Below-bandgap excitation of bulk semiconductors by twisted light

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Abstract. - I theoretically investigate the response of bulk semiconductors to excitation by twisted light below the energy bandgap. To this end, I modify a well-known model of light-semiconductor interaction to account for the conservation of the momentum of light. I predict that the excited states can be thought of as a superposition of slightly perturbed exciton states undergoing a complex center-of-mass 3D motion. In addition, other effects are found; first, the absorption of twisted light and plane-wave light occur at a slightly different energy; second, the absorption of twisted light produces complex spatial patterns in the polarization and electric current.

In the realm of semiconductor optics, a generalized practice is to neglect the photon's momentum in direct absorption/emission processes. This assumption, known as "vertical transitions"(VT) [1], has been applied with such success that sometimes the need to include the momentum is understimated. Exceptions are found in early and recent works, reporting theoretical and experimental results strongly dependending on the conservation of the linear momentum of the photon. In the late '50 Hopfield elaborated the theory of exciton-polaritons [2], and later he pointed out, in a different context, the existence of new selection rules not accounted for by VT [3]. Afterwards the photon drag effect [4] was proposed, experimentally verified and brought to the point of technological applications. More recently, the photon momentum has been incorporated in the equations of motion describing the dynamics of electrons and holes under the action of inhomogeneous light fields to treat lasers [5]; meanwhile, research in exciton-polaritons continue evolving [6].

In the last few decades there has been an increase in the number of studies on spatially inhomogeneous light fields, most important to this article is the case of optical vortices [7], and subsequently that of twisted light(TL) —light carrying orbital angular momentum(OAM) [8]. These have motivated studies of inhomogeneous light-matter interaction in several fields [9], and likely part of the work in semiconductors cited above. In particular, theoretical work by myself and collaborators show the need to include the OAM of TL in above-bandgap excitation of semiconductor structures. [10–12]

From the point of view of applications, the tendency to miniaturization requires the manipulation of states having few particles, and the consideration of small quantities. Thus, what has been judgicioulsly neglected in the past, may acquire relevance in present-days technology.

The present work is motivated by the combination of the current interest in optical vortices, the long-standing attention to the light-matter interaction problem, and finally the intention to gain control on the quantum level in semiconductor structures. In the following I explain the modifications introduced in a standard model of light-semiconductors interaction, to account for the linear momentum or OAM of the light. Using this extended model I work out the familiar situation of excitation of excitons by plane waves. I have in mind two aims; first, I intend to further support —in addition to the aforemention literature— the idea that the inclusion of the momentum of light is a must, for certain situations. Second, some simple new results and connection to other theories will emerge. The main part of the article treats the below-bandgap coherent excitation by TL; here I predict that the excitation may be understood as generating a superposition of states, that differ slightly from excitons, and undergo a complex center-of-mass 3D motion. In addition, other effects are described and compared to the excitation by plane waves.

The theory of optical excitation of semiconductors in the VT limit is well developed and known. An excellent treatment is given by Haug et al [1], for the cases without (inter-band transitions) and with (excitonic transitions) Coulomb interaction. In the following, this formalism will be extended to include the momentum of light.

The dynamics of electrons in a semiconductor having a valence(v) and a conduction(c) bands may be described by Heisenberg equations of motion [12] for intraband coherences (and populations) in each band plus the interband coherence between them; these equations form a coupled system. Under the condition of low excitation (low light field intensity/large detuning), the equations can be treated perturbatively and can be decoupled. [1] The interband coherence $\rho_{vi,cj} = \langle a^\dagger_{vi} a_{cj}\rangle$ is first order in the light field strength —$a^\dagger_{bk}/a_{bk}$ are creation/annihilation operators of electrons in a Bloch state $\mathbf{k}$ and band b: $\psi_{b\mathbf{k}}(\mathbf{r}) = \langle \mathbf{r}|\, b\mathbf{k}\rangle = L^{-3/2} e^{i\mathbf{k}\mathbf{r}} u_b(\mathbf{r})$, with system's length L, energy $\varepsilon_{b\mathbf{k}}$, and periodic Bloch function $u_b(\mathbf{r})$— whereas intraband coherences (and populations) are second order in the light field strength. Then, the linear response of the unexcited system (zero conduction band population) is obtained from

$$\left(i\hbar\frac{d}{dt} - \Delta_{c\mathbf{k},v\mathbf{k}'}\right)\rho_{v\mathbf{k}',c\mathbf{k}}(t) = \langle c\mathbf{k}|\, h_I(t)\, |v\mathbf{k}'\rangle - \sum_{\mathbf{q}\neq 0} V_{\mathbf{q}}\, \rho_{v\mathbf{k}'-\mathbf{q},c\mathbf{k}-\mathbf{q}}(t)\,, \quad (1)$$

where $\Delta_{c\mathbf{k},v\mathbf{k}'} = \varepsilon_{c\mathbf{k}} - \varepsilon_{v\mathbf{k}'}$ and the self-energy correction has been neglected. The first term in the RHS is the matrix element of the light-matter interaction, modeled by "minimal coupling" (for the case of TL see Quinteiro et al. [10]). The second term on the RHS is the Coulomb potential in the random phase approximation. I first note that, due to the conservation of the momentum of light, $\langle c\mathbf{k}|\, h_I(t)\, |v\mathbf{k}'\rangle$ vanishes unless $\mathbf{k}' = \mathbf{k} - \mathbf{q}_0$ for some vector $\mathbf{q}_0$. In the case of plane waves propagating along the z-axis $\mathbf{q}_0 = q_0\hat{z}$ is a constant or parameter —the linear momentum of the photon—, while for twisted light $\mathbf{q}_0 = q_r\cos\theta\,\hat{x} + q_r\sin\theta\,\hat{y} + q_z\,\hat{z}$ with θ a variable and $\{q_r, q_z\}$ parameters (see below for details). Therefore, to capture the relevant physics of the light-matter interaction I specialize and study Eq. (1) at $\mathbf{k}' = \mathbf{k} - \mathbf{q}_0$. I will treat the case of TL, since that of plane waves can be easily deduced from the former. The main contribution to the light-matter interaction arises from the transverse component $\mathbf{A}(\mathbf{r}, t) = \boldsymbol{\epsilon}_\sigma\, A_0(t) J_l(q_r\, r)\exp[i(q_z z + l\phi)] + c.c.$ of the vector potential, with $J_l(x)$ a Bessel function, $\hbar l$ the OAM, $\boldsymbol{\epsilon}_\sigma$ the vector for circular polarization $\sigma = +/-$, and $q_r < q_z$. A Fourier transform in $\mathbf{k}$-space $\{f(\mathbf{r}) = [L/(2\pi)]^3 \int d^3k\, e^{-i\mathbf{k}\mathbf{r}} f(\mathbf{k})\}$ and time $[f(t) = 1/(2\pi)\int d\omega\, e^{-i\omega t} f(\omega)]$ is applied to simplify each term: The light-matter interaction $\langle c\mathbf{k}|\, h_I(t)\, |v\mathbf{k}'\rangle = \xi(t)\,\delta_{(\mathbf{k}-\mathbf{k}')_r q_r}\delta_{(\mathbf{k}-\mathbf{k}')_z q_z}\exp(i\theta l)/(L\, q_r)$ becomes

$$\langle c\mathbf{k}|\, h_I(t)\, |v\mathbf{k} - \mathbf{q}_0\rangle \;\rightarrow\; L^3\,\xi(\omega)\,\frac{e^{i\theta l}}{L\, q_r}\,\delta(\mathbf{r})$$

with $\xi(t) = -(-i)^l\,(\boldsymbol{\epsilon}\cdot\mathbf{p}_{cv})\, Q\, A_0(t)/m$, $\mathbf{p}_{cv} = \langle u_c|\,\mathbf{p}\,|u_c\rangle$ the momentum-operator matrix element, Q the electron's charge, and m the electron's mass. Then,

$$\Delta_{c\mathbf{k},v\mathbf{k}-\mathbf{q}_0}\,\rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(\omega) \;\rightarrow\; \left[\left(E_g + \frac{\hbar^2\,\mathbf{q}_0^2}{2|m_v^*|}\right) - i\,\frac{\hbar^2\,\mathbf{q}_0}{|m_v^*|}\cdot\nabla - \frac{\hbar^2}{2\,\mu}\nabla^2\right]\rho_{\mathbf{q}_0}(\omega, \mathbf{r}) \quad (2)$$

with $1/\mu = 1/|m_v^*| + 1/|m_c^*|$, and the subscript v/c from ρ was eliminated to ease the notation. Finally,

$$\frac{L^3}{(2\,\pi)^3}\int d^3j\, V_{\mathbf{k}-\mathbf{j}}\,\rho_{v\mathbf{j}-\mathbf{q}_0,c\mathbf{j}}(\omega) \rightarrow V(\mathbf{r})\,\rho_{\mathbf{q}_0}(\omega, \mathbf{r})\,. \quad (3)$$

Assembling all terms, the transformed version of Eq. (1) becomes

$$\left[\hbar\omega - \left(E_g + \frac{\hbar^2\,\mathbf{q}_0^2}{2|m_v^*|}\right) + i\,\frac{\hbar^2\,\mathbf{q}_0}{|m_v^*|}\cdot\nabla + \frac{\hbar^2}{2\,\mu}\nabla^2 + V(\mathbf{r})\right]\rho_{\mathbf{q}_0}(\omega, \mathbf{r}) = L^3\,\xi(\omega)\,\frac{1}{L\, q_r}\, e^{i\theta l}\,\delta(\mathbf{r})\,. \quad (4)$$

To solve it one may first find the solutions $\psi_\nu(\mathbf{r})$ of the corresponding homogeneous equation (RHS set to zero), and then express the solution $\rho_{\mathbf{q}_0}(\omega, \mathbf{r})$ to the complete equation as an expansion in terms of $\psi_\nu(\mathbf{r})$. In the VT model ($\mathbf{q}_0 = 0$) the homogeneous equation is the Wannier equation, and its solutions are the wave-functions for the relative motion of excitons having energy E_ν and spatial extent a_B^*. In the general case, Eq. (4) exhibits two new terms: (1) $\hbar^2\,\mathbf{q}_0^2/(2|m_v^*|)$, and (2) $i\hbar^2/|m_v^*|\,\mathbf{q}_0\cdot\nabla$. Term (1) needs no special treatment, since it is a renormalization of the energy. Term (2) can be handled by either i) perturbation theory, or ii) completing squares and applying a unitary transformation (displacement in $\mathbf{k}$-space space) $U(\mathbf{r}) = \exp[i(\mu/|m_v^*|)\,\mathbf{q}_0\cdot\mathbf{r}]$. By method ii Eq. (4) becomes

$$\left[\hbar\,\omega - E_g - \frac{\hbar^2\mathbf{q}_0^2}{2M} - \frac{\mathbf{p}^2}{2\mu} + V(\mathbf{r})\right]\widetilde{\rho}_{\mathbf{q}_0}(\omega, \mathbf{r}) = L^3\,\xi(\omega)\,\frac{e^{i\theta l}}{L\, q_r}\, U(\mathbf{r})\,\delta(\mathbf{r})\,, \quad (5)$$

where $\widetilde{\rho}_{\mathbf{q}_0}(\omega, \mathbf{r}) = U(\mathbf{r})\rho_{\mathbf{q}_0}(\omega, \mathbf{r})$, and $M = |m_v^*| + |m_c^*|$. Redefining $E_g' = E_g + \hbar^2\mathbf{q}_0^2/(2M)$ the homogeneous part adquires the form of the equation for the relative motion of an exciton, where the renormalization of the energy signals its center-of-mass motion with momentum $\mathbf{q}_0$. One should keep in mind, that this resemblance to the Wannier equation does not imply, though, that the original states

[1] Perturbation theory breaks down when the quantization volumes for electrons and photons is the same and no additional dissipation channel exists. [2] On the other hand, the conservation of momentum requires that the system is larger than the wavelength λ of light; thus, for a system smaller that the photon's quantization box and larger than λ perturbation theory can be used.

[solutions to Eq. (1) without the light term] are exact excitons $\psi_\nu(\mathbf{r})$, since the inverse transformation $U(\mathbf{r})$ remains to be applied; this point will be clarified when discussing the perturbation theory solution. Back to $\mathbf{k}$-space [2]

$$\rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(\omega) = L^3\,\xi(\omega)\,\frac{e^{i\theta l}}{L\,q_r}\times \sum_\nu \frac{\psi^*_\nu(\mathbf{r}=0)}{\hbar\,\omega - E_g - \frac{\hbar^2\mathbf{q}_0^2}{2M} - E_\nu}\psi_\nu\left(\mathbf{k}-\frac{\mu}{|m^*_v|}\mathbf{q}_0\right). \quad (6)$$

It is instructive to apply perturbation theory [method i above)] to the homogeneous equation of Eq. (4). First, I show that the perturbation term $i\hbar^2/|m^*_v|\,\mathbf{q}_0\cdot\nabla$ is indeed small compared to the rest. The action of the perturbation onto the unperturbed solutions $\psi_\nu(\mathbf{r})$ yields $\hbar^2/|m^*_v|\,\mathbf{q}_0\cdot\nabla\psi_\nu(\mathbf{r}) \simeq \hbar^2 q_0/(a^*_B|m^*_v|)\,\phi_\nu(\mathbf{r})$, with $\phi_\nu(\mathbf{r})$ a function of roughly the same magnitude as $\psi_\nu(\mathbf{r})$. Typically (e.g. GaAs) $a^*_B \simeq 10$ nm and $q_0 \simeq 10^{-2}$ nm^{-1}, so $\hbar^2 q_0/(a^*_B|m^*_v|) \simeq 0.1$ meV which is smaller than $E_\nu \simeq 1$ meV, as required. In addition, the ratio of terms (2)/(1) $\simeq 8$; thus, an analysis that includes the renormalization term (1), should not neglect term (2). From perturbation theory a solution $\eta(\mathbf{r})$ to the homogeneous equation is built up by a superposition of unperturbed wave-functions $\eta_\alpha(\mathbf{r}) = \psi_\alpha(\mathbf{r}) + \sum_\nu a^{(1)}_\nu \psi_\nu(\mathbf{r}) + \ldots$, where $a^{(1)}_\nu$ is linear in the perturbation and the dots stand for higher order corrections. Besides, a Taylor expansion of $\psi_\nu(\mathbf{k}-\mu\mathbf{q}_0/|m^*_v|)$ is possible, since each term is of the order of $\psi_\nu(\mathbf{k})(q_0 a^*_B\mu/|m^*_v|)^n$, with $q_0 a^*_B \mu/|m^*_v| < 1$. The correspondence between this expansion and the perturbation theory approach, helps to clarify what was anticipated above, that the solutions to the original homogeneous equation are not excitons.

Equation (6) is the building-block of several quantities describing both the electrons' kinetics/dynamics and the effect that electrons have on the EM-field. Next, I provide the expectation values of the polarization, the electric current density, and the OAM.

As a consequence of the conservation of photon momentum, the global polarization of the system is zero; thus, a more correct quantity is a local or space-dependent polarization

$$\mathbf{P}(\mathbf{R},t) = 2\left(\frac{L_{\mathcal{R}}}{L}\right)^3 \sum_{\mathbf{k}\mathbf{q}_0} \Re\left\{e^{i\mathbf{q}_0\cdot\mathbf{R}}\,\mathbf{d}_{vc}\,\rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(t)\right\}, \quad (7)$$

with $\Re\{\ldots\}$ the real part, $\mathbf{d}_{vc}$ the dipole matrix element, and $\mathbf{R}$ (with coordinates $\{R,\Phi,Z\}$) pointing to a macroscopic cell of linear size $L_{\mathbf{R}}$ small compared to the scale of variation of the EM-field, but larger than the unit cell of the semiconductor.

In addition, a TL-field induces electric currents [10–12]

$$\mathbf{j}(\mathbf{R},t) = 2\frac{Q}{m}\frac{1}{L^3}\sum_{\mathbf{k}\mathbf{q}_0}\Re\left\{e^{i\mathbf{q}_0\cdot\mathbf{R}}\,\mathbf{p}_{vc}\,\rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(t)\right\}. \quad (8)$$

The similarity between the polarization and current is not surprising; the theory of macroscopic media relates the polarization charge to the electric current by the continuity equation [13]. Either one or the other may be used to study the effect that the interband coherence Eq. (6) has on the EM-field.

Due to the fact that the light beam carries OAM, it is of interest to calculate the OAM acquired by the electrons as a result of the interaction. For reasons similar to those that cause the global polarization to vanish, only an OAM in slices perpendicular to the propagation direction of light yields a non-zero result

$$L_z(Z,t) = \beta\,e^{iq_z Z}\sum_{\mathbf{k}\mathbf{q}_0}\Re\left\{(p_{-,b'b}\,e^{i\theta} + p_{+,b'b}\,e^{-i\theta})\times \rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(t)\right\}, \quad (9)$$

where $\beta = -(4\pi/q_r)\,J_2(q_r L)$, and $p_{\pm,b'b} = p_{x,b'b} \pm p_{y,b'b}$. Some general comments are in order: i) Because of their local character, all quantities exhibit spatial dependence; ii) to obtain explicit expressions, it only remains to insert the time-domain version of $\rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(\omega)$ into Eqs. (7)-(9), or to transform all quantities to the frequency domain and use $\rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(\omega)$ directly; iii) for plane waves all sums over $\{\mathbf{k},\mathbf{q}_0\}$ simplify to $\{\mathbf{k}\}$, while for TL they simplify to $\{\mathbf{k},\theta\}$.

With the tools developed so far, I analyze the cases of plane waves and twisted light.

Plane waves: I set $l = 0$ and $e^{i\theta l}/(L\,q_r) = 1$. The simplest situation is that of $q_0 = 0$, i.e. VT. By noting that $\sum_k \psi_\nu(\mathbf{k}) \propto \psi_\nu(\mathbf{r}=0)$ the expressions for the interband coherence and the polarization boil down to those of the standard result, that is the polarization is homogeneous and new absorption lines below the conduction-band edge appear due to the exciton's binding energy E_ν. Let us now turn to the general case $\mathbf{q}_0 = q_0\hat{z}$, and examine Eq. (6) after expanding $\psi_\nu(\mathbf{k}-\mu\mathbf{q}_0/|m^*_v|)$

$$\rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(\omega) = L^3\xi(\omega)\,\frac{e^{i\theta l}}{L\,q_r}\sum_\nu\frac{\psi^*_\nu(\mathbf{r}=0)}{\hbar\omega - E_g - \frac{\hbar^2\mathbf{q}_0^2}{2M} - E_\nu}\times \left[\psi_\nu(\mathbf{k}) + \sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{\mu q_0}{|m^*_v|}\right)^n \partial^n_{k_z}\psi_\nu(\mathbf{k})\right]. \quad (10)$$

The first term on the RHS is similar to what was obtained for the case of VTs; nevertheless, the presence of the kinetic energy term $\hbar^2 q_0^2/(2M)$ in the denominator signals the center-of-mass (COM) motion of the exciton, as has been known for long time. The next terms in Eq. (10) may be considered the result of the displacement in momentum space of the excitonic relative-motion wave-function, as the argument of $\psi_\nu(\mathbf{k}-\mu\mathbf{q}_0/|m^*_v|)$ reflects. From the point of view of perturbation theory, the term $i\hbar^2/|m^*_v|\,\mathbf{q}_0\cdot\nabla$ in Eq. (4) causes the eigenstates to be a superposition of unperturbed excitonic wave-functions. Both standpoints tell us that this correction affects the internal degree of freedom, in contrast to the effect $\hbar^2\,\mathbf{q}_0^2/(2|m^*_v|)$ has. Shifting

[2] To account phenomenologically for decoherence ω may be replaced by $\omega + i\gamma$ in the denominator.

to the analysis of derived quantities, we see that the polarization presents spatial dependency on the coordinate Z; from this expression an electric susceptibility in the direction of the electric field can be deduced. Expanding the real part in Eq. (7) and after some calculation

$$\chi(\omega) = -|d_{vc}|^2 \sum_\nu \frac{\psi_\nu^*(\mathbf{r}=0)}{\hbar\,\omega - E_g - \frac{\hbar^2 q_0^2}{2M} - E_\nu} \left[\psi_\nu\left(\mathbf{r}=0\right) + \sum_{n=1}^{\infty} \frac{1}{n!}\left(\frac{\mu q_0}{|m_v^*|}\right)^n \left(\frac{L}{2\,\pi}\right)^3 \int d^3\mathbf{k}\, \partial_{k_z}^n \psi_\nu\left(\mathbf{k}\right)\right] + \ldots, \quad (11)$$

where the dots indicate an extra term arising from $\left\{e^{i\mathbf{q}_0\cdot\mathbf{R}}\,\mathbf{d}_{vc}\,\rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(t)\right\}^*$. The susceptibility shows the expected features, i.e. no spatial dependence [3] but "spatial dispersion" (q_0 dependence). The COM-motion correction in the denominator introduces a tiny shift in the absorption line. Used in conjunction with Maxwell's equations, the susceptibility yields information about the effect that electrons has on the EM field, e.g. attenuation of the beam. I finally note that the effects introduced by a finite wave-vector q_0 is negligible when the semiconductor's z-length is smaller than $2\pi/q_0$, e.g. thin samples/quantum wells.

Twisted light: The full solution Eq. (6) having $e^{i\theta l}/(L\,q_r) \neq 0$, and variable θ is used. Let us first focus on the term $\hbar^2 q_0^2/(2M)$. Regarded as the COM kinetic energy of excitons, it indicates a complex motion connected to both parameters q_z and q_r. Although q_r is not a momentum (it relates to the inverse of the beam waist), its presence may be understood by thinking on the linear momentum ($\mathbf{P}^{(TL)} = \epsilon_0 \mathbf{E}\times\mathbf{B}$) at each point in space carried by the TL field: A preliminary calculation, under the assumption $q_r < q_z$, shows that the radial and angular components of $\mathbf{P}^{(TL)}$ are proportional to q_r, while —as expected— its z component is proportional to q_z. Despite this tells that q_r participates in the momentum, a deeper analysis remains to be done in order to confirm and expand this result. Once again, the argument of $\psi_\nu(\mathbf{k} - \mu\mathbf{q}_0/|m_v^*|)$ indicates that the TL-field excites states which are not exactly but slightly perturbed excitonic states. Additionally, the presence of the factor $e^{i\theta l}$ signals that a superposition of states —differing in their θ variable and thus having different phases— is created, in contrast to the case of the one-state excitation by plane waves. As for the macroscopic description of the system, the states contribute, by the term $E_\nu + \hbar^2 q_0^2/(2M)$, absorption lines below the band gap; the shift due to the COM kinetic energy is about 10μeV. The absorption could be studied more carefully by either calculating the local polarization or the electric current, and if possible deriving a susceptibility or conductivity (through Ohm's law) respectively. It is of interest to look at the electric current produced by the interband coherence; using Eq. (6) and defining $\rho_{v\mathbf{k}-\mathbf{q}_0,c\mathbf{k}}(t) = e^{i\theta l}\rho_0(t)$ Eq. (8) becomes

$$\mathbf{j}(Z,\Phi;t) = \frac{Q}{m}\frac{4\pi}{L^3}\sum_{\mathbf{k}} \Re\left\{i^l e^{iq_z Z} J_l(q_r R) e^{il\Phi}\mathbf{p}_{vc}\,\rho_0(t)\right\}.$$

The current perpendicular to the z-axis exhibits complex flow patterns: for $l=1$ one observes circular flows around the beam axis, for $l=2$ two electric current vortices appear at both sides of the beam axis, for $l>2$ the complexity increases and several vortices show up (see Fig. 2 in Quinteiro et al [10]), suggesting a transfer of the optical vortices to the solid. As a consequence, the OAM in z-direction only exists for the case of $l=\pm 1$. To conclude, it is my believe that the the correct physical description of the TL-semiconductor interaction requires the inclusion of $\mathbf{q}_0$; this is simply because common semiconductor structures are not shaped to avoid the angular inhomogeneity of a TL beam. [4]

In conclusion, a modified version of a well-established model has been presented and used to explain the below-bandgap excitation of semiconductors by inhomogeneous light beams, with special emphasis on twisted light. First, I show how the standard results for the case of vertical transitions induced by plane waves are obtained in the appropriate limit; this attests for the robustness of the extended model. Next, the case of plane waves when the wave vector of the light beam is taken into account is analyzed; I show how to recover the exciton's COM motion, and an additional correction to the relative-motion wave-function is found. Then, the theory is applied to the case of twisted light. I predict that the optical excitation produces a superposition of states that differ slightly from excitons and undergo complex center-of-mass motion. In addition, I show that the absorption energy is modified by the center-of-mass motion, and that the polarization and electric current induced by the transition present a complex spatial pattern. Given the current interest in the interaction of inhomogeneous EM-fields with semiconductors, and in particular the work in exciton-polaritons, further research on the generation of exciton-like states and their complex motion due to twisted light excitation may significantly impact the basic research and applications in these areas.

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[3] The $\exp(iq_0Z)$ was used to reconstruct the electric field in a formula of the type $P(Z) = \chi E(Z)$.

[4] An exception would be an arc-section sample, subtending a small angle, whose origin coincides with the the beam axis.