SCM, $\mathcal{C}_{JS}$ (Eq. (1)), are evaluated using the permutation probability distribution. Defined in this way, these quantifiers are usually known as *permutation entropy* and *permutation statistical complexity* [37]. They characterize the diversity and correlational structure, respectively, of the orderings present in the complex time series.

The complexity-entropy causality plane (CECP) was introduced in Ref. [38] as the representation space obtained with the permutation entropy of the system in the horizontal axis and the permutation statistical complexity in the vertical one. The term causality takes into consideration that the temporal correlation between successive samples is included through the Bandt and Pompe recipe used to estimate both information-theory quantifiers. This

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two-dimensional (2D) diagram was shown to be particularly efficient to distinguish between the deterministically chaotic and stochastic nature of a time series since the permutation quantifiers have distinctive behaviors for different type of motions. According to the findings obtained by Rosso *et al.* [38], chaotic maps have intermediate $\mathcal{H}_S$ values while $\mathcal{C}_{JS}$ reaches larger values, very close to the limit ones. For regular processes, both quantifiers have small values, close to 0. Finally, totally uncorrelated stochastic processes are located in the planar location associated with $\mathcal{H}_S$ and $\mathcal{C}_{JS}$ near 1 and 0, respectively. It has also been found that $1/f^\alpha$ correlated stochastic processes with $1 < \alpha < 3$ are characterized by intermediate permutation entropy and intermediate statistical complexity values. Within the econophysics framework, it has been recently shown that this information-theory-derived approach is an effective tool for distinguishing the stage of stock and commodity markets development [26, 27].

3. Data and results

In this paper we analyze the daily values of thirty bond indices, corresponding to twenty one developed and nine emerging markets, from 3rd January, 2000 until 7th September, 2011, giving a total of $N = 3047$ data points for each bond daily record. All data were collected from Datastream database. The codes and names of these indices are presented in Table 1. We worked with two different indices elaborated by Citigroup: World Government Bond Index (WGBI) and Global Emerging Market Sovereign Bond Index (ESBI). The selection of these indices is based on their general characteristics that guarantee a uniform calculation across countries and the availability of a sufficiently long time series. WGBI includes sovereign debts denominated in the domestic currency, with a minimum size of USD 20 billion and a minimum credit quality of Baa3/BBB- by Moody´s or Standard & Poor´s (S&P). ESBI includes US dollar-denominated emerging market sovereign debts issued in the global, US and Eurodollar markets with a minimum size of USD 500 million, and maximum credit rating of Baa1/BBB+ by Moody´s or S&P, excluding debts into default. An overview about the categories of government bonds by these agencies is shown in Table 2. Credit ratings is an appraisal about the credit risk of a debt instrument and/or an issuer and

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159 are provided by specialized firms. These ratings are relative rather than absolute opinions

160 about credit quality, i.e. about the ability of an issuer to fulfill its financial obligations on

161 time. These opinions are important to increase the information flow across the market and

162 are useful for the different participants in the market: investors, intermediaries and issuers.

163   We select indices that contain long maturity bonds (7-10 years) because, as explained

164 in Ref. [39], the returns of these bonds are not heavily influenced by short-term monetary

165 policy and home bias, but reflects global investor preferences, global savings trends and

166 international risk appetite. Among all the countries available we made a selection that

167 allows us to work with a large number of countries and a long time coverage.

168   Locations of the different sovereign bond markets in the CECP are estimated from the

169 daily indices for different embedding dimensions (pattern length) ($D = 4$, $D = 5$ and $D = 6$).

170 In Fig. 1 we can observe that developed and emerging bond markets are clearly discrimi-

171 nated in this representation space. In particular, we detect that developed markets exhibit

172 higher permutation entropy and lower permutation statistical complexity whereas emerging

173 markets present lower permutation entropy and higher permutation statistical complexity.

174 This indicates that bond indices corresponding to developed markets exhibit more random

175 behavior than those associated with emerging markets, which means higher informational ef-

176 ficiency in developed markets and, consequently, less predictability. Additionally, we observe

177 that developed markets conform a compact cluster, different from the pattern of emerging

178 markets that are more scattered on this representation space. It is worth remarking that

179 these findings appear to be independent of the pattern length selected for the symbolic

180 reconstruction of the original time series.

181   As can be seen in Fig. 2, we have also detected that, within developed markets, Eurozone

182 countries are more closed together, indicating that the price dynamics are very similar. This

183 situation could be caused by the existence of a common currency that avoids the exchange

184 rate risk, remaining only the credit and liquidity risks, as suggested in Ref. [40]. Note that

185 only Ireland (identified by the number 9 in Fig. 2) is not included in the Eurozone cluster.

186 Its permutation entropy is lower and its statistical complexity is higher due to a constant

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Table 1: Sovereign bond indices analyzed in this paper.

WGBI		ESBI	
Country	Datastream code	Country	Datastream code
1. Australia	SBAD70U	1. Argentina	CGESARL
2. Austria	SBAS70U	2. Brazil	CGESBRL
3. Belgium	SBBF70U	3. Chile	CGESCLL
4. Canada	SBCD70U	4. Malaysia	CGESMYL
5. Denmark	SBDK70U	5. Mexico	CGESMXL
6. Finland	SBFN71$	6. Philippines	CGESPHL
7. France	SBFF70U	7. Turkey	CGESTKL
8. Germany	SBDM70U	8. Uruguay	CGESUGL
9. Ireland	SBIR71$	9. Venezuela	CGESVZL
10. Italy	SBIT70U		
11. Japan	SBJY70U		
12. Netherlands	SBDG70U		
13. New Zealand	CGNZ71$		
14. Norway	CGNW71$		
15. Poland	SBPL7T$		
16. Singapore	CGSI71$		
17. Spain	SBSP70U		
18. Sweden	SBSK70U		
19. Switzerland	SBSZ70U		
20. United Kingdom	SBUK70U		
21. United States	SBUS70L		

Table 2: Bond rating by Moody's and Standard & Poor's. Source: Own elaboration based on information retrieved from http://www.moodys.com and http://www.standardandpoors.com.

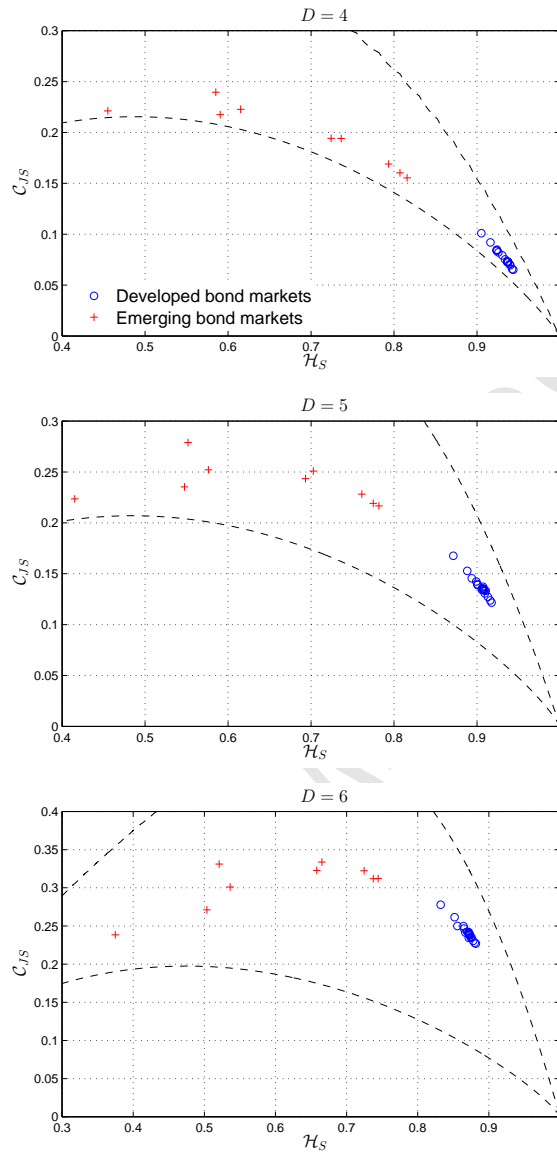|	Moody's	S&P
	Aaa	AAA
	Aa1	AA+
	Aa2	AA
	Aa3	AA-
	A1	A+
Investment grade	A2	A
	A3	A-
	Baa1	BBB+
	Baa2	BBB
	Baa3	BBB-
	Ba1	BB+
	Ba2	BB
	Ba3	BB-
	B1	B+
	B2	B
Speculative grade	B3	B-
	Caa1	CCC+
	Caa2	CCC
	Caa3	CCC-
	Ca	CC
	C	C

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Figure 1: (Color online) Location of the developed and emerging bond markets, according to the indices elaborated by Citigroup (daily data from 3rd January, 2000 until 7th September 2011, $N = 3047$ data points), in the CECP with embedding dimensions $D = 4$ (upper plot), $D = 5$ (central plot) and $D = 6$ (lower plot), and time delay $\tau = 1$. We also display the minimum and maximum possible values of the complexity measure (dashed lines). For further details about the range of possible SCM values see Ref. [32].

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187 behavior during the period from 31st May, 1999 to 28th April, 2000[2]. This highly regular

188 local dynamics strongly affects the global permutation quantifiers' estimations. Focusing on

189 developed government bond markets, we perform a similar analysis for the WGBI beginning

190 on 2nd April, 2002 ($N = 2462$ data points). In this way, the constant behavior observed

191 in the Ireland bond index is avoided. Moreover, a new index (SBPE71$) associated to

192 another country member of the Eurozone, Portugal (identified by the number 22 in Fig. 3),

193 is included because of data availability. As can be concluded from Fig. 3, the previous finding

194 is confirmed, i.e. the Eurozone countries conform a well-defined cluster in the CECP. It is

195 clear that the monetary policy harmonization within the Eurozone increases the financial

196 integration [8].

197   Another important result is that the classification derived from the CECP is coher-

198 ent with the qualification made by rating agencies. In fact, markets with better ratings

199 (Baa3/BBB- or better) are more random and behave more efficiently. On the other hand,

200 emerging countries (with a maximum qualification of Baa1/BBB+) have lower permutation

201 entropy values, which indicate a more regular behavior. This results allows us to confirm

202 that emerging and developed bond markets differ in their informational efficiency from a

203 information-theory-viewpoint.

204   In light of the results obtained, we investigate if the permutation entropy, that quantifies

205 the random behavior of the bond indices, is related to the developmental stage of the econ-

206 omy and/or to the market size. If bond markets were a pure random walk, their associated

207 entropy values would be maximized. On the other hand, if the bond indices were somewhat

208 correlated, then their entropy would not attain its maximal value [41]. Dependence of the

209 data generating process introduces patterns in the time series. Hence, the permutation en-

210 tropy decreases because the ordinal patterns are distant from sharing the same probability.

211 In order to assess the relationship between permutation entropy and the country develop-

212 ment we perform a non-parametric regression between the estimated values for the entropy

213 quantifier and the gross domestic product (GDP) per capita, measured in constant dollars

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[2]There were no trades on the bonds of WGBI Ireland index during this period of time and, consequently, the index remained constant and no returns were recorded.
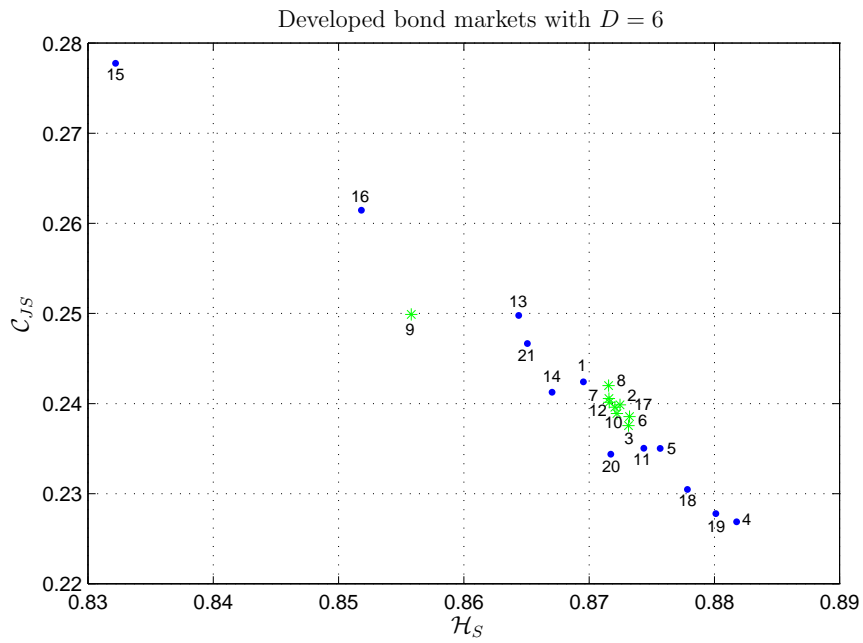
Figure 2: (Color online) Location of the different developed bond markets (daily data from 3rd January, 2000 until 7th September 2011, $N = 3047$ data points) in the CECP with embedding dimension $D = 6$ and time delay $\tau = 1$. A similar grouping is obtained for $D = 4$ and $D = 5$. Numbers indicate WGBI bond indices listed in Table 1. Eurozone sovereign bond markets are identified with green stars.
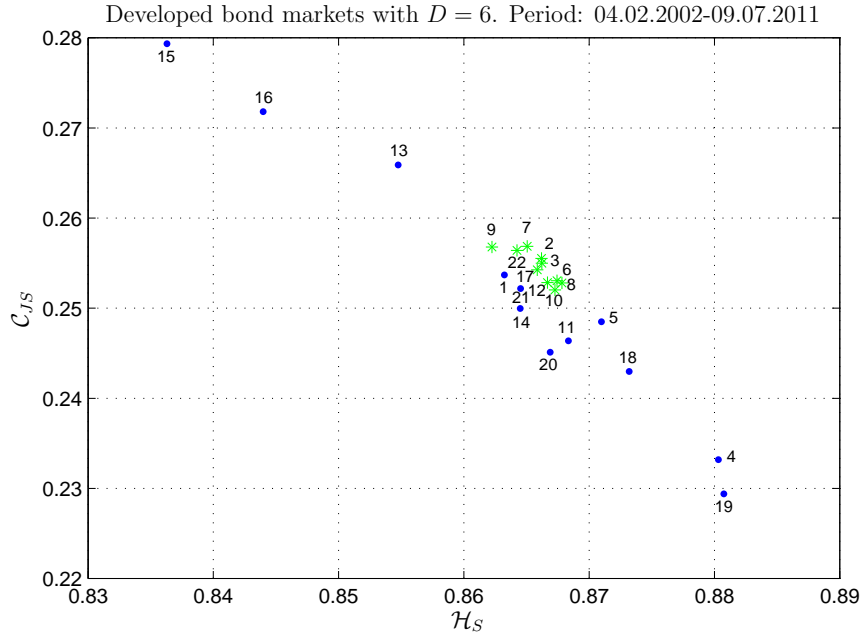
Figure 3: (Color online) Location of the different developed bond markets (daily data from 2nd April, 2002 until 7th September 2011, $N = 2462$ data points) in the CECP with embedding dimension $D = 6$ and time delay $\tau = 1$ for the WGBI beginning on 2nd April, 2002. Similar results are obtained for $D = 4$ and $D = 5$. Numbers indicate WGBI bond indices listed in Table 1. Portugal bond index is identified by the number 22. Eurozone sovereign bond markets are identified with green stars.

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Table 3: Non-parametric rank correlation between permutation entropy, economic development (GDP per capita) and bond market size (Public bond market capitalization/GDP).

Variable	Test	N	Coefficient	P-value
GDP per capita (constant 2000 USD)	Kendall's tau-b	30	0.513	0.000
	Spearman's rho	30	0.706	0.000
GDP per capita (PPP constant 2005 USD)	Kendall's tau-b	30	0.375	0.004
	Spearman's rho	30	0.582	0.001
Public bond market capitalization/GDP	Kendall's tau-b	29[a]	0.281	0.032
	Spearman's rho	29[a]	0.417	0.024

[a] The regresssion does not include Uruguay because the bond market capitalization corresponding to this country is not available at Ref. [42].

and at purchasing power parity (PPP). The results (see Table 3) show a moderate to strong relationship between permutation entropy and the development proxies. Additionally, and in order to study the effect of market size on the efficiency of the bond market, we perform a non-parametric regression between permutation entropy and a size proxy. We select the ratio of bond market capitalization to GDP as a variable that is representative of the market's depth [42]. Table 3 shows a moderate relationship between permutation entropy and market size, which highlights the usefulness of permutation entropy in financial time series analysis. In fact, these results are important in two aspects. The first one is that permutation entropy is positively related with the stage of economic development. The second one is that this quantifier is also affected by market size. These findings can be of great value for policy makers in order to set measures for improving the informational efficiency of bond markets.

4. Conclusions

We used the complexity-entropy causality plane in order to unveil the presence of correlations and hidden structures in the daily values of thirty bond indices. We detect that

²²⁸ the qualifications given by the main rating agencies are coherent with the location of the
²²⁹ associated time series in this representation space. In this sense, we expanded the literature
²³⁰ of EMH to a market that was not sufficiently studied in this aspect. Additionally, we find a
²³¹ link between the entropy measure, economic growth and market size. In fact, permutation
²³² entropy is higher for developed countries than for emerging ones, and market size is corre-
²³³ lated with permutation entropy, being the bigger markets the ones with higher permutation
²³⁴ entropy. In future works we would like to study the comovements and efficiency evolution
²³⁵ of government bond markets.

Acknowledgements

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