



Water sustainability: A systems engineering approach to restoration of eutrophic Lakes

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ABSTRACT

We address restoration of eutrophic lakes and reservoirs through the formulation of dynamic optimization problems subject to complex PDAE systems representing biogeochemical processes in the water bodies. The model includes phytoplankton, zooplankton, fish, nutrients, DO, particulate and dissolved carbon dynamics. The PDAE has been transformed into an ordinary differential algebraic equation system by spatial discretization into two water layers. An optimal control problem for the implementation of three different restoration techniques and their combinations has been formulated within a control vector parameterization approach. Numerical results for the different problems provide optimal profiles for tributary deviation flowrate through a nearby wetland, aeration rates and fish removal rates, as restoration strategies.

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1. Introduction

Access to clean, sustainable supplies of water is essential for humans, mainly for drinking water and for the operation and growth of major natural resource sectors, such as energy, mining, forest and agriculture. Alterations to the timing and volumes of flow, quality, and temperature of freshwater affect both aquatic and terrestrial ecosystems. To safeguard water resources for present and future uses, water bodies must be managed with maintenance of the integrity of ecosystems as a core principle.

During the last decades, eutrophication has become the most serious environmental problem in many lakes and reservoirs (Schindler, 2006), especially those located next to densely populated or agricultural areas. Increased nutrient loadings are most commonly due to excessive use of fertilizers, malfunctioning septic systems, poor aeration system and improper waste disposal within the watershed. Excessive growth of phytoplankton (algal blooms), associated to high nutrient levels, can produce blockage of water-filters, unpleasant odor and taste, aesthetics problems, decrease in biodiversity, fish mortality and the potential production of toxins. In a recent study (EPA 841-R-09-001, 2010), the US Environmental Protection Agency has investigated the overall condition of US

lakes. They have found that even though nearly 75% of the 800 lakes sampled 30 years ago showed either improvement or no change in phosphorus levels and trophic status under population increase, mainly due to wastewater treatment and other pollution control activities, 20% of the currently investigated lakes have high nutrient concentrations. Regarding their biological condition, 21% of the lakes are in fair condition and 22% are in poor biological condition, based on an index of phytoplankton and zooplankton taxa loss. This study has also revealed that microcystin, an algal toxin produced by cyanobacteria that can severely compromise human and animal health, was found in about one third of lakes and, at levels of concern, in 1% of the lakes.

Much effort has been devoted to reduce external nutrient loading from point sources, through the design and construction of wastewater treatment plants (Cristea, de Prada, Sarabia, & Gutiérrez, 2011; Faria & Bagajewicz, 2011; Hostrup, Harper, & Gani, 1999; San Román, Bringas, Ortiz, & Grossmann, 2007). This has led to the improvement of water quality and ecological state (Jeppesen et al., 2005). However, eutrophication in water bodies remains a major problem and the main pollution input is associated to nonpoint sources, mainly from agricultural activities, resulting in high nitrogen and phosphorus loading (Bøgestrand, Kristensen, & Krovang, 2005). Furthermore, increased eutrophication is a key feature in biofuel production from energy crops as compared to fossil fuels (Bringezu et al., 2009) and the life cycle wide emissions of nutrients critically depend on the application and losses of fertilizers during the agricultural production of biofuel feedstocks.

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Even though external loading could be reduced sufficiently, chemical and biological reactions that take place within water bodies might delay recovery. To accelerate recovery, several in-lake restoration strategies have been proposed and widely applied in the last decades (Cooke, Welch, Peterson, & Newroth, 1993). Aeration of the hypolimnion is a well-known in-lake strategy to reduce internal loading of phosphorus (release from the sediment in anoxic conditions), which may delay lake recovery for decades (Søndergaard, Jensen, & Jeppesen, 2003). Biomanipulation, especially by fish removal to allow the increase of zooplankton pressure on phytoplankton, is also an important part in many restoration programs and, since 1980 it has been applied with several successful results. Søndergaard et al. (2007) report results from 56 fish removal projects in Denmark and the Netherlands during the past 20 years. However, in-lake restoration techniques have not always been successful in the long term and this fact can be associated to both the lack of modeling tools to estimate their effects and the lack of monitoring. Experimental analysis of this process is time-consuming and monitoring at length is highly costly to carry out. The development of ecological water models can help to evaluate management strategies in both the short and the long term. In this sense, mechanistic models of different degrees of complexity have been proposed in the last years (Arhonditsis & Brett, 2005a, 2005b; Chen et al., 2002; Estrada, Parodi, & Diaz, 2009a, 2009b; Fragoso, Motta Marques, Collischonn, Tucci, & van Nes, 2008; Hamilton & Schladow, 1997; Omlin, Brun, & Reitchert, 2001). Regarding modeling of restoration strategies, Sagehashi, Sakoda, and Suzuki (2000, 2001) propose a water quality simulation model to study the long-term stability of an ecological system in a hypothetical water ecosystem, after biomanipulation, with data of mesocosms experiments, through repeated simulations. Krivtsov, Goldspink, Sigeo, and Bellionger, (2001) have extended the model “Rostherne” including zooplankton and fish dynamics to simulate the application of top-down control strategies. More recently, Gurkan, Zhang, and Jørgensen (2006) have applied a structurally dynamic simulation model for examining the effects of two different restoration approaches, aeration of the hypolimnion and biomanipulation, to improve water quality in Fure Lake (Denmark). Prokopkin, Gubanov, and Gladishev (2006) have studied fish removal effect on the biomass of cyanobacteria in a small reservoir in Russia through a simplified eutrophication simulation model. More recently, Estrada et al. (2009b) have developed a mechanistic ecological water quality model within an optimization framework for the determination of optimal policies for biomanipulation in a eutrophic reservoir.

In this work, we formulate a mechanistic eutrophication model for inclusion within an optimal control problem to plan different restoration actions and compare costs for the different in-lake restoration strategies. The model proposed by Estrada et al. (2009a) is extended to include state differential variables describing the behavior of higher links in the trophic chain; namely, two zooplankton groups and three size classes of zooplanktivorous fish. Additional state variables are introduced to better describe internal phosphorus and nitrogen content in phytoplankton groups, as well as particulate and dissolved organic carbon. These equations, together with mass balances for three phytoplankton groups, dissolved oxygen and nutrients, provide a detailed description of the main biogeochemical processes that take place within water bodies. The resulting model is a partial differential algebraic equations (PDAE) system, in which algebraic equations stand for forcing functions profiles, such as temperature, solar radiation, river inflows and concentrations. The PDAE is transformed into an ordinary differential equation system by spatial discretization in horizontal layers. The objective function is the minimization of a weighted sum of the difference between phytoplankton concentration and a desired value of phytoplankton concentration and of phosphorus concentration and a desired

value of phosphorous concentration in the water body along a time horizon of 1 year and a half. Clearly both desired concentrations should be below eutrophication limits. Optimization variables are associated to the reduction of nutrient loading by deviation to a nearby wetland (tributary flowrate to wetland) and to in-lake restoration strategies. The inclusion of additional mass balances allows direct handling of in-lake techniques through oxygen addition rate (aeration) and fish removal rate (biomanipulation) as optimization variables. The DAE optimization problem is formulated within a control vector parameterization framework (Process Systems Enterprise, 2009). The present study has been performed on Paso de las Piedras Reservoir, which is the drinking water source for more than a 450,000 population in Argentina. Numerical results provide optimal profiles for restoration actions, as well as associated costs and a quantitative estimation of restoration effects on the water body, along a middle term time horizon.

2. Process description

Reservoirs, which are created by damming streams, may be eutrophic because nutrients in the previous stream floodplain are released into the water column. This means that high phosphorus levels in these water bodies may be attributed to both high external loadings from tributaries and runoff from nearby lands, as well as to internal loadings from a sediment pool of phosphorus (Marsden, 1989; Søndergaard et al., 2003).

2.1. Case study

Our case study is Paso de las Piedras Reservoir, located in Argentina at 38°22'S and 61°12'W. This artificial water body was built by damming the Sauce Grande River to supply drinking water to more than 450,000 inhabitants of two cities in Argentina and for industrial purposes at a petrochemical complex nearby. It has two tributaries, Sauce Grande River and El Divisorio Stream, which run through an important agricultural area in the country, with a drainage basin area of 1620 km². The lake has a coastline perimeter of 60 km and a surface area of 36 km², while its mean depth is 8.2 m and its retention time is 4 years. Another distinctive feature of this water body is that there is no stratification (Intartaglia & Sala, 1989), mainly due to the fact that the reservoir is wind-exposed throughout the entire year. The high discharge of plant nutrients (mainly phosphorus, nitrogen and silica) by the tributaries into the reservoir, together with the release of nutrients from the previous stream's floodplain and high retention times has made this water body eutrophic. This fact causes recurrent algal blooms, which produce several problems to humans and natural ecosystem (Intartaglia & Sala, 1989; Parodi, Estrada, Trobbiani, & Argañaraz Bonini, 2004).

2.2. Restoration strategies

The most important corrective action for eutrophication control in lakes and reservoirs is the reduction of external nutrient loading (bottom-up control). However, different chemical and biological processes which take place in the water column and bottom sediments, may delay the water quality recovery. In the last decades, multiple in-lake restoration techniques have been developed and applied in order to combat the chemical and biological resistance (Cooke et al., 1993; Meijer, de Boois, Scheffer, Portielje, & Hosper, 1999; Søndergaard et al., 2007). Main in-lake restoration techniques are sediment fixation with iron or aluminum to decrease internal phosphorus release, sediment dredging (Van der Does, Verstraelen, Boers, Roijackers, & Moser, 1992), hydrological measures such as flushing (Jagtman, Van der Molen, & Vermij, 1992),

isolation (Bootsma, Barendregt, & van Alphen, 1999) and biomanipulation (Van Donk et al., 1990).

Generally, phosphate is considered as the limiting nutrient for primary productivity in freshwater ecosystems (Hecky & Kilham, 1988; Vollenweider, 1975) and its control is usually the best strategy for management purposes.

In this work we evaluate three of the most popular approaches for the restoration of water quality in lakes. One of them is the retention of phosphorus by an artificial wetland in order to decrease external nutrient loading. The remaining two techniques are in-lake ones and tend to control algal growth within a short term horizon: aeration and biomanipulation by fish removal.

2.2.1. Artificial wetland

One ecological engineering approach applied to address the problem of nonpoint nutrients sources for water bodies is the use of artificial wetlands to decrease nutrient loading. Because of the ability of natural and artificial wetlands to retain nutrients, heavy metals and suspended solids, they are becoming a key measure in water recourse management (Braskerud, 2000; Gale, Reddy, & Graetz, 1994). Retention of nutrients by wetland ecosystems can be defined as the ability to remove them from the water column through physical, chemical and biological processes (Gale et al., 1994). In this way, the external loading of nutrients to the reservoir can be reduced. Access to information on the wetland retention capacity allows further investigation on the need for additional restoration policies.

2.2.2. Aeration

Several studies have revealed that high internal loading of phosphorus from sediments is one of the most important mechanisms delaying lake recovery, after external loading reduction (Recknagel, Hosomi, Fukushima, & Kong, 1995; Søndergaard et al., 2007). Historically, phosphorus input from lake sediments has received lower attention than external loading. However, internal phosphorus loading can be a significant term in the annual phosphorus budget of many lakes (Gantzer, Bryant, & Little, 2009; Nowlin, Everts, & Vanni, 2005; Recknagel et al., 1995). The mechanism for nutrient release from the sediment–water interface in lakes and reservoirs is related to hypoxic/anoxic conditions, which lower the redox potential at the sediment–water interface. At low redox potentials, Fe (3+) is reduced to Fe (2+) (Boström, Ahlgren, & Bell, 1985). This process causes phosphorus associated with iron mineral become soluble and diluted in the water column, thus available for phytoplankton uptake.

The main dissolved oxygen-consuming processes in the water column are autotrophic and heterotrophic respiration, organic matter oxidation, nitrification and sediment oxygen demand. In stratified lakes, the hypolimnion layer becomes anoxic in summer due to the processes listed above. Non-oxidized organic matter settles through the hypolimnion and accumulates in upper sediments contributing to sediment oxygen demand.

In a hypolimnetic oxygenation or aeration approach, oxygen (or air) is pumped into the deep through aerators or diffusers in order to increase oxygen concentration in the hypolimnion, thus reducing phosphorus release from sediments. Other advantages of this method are: reduction in ammonium, iron, manganese and hydrogen sulfide accumulation, as well as increase in habitats and food supply for the benthic biota.

2.2.3. Biomanipulation

Increase of top-down control of algal growth by removal of planktivorous fish has become one of the most widely applied in-lake techniques. This technique is known as biomanipulation, a term coined by Shapiro et al. (1975) and Shapiro (1990). Biomanipulation is supported by the trophic chain theory and the basic idea

is to carry out zooplanktivorous fish removal or piscivorous fish stocking, or a combination of both, to keep a high grazing pressure on the phytoplankton community by the herbivore zooplankton (Jeppesen et al., 1990; Shapiro & Wright, 1984). The overall result is then a decrease in phytoplankton concentration.

Planktivorous fish are sometimes considered to promote eutrophication processes in lakes (Quirós, 1998). Visually feeding zooplanktivorous fish directly suppress large zooplankton, enhancing the development of phytoplankton biomass (Boveri & Quirós, 2002; Drenner, Threlkeld, & McCracken, 1986; Hurlbert & Mulla, 1981). Quirós (1995) reports that the planktivorous fish *Odon-testhes bonariensis* (pampean silverside) abundance showed an indirect relation to phytoplankton biomass, after nutrient effects have been taken into account. Several studies have reported the negative effects of *O. bonariensis* on the trophic cascade, associated to increased eutrophication trophic state (Boveri & Quirós, 2002; Grosman & Sanzano, 2003; Quirós, 1998; Sosnovsky & Quirós, 2009).

3. Mathematical model

We formulate a first principle-based ecological water quality model with dynamic mass balances for main components in the water body. In previous work (Estrada et al., 2009a), we have included balances for the three main phytoplankton groups (cyanobacteria, diatoms and chlorophytes), nutrients (nitrate, ammonium, organic nitrogen, phosphate and organic phosphate and cyanobacteria, diatoms and chlorophytes internal nitrogen and phosphorus) and dissolved oxygen. We have performed dynamic global sensitivity analysis to determine identifiable parameters (Estrada & Diaz, 2010) and have estimated the model main parameters with data collected from the lake throughout a 1 year time horizon (Estrada et al., 2009a). The model has been later formulated within an optimization framework to determine the application of biomanipulation strategies (Estrada et al., 2009b), giving optimal profiles for zooplankton concentration as the control variable.

In the present study, we include additional mass balance equations for dissolved and particulate carbon, two functional groups of zooplankton (copepoda and cladocera) and three size classes for zooplanktivorous fish (S_1 , S_2 and S_3). In this way, the optimal removal rate of each planktivorous fish size class is determined, which allows the systematic planning of biomanipulation actions and their associated costs. The model includes a global mass balance (Eqn. (1)), which takes into account inflows from tributaries (Q_{IN_k}) and rain (Q_{rain}) and outflows for drinking water and industrial purposes, Sauce Grande river (Q_{OUT_m}) and evaporation (Q_{evap}). Composition gradients have been considered along the water column, rendering partial differential algebraic equations (PDAE) system. To transform the resulting PDAE into a set of ordinary differential equations, the column height is discretized into two layers, according to available observed data from the lake (Eqs. (2a) and (2b)).

3.1. Mass balances

Mass balances in each spatial layer include component inputs from tributaries (Q_{IN}), outputs for both potabilization and industrial purposes (Q_{OUT}) and the river itself, generation and consumption, and mass transfer between layers, also accounting for lake volume variability (through upper layer height variability). Algebraic equations stand for the generation/consumption terms, as well as for seasonal climatological forcing functions. Forcing functions (temperature, solar radiation, rain, evaporation and river inflows and

corresponding concentrations) are represented by sinusoidal functions, based on data from Paso de las Piedras Reservoir.

Total mass balance

$$\frac{dh_T}{dt} = \frac{1}{\rho A} \left[\sum_{k=1}^{N_{IN}} Q_{IN} - \sum_{m=1}^{N_{OUT}} Q_{OUT_m} + Q_{rain} - Q_{evap} \right] \quad (1)$$

where set k includes inputs from Sauce Grande River and El Divisorio Stream, m stands for outputs to potabilization and industrial purposes and Sauce Grande River.

Upper layer

$$\frac{dC_{U,j}}{dt} = \sum_{k=1}^{N_{IN}} \frac{Q_{IN}}{V_U} C_{IN} - \frac{Q_{OUTU}}{V_U} C_{U,j} + r_{U,j} - \frac{k_d A}{\Delta h_U h_U} (C_{U,j} - C_{L,j}) - \frac{C_{U,j}}{h_U} \frac{dh_U}{dt} \quad (2a)$$

Lower layer

$$\frac{dC_{L,j}}{dt} = \frac{Q_{OUTL}}{V_L} C_{L,j} + r_{L,j} + \frac{k_d A}{\Delta h_U h_U} (C_{U,j} - C_{L,j}) - \frac{C_{L,j}}{h_L} \frac{dh_L}{dt} \quad (2b)$$

where U and L represent the upper and lower water layers, respectively, j stands for the three phytoplankton groups, the two zooplankton groups, the three sizes classes of fish, ammonium, nitrate, phosphate, organic nitrogen, organic phosphorus, internal phosphorus and nitrogen for the three phytoplankton groups, organic and dissolved carbon, dissolved oxygen. Q_{OUTU} and Q_{OUTL} are the outputs for Sauce Grande River and for potabilization and industrial purpose, respectively. Generation/consumption terms for each component in the upper and lower water layer are $r_{U,j}$ and $r_{L,j}$, respectively. They are described in Section 3.2. The fourth term in Eq. (2a) stands for mass transfer between layers. Finally, h_U and V_U are the upper layer height and volume. Table 1 shows model parameters for Paso de las Piedras Reservoir. Fig. 1 shows a detail of main state variables and processes that relate them.

3.2. Generation/consumption equations

3.2.1. Phytoplankton

In the present work, we have included additional state variables for internal nitrogen (IN) and phosphorus (IP) concentration for each phytoplankton group (cyanobacteria, diatoms and chlorophyta). In previous work (Estrada et al., 2009a), carbon to nitrogen and carbon to phosphorus ratios have been considered as time invariant parameters. The inclusion of these additional state variables allows modeling dynamic behavior of intracellular nutrients concentrations, which largely affect cellular growth rate. Mass balances for IP and IN are formulated as (Hamilton & Schladow, 1997):

$$\frac{dIP_{ij}}{dt} = UP \max_j \left(\frac{C_{i,PO_4}}{K_{Pj} + C_{i,PO_4}} \right) \left(\frac{P_{\max j} - IP_{ij}}{P_{\max j} - P_{\min j}} \right) - R_{ij,growth} IP_{ij} \quad (3)$$

$$\frac{dIN_{ij}}{dt} = UN \max_j \left(\frac{C_{i,NH_4} + C_{i,NO_3}}{K_{Nj} + C_{i,NH_4} + C_{i,NO_3}} \right) \left(\frac{N_{\max j} - IN_{ij}}{N_{\max j} - N_{\min j}} \right) - R_{ij,growth} IN_{ij} \quad (4)$$

where i = upper layer (U), lower layer (L); j = cyanobacteria (C), diatoms (D) and chlorophyta (G).

This model takes into account differences in maximum uptake rates and storage of nutrients between phytoplankton groups. Nutrient growth limitation function was modified to account for luxury uptake of phosphorus, which is considered the limiting nutrient for phytoplankton (Arhonditsis, Tsirtsis, & Karydis, 2002; Asaeda & Van Bon, 1997; Hamilton & Schladow, 1997). For diatoms,

silica limitation is also taken into account, since it is a requirement for most of the diatoms species for cell wall formation (Thamatrakoln & Hildebrand, 2008). Maximum physiological storage of internal nutrients and minimum levels required for growth are modeled by assigning maximum and minimum parameters for nutrient cellular content (Hamilton & Schladow, 1997). The nutrient concentration dependence in phytoplankton growth term is now calculated as:

$$f(N)_{ij} = \begin{cases} \frac{IP_{ij} - P_{\min j}}{P_{\max j} - P_{\min j}}, & i = U, L; j = C, G \\ \min \left(\frac{IP_{ij} - P_{\min j}}{P_{\max j} - P_{\min j}}, \frac{C_{i,Si}}{K_S + C_{i,Si}} \right), & i = U, L; j = D \end{cases} \quad (5)$$

For a detailed explanation of the biomass phytoplankton model, see Estrada et al. (2009a).

3.2.2. Herbivorous zooplankton

In the present model, we include mass balance equations for herbivorous zooplankton, represented by two functional groups, cladocera (Cl) and copepoda (Co), which differ in grazing rates, food preferences, feeding strategies, C:N:P ratios, optimal growth temperatures (Arhonditsis & Brett, 2005a; Zhao, Maryam, Cheng, George, & Arhonditsis, 2008). The net generation term (r_{ik}) related to biomass zooplankton groups are grazing ($R_{ik,Graz}$), basal metabolism ($R_{ik,bm}$, basal metabolism and natural mortality) and predation by zooplanktivorous fish ($R_{ik,pred}$):

$$r_{ik} = R_{ik,graz} - R_{ik,bm} - R_{ik,pred} \quad i = U, L; \quad k = Cl, Co \quad (6)$$

Zooplankton growth is limited by food availability (from the three phytoplankton groups and detritus (det)), food assimilation and temperature according to:

$$R_{ik,graz} = \left(\sum_{j=C,D,G} Graz_{kji} + Graz_{k, det} \right) A_k f(T)_k, \quad i = U, L; k = Cl, Co \quad (7)$$

where A_k is the assimilation rate parameter for zooplankton and the temperature dependent factor is calculated as:

$$f(T)_k = \exp(-KTgr_k(T_i - T_{opt_k})^2) \quad i = U, L; k = Cl, Co \quad (8)$$

The grazing term ($Graz_{ikj}$) is a Monod-type equation that takes into account zooplankton group preference among the four food items for zooplankton grazing (cyanobacteria, diatoms, chlorophyta and detritus).

$$Graz_{ikj} = k_{k,grazing} \frac{P_{ijk} C_{ij}}{KZ_k + \sum_{j=C,D,G} P_{ijk} + P_{i det k}} \quad i = U, L; \quad k = Cl, Co; j = C, D, G \quad (9)$$

$$Graz_{ik det} = k_{k,grazing} \frac{P_{i det k} C_{iPOC}}{KZ_k + \sum_{j=C,D,G} P_{ijk} + P_{i det k}}, \quad i = U, L; \quad k = Cl, Co; j = C, D, G \quad (10)$$

where P_{ijk} is the preference term of zooplankton on phytoplankton and detritus. From the two zooplankton groups, cladocera are filter-feeders with equal preferences for the four food items while copepods are selective-feeders, selecting their food by their quality and particle size. These different feeding strategies are modeled

Table 1
Parameters for eutrophication model.

Parameter	Description	Value	Units
Phytoplankton			
$k_{C,growth}$	Maximum growth rate for Cyanobacteria	0.350	1/day
$k_{D,growth}$	Maximum growth rate for Diatoms	0.472	1/day
$k_{G,growth}$	Maximum growth rate for Chlorophyta	0.473	1/day
$k_{C,bm}$	Basal metabolism for Cyanobacteria	0.029	1/day
$k_{D,bm}$	Basal metabolism for Diatoms	0.102	1/day
$k_{G,bm}$	Basal metabolism for Chlorophyta	0.022	1/day
$k_{C,settling}$	Settling rate for Cyanobacteria	0.02	1/day
$k_{D,settling}$	Settling rate for Diatoms	0.33	1/day
$k_{G,settling}$	Settling rate for Chlorophyta	0.25	1/day
K_{PC}	Half-saturation constant for P uptake for Cyanobacteria	0.001	mgP/l
K_{PD}	Half-saturation constant for P uptake for Diatoms	0.020	mgP/l
K_{PG}	Half-saturation constant for P uptake for Chlorophyta	0.030	mgP/l
K_{NC}	Half-saturation constant for N uptake for Cyanobacteria	0.025	mgN/l
K_{ND}	Half-saturation constant for N uptake for Diatoms	0.065	mgN/l
K_{NG}	Half-saturation constant for N uptake for Chlorophyta	0.045	mgN/l
UP_{maxC}	Maximum rate of P uptake for Cyanobacteria	0.090	mgP/mgCday
UP_{maxD}	Maximum rate of P uptake for Diatoms	0.077	mgP/mgCday
UP_{maxG}	Maximum rate of P uptake for Chlorophyta	0.012	mgP/mgCday
UN_{maxC}	Maximum rate of N uptake for Cyanobacteria	0.160	mgN/mgCday
UN_{maxD}	Maximum rate of N uptake for Diatoms	0.160	mgN/mgCday
UN_{maxG}	Maximum rate of N uptake for Chlorophyta	0.160	mgN/mgCday
P_{maxC}	Maximum internal P:C ratio for Cyanobacteria	0.025	mgP/mgC
P_{maxD}	Maximum internal P:C ratio for Diatoms	0.025	mgP/mgC
P_{maxG}	Maximum internal P:C ratio for Chlorophyta	0.025	mgP/mgC
N_{maxC}	Maximum internal N:C ratio for Cyanobacteria	0.18	mgN/mgC
N_{maxD}	Maximum internal N:C ratio for Diatoms	0.18	mgN/mgC
N_{maxG}	Maximum internal N:C ratio for Chlorophyta	0.18	mgN/mgC
P_{minC}	Minimum internal P:C ratio for Cyanobacteria	0.008	mgP/mgC
P_{minD}	Minimum internal P:C ratio for Diatoms	0.008	mgP/mgC
P_{minG}	Minimum internal P:C ratio for Chlorophyta	0.008	mgP/mgC
N_{minC}	Minimum internal N:C ratio for Cyanobacteria	0.08	mgN/mgC
N_{minD}	Minimum internal N:C ratio for Diatoms	0.08	mgN/mgC
N_{minG}	Minimum internal N:C ratio for Chlorophyta	0.08	mgN/mgC
θ_{bmC}	Temperature adjustment for basal metabolism of Cyanobacteria	1.010	Unitless
θ_{bmD}	Temperature adjustment for basal metabolism of Diatoms	1.071	Unitless
θ_{bmG}	Temperature adjustment for basal metabolism of Chlorophyta	1.170	Unitless
Zooplankton			
$k_{Cla,grazing}$	Maximum grazing rate for Cladocera	0.30	1/day
$k_{Co,grazing}$	Maximum grazing rate for Copepoda	0.26	1/day
$k_{Cla,bm}$	Basal metabolism for Cladocera	0.06	1/day
$k_{Co,bm}$	Basal metabolism for Copepoda	0.04	1/day
KZ_{Cla}	Half-saturation constant for grazing for Cladocera	0.12	mg/l
KZ_{Co}	Half-saturation constant for grazing for Copepoda	0.10	mg/l
A_{Cla}	Assimilation rate for Cladocera	0.95	Unitless
A_{Co}	Assimilation rate for Copepoda	0.90	Unitless
T_{optCla}	Optimal growth temperature for Cladocera	20.0	°C
T_{optCo}	Optimal growth temperature for Copepoda	18.0	°C
$KTgr1_{Cla}$	Effect of temperature below T_{opt} for Cladocera	0.007	1/°C ²
$KTgr1_{Cla}$	Effect of temperature above T_{opt} for Cladocera	0.007	1/°C ²
$KTgr1_{Co}$	Effect of temperature below T_{opt} for Copepoda	0.002	1/°C ²
$KTgr1_{Co}$	Effect of temperature above T_{opt} for Copepoda	0.002	1/°C ²
$pref_{CCla}$	Cladocera preference factor for Cyanobacteria	0.25	Unitless
$pref_{DCla}$	Cladocera preference factor for Diatoms	0.25	Unitless
$pref_{CCla}$	Cladocera preference factor for Chlorophyta	0.25	Unitless
$pref_{detCla}$	Cladocera preference factor for detritus	0.25	Unitless
$pref_{CCo}$	Copepoda preference factor for Cyanobacteria	0.20	Unitless
$pref_{DCo}$	Copepoda preference factor for Diatoms	0.30	Unitless
$pref_{CCo}$	Copepoda preference factor for Chlorophyta	0.25	Unitless
$pref_{detCo}$	Copepoda preference factor for detritus	0.25	Unitless
α_{PCCla}	P:C ratio for Cladocera	0.29	Unitless
α_{NCCla}	N:C ratio for Cladocera	0.17	Unitless
α_{PCCo}	P:C ratio for Copepoda	0.02	Unitless
α_{NCCo}	N:C ratio for Copepoda	0.20	Unitless
θ_{bmCla}	Temperature adjustment for basal metabolism of Cladocera	1.050	Unitless
θ_{bmCo}	Temperature adjustment for basal metabolism of Copepoda	1.050	Unitless
Fish			
$k_{S_1,predation}$	Maximum predation rate for S_1	0.15	1/day
$k_{S_2,predation}$	Maximum predation rate for S_2	0.15	1/day
$k_{S_3,predation}$	Maximum predation rate for S_3	0.15	1/day
$k_{S_1,bm}$	Basal metabolism for S_1	0.009	1/day
$k_{S_2,bm}$	Basal metabolism for S_2	0.024	1/day
$k_{S_3,bm}$	Basal metabolism for S_3	0.017	1/day
$k_{S_1,recruit}$	S_1 to S_2 recruitment rate	0.0002	1/day
$k_{S_2,recruit}$	S_2 to S_3 recruitment rate	0.0007	1/day
$k_{S_2,reprd}$	S_2 reproduction rate	0.0001	1/day

Table 1 (Continued)

Parameter	Description	Value	Units
$k_{S_3, \text{reprod}}$	S_3 reproduction rate	0.0001	1/day
spaw	Spawning efficiency	0.16	Unitless
$K_{F_{S_1}}$	Half-saturation constant for S_1 predation	0.75	mg/l
$K_{F_{S_2}}$	Half-saturation constant for S_2 predation	0.75	mg/l
$K_{F_{S_3}}$	Half-saturation constant for S_3 predation	1.75	mg/l
A_{S_1}	Assimilation rate for S_1	0.40	Unitless
A_{S_2}	Assimilation rate for S_2	0.40	Unitless
A_{S_3}	Assimilation rate for S_3	0.40	Unitless
$\text{pref}_{\text{Fito}S_1}$	S_1 preference factor for phytoplankton	0.20	Unitless
$\text{pref}_{\text{Cla}S_1}$	S_1 preference factor for Cladocera	0.40	Unitless
$\text{pref}_{\text{Co}S_1}$	S_1 preference factor for Copepoda	0.40	Unitless
$\text{pref}_{\text{det}S_1}$	S_1 preference factor for detritus	0.00	Unitless
$\text{pref}_{\text{Fito}S_2}$	S_2 preference factor for phytoplankton	0.05	Unitless
$\text{pref}_{\text{Cla}S_2}$	S_2 preference factor for Cladocera	0.425	Unitless
$\text{pref}_{\text{Co}S_2}$	S_2 preference factor for Copepoda	0.425	Unitless
$\text{pref}_{\text{det}S_2}$	S_2 preference factor for detritus	0.10	Unitless
$\text{pref}_{\text{Fito}S_3}$	S_3 preference factor for phytoplankton	0.00	Unitless
$\text{pref}_{\text{Cla}S_3}$	S_3 preference factor for Cladocera	0.35	Unitless
$\text{pref}_{\text{Co}S_3}$	S_3 preference factor for Copepoda	0.35	Unitless
$\text{pref}_{\text{det}S_3}$	S_3 preference factor for detritus	0.10	Unitless
$\text{pref}_{S_1S_3}$	S_3 preference factor for S_1	0.20	Unitless
α_{PCFish}	P:C ratio for fish	0.024	mgP/mgC
α_{NCFish}	N:C ratio for fish	0.180	mgN/mgC
$\theta_{\text{bms}S_1}$	Temperature adjustment for basal metabolism of S_1	1.020	Unitless
$\theta_{\text{bms}S_2}$	Temperature adjustment for basal metabolism of S_2	1.020	Unitless
$\theta_{\text{bms}S_3}$	Temperature adjustment for basal metabolism of S_3	1.020	Unitless
$\theta_{\text{pred}S_1}$	Temperature adjustment for predation of S_1	1.020	Unitless
$\theta_{\text{pred}S_2}$	Temperature adjustment for predation of S_2	1.020	Unitless
$\theta_{\text{pred}S_3}$	Temperature adjustment for predation of S_3	1.020	Unitless
Carbon			
k_{mc}	Mineralization rate for organic carbon	0.7	1/day
K_{POC}	Half-saturation constant for organic carbon mineralization	0.008	mg/l
$K_{\text{POC,settling}}$	Settling rate for organic carbon	0.03	mg/day
θ_{mc}	Temperature adjustment for organic carbon mineralization	1.05	Unitless
f_{POC}	Fraction of dead and respired organism recycled to POC pool	0.5	Unitless
Nitrogen			
k_{nit}	Nitrification rate	0.037	1/day
k_{denit}	Denitrification rate	0.3	1/day
K_{nit}	Half-saturation constant for nitrification rate	3.0	mg/l
K_{denit}	Half-saturation constant for denitrification rate	0.5	mg/l
k_{mn}	Mineralization rate for organic nitrogen	0.03	1/day
$K_{\text{ON,settling}}$	Settling rate for organic nitrogen	0.03	mg/day
θ_{ni}	Temperature adjustment for nitrification rate	1.080	Unitless
θ_{deni}	Temperature adjustment for denitrification rate	1.080	Unitless
θ_{mn}	Temperature adjustment for mineralization rate for ON	1.080	Unitless
f_{ON}	Fraction of dead and respired organism recycled to ON pool	0.5	Unitless
Phosphorus			
k_{mp}	Mineralization rate for organic phosphorus	0.038	1/day
$K_{\text{OP,settling}}$	Settling rate for organic phosphorus	0.03	mg/day
θ_{mp}	Temperature adjustment for mineralization rate for OP	1.080	Unitless
f_{OP}	Fraction of dead and respired organism recycled to OP pool	0.5	Unitless
Sediment			
S_{N}	Release rate of NH_4 from the sediment	0.4	mgN/mday
S_{P}	Release rate of PO_4 from the sediment	0.0013	mgN/mday
θ_{N}	Temperature adjustment for release of NH_4 from sediments	1.080	Unitless
θ_{P}	Temperature adjustment for release of PO_4 from sediments	1.080	Unitless
K_{DOS}	Half-saturation const. for nutrient sediment fluxes 0.4	0.4	mg/day
DO			
k_{a}	Re-aeration rate	0.38	1/day
k_{SOD}	Sediment oxygen demand rate	0.65	1/day
K_{SOD}	Half-saturation constant for sediment oxygen demand	0.4	mg/day
α_{oc}	O:C ratio	2.67	mgO/mgC
α_{on}	O:N ratio	4.57	mgO/mgC
θ_{sod}	Temperature adjustment for oxygen sediment demand rate	1.050	Unitless
General			
A	Lake area	36.0	km ²
D_{L}	Lower layer depth	3.5	m

defining prior weighted preferences. It must be noted that the preference term depends on the food items concentration for the two zooplankton groups as:

$$P_{ijk} = \frac{\text{pref}_{jk} C_{ij}}{\sum_{j=C,D,G} \text{pref}_{jk} C_{ij} + \text{pref}_{\text{det}k} C_{i\text{POC}}}, \quad i = U, L; \quad (11)$$

$$j = C, D, G; \quad k = \text{Cla}, \text{Co}$$

$$P_{i\text{det}k} = \frac{\text{pref}_{\text{det}k} C_{i\text{POC}}}{\sum_{j=C,D,G} \text{pref}_{jk} C_{ij} + \text{pref}_{\text{det}k} C_{i\text{POC}}}, \quad i = U, L; \quad k = \text{Cla}, \text{Co} \quad (12)$$

where $C_{i\text{POC}}$ stands for concentration of particulate organic carbon (detritus) in layer i and pref_{jk} is the preference parameter of zooplankton group k for phytoplankton group j (see Table 1).

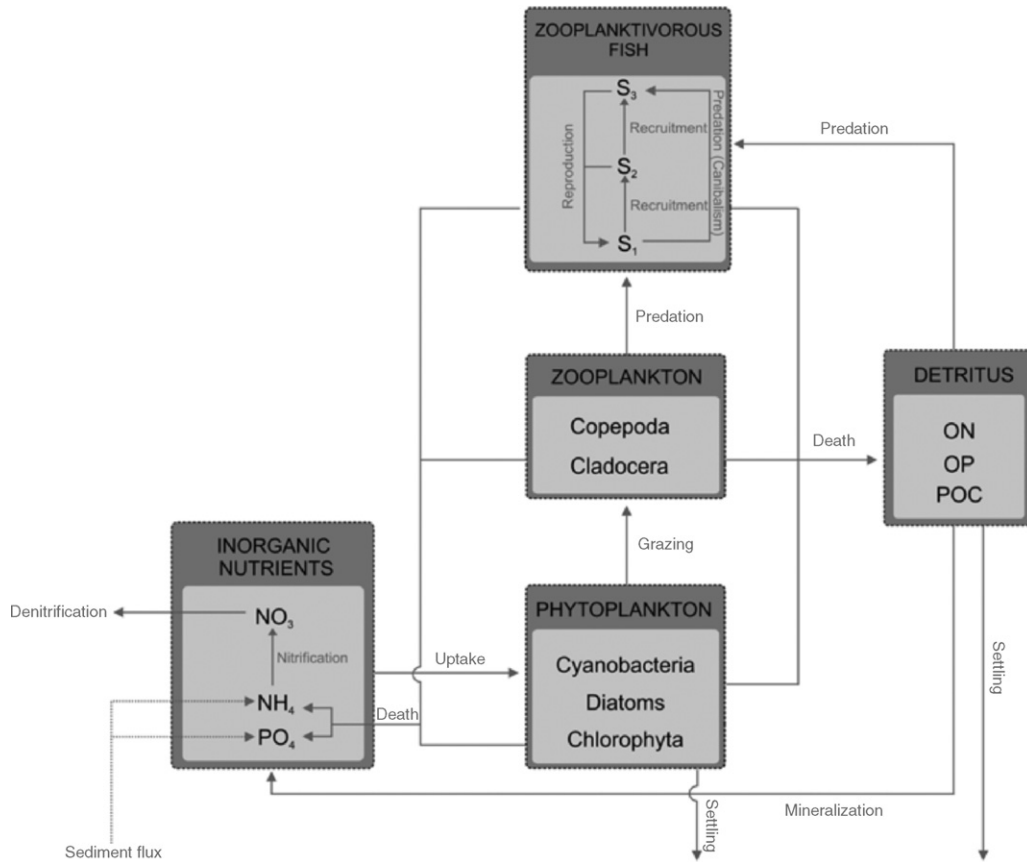


Fig. 1. State variables and processes.

The remaining terms in zooplankton mass balance, $R_{ik,bm}$ and $R_{ik,pred}$, stand for biomass loss due to basal metabolism and zooplanktivorous fish predation, respectively, and they are calculated as functions of temperature (T) and zooplankton concentration (C_{ik}) as:

$$R_{ik,bm} = k_{k,bm} \theta_{bm}^{(T-20)} C_{ik} \quad i = U, L; k = Cla, Co \quad (13)$$

$$R_{ik,pred} = \sum_{m=S_1, S_2} Pred_{ikm} \theta_{pred_m}^{(T-20)} C_{ik} \quad i = U, L; k = Cla, Co \quad (14)$$

where $Pred_{ikm}$ stands for predation rate of fish size class m on zooplankton group k .

3.2.3. Planktivorous fish

In this study, we consider three stages of zooplanktivorous fish life cycle (S_1, S_2, S_3) based on size classes. S_1 stands for a size range between 0 and 100 mm, S_2 , for 100 and 200 mm and S_3 for sizes greater than 200 mm. Rate equations for carbon dynamics are composed of terms that stand for growth by predation ($R_{im,pred}$), basal metabolism ($R_{im,bmetab}$, basal metabolism and natural mortality), recruitment ($R_{im,recruit}$) and spawning ($R_{im,spaw}$) terms, as follows.

$$r_{iS_1} = R_{im,pred} - R_{im,bmetab} - R_{iS_3,canib} - R_{im,recruit} + R_{im,spaw} \quad i = U, L \quad (15)$$

$$r_{im} = R_{im,pred} - R_{im,bmetab} - R_{im,recruit} - R_{im,spaw} \quad i = U, L; m = S_2, S_3 \quad (16)$$

In the proposed model, S_1 and S_2 are assumed strictly phyto and zooplanktivorous while S_3 also predate on detritus. The predation

term for the third size class (S_3) is modeled taking into account zooplankton (copepoda and cladocera) and detritus items, and as well as the first size class (S_1), to incorporate the adult cannibalism phase within this term:

$$R_{iS_3,canib} = k_{S_3,canib} \theta_{m_s}^{(T-20)} C_{iS_3}, \quad i = U, L \quad (17)$$

Growth rate for fish ($R_{im,pred}$) is calculated as the sum of predation rates on the different plankton groups and detritus, affected by a temperature dependent factor and a food assimilation rate parameter (A_m). Temperature adjustment for predation is modeled as an Arrhenius law. As in zooplankton dynamics, the predation term is a Monod-type equation and it is dependent on the fish state food preference and food concentration.

$$R_{im,pred} = \left(\sum_{j=C,D,G} Pred_{imj} + \sum_{k=Cl,Co} Pred_{imk} + Pred_{im det} \right) A_m \theta_{pred_m}^{(T-20)}, \quad i = U, L; m = S_1, S_2 \quad (18)$$

$$R_{iS_3,pred} = \left(\sum_{j=C,D,G} Pred_{imj} + \sum_{k=Cl,Co} Pred_{imk} + Pred_{im det} + Pred_{iS_3S_1} \right) A_m \theta_{pred_{S_3}}^{(T-20)}, \quad i = U, L \quad (19)$$

where the predation terms are calculated as follows:

Predation on phytoplankton

$$Pred_{imj} = p_{\max m} \frac{P_{jmi} C_{ij}}{KF_m + \sum_{j=C,D,G} P_{ijm} + \sum_{k=Cl a, Co} P_{ikm} + P_{i det m}},$$

$$i = U, L; j = C, D, G; m = S_1, S_2 \quad (20)$$

Pred_{iS₃j}

$$= p_{\max S_3} \frac{P_{ijS_3} C_{ij}}{KF_{S_3} + \sum_{j=C,D,G} P_{ijS_3} + \sum_{k=Cl a, Co} P_{ijS_3} + P_{i det S_3} + P_{iS_1 S_3}}$$

$$i = U, L; j = C, D, G \quad (21)$$

Predation on zooplankton

$$Pred_{imk} = p_{\max m} \frac{P_{ikm} C_{ik}}{KF_m + \sum_{j=C,D,G} P_{ijm} + \sum_{k=Cl a, Co} P_{ikm} + P_{i det m}}$$

$$i = U, L; k = Cl a, Co; m = S_1, S_2 \quad (22)$$

Pred_{iS₃k}

$$= p_{\max S_3} \frac{P_{ikS_3} C_{ik}}{KF_{S_3} + \sum_{j=C,D,G} P_{ijS_3} + \sum_{k=Cl a, Co} P_{ijS_3} + P_{i det S_3} + P_{iS_1 S_3}}$$

$$i = U, L; k = Cl a, Co \quad (23)$$

Predation on detritus (POC)

$$Pred_{im det} = p_{\max m} \frac{P_{i det m} C_{i POC}}{KF_m + \sum_{j=C,D,G} P_{ijm} + \sum_{k=Cl a, Co} P_{ikm} + P_{i det m}},$$

$$i = U, L; m = S_1, S_2 \quad (24)$$

Pred_{iS₃ det}

$$= p_{\max S_3} \frac{P_{i det S_3} C_{i POC}}{KF_{S_3} + \sum_{j=C,D,G} P_{ijS_3} + \sum_{k=Cl a, Co} P_{ijS_3} + P_{i det S_3} + P_{iS_1 S_3}}$$

$$i = U, L \quad (25)$$

Predation of S₃ on S₁ (cannibalism)

$$Pred_{iS_3 S_1} = p_{\max S_3} \frac{P_{iS_1 S_3} C_{iS_1}}{KF_{S_3} + \sum_{j=C,D,G} P_{ijS_3} + \sum_{k=Cl a, Co} P_{ijS_3} + P_{i det S_3} + P_{iS_1 S_3}}$$

$$i = U, L \quad (26)$$

where the preference factors (P_{ijk}) associated to the different predation terms are calculated as:

Preference for phytoplankton

$$P_{ijm} = \frac{pref_{jm} C_{ij}}{\sum_{j=1}^3 pref_{jm} C_{ij} + \sum_{k=1}^2 pref_{km} C_{ik} + pref_{det m} C_{i POC}}$$

$$i = U, L; j = C, D, G; m = S_1, S_2 \quad (27)$$

$$P_{ijS_3} = \frac{pref_{jm} C_{ij}}{\sum_{j=C,D,G} pref_{jS_3} C_{ij} + \sum_{k=Cl a, Co} pref_{kS_3} C_{ik} + pref_{det S_3} C_{i POC} + pref_{S_1 S_3} C_{S_1}}$$

$$i = U, L \quad (28)$$

Preference on zooplankton

$$P_{ikm} = \frac{pref_{km} C_{ik}}{\sum_{j=C,D,G} pref_{jm} C_{ij} + \sum_{k=Cl a, Co} pref_{km} C_{ik} + pref_{det m} C_{i POC}}$$

$$i = U, L; k = Cl a, Co; m = S_1, S_2 \quad (29)$$

$$P_{ikS_3} = \frac{pref_{kS_3} C_{ik}}{\sum_{j=C,D,G} pref_{jS_3} C_{ij} + \sum_{k=Cl a, Co} pref_{kS_3} C_{ik} + pref_{det S_3} C_{i POC} + pref_{S_1 S_3} C_{S_1}}$$

$$i = U, L; k = Cl a, Co \quad (30)$$

Preference on detritus

$$P_{i det m} = \frac{pref_{det m} C_{i POC}}{\sum_{j=C,D,G} pref_{jk} C_{ij} + \sum_{k=Cl a, Co} pref_{km} C_{ik} + pref_{det m} C_{i POC}}$$

$$i = U, L; m = S_1, S_2 \quad (31)$$

$$P_{i det S_3} = \frac{pref_{det S_3} C_{i POC}}{\sum_{j=C,D,G} pref_{jS_3} C_{ij} + \sum_{k=Cl a, Co} pref_{kS_3} C_{ik} + pref_{det S_3} C_{i POC} + pref_{S_1 S_3} C_{S_1}}$$

$$i = U, L \quad (32)$$

Preference on S₁ (S₃)

$$P_{iS_3 S_1} = \frac{pref_{S_1 S_3} m C_{iS_1}}{\sum_{j=C,D,G} pref_{jm} C_{ij} + \sum_{k=Cl a, Co} pref_{km} C_{ik} + pref_{det m} C_{i POC} + pref_{S_1 S_3} C_{S_1}}$$

$$i = U, L \quad (33)$$

The term $R_{im,bm}$ accounts for biomass losses due to basal metabolism. (respiration + natural mortality):

$$R_{im,bm} = k_{m,bm} \theta_{bm}^{(T-20)} C_{im} \quad i = U, L; m = S_1, S_2, S_3 \quad (34)$$

The recruitment term, $R_{im,recruit}$, is used to model the transference from one size class to the next one (by growth). Thus, S₂ class recruits biomass from S₁ and S₃, from S₂.

$$R_{im,recruit} = k_{m,recruit} C_{im} \quad i = U, L; m = S_1, S_2 \quad (35)$$

The spawning term, $R_{im,spaw}$, stands for reproduction. Spawning is modeled as a temperature dependent process and takes place in two time periods throughout the year March–April and September–October when the water temperature is between 10 and 20 °C (in our case study). The biomass re-allocation takes place from S₂ and S₃ to S₁ because S₁ is the first stage of the life cycle and it is considered as not sexually mature (Calvo & Dadone, 1972; Calvo & Morriconi, 1972).

R_{im,spaw}

$$= \begin{cases} k_{m,spaw} C_{im}, & 10 < temp < 20 \text{ } ^\circ\text{C} \\ 0, & \text{otherwise} \end{cases} \quad i = U, L; m = S_2, S_3 \quad (36)$$

3.2.4. Dissolved oxygen

In the present work, mass balance equations for dissolved oxygen are modified to include terms involved in zooplankton and fish respiration, as well as to account for the oxidation of particulate organic carbon (POC). Rate equations that describe the major processes governing dissolved oxygen (DO) dynamics are given as Priyantha, Asaeda, Saitoh, and Gotoh (1997). Fig. 2 shows the most important processes involved in dissolved oxygen generation/consumption. Main sources of dissolved oxygen in the water column are the saturation concentration dependent atmospheric

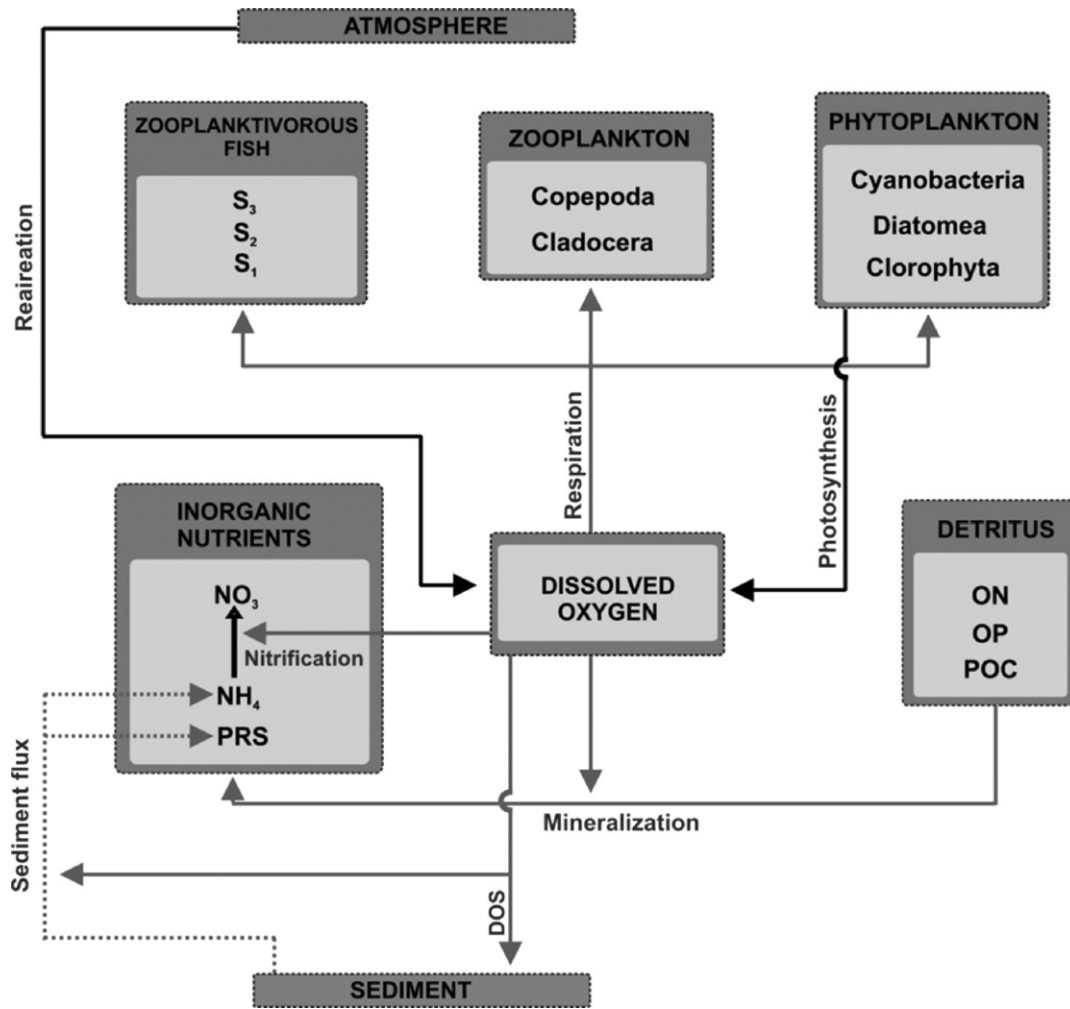


Fig. 2. Main processes involved in generation (black arrows)/consumption (gray arrows) of dissolved oxygen.

re-aeration ($R_{U,DO,repr}$), and photosynthetic oxygen production ($R_{U,DO,resp fot}$), in the upper water layer. The dissolved oxygen sinks are autotrophic and heterotrophic respiration ($R_{i,DO,resp}$), organic matter oxidation ($R_{i,POC,miner}$), nitrification ($R_{i,DO,nitr}$), and sediment oxygen demand ($R_{L,DO,sediment}$) in the lower water layer, as follows:

$$r_{U,DO} = R_{U,DO,repr} - R_{U,DO,nitr} + R_{U,DO,resp fot} - R_{U,DO,resp} - R_{U,POC,miner} \quad (37)$$

$$r_{L,DO} = -R_{L,DO,nitr} - R_{L,DO,resp fot} - R_{U,DO,resp} - R_{L,POC,miner} - R_{L,DO,sediment} \quad (38)$$

$$R_{U,DO,repr} = \frac{k_a \theta_a^{(T-20)}}{D_U} (C^* - C_{U,DO}) \quad (39)$$

where C^* is the saturated oxygen concentration at the surface temperature and is given by:

$$C^* = 16.5 - \frac{8}{22} T \quad (40)$$

$$R_{i,DO,nitr} = R_{i,NH_4,nit} \alpha_{on} \quad i = U, L \quad (41)$$

$$R_{i,DO,resp fot} = \sum_{j=C,D,G} (R_{ij,growth} C_{ij} - R_{ij,bm} C_{ij}) \alpha_{oc} \quad i = U, L \quad (42)$$

$$R_{i,DO,resp} = \left(\sum_{k=Cl a, Co} R_{ik,bm} C_{ik} + \sum_{m=S1,S2,S3} R_{im,bm} C_{im} \right) \alpha_{oc} \quad i = U, L \quad (43)$$

$$R_{i,POC,miner} = \left(k_{mc} \theta_{mc}^{(T-20)} \frac{C_{iDO}}{K_{POC} + C_{iDO}} C_{iPOC} \right) \alpha_{oc}, \quad i = U, L \quad (44)$$

$$R_{L,DO,sediment} = \frac{k_{sod} \theta_{sod}^{(T-20)}}{D_L} \frac{C_{L,DO}}{K_{sod} + C_{L,DO}} \quad (45)$$

3.2.5. Biogeochemical cycles

In this work, we incorporate new state variables corresponding to carbon cycle and we modify equations corresponding to nitrogen and phosphorus cycles to take into account the addition of zooplankton and fish balances.

Carbon cycle: The present model has additional mass balances to account for dissolved (*DOC*) and particulate organic carbon (*POC*). Processes that provide dissolved and particulate organic carbon to the water column include basal metabolism ($R_{i,POC,bm}$, $R_{i,DOC,bm}$, respiration and mortality for phytoplankton, zooplankton and fish), zooplankton egestion and not assimilated food of zooplankton and fish. Particulate organic carbon concentration decreases by hydrolyzation of oxygen-dependent mineralization rate to dis-

solved organic carbon ($R_{i,POC,miner}$), zooplankton and fish detritus feeding ($R_{i,POC,pred}$) and a fraction that settles to the lake bottoms ($R_{i,POC,settling}$) as follows:

$$r_{i,POC} = R_{i,POC,bm} - R_{i,POC,miner} - R_{i,POC,settling} - R_{i,POC,pred}, \quad i = U, L \quad (46)$$

$$r_{i,DOC} = R_{i,DOC,bm} + R_{i,POC,miner}, \quad i = U, L \quad (47)$$

Particulate organic carbon due to basal metabolism (bm)

$$R_{i,POC,bm} = \sum_{j=C,D,G} (k_{j,bm} \theta_{bm_j}^{(T-20)} f_{POC} C_{ij}) + \sum_{k=Cl a, Co} (k_{k,bm} \theta_{bm_k}^{(T-20)} f_{POC} C_{ik}) + \left(\sum_{j=C,D,G} Graz_{ikj} + Graz_{ik det} \right) f(T)_k (1 - A_k) + \sum_{m=S1,S2,S3} (k_{m,bm} \theta_{bm_m}^{(T-20)} f_{POC} C_{im}) + \left(\sum_{m=S1,S2,S3} \left(\sum_{j=C,D,G} Pred_{imj} + \sum_{k=Cl a, Co} Pred_{imk} + Pred_{im det} \right) + Pred_{iS_3S_1} \right) \theta_{pred_m}^{(T-20)} (1 - A_m), \quad i = U, L \quad (48)$$

$$R_{i,POC,miner} = k_{mc} \theta_{mc}^{(T-20)} \frac{C_{iDO}}{K_{POC} + C_{iDO}} C_{iPOC}, \quad i = U, L \quad (49)$$

$$R_{i,POC,settling} = \frac{k_{POC,settling}}{D_i} C_{iPOC} \quad i = U, L \quad (50)$$

$$R_{i,POC,pred} = \left(\sum_{k=Cl a, Co} Graz_{ik det} f(T)_k + \sum_{m=S1,S2,S3} Pred_{im det} \theta_{pred_m}^{(T-20)} \right) \times C_{iPOC} \quad i = U, L \quad (51)$$

Nitrogen: Three state variables describe nitrogen cycle: ammonium (NH_4), nitrate (NO_3) and organic nitrogen (ON). Phytoplankton is able to uptake both ammonium ($R_{i,NH_4,uptake}$) and nitrate ($R_{i,NO_3,uptake}$) for growth. Ammonium is oxidized to nitrate in an oxygen-temperature-dependent nitrification rate ($R_{i,NH_4,nit}$) and its concentration increases by organic nitrogen hydrolysis at a temperature-dependent mineralization rate ($R_{i,ON,miner}$) and phytoplankton death ($R_{i,NH_4,death}$). For the lower layer, the model includes a term of ammonium release from the sediments ($R_{i,NH_4,sed}$). Nitrate can be reduced to molecular nitrogen by a process requiring low oxygen conditions, i.e. denitrification ($R_{i,NO_3,denit}$). The organic nitrogen pool is reduced by mineralization ($R_{i,ON,miner}$), zooplankton and fish grazing ($R_{i,ON,pred}$) and settling ($R_{i,ON,settling}$). It increases with phytoplankton, zooplankton and fish mortality ($R_{i,ON,bm}$). In this work, rate equations for organic nitrogen are modified to take into account zooplankton and planktivorous fish dynamics. The resulting governing equations are as follows:

$$r_{U,NH_4} = -R_{U,NH_4,uptake} - R_{U,NH_4,nit} + R_{U,NH_4,bm} + R_{U,ON,miner} \quad (52)$$

$$r_{L,NH_4} = -R_{L,NH_4,uptake} - R_{L,NH_4,nit} + R_{L,NH_4,bm} + R_{L,ON,miner} + R_{L,NH_4,sed} \quad (53)$$

$$r_{i,NO_3} = -R_{i,NO_3,uptake} + R_{i,NH_4,nit} - R_{i,NO_3,denit}, \quad i = U, L \quad (54)$$

$$r_{i,ON} = R_{i,ON,bm} - R_{i,ON,miner} - R_{i,ON,settling} - R_{i,ON,pred}, \quad i = U, L \quad (55)$$

$$R_{i,NH_4,uptake} = \sum_{j=C,D,G} (C_{iINj} R_{j,growth} P_{iNH_4j}), \quad i = U, L \quad (56)$$

where P_{iNH_4j} is the preference factor for ammonium uptake, calculated as:

$$P_{iNH_4j} = \frac{C_{iNH_4} C_{iNO_3}}{(K_{Nj} + C_{iNH_4})(K_{Nj} + C_{iNO_3})} + \frac{C_{iNH_4} K_{Nj}}{(K_{Nj} + C_{iNH_4})(K_{Nj} + C_{iNO_3})}, \quad i = U, L; j = C, D, G \quad (57)$$

$$R_{i,NH_4,nit} = k_{nit} \theta_{nit}^{(T-20)} \frac{C_{iDO}}{K_{nit} + C_{iDO}} C_{iNH_4}, \quad i = U, L \quad (58)$$

$$R_{i,ON,miner} = k_{mn} \theta_{mn}^{(T-20)} \frac{\sum_{j=C,D,G} C_{ij} + \sum_{k=Cl a, Co} C_{ik} + \sum_{m=S1,S2,S3} C_{im}}{K_{mpc} + \sum_{j=C,D,G} C_{ij} + \sum_{k=Cl a, Co} C_{ik} + \sum_{m=S1,S2,S3} C_{im}} \times C_{i,ON}, \quad i = U, L \quad (59)$$

$$R_{i,ON,miner} = k_{mn} \theta_{mn}^{(T-20)} \frac{\sum_{j=C,D,G} C_{ij}}{K_{mpc} + \sum_{j=C,D,G} C_{ij}} C_{i,ON}, \quad i = U, L \quad (60)$$

$$R_{L,NH_4,sed} = S_N \left(1 - \frac{C_{L,DO}}{K_{DOS} + C_{L,DO}} \right) A \quad (61)$$

$$R_{i,NO_3,uptake} = \sum_{j=C,D,G} (C_{iINj} R_{j,growth} (1 - P_{iNH_4j})), \quad i = U, L \quad (62)$$

$$R_{i,NO_3,denit} = k_{denit} \theta_{denit}^{(T-20)} \frac{K_{denit}}{K_{denit} + C_{iDO}} C_{iNO_3} \quad i = U, L \quad (63)$$

$$R_{i,ON,bm} = \sum_{j=C,D,G} (k_{j,bm} \theta_{bm_j}^{(T-20)} C_{iINj} f_{OP} C_{ij}) + \sum_{k=Cl a, Co} (k_{k,bm} \theta_{bm_k}^{(T-20)} \alpha_{NC_k} f_{OP} C_{ik}) + \left(\sum_{j=C,D,G} Graz_{ikj} + Graz_{ik det} \right) \alpha_{NC_k} f(T)_k (1 - A_k) + \sum_{m=S1,S2,S3} (k_{m,bm} \theta_{bm_m}^{(T-20)} \alpha_{NC_m} f_{OP} C_{im}) + \sum_{m=S1,S2,S3} \left(\sum_{j=C,D,G} Pred_{imj} + \sum_{k=Cl a, Co} Pred_{imk} + Pred_{im det} \right) \alpha_{NC_m} \theta_{pred_m}^{(T-20)} (1 - A_m) + Pred_{iS_3S_1} \alpha_{NC S_3} \theta_{pred S_3}^{(T-20)} (1 - A_{S_3}) \quad i = U, L \quad (64)$$

$$R_{i,ON,pred} = \left(\sum_{k=Cl a, Co} Graz_{ik det} \alpha_{NC_k} f(T)_k + \sum_{m=S1,S2,S3} Pred_{im det} \alpha_{NC_m} \theta_{pred_m}^{(T-20)} \right) C_{i,ON}, \quad i = U, L \quad (65)$$

$$R_{i,ON,settling} = \frac{k_{ON,settling}(1-f_{DON})}{D_i} C_{i,ON}, \quad i = U, L \quad (66)$$

Phosphorus: Variables describing the phosphorus cycle are phosphate (PO₄) and organic phosphorus (OP). Processes within the phosphorus cycle are similar to those from the nitrogen cycles. Phosphorus is uptaken by phytoplankton in phosphate form ($R_{i,PO_4,uptake}$). The increase in phosphate concentration is due to phytoplankton, zooplankton and fish basal metabolism ($R_{i,PO_4,bm}$), temperature-dependent mineralization of organic phosphorus ($R_{i,OP,miner}$) and oxygen-dependent release from sediments ($R_{i,PO_4,sed}$). Organic phosphorus pool is augmented with algal, zooplankton and fish basal metabolism ($R_{i,OP,bm}$) and decline by mineralization ($R_{i,OP,miner}$), zooplankton and fish grazing ($R_{i,OP,pred}$) and settling process ($R_{i,OP,settling}$). The following rate equations stand for the main processes in phosphorus biogeochemical cycle:

$$r_{i,PO_4} = \begin{cases} -R_{i,PO_4,uptake} + R_{i,PO_4,bm} + R_{i,OP,miner}, & i = U \\ -R_{i,PO_4,uptake} + R_{i,PO_4,bm} + R_{i,OP,miner} + R_{i,PO_4,sed}, & i = L \end{cases} \quad (67)$$

$$r_{i,OP} = R_{i,OP,bm} - R_{i,OP,miner} - R_{i,OP,settling} - R_{i,OP,pred}, \quad i = U, L \quad (68)$$

$$R_{i,PO_4,uptake} = \sum_{j=C,D,G} (C_{i,IPj} R_{j,growth}), \quad i = U, L \quad (69)$$

$$R_{i,PO_4,bm} = \sum_{j=C,D,G} (k_{j,bm} \theta_{bmj}^{(T-20)} C_{i,IPj} (1-f_{OP}) C_{ij}) + \sum_{k=Cl,Co} (k_{k,bm} \theta_{bm_k}^{(T-20)} \alpha_{PC_k} (1-f_{OP}) C_{ik}) + \sum_{m=S_1,S_2,S_3} (k_{m,bm} \theta_{bmm}^{(T-20)} \alpha_{PC_m} (1-f_{OP}) C_{im}), \quad i = U, L \quad (70)$$

$$R_{L,PO_4,sediment} = S_P \left(1 - \frac{C_{L,DO}}{K_{DOS} + C_{L,DO}} \right) A \quad (71)$$

$$R_{i,OP,bm} = \sum_{j=C,D,G} (k_{j,bm} \theta_{bmj}^{(T-20)} C_{i,IPj} f_{OP} C_{ij}) + \sum_{k=Cl,Co} \left((k_{k,bm} \theta_{bm_k}^{(T-20)} \alpha_{PC_k} f_{OP} C_{ik}) + \left(\sum_{j=C,D,G} \text{Graz}_{ikj} + \text{Graz}_{ik} \det \right) \alpha_{PC_k} f(T)_k (1-A_k) \right) + \sum_{m=S_1,S_2,S_3} (k_{m,bm} \theta_{bmm}^{(T-20)} \alpha_{PC_m} f_{OP} C_{im}) + \left(\sum_{j=C,D,G} \text{Pred}_{imj} + \sum_{k=Cl,Co} \text{Pred}_{imk} + \sum_{m=S_1,S_2,S_3} \text{Pred}_{im} \det + \text{Pred}_{iS_3S_1} \right) \times \alpha_{PC_m} \theta_{pred_m}^{(T-20)} (1-A_m), \quad i = U, L \quad (72)$$

$$R_{i,OP,pred} = \left(\sum_{k=Cl,Co} \text{Graz}_{ik} \det \alpha_{PC_k} f(T)_k + \sum_{m=S_1,S_2,S_3} \text{Pred}_{im} \det \alpha_{PC_m} \theta_{pred_m}^{(T-20)} \right) C_{i,OP}, \quad i = U, L \quad (73)$$

$$R_{i,OP,miner} = k_{mp} \theta_{mp}^{(T-20)} \frac{\sum_{j=C,D,G} C_{ij} + \sum_{k=Cl,Co} C_{ik} + \sum_{m=S_1,S_2,S_3} C_{im}}{K_{mpc} + \sum_{j=C,D,G} C_{ij} + \sum_{k=Cl,Co} C_{ik} + \sum_{m=S_1,S_2,S_3} C_{im}} C_{i,OP}, \quad i = U, L \quad (74)$$

$$R_{i,OP,settling} = \frac{k_{OP,settling}(1-f_{DOP})}{D_i} C_{i,OP} \quad i = U, L \quad (75)$$

3.3. Optimal control problem formulation

The detailed mechanistic eutrophication model described in Sections 3.1 and 3.2 is formulated within a dynamic optimization framework to allow the determination of restoration actions through three different techniques, which have been described in Section 2, and their combinations, as well as associated costs. In all cases, the objective function is the minimization of the weighted sum of the squared differences between cyanobacteria concentration in the upper water layer and a desired value of 0.25 mg/l and the squared differences between phosphate concentration and a desired value of 0.02 mg PO₄/l, both of them below eutrophication limits (Wetzel, 1983).

3.3.1. Optimization model for restoration through an artificial wetland

The control variable representing phosphorus retention through the wetland is the fraction of tributary flowrate (El Divisorio) that is derived through the wetland. The dynamic optimization problem is formulated as:

$$\min \left[0.7 \int_0^{t_f} \left(\sum_{j=1}^3 C_{(t)} - 0.25 \right)^2 dt + 0.3 \int_0^{t_f} (C_{(t)U,PO_4} - 0.02)^2 dt \right]_{st} \quad (76)$$

DAE Water Quality Model

$$0 \leq F_{wetland} \leq 0.5 F_{Divisorio} (\text{m}^3/\text{day})$$

3.3.2. Optimization model for restoration by aeration and artificial wetland

In this work, we include the possibility of applying an aeration strategy in the lower water layer, as an additional restoration technique. As it can be seen in Eqs. (67) and (71), phosphate rate equations include an oxygen dependent phosphorus release term ($R_{L,PO_4,sediment}$) from the sediment. These equations show that the higher dissolved oxygen concentration, the lower phosphate concentration in the water column. To represent the combination of reducing external nutrient loading and the in-lake restoration strategy consisting of hypolimnetic oxygenation, the optimization problem has two time dependent degrees of freedom, corresponding to the fraction of tributary that is derived to the wetland ($F_{wetland}$) and the aeration term (R_{Air}) that is added to the dissolved oxygen equation rate in the lower layer (Eq. (38)):

The dynamic optimization problem (76) is extended to:

$$\min \left[0.7 \int_0^{t_f} \left(\sum_{j=1}^3 C_{(t)} - 0.25 \right)^2 dt + 0.3 \int_0^{t_f} (C_{(t)U,PO_4} - 0.02)^2 dt \right] \quad (77)$$

DAE Water Quality Model (1)–(37), (39)–(75)

$$r_{L,DO} = -R_{L,DO,nitr} - R_{L,DO,resp/fof} - R_{U,DO,resp} - R_{L,POC,miner} - R_{L,DO,sediment} + R_{Air}$$

$$0 \leq F_{wetland} \leq 0.5F_{Divisorio}(\text{m}^3/\text{day})$$

$$R_L \leq R_{Air} \leq R_U$$

Standardized investment costs on a per kg oxygen basis are approximately \$2.50/kgO₂ with an operating cost of \$0.072/kgO₂ (Wagner, 2004). Shape and depth of the lake affect cost terms, with deep, single basin lakes requiring the simplest and least expensive systems.

3.3.3. Optimization model for restoration by biomanipulation and artificial wetland

In this work, the addition of state variables associated to zooplankton and planktivorous fish concentrations allows the definition of three control variables to represent biomanipulation within the water body. They correspond to the rates of fish removal for each of the three zooplanktivorous size classes (S₁, S₂, S₃). In this way, to model external nutrient loading reduction through a wetland combined with biomanipulation associated to fish removal, four time dependent optimization variables have been considered, including the fraction of tributary that is derived to the wetland. A removal term is added to the generation/consumption equation for each size class of planktivorous fish (S₁, S₂ and S₃, Eqs. (15) and (16)). The dynamic optimization problem is as follows:

$$\min \left[0.7 \int_0^{t_f} \left(\sum_{j=1}^3 C_{(t)} - 0.25 \right)^2 dt + 0.3 \int_0^{t_f} (C_{(t)} - 0.02)^2 dt \right] \quad (78)$$

DAE Water Quality Model (1)–(14), (17)–(75)

$$r_{im} = R_{im,pred} - R_{im,bmetab} - R_{im,recruit} - R_{im,spaw} - S_m \text{Removal}, \quad m = S_1, S_2$$

$$r_{iS_1} = R_{im,pred} - R_{im,bmetab} - R_{iS_3,canib} - R_{im,recruit} + R_{im,spaw} - S_3 \text{Removal}$$

$$0 \leq F_{wetland} \leq 0.5F_{Divisorio}(\text{m}^3/\text{day})$$

$$LB \leq C_{im} \leq UB,$$

$$i = U, L, m = S_1, S_2, S_3$$

Biomanipulation can be an economic way to reduce eutrophication symptoms. Håkanson (2000) reported that a typical cost for fish removal would be about 40–120 US\$/kg (wet weight) removed fish. Wagner (2004) reports costs between US\$/ha 2.500 and 12.500 for Massachusetts lakes and Liboriussen, Søndergaard, and Jeppesen (2007) reports 2586 US\$/ha for Danish lakes.

4. Discussion of results

The DAE model has 42 state differential variables and 110 algebraic ones. It has been formulated within a dynamic optimization environment and solved with a control vector parameterization strategy (Process Systems Enterprise, 2009). In all cases, a time horizon of 548 days (1 year and a half) has been considered and control variables have been discretized in time periods of 1 day length. Constant values have been considered for desired values of total phytoplankton and phosphorus concentrations (0.25 and 0.02 mgC/l, respectively), as they correspond to eutrophication limits.

4.1. Restoration through an artificial wetland

As a first step, we have solved Problem (76) to determine the fraction of tributary stream that is deviated through an arti-

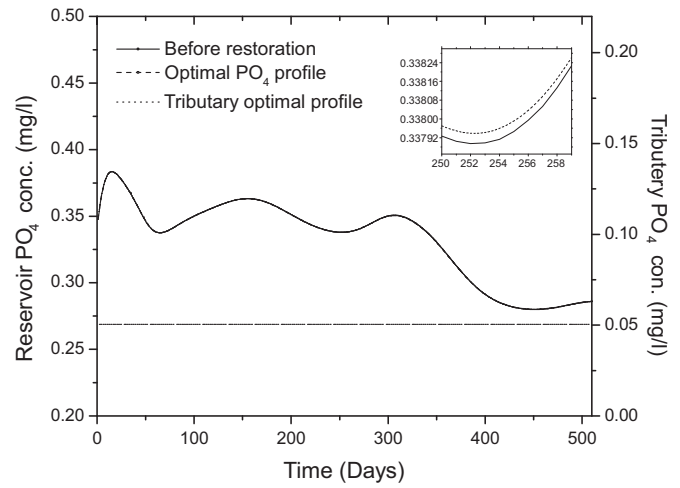


Fig. 3. Profiles for phosphate concentration prior and after nutrient loading reduction and tributary deviation through a wetland.

cial wetland, to reduce nutrient loading to the lake. In our case study, a pilot plant wetland has been built next to Paso de las Piedras Reservoir, for phosphate and nitrate retention from the Divisorio Stream (López et al., 2007), which is the tributary with highest nutrient concentration (Estrada et al., 2009a). Collected data from this artificial wetland show that the global retention of phosphate is 50%, which has been included in the model. When solving the optimal control problem with this single optimization variable, the optimal profile for the control variable remains at its upper bound throughout the entire time horizon. This causes an average decrease in the water column phosphate concentration from 0.2237 to 0.2223 mg/l (Fig. 3). Total phytoplankton concentration (cyanobacteria + diatoms + chlorophyta) in the upper layer of Paso de las Piedras Reservoir has a slight decrease from 1.349 to 1.351 mgC/l mean value over the 548 days (Fig. 4). These results confirm that even though reduction of external loading of phosphorus is necessary for restoration in the long-term, it is not enough to radically control phytoplankton growth, mainly due to internal nutrient recycles in the reservoir, as described by biogeochemical cycle equations (Eqs. (46)–(75)).

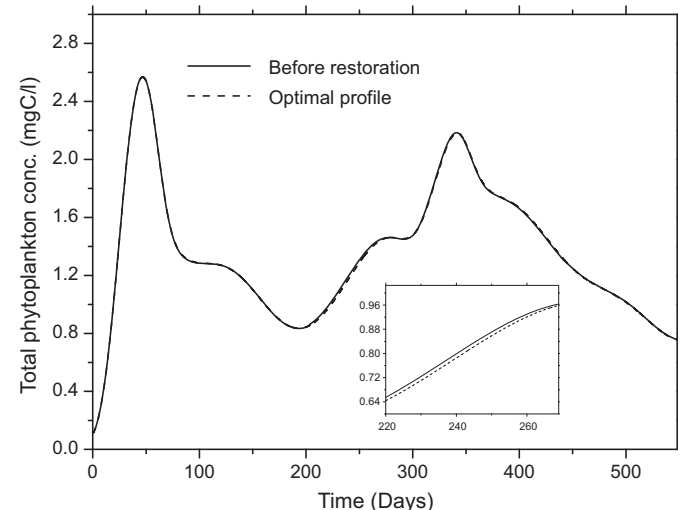


Fig. 4. Profiles for total phytoplankton concentration before and after tributary deviation through a wetland for algae growth control.

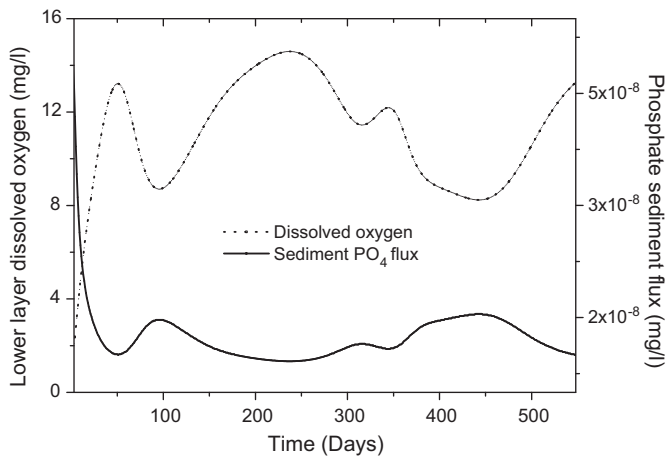


Fig. 5. Lower layer dissolved oxygen and phosphate sediment flux profiles.

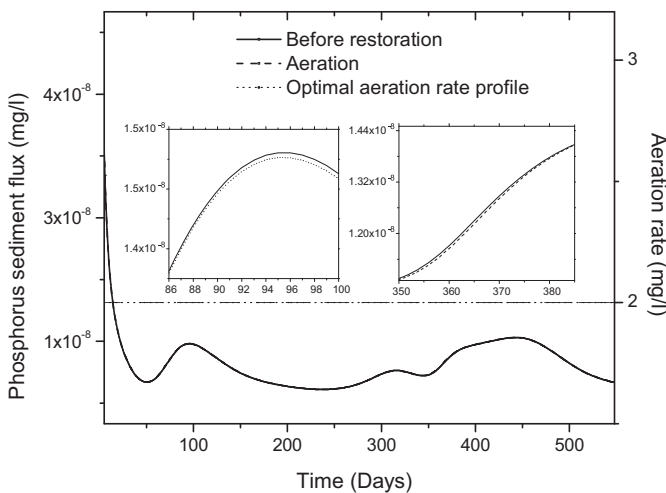


Fig. 6. Profiles for aeration rate and phosphate sediment flux profile before and after aeration.

4.2. Restoration by aeration

Figs. 5–7 show numerical results for this case. The optimal profile for aeration rate, the control variable, has a constant value of 2 mgO/l (Fig. 5), which corresponds to its lower bound. This value

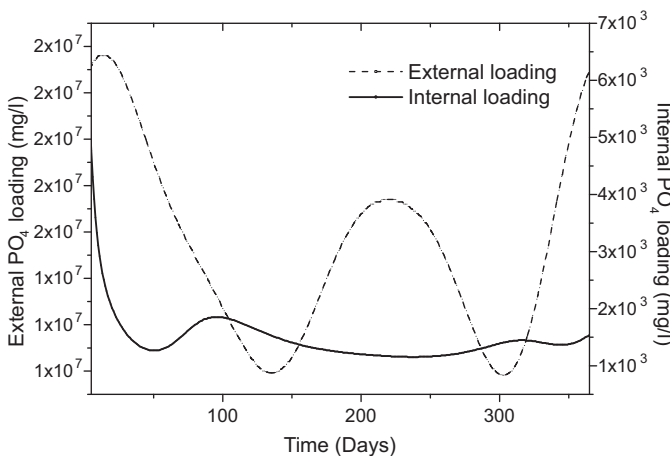


Fig. 7. Comparison between external (from tributaries) and internal (from sediments) phosphate loading in Paso de las Piedras Reservoir.

has been taken from the literature, as a minimum value to justify aeration investment costs. Fig. 5 also shows profiles for dissolved oxygen concentration and phosphate flux from sediments in Paso de las Piedras Reservoir. It can be clearly noted that as dissolved oxygen concentration increases, phosphate flux from sediments decreases, following Eq. (71). However, in Fig. 6, it can be seen that there is only a slight decrease of internal phosphate release (phosphorus sediment flux) when aeration of the lower layer of the water column is applied, with values ranging between 0 and 0.59% and an average value of 0.085% of reduction of internal phosphate release. These results are in agreement with collected data from Paso de las Piedras reservoir that show there is no stratification in the water body (Intartaglia & Sala, 1989) and, therefore, no need for aeration in the bottom. Furthermore, Gurkan et al. (2006) found that aeration by itself reduced total phosphorus about 62% from the beginning level after 10 years in anoxic lower layers. Finally, as it can be seen in Fig. 7, the external phosphorus loading in Paso de las Piedras Reservoir is significantly higher than the internal loading from sediments, which enforces the fact that external restoration strategies are required.

Based on these results, the implementation of an aeration restoration method could involve a cost of US\$647 million, without rendering a proper control on nutrient content and phytoplankton concentration in the short and middle term. The main reason for the failure of an aeration approach for the restoration of Paso de las Piedras water quality, is that dissolved oxygen concentration in the lower layer is close to the saturation value almost throughout all the year. This fact is due to both the non-stratified condition of the water body and the large retention time, which allow the oxygenation of the deeper water layers of the reservoir. Model results also confirm that the application of any alternative methods for internal phosphorus loading reduction (e.g. dredging, alum treatment) may not be useful, at least in a short-term horizon, for Paso de las Piedras Reservoir.

4.3. Restoration by biomanipulation

In our case study, we have considered *O. bonariensis* (pampean silverside) as the zooplanktivorous fish, as it is the most abundant fish in large pampean lakes (Quirós, Rosso, Rennella, Sosnovsky, & Boveri, 2002). It is a visually guided planktivorous (Boveri & Quirós, 2002) and benthivorous fish. In the first years of its life cycle, *O. bonariensis* is strictly planktivorous. Numerous studies have shown that copepoda and cladocera are two primary food items in the silverside diet (Boveri & Quirós, 2002; Escalante, 2001; Grosman & Sanzano, 2003; Grosman, Sanzano, Agüera, & Gonzalez, 2001). Last juvenile and adult stages also incorporate detritus and occasionally gastropods, and generally become cannibals around the fourth year (Escalante, 2001). Numerical results for the optimal control problem that takes into account the use of an artificial wetland combined with biomanipulation (Eq. (78)) are showed in Figs. 8–12. Fig. 8 shows a comparison of cyanobacteria profile before and after restoration, throughout the time horizon of 548 days. The maximum concentration value in both cyanobacteria blooms can be reduced by biomanipulation from 1.911 to 0.185 mgC/l (day 61) and from 2.168 to 0.346 mgC/l (day 347), respectively, which is an acceptable value considering the implications that high concentrations of this algae group may have on human and animal health (potential toxins). The removal of zooplanktivorous fish allows zooplankton concentration average increase from 0.157 to 0.702 mgC/l for copepoda and from 0.1190 to 0.6144 mgC/l for cladocera (Fig. 9). It can be seen that the increase in zooplankton concentration is not the same for the two functional groups (copepoda and cladocera) and this fact reflects the difference in their feeding habits and food preferences, included in the model for each zooplankton group.

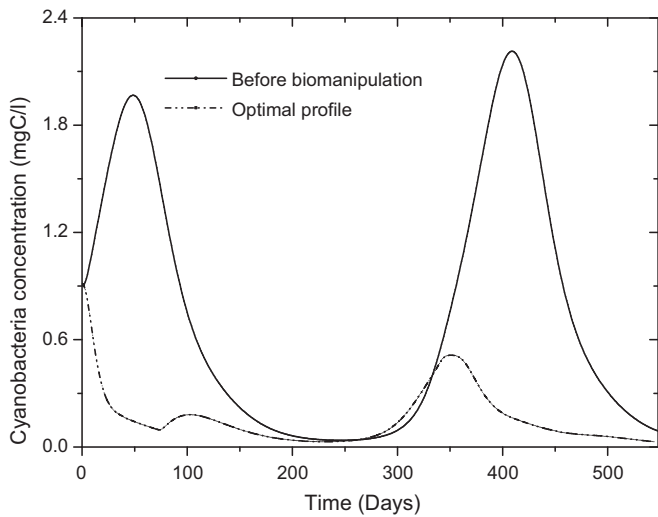


Fig. 8. Cyanobacteria concentration profile prior to biorestation, as compared to optimal profile after biomanipulation.

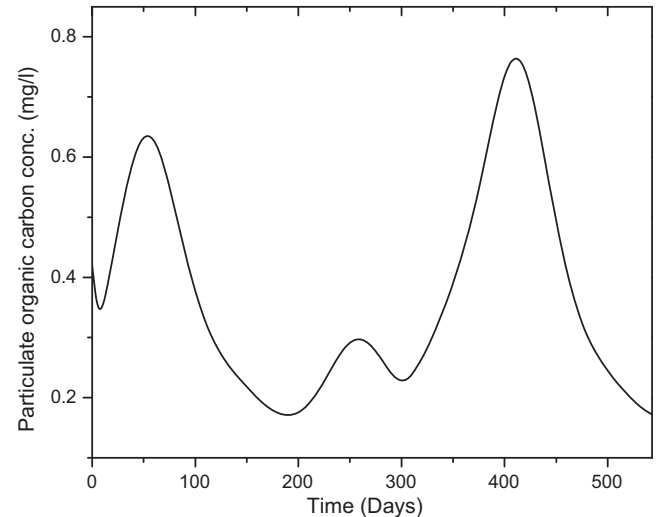


Fig. 11. Optimal particulate organic carbon concentration profiles.

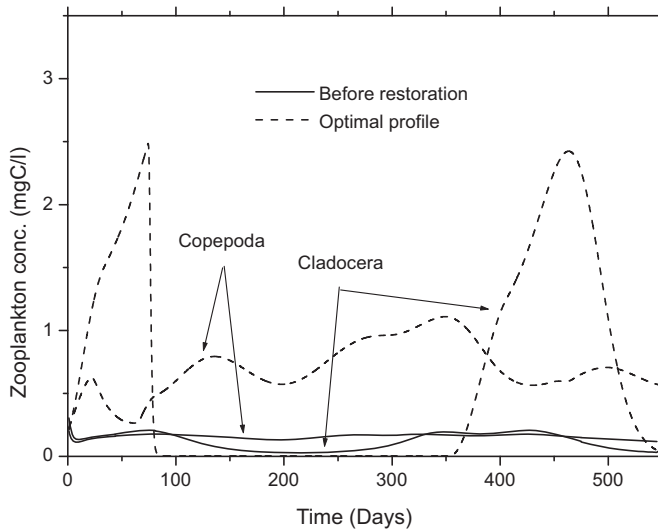


Fig. 9. Optimal profiles for zooplankton (copepoda and cladocera) before and after zooplanktivorous fish removal.

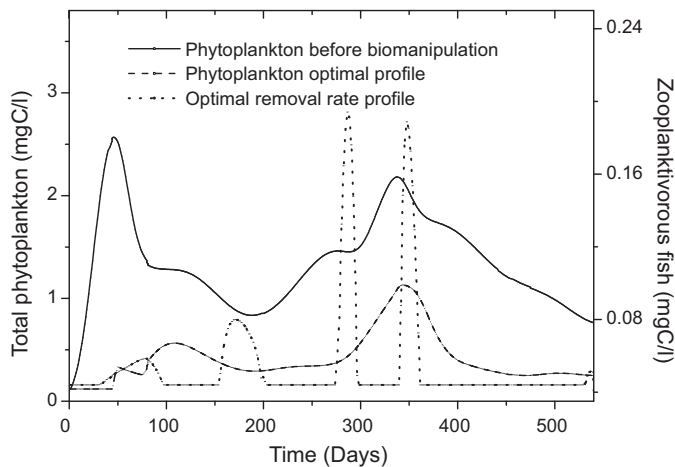


Fig. 10. Optimal profiles for zooplanktivorous fish removal rate and total phytoplankton concentration before and after biomanipulation.

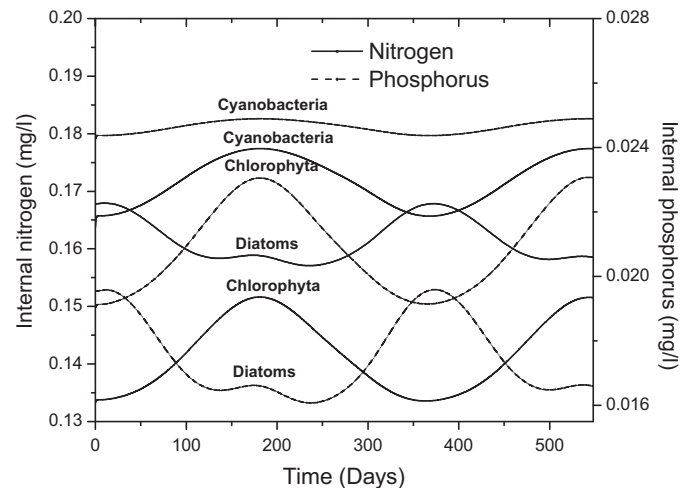


Fig. 12. Optimal concentration profiles for phytoplankton internal phosphorus and nitrogen.

Optimal fish removal rate profile (Fig. 10) shows four peaks, the first one during phytoplankton summer–autumn blooms, the second before the maximum diatoms concentration and the last two before and during the second summer bloom, respectively. A constant removal rate of 4.4×10^{-2} mgC/l day is determined for the rest of the time horizon. To calculate wet weight (WW) for fish biomass, we consider that carbon mass is 50% of fish biomass dry weight (DW). Furthermore, the following relationship holds for *O. bonariensis* (Tsuzuki, Aikawa, Strüssmann, & Takashima, 2000): $3.1 \text{ mgDW} = 17.1 \text{ mg WW}$. Fig. 11 shows profiles for particulate organic carbon (POC), which represents the carbon portion of detritus and is a food item in zooplankton and fish diet. Finally, Fig. 12 shows optimal profiles for internal phosphorus and nitrogen in each phytoplankton group.

Based on the optimal fish removal rate profile, shown in Fig. 10, the total biomass of *O. bonariensis* that should be removed is calculated for the entire time horizon, giving a total value of 328 ton WW. The associated removal cost is between US\$ 15.26 and 45.78 million, taking into account values of US\$ 40 and 120 per kg of wet weight (Håkanson, 2000). The inclusion of model equations for the three *O. bonariensis* size classes allows considering the differences in growth kinetics and food preferences in the silverside life cycle and provides valuable information that can be used to

select the appropriate fishery technique, such as the gillnet mesh size required for fish removal. Numerical results for biomanipulation of *O. bonariensis* in Paso de las Piedras Reservoir reveal that 45% of stage S_1 , 38% of S_2 and 17% of S_3 , respectively, have to be removed. As numerical results have shown, biomanipulation (combined with an artificial wetland) is the most cost effective strategy to achieve essential improvement of water quality in the short and middle term, for Paso de las Piedras Reservoir.

5. Conclusions

In this paper, we address the restoration of water bodies for which nonpoint nutrient sources are dominant. We formulate a detailed mechanistic model that represents most important processes in a lake whose parameters have been tuned to the site under study (Estrada et al., 2009a), as a highly complex differential algebraic system. We formulate an optimal control problem subject to this DAE to determine the application of different restoration strategies, their predicted effects in the short and middle term and their associated costs. Numerical results show that this approach can be a valuable tool for the selection and planning of restoration techniques, providing not only estimations of restoration results, but useful information on the restoration technique itself. For example, the size of the required wetland for nutrient loading decrease can be estimated, as well as the proper selection of the fishery technique, such as the gillnet mesh size, in the case of manipulation by fish removal. Numerical results also show that the alternative in-lake strategy of hypolimnion aeration is neither adequate nor cost effective for this ecosystem.

Process systems engineering techniques provide important insights on the sustainable management of water resources, handled as complex dynamic ecological systems. This information can be used to provide recommendations on policies, approaches and mechanisms through which water can be better managed to maintain both the ecosystem health and the natural resource sectors' economic sustainability.

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