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Local and effective temperatures of quantum driven systems

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We introduce thermometers to define the local temperature of an electronic system driven out of equilibrium by local ac fields. We also define the effective temperature in terms of a local fluctuation-dissipation relation. We show that within the weak driving regime these two temperatures coincide. We also discuss the behavior of the local temperature along the sample. We show that it exhibits spatial fluctuations following an oscillatory pattern. For weak driving, regions of the sample become heated, while others become cooled as a consequence of the driving.

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The study of heat transport at the mesoscale and nanoscale is being the subject of intense interest at present. Motivation in this field is twofold. On one side, the technological trend toward miniaturization of electronic circuits pushes for a better understanding of heat dissipation at this scale. From a more general point of view, one is often faced with situations, in which the very fundamental concepts of standard statistical mechanics and thermodynamics are put into test. This is for instance the case when the system under consideration is driven out of equilibrium.

Dynamical evolution out of thermodynamical equilibrium takes place in a great variety of physical situations and many efforts have been devoted during the last decade toward the extension of standard thermodynamical concepts to this domain. Well-known examples in this area include the aging regime of glassy systems, granular materials and colloids. A break through in this field has been the identification of *effective temperatures*, that is, even when the system evolves out of the equilibrium, it is possible to identify a parameter that has the same properties of the temperature of a system at equilibrium. Even more, it is sometimes possible to formulate a generalization of the equilibrium fluctuation dissipation relations (FDR), with this new parameter playing the role in an effective temperature.[1–5]

In the context of quantum transport, electronic devices driven under ac potentials offer an ideal playground to explore these fundamental issues. The study of heat transport in these systems has captured increasing attention during the last years.[6–8] Particularly appealing in this sense are setups where the ac fields act locally within some region of the sample, that we define as the "central system." In practical configurations, this central (out of equilibrium) system is in contact with macroscopic wires which remain at thermodynamical equilibrium and act as particle and thermal reservoirs. A paradigmatic example is a quantum dot driven at its walls by two voltages oscillating with a phase-lag named "quantum pump."[9–11] Another example corresponds to arrays of driven quantum capacitors coupled to the edge state of an electronic gas in the Hall regime.[12,13]

The aim of this work is to introduce the concept of *local temperature* along the central system. To this end we follow a procedure inspired in a pioneer work by Engquist and Anderson.[14] The idea is to include in the microscopic description of the driven system a *thermometer*, namely, a macroscopic system, which is in local thermodynamic equilibrium with the sample. This theoretical construction enable us the investigation of interesting features on the behavior of the energy propagation along the sample. One of the most remarkable features is the development of spatial fluctuations of the local temperature, which leads to local cooling of regions of the sample within the weak driving regime. We also make a step further by identifying a FDR, which for weak driving casts an effective temperature that is shown to exactly coincide with the local one measured by the thermometer.

Our setup, including the device with the driven system in contact to reservoirs and thermometer is described by the Hamiltonian

$$H(t) = H_{sys}(t) + H_{cP} + H_P,$$

$$H_{sys}(t) = H_L + H_{cL} + H_C(t) + H_{cR} + H_R. \quad (1)$$

The piece $H_{sys}(t)$ contains the term describing the central system (C) with the ac fields, $H_C(t)=H_0+H_V(t)$ as well as terms corresponding to left (L) and right (R) reservoirs with the ensuing contacts H_{cL} and H_{cR}. The term H_P represents the thermometer. It consists in a macroscopic system weakly coupled to a given point lP of C, through a contact described by H_{cP}. It behaves like a reservoir with a temperature T_{lP} that is determined by the condition of a vanishing heat flow between it and C. This is the thermal counterpart of a voltage probe (see Ref. 14–16). All the reservoirs are modeled by systems of noninteracting electrons with many degrees of freedom, $H_\alpha=\Sigma_{k\alpha}\varepsilon_{k\alpha}c^\dagger_{k\alpha}c_{k\alpha}$, being $\alpha=L,R,P$. The corresponding contacts are $H_{c\alpha}=w_{c\alpha}(c^\dagger_{k\alpha}c_{l\alpha}+c^\dagger_{l\alpha}c_{k\alpha})$, where $l\alpha$ denotes the coordinate of C at which the reservoir α is connected. We take into account the noninvasive property of the thermometer[14] by treating w_{cP} at the lowest order of perturbation theory when necessary. We leave for the moment H_C undetermined as much of the coming discussion is model independent.

The dynamics of the system is best described within the Schwinger-Keldysh Green functions formalism. This involves the calculation of the Keldysh and retarded Green's functions,

$$G^K_{l,l'}(t,t') = i\langle c^\dagger_{l'}(t')c_l(t) - c_l(t)c^\dagger_{l'}(t')\rangle,$$

$$G^R_{l,l'}(t,t') = -i\Theta(t-t')\langle c_l(t)c^\dagger_{l'}(t') + c^\dagger_{l'}(t')c_l(t)\rangle, \quad (2)$$

where the indexes l,l' denote spatial coordinates of the central system. These Green functions can be evaluated after

solving the Dyson equations. For ac driven systems, it is convenient to use the Floquet Fourier representation of these functions,[11]

$$G^{K,R}_{l,l'}(t,t-\tau)=\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i(k\Omega_0 t+\omega\tau)}G^{K,R}_{l,l'}(k,\omega), \quad (3)$$

where Ω_0 is the frequency of the ac fields.

We determine the local temperature by requiring that the heat current from the system to the thermometer vanishes. We will assume here that the L and R leads are at the same temperature T and that both leads and the thermometer have the same chemical potential μ. We work in units where $\hbar=e=k_B=1$. As shown in,[7] given $H_C(t)$ without many-body interactions, the heat current from the central system to the thermometer can be expressed as

$$J^Q_P(T_{lP})=\sum_{\alpha=L,R,P}\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\{[f_\alpha(\omega)-f_P(\omega_k)]\times(\omega_k-\mu)\Gamma_P(\omega_k)\Gamma_\alpha(\omega)|G^R_{lP,l\alpha}(k,\omega)|^2\}, \quad (4)$$

where $\omega_k=\omega+k\Omega_0$, $\Gamma_\alpha(\omega)=-2\pi|w_\alpha|^2\Sigma_{k\alpha}\delta(\omega-\varepsilon_{k\alpha})$ is the spectral function that determines the escape to the reservoir α, and $f_\alpha(\omega)=1/[e^{\beta_\alpha(\omega-\mu)}+1]$, is the Fermi function, which depends on the temperature $T_\alpha=1/\beta_\alpha$ and the chemical potential of the reservoir α. Thus, the local temperature T_{lP} corresponds to the solution of the equation $J^Q_P(T_{lP})=0$. In general, the exact solution must be found numerically, however, an exact analytical expression can be obtained within the weak-coupling and low-temperature T regime.

Before doing so, let us analyze a FDR between the local Green functions $G^{K,R}_{l,l}(t,t')$. Let us recall that for systems in equilibrium, the fluctuation dissipation theorem establishes a relation between the Keldysh (correlation) and Retarded Green functions. Indeed, for a system such as the one under consideration, but without the time-dependent fields, it can be shown that the relation between the fluctuations in the system, $iG^{0,K}_{l,l}(\omega)$, with the dissipation term of the bath, $\Gamma_\alpha(\omega)$, is[3,4]

$$iG^{0,K}_{l,l}(\omega)=\tanh\left[\frac{\beta(\omega-\mu)}{2}\right]\varphi^0_l(\omega), \quad (5)$$

$$\varphi^0_l(\omega)=-2\,\mathrm{Im}[G^{0,R}_{l,l}(\omega)]=\sum_{\alpha=L,R}|G^{0,R}_{l,l\alpha}(\omega)|^2\Gamma_\alpha(\omega), \quad (6)$$

where the supraindex zero indicates that we are considering $H_V(t)=0$ and all the reservoirs at the same temperature T. When the time-dependent term is turned on, identities between Green functions[11] generalize to

$$iG^K_{l,l}(0,\omega)=\sum_{k=-\infty}^{\infty}\tanh\left[\frac{\beta(\omega_{-k}-\mu)}{2}\right]\varphi_l(k,\omega_{-k}), \quad (7)$$

$$\varphi_l(k,\omega)=\sum_{\alpha=L,R}|G^R_{l,l\alpha}(k,\omega)|^2\Gamma_\alpha(\omega). \quad (8)$$

We will show below that within the weak driving-adiabatic regime, where the term $H_V(t)$ is treated as a perturbation and driving frequency is smaller than the dwell time of the electrons within the central system,[10] it is possible to define an effective temperature $T^{eff}_l=1/\beta^{eff}_l$ through the following relation:

$$iG^K_{l,l}(0,\omega)-iG^K_{l,l}(0,\mu)=\tanh\left[\frac{\beta^{eff}_l(\omega-\mu)}{2}\right]\overline{\varphi}_l(\omega), \quad (9)$$

with $\overline{\varphi}_l(\omega)=-2\,\mathrm{Im}[G^R_{l,l}(0,\omega)]=\Sigma_k\varphi_l(k,\omega_{-k})$. A similar relation in the time domain has been studied numerically for a driven ring in contact to a reservoir.[4] In the present problem, we are able to demonstrate that for weak driving and low temperature, T^{eff}_l coincides with the temperature T_{lP} determined by the thermometer.

We now turn to analyze in detail the weak driving regime where we consider $H_V(t)$ as a perturbation in evaluating J^Q_P (see Refs. 7, 11, and 16). For reservoirs at low-temperature T (compared with the Fermi energy), a Sommerfeld expansion may be applied in Eq. (4) leading to

$$T^2_{lP}\sim\frac{6}{\pi^2}\frac{\sum_k\Phi_{lP}(k)}{\sum_k F_{lP}(k,\mu_{-k})}+T^2\frac{\sum_k F_{lP}(k,\mu)}{\sum_k F_{lP}(k,\mu_{-k})}, \quad (10)$$

where

$$\Phi_l(k)=\int_{\mu_{-k}}^{\mu}d\omega(\omega_k-\mu)\varphi_l(k,\omega), \quad (11)$$

$$F_l(k,\omega)=\frac{d}{d\omega}[(\omega_k-\mu)\varphi_l(k,\omega)], \quad (12)$$

encode the dependence on the driving field and the geometry of the central system. This expression makes it explicit the fact that the local temperature is different from the temperature of the leads.

In order to be more specific, let us consider a driving term of the form,

$$H_V(t)=\sum_{j=1}^{M}V_j(t)c^\dagger_{lj}c_{lj}, \quad (13)$$

with $V_j(t)=V_0\cos(\Omega_0 t+\delta_j)$, being lj the positions at where the ac fields are applied. For small V_0 the Dyson equation is solved to lowest order in this amplitude and the only nonvanishing Floquet components of $G^R_{l,l'}(k,\omega)$ are those with $k=-1,0,1$.[7,11] The adiabatic condition is introduced by expanding all terms of (10) in powers of Ω_0. Keeping terms up to Ω_0^2,

$$T^2_{lP}\sim T^2+\frac{3}{\pi^2}\lambda^{(0)}_{lP}(\mu)\Omega_0^2+2\lambda^{(1)}_{lP}(\mu)T^2\Omega_0-\frac{1}{2}\lambda^{(2)}_{lP}(\mu)T^2\Omega_0^2, \quad (14)$$

being

$$\lambda^{(n)}_l(\omega)=\frac{1}{\sum_{k=-1}^{1}\varphi_l(k,\omega)}\sum_{k=-1}^{1}(k)^{n+2}\frac{d^n[\varphi_l(k,\omega)]}{d\omega^n}. \quad (15)$$

We now carry out a similar analysis with the effective

temperature. For weak driving, the relevant electronic energies ω are such that $|\omega-\mu| \lesssim \max(T,\Omega_0)$. For small T and Ω_0 we, thus, expand both sides of Eq. (9) around $\omega=\mu$. Keeping terms up to first order in ω, we find,

$$T_l^{eff} = \frac{1}{2}\frac{\bar{\varphi}_l(\mu)}{\frac{d}{d\omega}\left\{\sum_{k=-1}^{1}\varphi_l(k,\omega_{-k})\tanh\left[\frac{\beta(\omega_{-k}-\mu)}{2}\right]\right\}_{\omega=\mu}}, \tag{16}$$

which, for low-driving frequency Ω_0, reduces to

$$T_l^{eff} = T[1+\lambda_l^{(1)}(\mu)\Omega_0]. \tag{17}$$

By comparing this expression with Eq. (14) we find that $T_{lP}^{eff}=T_{lP}$ when $\Omega_0 \ll T$. We would like to emphasize that our definition of local temperature is independent of the weak driving-adiabatic assumption. However, it is within this regime where the system is slightly out of equilibrium and the equivalence with the effective temperature defined from the FDR Eq. (9) is expected.

In order to show other explicit results we will choose a simple set up composed of a central system with two local ac fields oscillating with a phase lag, i.e., $M=2$, $\delta_1=0$, and $\delta_2=\delta$ in Eq. (13), a simple model for a quantum pump. As central system we take a one-dimensional lattice of N sites, with the first one connected to L and the site N connected to R ($l\alpha=1,N$ for $\alpha=L,R$), and $H_0=-w\Sigma_{l=1}^{N-1}(c_l^\dagger c_{l+1}+\text{H.c.})$. All energies and temperatures are expressed in units of w.

We first discuss the behavior of the local temperature. One would expect that the driving fields heat the sample giving rise to dissipation of energy from the system to the reservoirs and leading to a local temperature of the sample which is higher than that of the reservoirs. At weak driving, however, it has been shown that it is possible to coherently transport energy along the sample in a way that some of the ac fields develop power against other ones with a low dissipation of heat to the reservoirs.[7] Actually, the mechanism of energy exchange between the fields behaves $\propto\Omega_0$, while the rate at which energy is dissipated as heat is $\propto\Omega_0^2$. A non trivial behavior of the local temperature, could, thus, take place within this regime. This is, in fact, the case shown in Fig. 1, where parameters are chosen within the weak driving and low temperature regime. In agreement with our previous discussion we show that $T_{lP}\equiv T_{lP}^{eff}$. Remarkably, as a function of the position at which the thermometer is connected or, equivalently, the FDR (9) is evaluated, the local temperature varies along the sample, being lower than T in some places, while higher in others. One end of the system is hotter than the other because the presence of the quantum pump breaks time-inversion and space-inversion symmetries being the value of δ that determines which one is hotter.[7,10,11] Between the two pumping centers, the local temperature displays oscillations with a spatial period $\sim 2k_F$. These oscillations are due to processes $\propto V_0^2\Omega_0\sin(\delta)$ and have a similar origin as the Friedel oscillations detected by voltage probes.[15] Similar oscillations in the local temperature were also reported in other mesoscopic systems.[8] In the present system, the latter are a consequence of interference processes in the charge

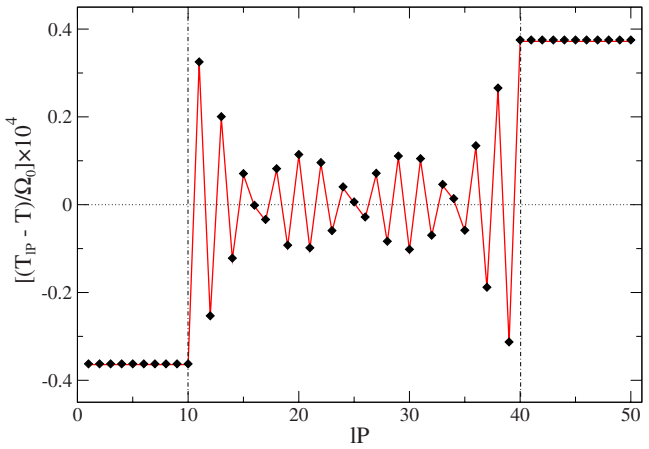

FIG. 1. (Color online) Local (solid red) and effective (black diamonds) temperatures along a one-dimensional model of $N=50$ sites with two ac fields operating with a phase-lag $\delta=\pi/2$ at the positions indicated in dotted lines. The system is in contact to reservoirs with chemical potentials $\mu=0.2$, and temperature $T=0.001$. The driving frequency is $\Omega_0=10^{-6}$ and the amplitude is $V_0=0.05$.

transport that take place between the two pumping centers, which act as dynamical local impurities.[16] Therefore, the oscillatory behavior in T_{lP} is a signature of the coherence of at least some component of energy transport along the sample.

In Fig. 2 we show results for the local temperature as a function of the position of the thermometer beyond the regime of validity of the weak driving and/or adiabatic approximation. Oscillations of the local temperature between the two pumping centers are apparent, pointing to the survival of a coherent component in the energy propagation. At fixed T, the mean temperature of the sample, $T_m=(\Sigma_{lP=1}^{N}T_{lP})/N$ grows as Ω_0 increases and becomes soon higher than the temperature of the reservoir.

Finally, we analyze the behavior of the mean temperature T_m as a function of T. Results are shown in Fig. 3 for a driving frequency beyond the adiabatic regime and different values of the pumping amplitude V_0. As expected, for T fixed, the mean temperature of the sample increases for in-

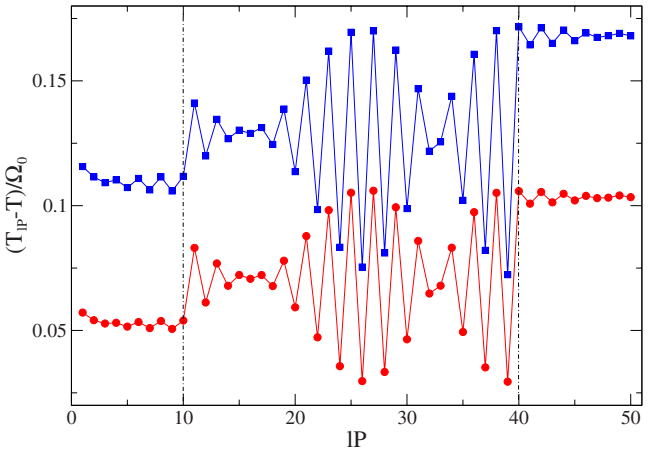

FIG. 2. (Color online) Local temperature along the sample for $\mu=0.2$, $\delta=\pi/4$, $V_0=0.25$, $\Omega_0=0.1$, and $T=0.001$ (blue squares) and $T=0.01$ (red circles). All other parameters are the same as in Fig. 1.

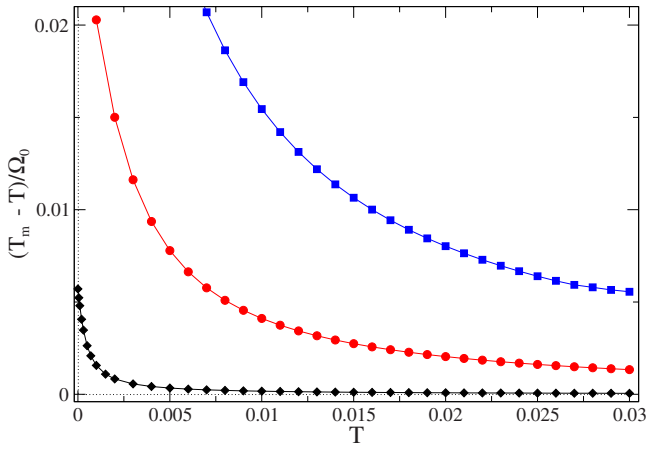

FIG. 3. (Color online) Departure of the mean temperature of the central system from the temperature of the reservoirs for $\mu=0.2$, $\Omega_0=0.1$, $\delta=\pi/4$, $V_0=0.1$ (blue squares), $V_0=0.05$ (red circles), and $V_0=0.01$ (black diamonds). Other parameters are the same as in Fig. 1.

creasing V_0. Instead, T_m-T is a decreasing function of T. This reflects the fact that for reservoirs at a high temperature, the effect of the driving becomes washed up and the sample becomes mainly heated due to the contact with a high temperature environment.

To conclude, we have defined local temperature for a quantum system driven out of equilibrium. The behavior of this quantity indicates a global heating, which manifests itself in the form of a mean temperature T_m higher than the one of the reservoirs. A more striking feature is the occurrence of $2k_F$ oscillations in the local temperature, similar to the ones predicted under a stationary situation.[8] This is an indication of quantum interference, i.e., coherence in the energy propagation along the sample. At weak driving, these oscillations give place to the local cooling of the sample. We have also defined an effective temperature from a local fluctuation dissipation relation. We have shown that for weak driving and for temperatures smaller than the Fermi energy of electrons, the latter coincides with the one defined by the thermometer. This equivalence has been previously established only for classical spin systems.[1] The fact that such a kind of equivalence holds for quantum fermionic systems is an important conceptual issue and its scope for other systems worth further future investigation.

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