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Polynomial Functions Resulting from the Multiplication of Curves in the Framework of the Research and Study Paths

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Date of publication: February 24th, 2015

Edition period: February 2015-June 2015

To cite this article: Llanos, V.C., Otero, M.R., & Rojas, E.C. (2015). Polynomial Functions Resulting from the Multiplication of Curves in the Framework of the Research and Study Paths. *REDIMAT*, Vol 4(1), 80-98. doi: 10.4471/redimat.2015.60

To link this article: <http://dx.doi.org/10.4471/redimat.2015.60>

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(Received: 28 July 2014; Accepted: 3 January 2015; Published: 24 February 2015)

Abstract

This paper presents the results of a research, which proposes the introduction of the teaching by Research and Study Paths (RSPs) into Argentinean secondary schools within the frame of the Anthropologic Theory of Didactics (ATD). The paths begin with the study of Q_0 : *How to operate with any curves knowing only its graphic representation and the unit of the axes?*, which was implemented for two consecutive years by 14-17 year-old students. The results of part of the paths that allows the study of the polynomial functions in the frame of RPS are hereby described. These results justify the introduction of a new pedagogy into schools as proposed by Yves Chevallard in order to study Mathematics in conventional courses of Secondary Education schools in Argentina. The scopes relative to the attitudes of the students, which are necessary to face a questions-based education, are outlined in this work.

Keywords: Anthropologic Theory of Didactics (ATD), Research and Study Paths (RSPs), Polynomial functions, Secondary school

Las Funciones Polinómicas como Resultado de Multiplicar Curvas en el Marco de un Recorrido de Estudio y de Investigación

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(Recibido: 28 Julio 2014; Aceptado: 3 Enero 2015; Publicado: 24 Febrero 2015)

Resumen

Se presentan resultados de una investigación que propone introducir en la escuela secundaria Argentina una enseñanza por *Recorridos de Estudio y de Investigación* (REI) en el marco de la Teoría Antropológica de lo Didáctico (TAD). El recorrido inicia con el estudio de Q_0 : *¿Cómo operar con curvas cualesquiera, si sólo se conoce su representación gráfica y la unidad en los ejes?* y se implementa durante dos años consecutivos con estudiantes entre 14 y 17 años. Se describen aquí los resultados de una parte del recorrido que permite estudiar las funciones polinómicas en el marco de un REI mono disciplinar. Los resultados obtenidos justifican la inserción de la nueva pedagogía escolar propuesta por Yves Chevallard para estudiar Matemática en clases regulares de la Escuela Secundaria en Argentina. Se destacan en este trabajo, los alcances relativos a las actitudes de los estudiantes para enfrentar una enseñanza basada en preguntas.

Palabras clave: Teoría Antropológica de lo Didáctico (TAD), Recorridos de Estudio y de Investigación (REI), funciones polinómicas, Escuela Secundaria

The teaching of mathematics in secondary school is characterized by the study of answers rather than questions. The main activity and major responsibility during the lesson is in the teacher's hands. Therefore, the teacher is in charge of selecting, introducing and communicating the Mathematical Organizations (MOs), which constitute the curriculum. Students do not take any responsibility in this process more than the reproduction of knowledge, which is presented as finished, transparent, unquestionable and, to some extent, dead. This phenomenon has been called *monumentalism of knowledge* (Chevallard, 2004, 2007, 2009) and one of its main characteristics is that it presents the MOs as finished works, i.e. as already created and meaningless objects.

The absence of questions and of genuine mathematical activity at school is one of the biggest difficulties in reversing the current teaching of mathematics, and therefore, it is the problem that we seek to address. For that reason, the *Research and Study Paths* (RSPs) are proposed in the frame of the Anthropologic Theory of Didactics (ATD) as a tool to face to the problem of the *monumentalism* and its unavoidable consequences (Ibid, 2009, 2013). The RSPs take the study of questions in the first place. One of the objectives of this research is to introduce and develop these RSPs in real mathematical lessons in secondary education in Argentina.

Some pioneering researches in ATD had developed RSP in specially designed classes in college, which were far from the traditional university lessons, as them of Barquero (2009), Barquero, Bosch & Gascón (2011, 2013), Ladage & Chevallard, (2011), Rodríguez, E.; Hidalgo, M.; Sierra, T. (2013), Serrano, Bosch & Gascón, (2010), Sierra, Bosch & Gascón, (2012). On the other hand, Fonseca (2011a, 2011b), Fonseca & Casas (2009) they analysed the problems in the transition from secondary school to college from a RSP that begins in secondary school and continues in the following year in college. The research experiences carried out by Fonseca, Pereira & Casas (2011), García, Bosch, Gascón & Ruiz (2005) are set in secondary schools. The researches study various mathematical organizations and other disciplines, for being co-disciplinary paths. In the previously mentioned cases, the RSP were extra-curricular activities performed in special workshops.

The RSP instruction in conventional mathematical courses in both secondary school and college is more recent, though some results are available. In the University, Costa, Arlego & Otero (2014) they elaborated a

proposal for vector calculus study in a context of a Faculty of Engineering in the conventional courses during four months. The study of Parra, Otero & Fanaro (2013a, 2013b) presents the results of a bi-disciplinary RSP in Mathematics and Economy, which was developed during eight months in conventional Mathematics lessons in the last year of Secondary School. The research of Donvito, Sureda & Otero (2013) describes the results of a bi-disciplinary RSP also in Mathematics and Economy but implemented in three different institutions in conventional classes: a private management state school, a state school for adults and a private school.

Contrary to all previous research on the issue, this work suggests the study of Mathematics by means of an RSP in a typical mathematics lesson including the same group of students during a long period of time, i.e. for two years. The whole path develops from a generative question Q_0 : *How to operate with any curves knowing only its graphic representation and the unit of axes?* (Llanos, Otero, 2012, 2013a, 2013b). In this work, there is only a description of the results generated in part of the SRP, arising from a derived question of Q_0 , called Q_2 : *How to multiply several straight line or straight lines and parabolas knowing only its graphic representations and the union in the axes?* The construction of an answer to this question in this part of the SRP leads to the study of the MOs of the polynomial functions.

Theoretical Framework

The Research and Study Paths (RSP) introduce a new epistemology, which returns the meaning and functionality to school mathematics (Chevallard, 2013), while replacing the pedagogy of “*visiting works.*”

Students (X) research and study a question Q guided by a teacher (y) or a set of teachers (Y) with the aim of providing an answer R^\heartsuit to Q . The didactic system S requires tools, resources, works, i.e. it needs to generate a didactic environment M, to produce R^\heartsuit . Chevallard (2009) describes it by means of Herbartian Schema:

$$[S(X; Y; Q) \leftrightarrow M] \leftrightarrow R^\heartsuit$$

The didactic system produces and organizes (\leftrightarrow) the medium M, that will allow to produce (\leftrightarrow) a response R^\heartsuit . The medium M contains the questions generated from Q , pre-built responses accepted by the school

culture, noted as R_i^\bullet (included within the responses could be a book, the Web, a teacher's course, etc.) - Also belonging to M , entities such as O_j , such as works - theories, experimental works, praxeologies, etc.- can be found, considered potentially useful for the formulation of responses R and obtaining from it something useful to generate R^\bullet . A Research and Study Path (RSP) is represented by what Chevallard (2004, 2009, 2013) calls Herbartian Scheme (extended):

$$[S(X, Y, Q) \leftrightarrow \{R_1^\bullet, R_2^\bullet, R_3^\bullet, \dots, R_n^\bullet, Q_{n+1}, \dots, Q_m, O_{m+1}, \dots, O_p\}] \leftrightarrow R^\bullet$$

Within any RSP, knowledge is produced by answering a mathematical question Q_0 that is meaningful to the study community; and providing that such an answer will require the elaboration of new Q_i derivative questions. At the beginning of RSP, the teacher presents a question Q , whose study will lead to encounter various mathematical organizations and / or other disciplines, depending on whether it is a monodisciplinary or co-disciplinary path. Therefore, a chain of questions and answers which are the core of the study process is established $P = (Q_i, R_i)$ (Otero, 2013). In the present research, the Q_i are mathematical questions.

The implementation of one RSP will need important changes in the dominant pedagogy, which affects the process of study, i.e. its ecology. These modifications impact on the *topogenesis*, *mesogenesis* and *chronogenesis* processes which are central to the didactic analysis proposed by the ATD (Chevallard, 1985, 2011).

The *mesogenesis* is the "construction" of M , using both *external* (internet, books, etc.) and *internal* productions (Otero, 2013). The M medium is constituted by what has been done in the class by the teacher and the students, and not only by the teacher. The place each one takes is known as *topogénesis*. The students' *topos* is broadened as they do not only contribute with their R_x answer, but also with the introduction of any other work they wish to include in M . This change produces other important modifications in the teacher's *topos*, who takes over the role of *study director* of Q , or *research director*. These modifications also produce changes in the *chronogenesis*, in the management and "control" of the didactic times.

The *chronogenesis*, *mesogenesis* and *topogenesis* didactic functions (Chevallard, 1985, 2009) are used to describe the functioning of RSP in the selected courses.

The research questions are: What characteristics of the graphical representation of the polynomial functions can be studied in the RSP? What characteristics of the MO of the polynomial functions can be reconstructed? Which are the scopes and limitations of RSP in relation to the didactic functions?

Research Methodology

The research is a qualitative, ethnographic and exploratory one. The RSP is introduced in a controlled way in secondary school courses in Argentina. A longitudinal study is carried out for two years, i.e. students begin the RSP in the 4th year continuing the following year, in the 5th year. In total, four implementations were done: two of them simultaneously in each year, in two cohorts. In total, 121 students took part in the research and in all the cases the teachers were the researchers. In each class, there were between 25-30 students, so that in the four implementations the number of students increased as was indicated. The groups are stable, that is to say they do not change during the longitudinal study.

Every class the teacher proposes a new question to study. Students construct possible answers to the question, and each group proposes other questions deriving from the initial question. The class decides that it is studied and everything remains registered by the students in your folders. At the end of every class, the teacher obtains all the written productions of the students, organized in the study groups. Later, the different works are scanned and are analyzed, and returned to the students on the following meeting. Thus, the records obtained out of the students-generated answers, provide an important empirical basis for the research team. The lessons are recorded in audios and field notes are taken. The audio recordings enable the teacher/researcher to recover information from the on-going processes in the class which have not been recorded by any other written means, i.e. by the students' protocols or the researchers' notes. Given the total number of audio recordings obtained, only those ones allowing the researcher to gain information which has not been duly justified in the student-written protocols, are transcribed.

Participant and non-participant observation was carried out from the collaboration with colleagues from the research team. In addition, the teacher/researcher elaborated field notes before and after the class. These records allow not only to know to what extent the objectives proposed in each meeting have been achieved, but also to describe the decisions that have been taken throughout the proposed course of study, which had not been previously taken into consideration in the class. Then, the results obtained from the implementation of RSP are analyzed out of the volume generated by the written protocols, the audio recordings and the observations made.

Characteristics of the RSP

The study involved the design, implementation and evaluation of a monodisciplinary RSP (Chevallard, 2009), in which the generative question was specifically mathematical. Therefore, the starting point was Q_0 : *How to operate with any curves, knowing only its graphical representation and the unit of the axes?* (Llanos, Otero, 2013a, 2013b). Possible answers to Q_0 generate potential paths, according to the questions arising from Q_0 , which is called Q_i . They allow a relatively complete coverage of the Mathematics curriculum in the last three years in secondary schools.

The Q_0 question was adapted from a problem proposed by Douady (1999) to study the signs of polynomial functions. According to the type of curve and possible operations between them, different Mathematical Organizations (MOs) can be found in the Mathematics program of secondary education (Llanos, Otero, 2013b). The RSP was developed in secondary school Mathematics lessons, with students attending 4th and 5th year, in which the teachers were the researchers. Its application started in the 4th year and continued in the 5th year, with the same group of 14-17 year-old students. Once Q_0 was introduced, the initial responses of the students determined the possible paths, depending on the previous knowledge of the pupils. Throughout all the longitudinal study, three questions deriving from the generative question Q_0 were developed. These questions depended on the curves and operations considered in each case. Consequently, the starting point has been the multiplication of two straight lines, as the students are more familiar with these curves when the RSP begins. After, operations with parabolas are performed. The elaboration of

possible answers to these questions has allowed the reconstruction of different MOs:

- of polynomial functions of the second degree, developed from the Q_1 question: *How to multiply two straight lines knowing only its graphic representations and the union of the axes?* (Llanos, Otero, 2013a, 2013b);
- of the polynomial functions in general developed from the Q_2 question: *How to multiply several straight line or straight lines and parabolas knowing only its graphic representations and the union in the axes?*, whose results are described hereafter;
- of the MOs of the rational functions developed from the Q_3 question: *How can the quotient between polynomial functions be calculated knowing only its graphic representations and the unit in the axes?* (Otero, Llanos, Gazzola, 2012).

The study develops in three parts (P1, P2 and P3). There is a description of the study of the polynomial functions from Q_2 , which corresponds to the second one (P_2).

The Polynomial Functions in the Framework of a RSP

Questions Q_1 , Q_2 and Q_3 were developed in this order. The results reached by the first situations of study from Q_2 are described in this work. The geometric calculation techniques needed to answer each situation have been previously developed when question Q_1 was studied, and have been adapted by the students in this part of the study. These can be found in the research carried out by Llanos & Otero (2013a). In the three initial situations of Q_2 , we propose:

- a. Which are the “safe points” and the signs of p?
- b. Which could be the most reasonable graphic for p? What features of the graph of p could you justify?

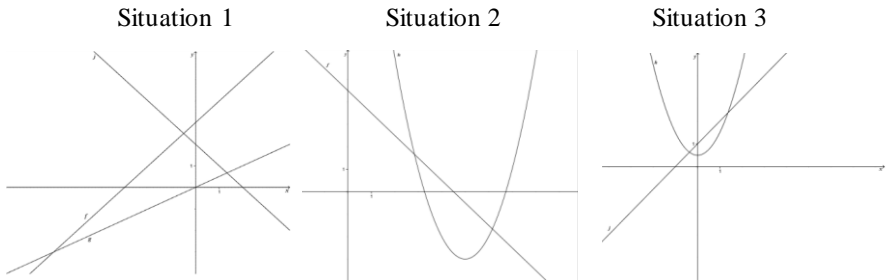


Figure 1. Graphics corresponding to the curves f, g and h respectively

In order to obtain the curve for p it is necessary to identify the signs (C^+ and C^-) and the outstanding points: zeros, ones, minus ones and other multiples of the union. Points are represented in red in Figures 2, 3 and 4.

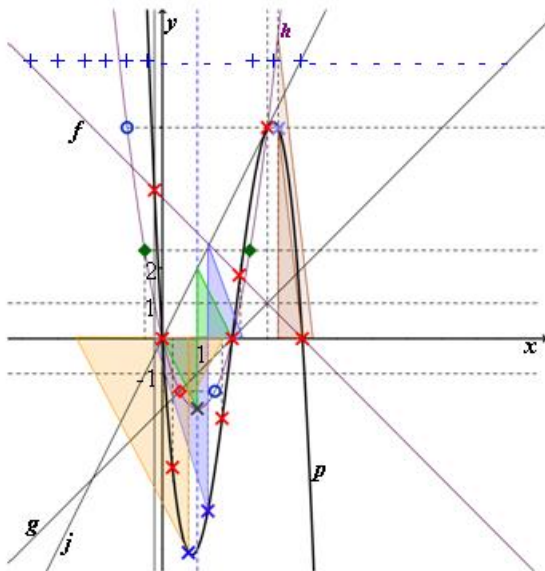


Figure 2. Graphical representation for p , corresponding to the situation 1

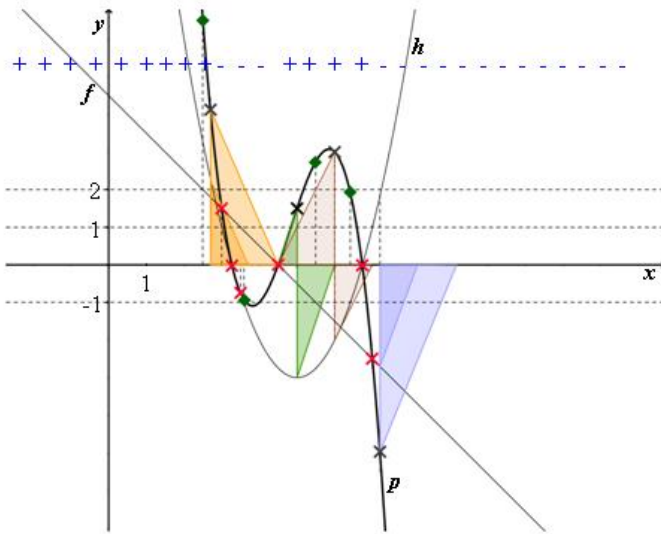


Figure 3. Graphical representation for p , corresponding to the situation 2

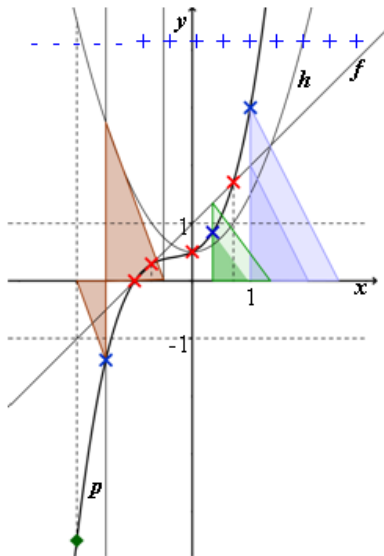
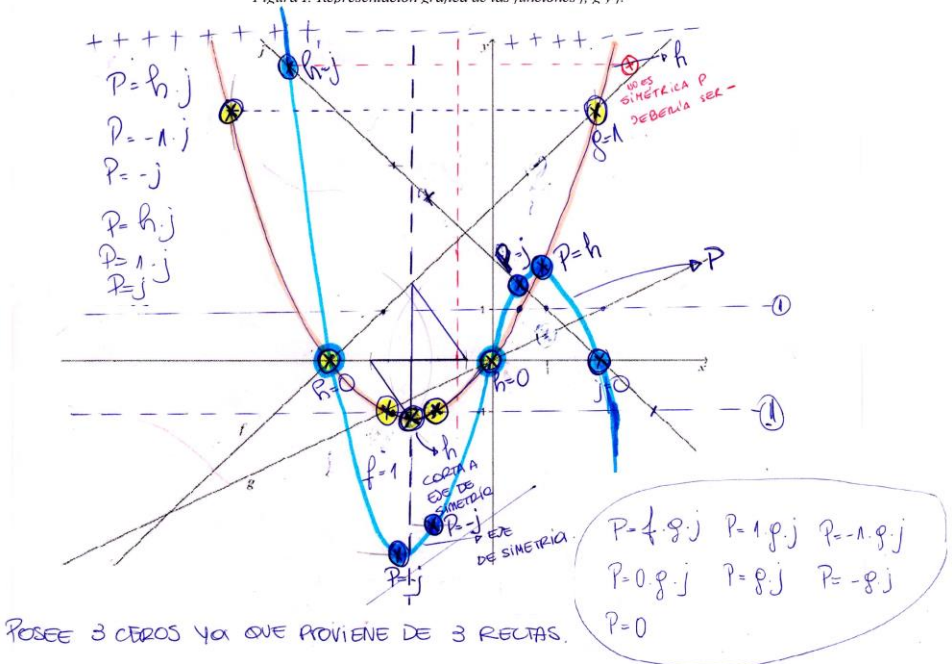


Figure 4. Graphical representation for p , corresponding to the situation 3

The Implementation of RSP: the Study of Q_2

Three protocols from the students participating in the RSP were selected to describe the results of the three first situations. In all of the cases, the students obtained a graphical representation for p by adapting the techniques constructed before, in order to multiply two straight lines. From Q_2 they obtained the generalizations to multiply three straight lines or straight lines with paraboles.

Las funciones f, g y h están dadas por el gráfico de la Figura 1. La función $p=f \cdot g \cdot j$
 Figura 1: Representación gráfica de las funciones f, g y j .



- a) ¿Cuáles son los puntos seguros y los signos de p ?
- b) ¿Cuál podría ser la gráfica más razonable para p ? ¿Qué características de la gráfica de p podrías justificar?

SIEMPRE VA A TENER CEROS PORQUE PROVIENE DE LA MULT. DE 3 RECTAS (SIEMPRE TENDRÁ AL MENOS 3 CEROS).

Figure 5. Protocol corresponding to student A128. Situation 1

In *situation 1* three straight lines f , g and j are multiplied. Student A128 protocol presented in Figure 5 shows how to get the curve of a third degree polynomial function when multiplying three straight lines, only when its graphical representation and the unit of the axis are known. Student A128 first multiplied two straight lines f and g , and obtained the parabola h . Then, he multiplied parabola h with the straight-line j obtaining an approximate representation of p . In order to achieve the graph of p , he analyzed the signs and the "safe" points: zeros, ones and minus ones. Also, the student represented an axis of symmetry in the point between the zeroes (shown in red color in the figure), which shows an important result in Q_1 by means of the proof of the symmetry of the parabola.

Given that the polynomial functions are not symmetrical, the student justified the result by applying the recurrence to the analysis of the signs. Consequently, the study of this problem was disqualified by an inappropriate decision at the *topogenesis* level. The teacher therefore, should have introduced to the students the problem of justifying the symmetry by adapting the techniques developed for Q_1 .

Student A98 is selected to describe the results of *situation 2*. In this case p is the result of multiplying one parabola h with two real zeros and the straight-line f . In Figure 6 the outstanding points and signs appeared. In addition, A98 represents the other points by means of geometrical construction with similar triangles in order to improve the characteristics of the graphical representation for p . Also, the construction of other points of the curve allows the student to analyze the behavior of the branches which are infinite, as indicated in the protocol with arrows.

The protocol of the Figure 7 belongs to student A103. It has been selected to describe the results of *situation 3*. The graphical representation of p corresponds to a function of degree three with a real zero. This student shows to have more control on the identification of the outstanding points, signs and also on the geometric construction that repeats in several points selected by himself. At this stage, the construction of any points by means of similar triangles gains relevance, given that the quantity of "outstanding points" likely to be constructed by the students is insufficient for any approximate graphic representation.

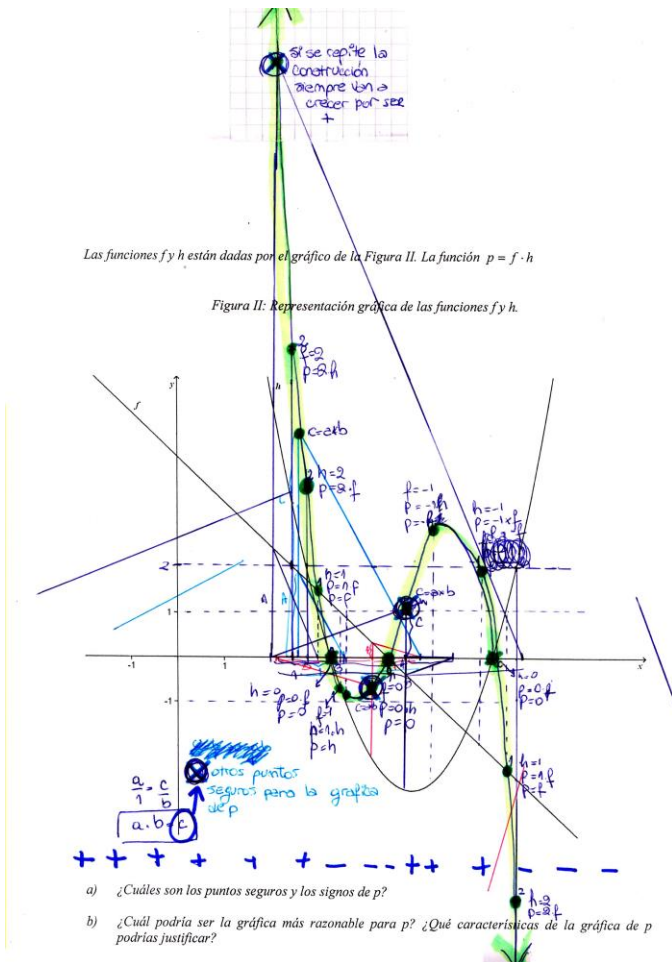


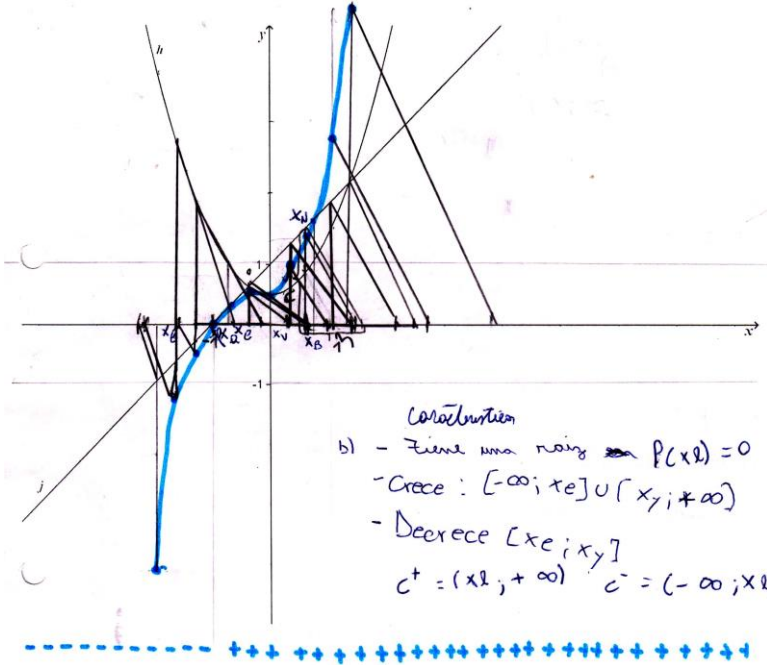
Figure 6. Protocol corresponding to student A98. Situation 2

The paths generated from Q_1 also allow reconstructing the idea that polynomial functions are the product of another function of the same type of a smaller degree. In Q_2 , this idea is regained. In *situation 1* the polynomial function of third degree is the result of multiplying three straight lines; in *situation 2* of the multiplication between one parabola with

two different real zeros and one straight line; and in *situation 3* p is the result of multiplying one parabola that does not have real zeros and one straight line. This analysis allows the study of the parity of the zeros in a natural way.

Las funciones j y h están dadas por el gráfico de la Figura III. La función $p = j \cdot h$

Figura III: Representación gráfica de las funciones j y h .



- a) ¿Cuáles son los puntos seguros y los signos de p ?
- b) ¿Cuál podría ser la gráfica más razonable para p ? ¿Qué características de la gráfica de p podrías justificar?

Puntos seguros:

$$P(x_2) = 0 \cdot h = 0$$

$$P(x_4) = 1 \cdot h = h$$

$$P(x_6) = j \cdot 1 = j$$

$$P(x_8) = j \cdot 1 = j$$

$$P(x_8) = -1 \cdot h = -h$$

$$P(x) = j \cdot h$$

puntos por construcción.

$$h(x_n) = j(x_n) \cdot h(x_n)$$

$$\frac{j(x_n)}{1} = \frac{c}{h(x_n)} = j(x_n) \cdot h(x_n) = 1 \cdot c$$

Figure 7. Protocol corresponding to student A103. Situation 3

The study of Q_2 goes beyond the analysis of the different graphs of the polynomial functions. Also, the algebraic multiplication between straight lines, parabolas and straight lines or among parabolas requires the adaptation of the techniques studied in Q_1 which allowed the construction of algebraic expressions for any polynomial functions in an almost natural way (Llanos, Otero, Bilbao, 2011). The problem of the zeros multiplicity and the property of the existence of at least one real zero in the polynomial functions of odd degree is justified by the multiplication between one straight line and another polynomial function. In addition, operations with polynomials were studied, recovering the characteristics of the graphical representation for the operations when the algebraic expression of p is known. These results are not displayed in the present work, though they are mentioned as a means to justify the relevance of this part of the path.

Some Results

The viability of RSP in the study of polynomial functions in secondary school is justified by the results presented in this work. All the mathematical techniques necessary to construct the characteristics of the graphical and analytic representation are an adjustment of those obtained in the first part of the path. The initiation generated by the geometrical multiplication of the curves, displayed the importance and quality of the mathematical work produced.

As regards the didactic functions in this second part of the longitudinal study, it is possible to highlight:

- In the *topogenesis* level, the distribution of spaces is improved between the teacher and the students. Contrary to the beginning, students did not resist the path, even when the teacher did not provide further explanations. Students took over their place in the process of construction of responses, but the generation of questions was the teacher's responsibility.
- In the *mesogenesis* level, students constructed mathematical concepts studied with the teacher (director of the study), and they also introduced modifications in the milieu. For example, in Situation 1 they proposed the problem of symmetry of the curve.

Unfortunately, one inappropriate decision of the teacher cut off this path rather than promoting it.

- As regards the *chonogenesis*, the time invested in this part has been recovered. Therefore, in order to construct the characteristics of the graphical and analytical representations, the students have adapted the techniques developed during the first part of the RSP, into those of a higher degree –in an almost natural way–.

In spite of the limitations presented, the results obtained have been encouraging compared to the ones shown within the traditional pedagogy. This analysis shows how the techniques and strategies of resolution that are constructed in a path can be generalized and re-used in others. Furthermore, these results justify the importance of continuing the study of other OMs of the program; and within the present research, the students made progress regarding the rational functions.

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