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To cite this article: C. I. Paulo, E. Borsa & M. R. Barbosa (2017): Non-linear optimisation and energy integration of a particles separation–classification process, International Journal of Sustainable Engineering, DOI: [10.1080/19397038.2017.1387185](https://doi.org/10.1080/19397038.2017.1387185)

To link to this article: <http://dx.doi.org/10.1080/19397038.2017.1387185>



Published online: 27 Oct 2017.



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Non-linear optimisation and energy integration of a particles separation–classification process

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ABSTRACT

The gas–solid separation has several industrial applications in engineering process. The collection efficiency and the energy consumption are critical variables in particles separation process. In this study, a non-linear programming problem is proposed to maximise the cyclone efficiency in a 34,560.0 ton/year particles separation–classification plant. A new process integration scheme is proposed to energy consumption minimisation. The units included in the model are a powder feeder, a high efficiency cyclone, a bag filter and a blower. Stairmand and Swift's high efficiency cyclones models are evaluated and compared. Four particulate materials are considered: limestone, dolomite, sawdust and toner. The particles diameters range studied is 0.1–1000 μm . Mass and energy balances are solved with an initial solid concentration of 500 g/m^3 . Stairmand's cyclone proved to be more efficient than Swift for all materials analysed. The highest cyclone efficiency is achieved for sawdust (> 99%). The cyclone separates more than 34,000 ton/year of particles > 10 μm . The energy integration scheme consumes 43% less energy than the basic one for all cases studied. The proposed optimisation model results a valuable tool for optimal process design, reducing the environment process impact and adaptable to operate at high temperatures values with different materials.

ARTICLE HISTORY

Received 5 August 2016
Accepted 23 August 2017

KEYWORDS

Energy integration; particles separation processes; non-linear optimisation; cyclone

1. Introduction

The demand for particulate materials follows closely the business cycle, and grows in the industrialised and industrializing countries. Industries such as plastics, rubber, paper, paints, cement-based, ceramic and aliphatic materials are some classic examples of industrial requirement of fine powders. These materials are expensive due to the sophistication involved in the particles separation and classification process. Equipments modelling and optimisation constitute a present objective in the gas–solid separation field. Cyclones are the most popular particles separators. They are simple and robust units, with low operation cost. For air purification process, the conventional cyclones are commonly used, while for particulate material of a specific size, it is more appropriate to use high efficiency cyclones. In the cement industry, to achieve high resistance concrete using crushed rocks wastes, it is necessary to control specifically the particles distribution size and shape. Concrete production required particles sizes that are generally not satisfied by conventional milling processes. In general, there is an abundance of coarse particles and lack of essential material, i.e. of particles in the range of 4–150 μm -diameter, Stone (2012) and Pettingell (2008).

Several studies have recently explored the simulation and optimisation of cyclone separators, Zou et al. (2015), Elsayed (2015), Sgrott et al. (2015) and Shaaban (2011). A high efficiency stairmand cyclone was designed by Fathizadeh et al. (2015) to separate the black powder from natural gas. Funk et al. (2015) proposed a energy saving reducing the pressure drop and the inlet velocity in the cyclone, but they do not report energy savings

values. Zhang et al. (2014) proposed a cascade of cyclonic pre-heaters to energy optimise cement kilns; and Elsayed and Lacor (2010) deeply studied the optimisation of cyclone dimensions through computational fluid dynamics. However, is not found in the literature energy integration reports for particles separations process. The development of integrated mathematical models that optimise the separation–classification process and consider energy integration is an uncompleted issue in the area.

In this study, an energy efficiency approach is proposed through a new process scheme for particles separation. The formulation of a non-linear mathematical problem is introduced to optimal process design. The cyclone efficiency is selected as the objective function to maximise. Mass and energy balances, equations for the design of high efficiency cyclones, filters equations designs, pressure drops restrictions and operating conditions are included as linear and non-linear constraints in the optimisation model. The problem is solved for four solid materials, two cyclones models and two process schemes. The paper is structured as follows. Section 2 describes the materials properties and the particles size distributions. Special description of the cyclone main dimensions is explained. Basic and energy integrations process schemes are presented and analysed. Mass and energy balances are listed in Section 3, including the enthalpies calculations. The models considered for the design of high efficiency cyclones and the optimisation problem solved by the algorithm implemented on CONOPT are also described in Section 3. Section 4 summarises the model results and discussions. Finally, conclusions are presented in Section 5.

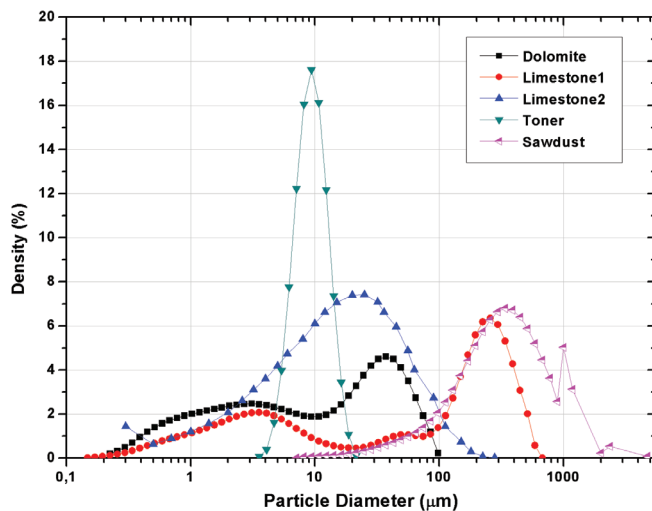


Figure 1. Particle size distributions for studied materials.

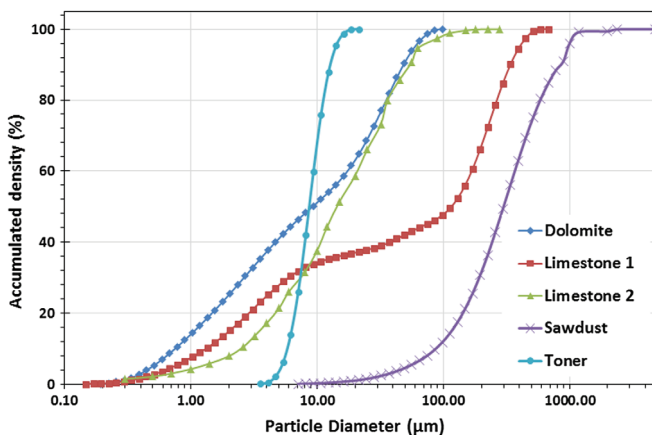


Figure 2. Particle size distributions accumulation for studied materials.

2. Material and methods

2.1. Materials properties

Four different particulate materials are considered: dolomite, limestone, sawdust and toner. The particle size distributions are analysed by laser diffraction (Analyser Malvern, Mastersizer 2000E, dry dispersion module Sirocco 2000M). Figure 1 shows the particles distributions obtained, and Figure 2 the accumulate distributions. Limestone 1 and 2 distinguishes between two different grinding times for the same material. To evaluate the molecular weights, dolomite is considered as a mineral composed of calcium and magnesium carbonate ($\text{CaMg}(\text{CO}_3)_2$); limestone is considered as CaCO_3 ; toner is 50% butyl acrylate and 50% iron oxide; and sawdust is considered as oak from bolivia (cerejeira) 48.9% C, 44.5% O and 6.6% H, Himmelblau (1997), Jiménez, Baquero, and Diaz (2006), XEROX (2002) and Gupta (2011).

Table 1 summarises the materials properties. For air, dolomite and limestone, the specific heat is computed as a temperature function; for sawdust and toner a mean value is used.

2.2. Separation–classification process

Dust separators are the classical equipments to obtain particulate material. Technological developments in cyclone separators permit its use as classifiers with a good particles separation performance up to $5\ \mu\text{m}$ -diameter. When industry requirements involve even smaller sizes, the use of more complex equipment is needed to obtain the desired separation. In these cases, bag filters are generally used. The optimal combination cyclone-filter allows to separate particles up to $0.01\ \mu\text{m}$ -diameter, yielding technical and economic advantages for the entire separation–classification process, Hoffmann and Stein (2008). The process considered in this work consists of a powder feeder that provides the solid material, a high efficiency cyclone where the main separation of particles occurs, a filter for collecting small particles and a blower device as a flow driver. The materials to be separated and classified may contain moisture initially. To avoid problems in the operation of the equipments due to the presence of water, the air entering the process is pre-heated in a heat exchanger. Figure 3 shows a Basic Scheme (BS) of the separation–classification process. CV_1 and CV_2 are the control volumes where energy consumption evaluation are performed. The powder feeder is used to regulate the solid input to the process. These types of units consist of electric motors, solenoids, pneumatic cylinders and regulators. To achieve a proper dosage is necessary to control the material input to the unit and ensure a homogeneous mix of the output material. Depending on variables such as nature and characteristics of the solid material, dynamics required for the discharge and desired accuracy, the feeders are commonly classified into volumetric and gravimetric, Baxter and Prescott (2009). For the process studied in this work the selection of a gravimetric feeder is preferred due to the precision required.

The cyclone is the most important process equipment, where the separation of the larger fraction of particles occurs. Cyclones are robust units, easy handling and low maintenance costs. Figure 4 shows the main dimensions of a tangential input cyclone.

Filters are commonly used for the separation of fine powders. A common filter type consists of closed metal structures with textile filtering elements arranged vertically. According to the filter design, the elements can be tubular or rectangular known as sleeves or bags, respectively. Filtration occurs as a result of the formation of a primary powder layer on the surface of the sleeves and a subsequent accumulation of dust. This accumulated material forms a ‘cake’ on the sleeves. Once formed the primary layer, the filtration occurs by sieving. Sleeve filters are highly efficient, they can reach efficiencies of 99% for particles of 0.5 microns, and even acceptable values for particles under 0.01 microns in size. The blower is the process flow driver. It is responsible for the whole movement of air and particles from the powder feeder to the cyclone and filter. The location of the blower is at the end of the process, sucking air. This arrangement is convenient because the unit operates with lower dust density, reducing pressure drop and increasing its useful life.

To recover the heat loss in the process, a recirculation of air is proposed as a new energy integration scheme (EI). The air and a few particles that are not separated by the combination cyclone-filter are recirculated to the mixer joining the input of clean air. Figure 5 shows the energy integration scheme. A purge

Table 1. Materials properties.

Material	Molecular weight MW_j	Density ^{293K} Kg.m^{-3}	Specific heat ^a $\text{J.gmol}^{-1}.\text{K}^{-1}$	Reference
Air	28.96	1.2	$a = 28.09, b = 1.97E - 3,$ $c = 4.80E - 6, d = -1.97E - 9$	Himmelblau (1997)
Dolomite	184.42	2770 grain density	$a = 385.80, b = -0.02, c = 5.12E5,$ $d = 6.49E - 6, e = -1.55E7,$ $g = -4.39E3, h = 8.32E3$	Gupta (2011)
Limestone	100.09	2680 grain density	$a = 82.34, b = 4.98E - 2, c = -1.29E6$	Himmelblau (1997)
Sawdust	13.1	530 with 12% moisture content	$C_{p\text{mean}} = 5,906.7$	Jiménez, Baquero, and Diaz (2006)
Toner	100.1	568 bulk density	$C_{p\text{mean}} = 295.9$	XEROX (2002)

$$^a C_{p\text{Air}} = a + bT + c.T^2 + d.T^3; C_{p\text{Dolomite}} = a + b.T + c.T^{-2} + d.T^2 + e.T^{-3} + g.T^{-1/2} + h.T^{-1}; C_{p\text{Limestone}} = a + b.T + c.T^{-2}.$$

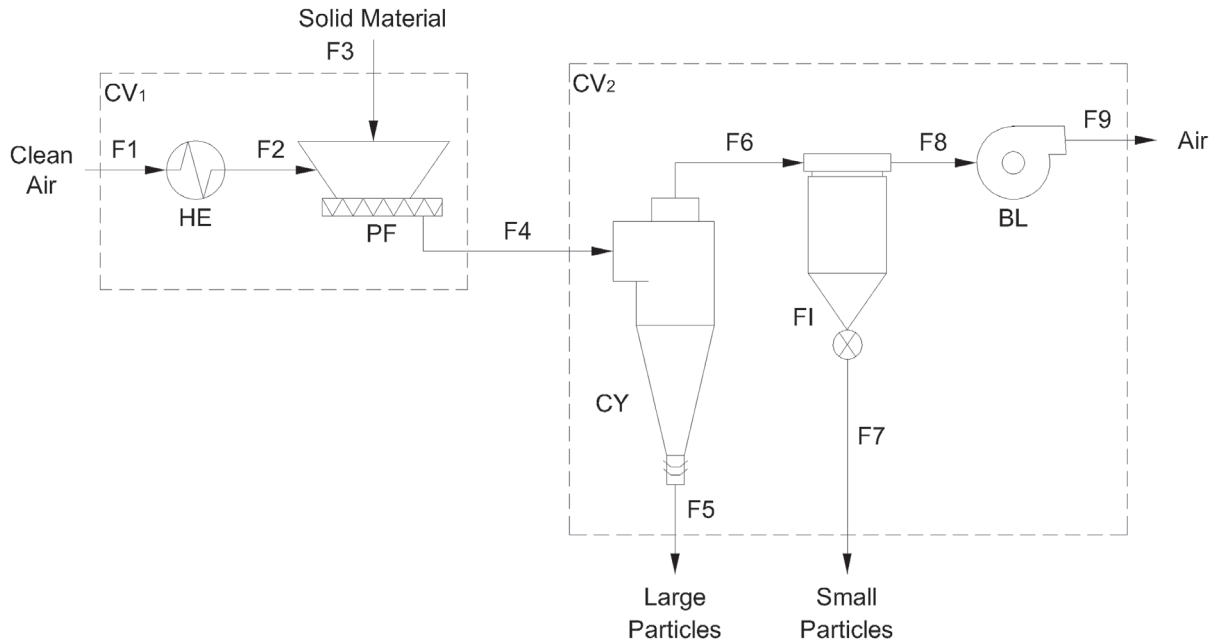


Figure 3. Basic scheme for particles separation-classification process (BS). HE: heat exchanger; PF: powder feeder; CY: cyclone; FI: filter; BL: blower; $F_{i:1...9}$: process streams. CV_1 and CV_2 : control volumes for energy purpose.

is necessary (stream 13), to prevent the accumulation of fine particles in the process. CV_1 and CV_2 are the control volumes to evaluate process energy balances.

3. Calculation

3.1. Mass and energy balances

Global and component mass balances are formulated for both, the basic and the energy integration schemes. For process analysis, the index i indicates the numbering of the mass flow in each process. That is, the subscript i indicates the number of each stream in the process, $I_{BS} = [1, 2, 3, \dots, 9]$; $I_{EI} = [1, 2, 3, \dots, 14]$, according to Figures 3 and 5.

The index j indicates the chemical component, either air ($j = \text{'air'}$) or the solid material to be separated ($j = \text{'particles'}$). The optimisation program does not run for all components simultaneously, for each material the non-linear programming problem

was coded. In each program the set 'particles' correspond to 'limestone 1', 'limestone 2', 'dolomite', 'sawdust' or 'toner'.

The set of equations relating each component flow $f_c(i, j)$, with total flow $F(i)$ and the mass fractions $x(i, j)$ are expressed as:

$$f_c(i, j) = x(i, j) \cdot F_i \quad \forall j \in J, \quad \forall i \in I_{BS, EI} \quad (1)$$

$$F_i = \sum_j f_c(i, j) \quad \forall i \in I_{BS, EI} \quad (2)$$

Considering e process units, $E = [\text{powder feeder, heat exchanger, cyclone, filter, blower, splitter 1, splitter 2}]$, global and component mass balances are performed. For the whole process the balances are also added:

$$\sum_{\text{input } i \text{ to } e} F(i) = \sum_{\text{output } i \text{ from } e} F(i) \quad \forall e \in E \quad (3)$$

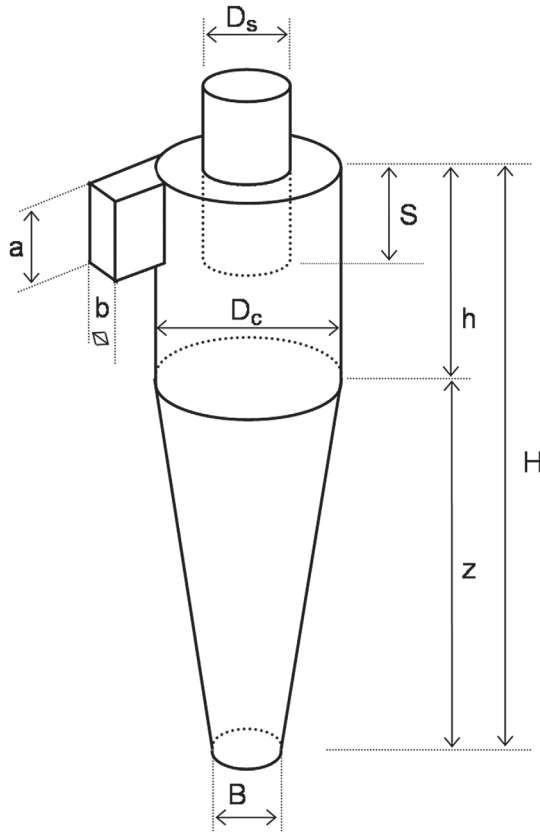


Figure 4. Cyclone main dimensions. Tangential input. a : input height; b : input width; D_c : diameter; B : bottom outlet diameter, D_s : top outlet diameter; S : output height; h : cylindrical height; z : conical height, H : total height.

$$\sum_{\text{input } i \text{ to } e} f_c(i,j) = \sum_{\text{output } i \text{ from } e} f_c(i,j) \quad \forall j \in J, \forall e \in E \quad (4)$$

$$\sum_{\text{input } i \text{ to Process}} F(i) = \sum_{\text{output } i \text{ from Process}} F(i) \quad (5)$$

$$\sum_{\text{input } i \text{ to Process}} f_c(i,j) = \sum_{\text{output } i \text{ from Process}} f_c(i,j) \quad \forall j \in J \quad (6)$$

Mass fractions are calculated for each stream considering cyclone and filter efficiencies. Table 2 shows the values obtained and the final expressions for $x(i,j)$. The corresponding equations are incorporated to the mathematical model as non-linear constraints.

The enthalpies values for each stream are expressed as:

$$H_{i,j} = \int_{T_{ref}}^{T_i} C_{p_j}(T) dT \quad \forall j \in J, \forall i \in I_{BS,EI} \quad (7)$$

T_{ref} is the reference temperature adopted (293K). The $C_{p_j}(T)$ [J/gmol K] are the expressions listed in Table 1 for each material. For air, limestone and dolomite the integrated expressions for enthalpies evaluations [J/kg] are:

$$H_{i,air} = \frac{1000}{MW_{air}} \cdot [a_{air} \cdot (T_i - T_{ref}) + \frac{b_{air}}{2} \cdot (T_i^2 - T_{ref}^2) + \frac{c_{air}}{3} \cdot (T_i^3 - T_{ref}^3) + \frac{d_{air}}{4} \cdot (T_i^4 - T_{ref}^4)] \quad \forall i \in I_{BS,EI} \quad (8)$$

$$H_{i,lim} = \frac{1000}{MW_{lim}} \cdot [a_{lim} \cdot (T_i - T_{ref}) + \frac{b_{lim}}{2} \cdot (T_i^2 - T_{ref}^2) + c_{lim} \cdot (\frac{1}{T_{ref}} - \frac{1}{T_i})] \quad \forall i \in I_{BS,EI} \quad (9)$$

$$H_{i,dol} = \frac{1000}{MW_{dol}} \cdot [a_{dol} \cdot (T_i - T_{ref}) + \frac{b_{dol}}{2} \cdot (T_i^2 - T_{ref}^2) + c_{dol} \cdot (\frac{1}{T_{ref}} - \frac{1}{T_i}) + \frac{d_{dol}}{3} \cdot (T_i^3 - T_{ref}^3) + \frac{e_{dol}}{2} \cdot (\frac{1}{T_{ref}^2} - \frac{1}{T_i^2}) + 2 \cdot g_{dol} \cdot (\sqrt{T_i} - \sqrt{T_{ref}}) + h_{dol} \cdot (\ln T_i - \ln T_{ref})] \quad \forall i \in I_{BS,EI} \quad (10)$$

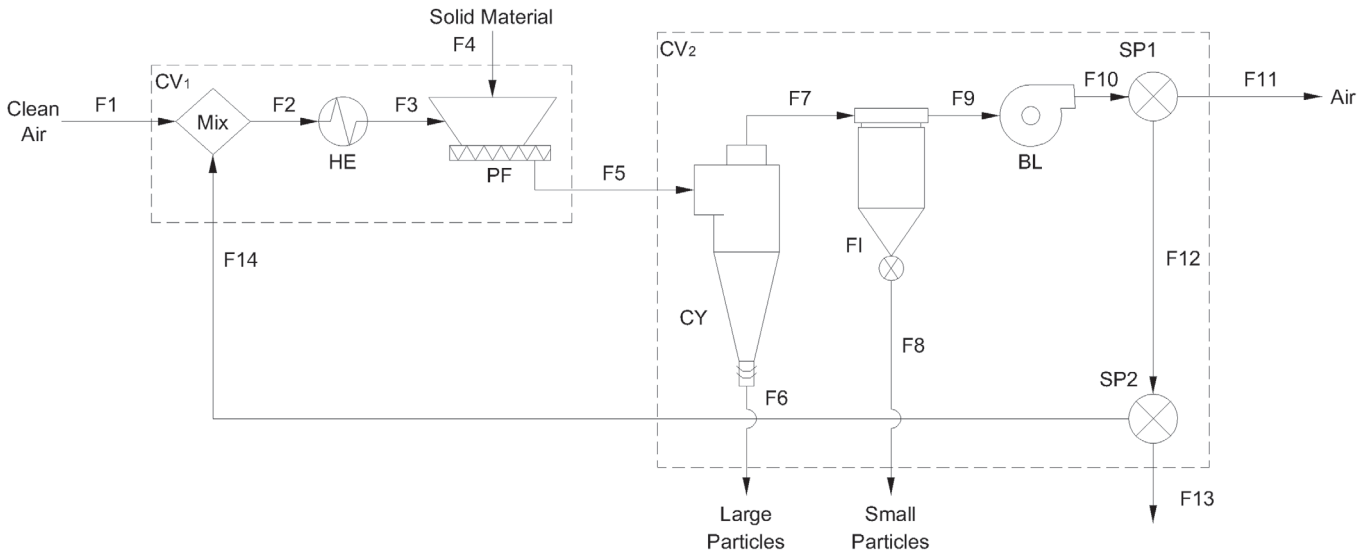


Figure 5. Energy integration scheme (EI). *Mix*: mixer; *SP1*: splitter 1; *SP2*: splitter 2; $F_{i;1...14}$: process streams. CV_1 and CV_2 : control volumes for energy purpose.

Table 2. Mass fractions $x(i,j)$ for BS and EI process streams.

Stream (i)	$x(i,1)$ BS scheme	$x(i,2)$ BS scheme	$x(i,1)$ EI scheme	$x(i,2)$ BS scheme
1	1	0	1	0
2	1	0	$x(3,1)$	$x(3,2)$
3	0	1	$1-x(3,2)$	< Tol.*
4	$1-x(4,2)$	$\frac{F(3)}{F(2)+F(3)}$	0	1
5	0	1	$1-x(5,2)$	$\frac{x(3,2)*F(3)+F(4)}{F(3)+F(4)}$
6	$1-x(6,2)$	$\frac{fc(4,2)-F(5)}{F(4)-F(5)}$	0	1
7	0	1	$1-x(7,2)$	$\frac{x(3,2)*F(3)+F(4)-F(6)}{F(5)-F(6)}$
8	$1-x(8,2)$	$\frac{fc(4,2)-F(5)-F(7)}{F(6)-F(7)}$	0	1
9	$x(8,1)$	$x(8,2)$	$1-x(9,2)$	$\frac{x(3,2)*F(3)+F(4)-F(6)-F(8)}{F(7)-F(8)}$
10	-	-	$x(9,1)$	$x(9,2)$
11	-	-	$x(10,1)$	$x(10,2)$
12	-	-	$x(11,1)$	$x(11,2)$
13	-	-	$x(12,1)$	$x(12,2)$
14	-	-	$x(13,1)$	$x(13,2)$

* Tol.: maximum tolerance of fine particles at cyclone input.

For sawdust and toner the integrated enthalpies [J/kg] are evaluated as:

$$H_{i,saw} = \frac{1000}{MW_{saw}} \cdot Cp_{mean,saw} \cdot (T_i - T_{ref}) \quad \forall i \in I_{BS,EI} \quad (11)$$

$$H_{i,ton} = \frac{1000}{MW_{ton}} \cdot Cp_{mean,ton} \cdot (T_i - T_{ref}) \quad \forall i \in I_{BS,EI} \quad (12)$$

Energy balances were evaluated over the control volumes (CV₁ and CV₂), using the enthalpies expression developed before and the component flow streams (fc(i,j)). The balances for the BS scheme are:

$$Q_{VC_1} = fc(4,1) \cdot H_{4,1} + fc(4,2) \cdot H_{4,2} - F(1) \cdot H_{1,1} - fc(3,2) \cdot H_{3,2} \quad (13)$$

$$Q_{VC_2} = fc(4,1) \cdot H_{4,1} + fc(4,2) \cdot H_{4,2} - fc(5,2) \cdot H_{5,2} - fc(7,2) \cdot H_{7,2} - (fc(9,1) \cdot H_{9,1} + fc(9,2) \cdot H_{9,2}) \quad (14)$$

The corresponding energy balances for EI scheme are:

$$Q_{VC_1} = fc(5,1) \cdot H_{5,1} + fc(5,2) \cdot H_{5,2} - F(1) \cdot H_{1,1} - fc(4,2) \cdot H_{4,2} - (fc(14,1) \cdot H_{14,1} + fc(14,2) \cdot H_{14,2}) \quad (15)$$

$$Q_{VC_2} = fc(5,1) \cdot H_{5,1} + fc(5,2) \cdot H_{5,2} - fc(6,2) \cdot H_{6,2} - fc(8,2) \cdot H_{8,2} - (fc(11,1) \cdot H_{11,1} + fc(11,2) \cdot H_{11,2}) - (fc(13,1) \cdot H_{13,1} + fc(13,2) \cdot H_{13,2}) - (fc(14,1) \cdot H_{14,1} + fc(14,2) \cdot H_{14,2}) \quad (16)$$

The process energy consumption are represented by the Q_{VC_1} [J/s] values, while Q_{VC_2} [J/s] represents the energy losses.

3.2. Models for the design of high efficiency cyclones

The cyclone efficiency is defined as the mass percentage of incoming particles that are separated or retained at the bottom of the cyclone. There is not a simple theoretical method for its calculation, so different theories have been developed

based on experimental approaches, Dirgo and Leith (1985), Leith and Licht (1972) and Gil, Romeo, and Corts (2002). These theories establish different relations between variables such as cyclone efficiency, particles size, geometric ratios, gas flow rate and operating temperatures. The particle removal efficiency is based on the number of turns or external vortices that the gas stream takes inside the cyclone. One of the relationship between cyclone fractional efficiency and some parameters and operating conditions is given by:

$$\eta(k) = 1 - \exp\left[-\frac{\pi \cdot N \cdot \rho_p \cdot D_p^2 \cdot V_e}{9 \cdot \mu \cdot b}\right] \quad (17)$$

where N is the number of turns or vortices, ρ_p the particle density (kg/m³), D_p the particle diameter (m), V_e the air inlet velocity (m/s) and μ the viscosity of air (Pa·s). The number of turns or vortices N , is defined as the turns present in the gas stream inside the cyclone. This parameter can be estimated from the dimensions of the cyclone as follows:

$$N = \frac{1}{a} \cdot \left[h + \frac{z}{2}\right] \quad (18)$$

Equation 17 provides the cyclone efficiency for a given particle size k . To obtain the global cyclone efficiency the particles distributions (Figure 1) and the specific cutting size must be considered. A reasonable value for the cutting size is 10 μm-diameter, meaning that at least 50% of the particles greater than this value will be retained in the cyclone. The overall efficiency of the cyclone is expressed as:

$$\eta_{C1} = \sum_k \eta(k) \cdot m(k) \quad \forall k > 10 \mu m \quad (19)$$

where $m(k)$ is the mass percentage of particles of size k according to the corresponding particle distribution (Figure 1). This efficiency is valid for low solids contents, because ignores the particle-particle interaction and should be used for dust concentrations not exceeding 2 g/m³ (C1), Hoffmann and Stein (2008), Gil, Romeo, and Corts (2002) and Echeverri Londoño (2006). Previous work showed that the cyclone separation

efficiency increases with the initial powder concentration, Paulo, Diaz, and Barbosa (2013); Trefz and Muschelknautz (1993). The highest solid concentration tested on Paulo, Diaz, and Barbosa (2013) is considered for this paper ($C_2 : 500 \text{ g/m}^3$). This operating value is an equality constraints in the mathematical model. To correct the cyclone efficiency for higher particle concentrations (C_2) the Smolik relation is necessary, Hoffmann and Stein (2008), Svarovsky (1981) and Gil, Romeo, and Corts (2002):

$$\eta_{C2} = 1 - (1 - \eta_{C1}) \cdot (C1/C2)^{0.182} \quad (20)$$

Temperature changes affect the gas density and viscosity. These changes also affect the cyclone efficiency, Echeverri Londoño (2006), Gimbut et al. (2005), Magwai and Bosman (2008), Chuah, Gimbut, and Choong (2006) and Bohnet (1995). The corrections are expressed between the reference temperature T_{ref} and the cyclone inlet temperature T_c (363K):

$$\eta_{C3} = 1 - ((1 - \eta_{C2}) \cdot (\mu_{T_c}/\mu_{T_{ref}})^{0.5}) \quad (21)$$

$$\eta_{C4} = 1 - ((1 - \eta_{C3}) \cdot ((\rho_p - \rho_{T_c})/(\rho_p - \rho_{T_{ref}}))^{0.5}) \quad (22)$$

where ρ_{T_c} is the air density evaluated at the cyclone temperature, and $\rho_{T_{ref}}$ is the air density evaluated at the reference temperature.

The air input velocity (V_e) is a critical variable. Low values can produce sedimentation of larger particles, whereas large velocities can re-suspend the collected material. The second situation decreases the operation cyclone efficiency. This phenomenon is considered by the saltation velocity (V_s). A semi-empirical expression for this variable is considered, Kalen and Zenz (1974), Berezowski and Warmuziski (1993) and Chen and Chen (1995):

$$V_s = \frac{4.913 \cdot W \cdot K_b^{0.4} \cdot D_c^{0.067} \cdot \sqrt{V_e^2}}{\sqrt[3]{1 - K_b}} \quad (23)$$

where $K_b = b/D_c$. W is the equivalent velocity, parameter that depends on the fluid and the particle properties:

$$W = \sqrt[3]{\frac{4 \cdot g \cdot \mu \cdot (\rho_p - \rho)}{3 \cdot \rho^2}} \quad (24)$$

Experimental correlations show that for maximum cyclone efficiency the gas inlet velocity should be 1.25 times the saltation velocity, this limit can be extended up to 1.35 to avoid re-suspension of collected particles, Salcedo and Cándido (2001):

$$V_s \cdot 1.25 \leq V_e \leq V_s \cdot 1.35 \quad (25)$$

The heat exchanger preheats the air entering the process to reach a temperature of 363K at the entrance of the cyclone. The flow rate input to the process is constant ($Q_e : 2.4 \text{ m}^3/\text{s}$).

$$Q_e = V_e \cdot a \cdot b \quad (26)$$

Diverse models to estimate cyclones pressure drops are available in literature. These models involve different degrees of complexity and vary in terms of the aspects that they account for. In this study, the task is to estimate the pressure drop in a smooth-walled cyclone operation at low solids loading, so the simple empirical model of Shepherd–Lapple is consider. Hoffmann and

Table 3. Geometric constraints for different cyclones models.

Dimensionless ratio	Stairmand (ST)	Swift (SW)
a/Dc	0.50	0.44
b/Dc	0.20	0.21
S/Dc	0.50	0.50
Ds/Dc	0.50	0.40
h/Dc	1.50	1.40
z/Dc	2.50	2.50
H/Dc	4.00	3.90
B/Dc	0.38	0.40

Stein (2008) listed the models than can be used when the cyclone has rough walls, e.g. due to refractory lining, or is operating at high loading. In these more complex cases the models of Barth, Casal-Martinez, Muschelknautz or Stairmand would be more appropriate, Shepherd and Lapple (1939), Hoffmann and Stein (2008) and Cortés and Gil (2007).

The pressure drop in the cyclone studied is evaluated by Shepherd–Lapple model:

$$\Delta P = 1/2 \cdot \rho \cdot V_e^2 \cdot NH \quad (27)$$

where ΔP is the pressure drop in pascals and NH the number of speed heads at the cyclone inlet:

$$NH = K \cdot \frac{a \cdot b}{D_s^2} \quad (28)$$

where K is a constant, equal to 16 for tangential entries cyclones.

Different models of high efficiency cyclones are analysed and compared. Stairmand (ST) and Swift (SW) models are tested in this paper regarding previous results, Paulo, Diaz, and Barbosa (2013). Table 3 shows the different geometric ratios. They are included into the non-linear model as design constraints.

3.3. Optimisation algorithm

Considering the equations and constraints presented in the previous sections the following non-linear optimisation problem (NLP) is formulated for maximizing the cyclone efficiency.

Maximise $Z_{objective} = \eta_{C4}$

s.t.

- Equation (1) – Equation (28)
- Operating values(equality constraints)
- $x(i,j)$ non – linear expressions from Table 2
- Design constraints from Table 3

The cyclone efficiency η_{C4} is selected as the objective variable. The independent variables in the model are the cyclone diameter (D_c), the inlet gas velocity (V_e), and the stream flows ($fc(i,j)$). More than hundred dependent variables are necessary depending on the process scheme. Air and solid process input are kept as constant. Some operating conditions, such as the cyclone inlet temperature, are added as equality constraints. A maximum pressure drop restriction is added (2,488.2 Pa).

Table 4. Optimal process variables for BS scheme. C2:500 g/m³.

Variables	Limestone-1		Limestone-2		Dolomite		Sawdust		Toner	
	ST	SW	ST	SW	ST	SW	ST	SW	ST	SW
$Z_{objective} = \eta_{C4}$ (%)	90.66	90.45	92.97	92.66	85.89	85.68	99.90	99.87	81.50	78.27
D_c (m)	0.99	1.06	0.99	1.06	1.00	1.06	0.93	1.06	0.93	1.06
V_e (m/s)	24.79	23.30	24.79	23.30	24.36	23.28	28.00	23.30	28.00	23.30
ΔP (Pa)	1,939.8	2,488.2	1,939.8	2,488.2	1,873.5	2,483.5	2,475.6	2,488.2	2,475.6	2,488.2
a (m)	0.49	0.46	0.49	0.46	0.50	0.46	0.46	0.46	0.46	0.46
b (m)	0.20	0.22	0.20	0.22	0.20	0.22	0.18	0.22	0.18	0.22
H (m)	3.95	4.12	3.95	4.12	3.98	4.12	3.72	4.12	3.72	4.12
$x(6,1)_{cyclon\ outlet}$ (%)	95.51	95.41	96.58	96.43	93.36	93.26	99.95	99.93	91.47	90.12
$x(8,1)_{filter\ outlet}$ (%)	99.77	99.76	99.82	99.82	99.65	99.64	99.998	99.997	99.54	99.46

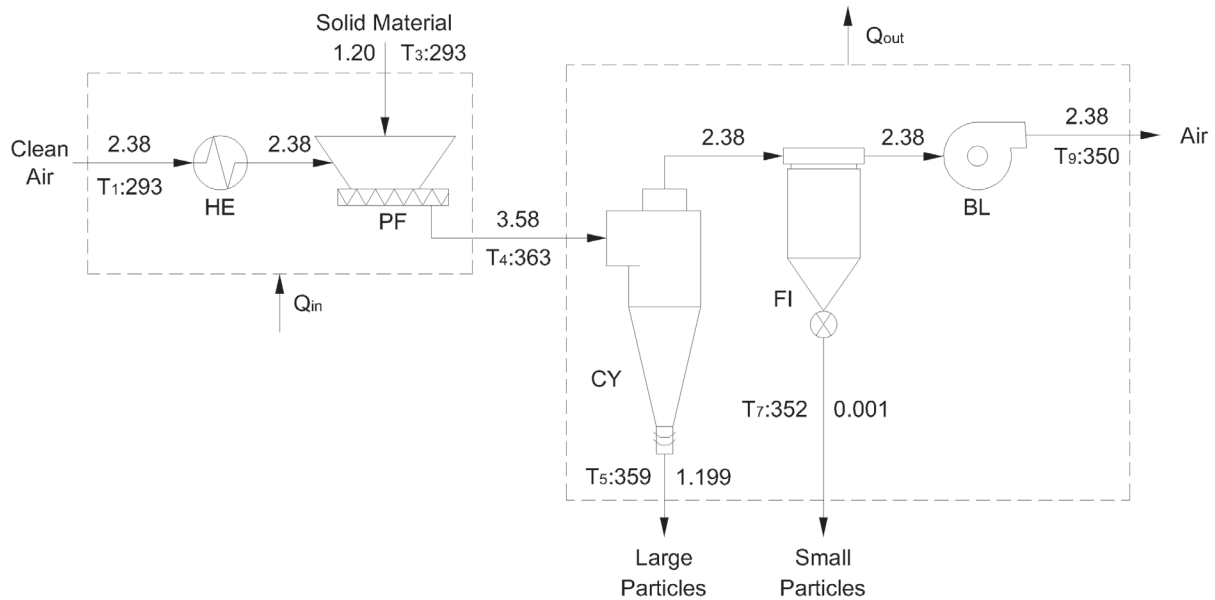
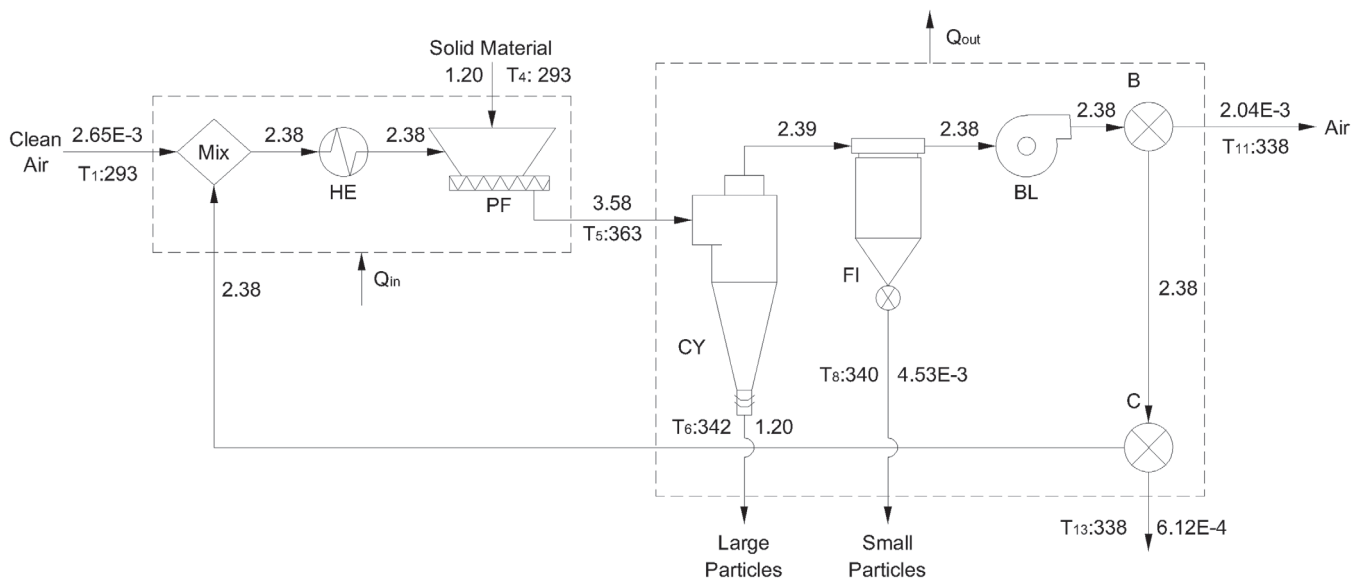

Figure 6. Optimal process flows. BS scheme for sawdust particles. Flows [kg/s].

Figure 7. Optimal process flows. EI scheme for sawdust particles. Flows [kg/s].

Table 5. Energy consumptions for different process schemes and materials.

Variables	Dolomite		Limestone-1		Limestone-2		Sawdust		Toner	
	BS	EI	BS	EI	BS	EI	BS	EI	BS	EI
Q_{CV1} (MWh)	865.85	375.02	865.85	375.05	878.11	387.41	18,450.62	7,815.22	990.13	499.74
Q_{CV2} (MWh)	130.91	130.87	130.28	131.13	133.20	133.15	1,134.24	557.53	142.01	142.29
Energy saving (MWh)	490.83		490.81		490.70		10,635.54		490.39	
Energy saving (US\$/h)	392.66		392.65		392.56		8,508.43		392.31	

Lower and upper variable bounds are included as inequality constraints. The purpose is to find the optimal flux distributions, the energy consumptions and the design variables that maximise the cyclone efficiency.

Twenty cases are solved, varying the solids materials, the cyclone models, and the process schemes. For the BS, each NLP includes 188 equations and 167 variables. For the EI, the problems solved include 246 equations and 211 variables. The non linear programming problems are solved through the Generalised Reduced Gradient Method, implemented in CONOPT, GAMS (Brooke, Kendrick, and Meeraus 2005; Edgar, Himmelblau, and Lasdon 2001). The gradients of the constraints and the objective function are employed in the non-linear problem to determine the search direction in the space of variables to estimate (cyclone diameter, height and width of the tangential inlet, input velocity, streams flows, etc.).

4. Results and discussions

Table 4 shows the key process variables obtained for the basic scheme.

The highest cyclone efficiency is achieved for sawdust (>99%), and the lowest for toner (78%). These results are consistent and concordant with Figure 1, showing that sawdust has the largest particles, and toner has the lowest particle diameters. For limestone-1 and limestone-2 high efficiencies values are achieved (>90%), whereas for dolomite an acceptable efficiency value is obtained (86%). ST cyclone model is slightly more efficient than SW model, being the difference much appreciable for toner. The results for the principal cyclone dimensions are consistent for each model. ST cyclones have a mean diameter value of 0.97 m over the materials studied, and 1.06 m for SW models. The total mean cyclone height is 3.86 m for ST and 4.12 m for SW. These results can project the use of the same cyclone size for the different materials, providing versatility to the process. The ST cyclone result more efficient and smaller than the SW model.

For all the cases studied, the combination cyclone-filter proved to be highly efficient to classify particulate matter, achieving a final purity of air >99.5% at the end of the process. Figures 6 and 7 present the optimal stream flows obtained for sawdust for BS and EI schemes.

The flow of air to the process is 68,548.9 ton/year, and the input of solid material is 34,560.0 ton/year for both schemes. The plant is considered continuous, operating 8,000.0 h per year. The cyclone separates 34,527.1 ton/year of large particles for the BS scheme, and 34,367.3 ton/year for the EI scheme. The difference is <0.5%. The savings in the clean air stream between the BS and EI schemes is greater than 99%. The filter separates 31.3 ton/year of small particles for the BS scheme. For the EI

scheme, the filter is six time more efficient and separates 192.7 ton/year.

Table 5 shows the result of the energy balances and the cost estimation of the energy savings. The values listed correspond to ST model cyclone. The plant energy consumptions (Q_{VC1}) and energy losses (Q_{VC2}) are represented in MWh, while the energy savings values are expressed in MWh and US\$/h. For the energy cost evaluations, the considered industrial electricity tariff is 800 US\$/MWh, supplied by a local cement plant.

In all cases studied, EI scheme consumes 43% less energy than the BS scheme. The energy savings are calculated comparing the consumptions (Q_{CV1}) between BS and EI schemes. A mean energy saving of 393 US\$/h is obtained for limestone 1 and 2, dolomite and toner. For the same materials, the energy losses (Q_{CV2}) remain constant between BS and EI schemes (130–142 MWh). It is important to note that sawdust has a better performance compared with the others materials. Its energy saving value is substantially higher (8,508 US\$/h), and its energy losses are reduced a 49% comparing the EI and BS schemes.

5. Conclusions

A non-linear programming problem is developed to optimise the efficiency collection in a 34,560.0 ton/year particles separation–classification plant. The generalised reduced gradient method, is used to find the optimal process design variables. Four different materials with a wide diameters range are studied (0.1–1000 μ m). A high process particle loading is used (500 g/m³). The objective function is defined to maximise the cyclone collection efficiency. Six operation and geometric variables are selected as manipulated variables. Stairmand's cyclone proved to be more efficient than Swift, for all materials studied. The highest cyclone efficiency is achieved for sawdust (> 99%). The cyclone separates more than 34,000 ton/year of particles >10 μ m which represent and overall process efficiency superior to 99.9%. The integration scheme consumes 43% less energy than the basic scheme for all cases studied, reducing the environment process impact. Future extension of this work includes a computational fluid dynamics simulation to correlate the numerical model results.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

The authors gratefully acknowledge financial support from the Laboratory of Microparticles of the Universidad Nacional del Centro de la Provincia de Buenos Aires (UNICEN), from the Centro de Investigaciones en Física e Ingeniería of the National Research Council (CIFICEN-CONICET), and

from the Planta Piloto de Ingeniería Química (PLAPIQUI-CONICET-UNS).

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