ISSN 1547-4771, Physics of Particles and Nuclei Letters, 2017, Vol. 14, No. 6, pp. 799–810. © Pleiades Publishing, Ltd., 2017.

PHYSICS OF ELEMENTARY PARTICLES AND ATOMIC NUCLEI. THEORY

Dynamo Effects and Geometrical Origin of the Alpha Term in Affine Theory of Gravity[1]

Diego Julio Cirilo-Lombardo[a, b]

[a] *Universidad de Buenos Aires, Consejo Nacional de Investigaciones Cientificas y Tecnicas (CONICET), National Institute of Plasma Physics(INFIP), Facultad de Ciencias Exactas y Naturales, Cuidad Universitaria, Buenos Aires, 1428 Argentina*

[b] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, 141980 Russia*

e-mail: E-maildiego777jcl@gmail.com

Received March 8, 2017

Abstract—Dynamo effect is considered in the context of an Unified field theoretical model based in affine geometries. We show that there exists an analog “α-term” in the equations that has a purely geometric origin, in sharp contrast with the turbulent one. Some high energy and astrophysical implicancies (primordial magnetic field, compact objects dynamics, chiral magnetic effects, etc) coming from this type of alternative model of gravitation are briefly discussed.

DOI: 10.1134/S1547477117060097

1. INTRODUCTION

Dynamo effect is considered in the context of an Unified field theoretical model based in affine geometries. Due that the common standard approaches based only in General Relativity (GR) are not based completely in symmetry principles: if this were the case, an harmonious interplay “matter-energy ↔ space-time” must be automatically fulfilled [10, 11]. However in the non-Riemannian case [12–16], the corresponding affine geometrical structure induces naturally the Lagrangian function. This Lagrangian function or geometrical action is a measure or the square root of the determinant of a particular combination of the fundamental tensors of the geometry: $\sqrt{\left|\det f\left(g_{\mu\nu}, F_{\mu\nu,} R_{\mu\nu}\right)\right|}$ with the (0,2) tensors $g_{\mu\nu}, F_{\mu\nu,} R_{\mu\nu}$: the symmetric metric, the antisymmetric (that acts as potential of the torsion field) and the generalized Ricci tensor (proper of the non Riemmanian geometry). The three tensors are related with a Clifford structure of the tangent space (for detaills see [17]) and the explicit simplest choice for $f\left(g_{\mu\nu}, F_{\mu\nu,} R_{\mu\nu}\right)$ is given by :

$$\mathcal{L}_{gs} = \sqrt{\det\left[\lambda g_{\alpha\beta}\left(1+\frac{R_s}{4\lambda}\right)+\lambda F_{\alpha\beta}\left(1+\frac{R_A}{4\lambda}\right)\right]}, \quad (1)$$

$$R_s \equiv g^{\alpha\beta}R_{(\alpha\beta)}; \quad R_A \equiv f^{\alpha\beta}R_{[\alpha\beta]} \quad (2)$$

with $f^{\alpha\beta} \equiv \dfrac{\partial \ln\left(\det F_{\mu\nu}\right)}{\partial F_{\alpha\beta}}, \det F_{\mu\nu} / \tilde{F}^{\mu\nu}$ and λ an arbitrary parameter). We pointed out that this particular

[1] The article is published in the original.

choice for the Larangian function is only for simplicity. Due the basic structure of the theory, the induced energy momentum tensor and fundamental constants (as the Newton G now are functions in reality) emerge naturally from the same geometry. Dynamo effect is considered in this letter in the context of an Unified field theoretical model based in affine geometries. We show that there exists an analog "α-term" in the equations that has a purely geometric origin, in sharp contrast with the turbulent one. Some high energy and astrophysical implicancies (primordial magnetic field, compact objects dynamics [30, 31], chiral magnetic effects, etc) coming from this type of alternative model of gravitation are briefly discussed and some concluding remarks in Section XII.

2. FIELD EQUATIONS

The variational process is a crucial point both: from the mathematical and from the physical viewpoint. As we have been analyzed before [21, 22], there exist several difficulties concerning the starting physical assumptions involving the variational procedure. The geometry of the spacetime Manifold is to be determined by the Noether symmetries

$$\frac{\delta L_G}{\delta g^{\mu\nu}} = 0, \quad \frac{\delta L_G}{\delta f^{\mu\nu}} = 0 \tag{3}$$

where the functional (Hamiltonian) derivatives in the sense of Palatini (in this case with respect to the potentials), are understood. The choice "measure-like" form for the geometrical Lagrangian L_G (reminiscent of a nonlinear sigma model), as is evident, satisfy the following principles:

(i) the principle of the natural extension of the Lagrangian density as square root of the fundamental line element containing also $F_{\mu\nu}$.

(ii) the symmetry principle between $g_{\mu\nu}$ and $F_{\mu\nu}$ (e.g. $g_{\mu\nu}$ and $F_{\mu\nu}$ should enter into L_G symmetrically)

(iii) the principle that the spinorial symmetry, namely

$$\nabla_\mu g_{\lambda\nu} = 0, \quad \nabla_\mu \sigma_{\lambda\nu} = 0 \tag{4), (5}$$

with

$$g_{\lambda\nu} = \gamma_\lambda \cdot \gamma_\nu, \quad \sigma_{\lambda\nu} = \gamma_\lambda \wedge \gamma_\nu \sim {}^*F_{\lambda\nu} \tag{6), (7}$$

should be derivable from $L_G(3)$.

The last principle is key because it states that the spinorial invariance of the fundamental spacetime structure (kinematic symmetry of the world picture) should be derivable from the dinamical simmetries given by (3). The fact that the L_G satisfies the 3 principles will be demonstrated below showing also that it has the simpler form.

Notice that the action density proposed by Einstein in [27] in his nonsymmetric field theory satisfies (i) and (ii) but not (iii).

Remark 1. Due the totally antisymmetric character of the torsion field it is completely determined by the fundamental (structural 2-form) antisymmetric tensor, and consequently the variations must adquire the form given by expression (3): **metric and torsion have each one their respective potentials that are in coincidence with the fundamental structure of the geometry.**

2.1. $\delta_g L_G$

The starting point for the metrical variational procedure is in the same way as in the standard Born-Infeld theory: from the following factorization of the geometrical Lagrangian:

$$\mathscr{L} = \sqrt{|g|}\sqrt{\det(\alpha\lambda)}\sqrt{1 + \frac{1}{2b^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16b^4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2} \equiv \sqrt{|g|}\sqrt{\det(\alpha\lambda)}\mathbb{R}, \tag{8}$$

where

$$b = \frac{\alpha}{\beta} = \frac{1 + (R_S/4\lambda)}{1 + (R_A/4\lambda)}, \quad R_S = g^{\alpha\beta}R_{\alpha\beta}, \quad R_A = f^{\alpha\beta}R_{\alpha\beta}, \tag{9), (10), (11}$$

and λ an arbitrary constant we perform the variational metric procedure with the following result (details see Appendix I)

$$\begin{aligned}\delta_g\mathscr{L} = 0 \Rightarrow R_{(\alpha\beta)} - \frac{g_{\alpha\beta}}{4}R_s &= \frac{R_s}{2\mathbb{R}^2\alpha^2}\left[F_{\alpha\lambda}F^\lambda_\beta - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right]\\ &+ \frac{R_s}{4\mathbb{R}^2\alpha^2 b^2}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\left(\frac{F_{\eta\rho}\tilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\tilde{F}^\lambda_\beta\right) + \frac{F_{\eta\rho}\tilde{F}^{\eta\rho}}{2}\frac{R_{(\alpha\beta)}}{R_s}\right]\\ &+ 2\lambda\left[g_{\alpha\beta} + \frac{1}{\mathbb{R}^2\alpha^2}\left(F_{\alpha\lambda}F^\lambda_\beta + \frac{F_{\mu\nu}\tilde{F}^{\mu\nu}}{2b^2}\left(\frac{F_{\eta\rho}\tilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\tilde{F}^\lambda_\beta\right)\right)\right],\end{aligned} \tag{12), (13}$$

Remark 2. Notice that:

(1) The Eq. (13) is *trace-free* type [1–5], consequently the trace of the third term of the above equation (that is the cosmological one) is equal to zero. This happens trivially if $\lambda = 0$ or $4\mathbb{R}^2\alpha^2 = -\left(F_{\alpha\lambda}F^{\alpha\lambda} - \frac{\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2}{4b^2}\right)$. In terms of the Maxwell lagrangian we have $(\mathbb{R}\alpha)^2 = \left(L_{\text{Maxwell}} + \frac{\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2}{16b^2}\right) \equiv \mathcal{W}\left(I_S, I_P, b\right)$ that allow us to simplify the Eq. (13) once more as follows

$$R_{(\alpha\beta)} - \frac{g_{\alpha\beta}}{4}R_s = \frac{R_s}{2\mathcal{W}}\left[F_{\alpha\lambda}F_\beta^\lambda - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right] + \frac{R_s}{4\mathcal{W}b^2}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\left(\frac{F_{\eta\rho}\tilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\tilde{F}_\beta^\lambda\right) + \frac{F_{\eta\rho}\tilde{F}^{\eta\rho}}{2}\frac{R_{(\alpha\beta)}}{R_s}\right]$$
$$+ 2\lambda\left[g_{\alpha\beta} + \frac{1}{\mathcal{W}}\left(F_{\alpha\lambda}F_\beta^\lambda + \frac{F_{\mu\nu}\tilde{F}^{\mu\nu}}{2b^2}\left(\frac{F_{\eta\rho}\tilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\tilde{F}_\beta^\lambda\right)\right)\right],$$

(2) b takes the place of limiting parameter (maximum value) for the electromagnetic field strenght.

(3) b is not a constant in general, in sharp contrast with the Born-Infeld or string theory cases.

(4) Because b is the ratio $\frac{\alpha}{\beta} = \frac{1 + \left(R_S/4\lambda\right)}{1 + \left(R_A/4\lambda\right)}$ involving both curvature scalars from the contractions of the generalized Ricci tensor: it is preponderant when the symmetrical contraction of $R_{\alpha\beta}$ is greater than the skew one.

(5) The fact pointed out in (ii), namely that the curvature scalar plays the role as some limiting parameter of the field strenght, was conjectured by Mansouri in [19] in the context of gravity theory over group manifold (generally with symmetry breaking). In such a case, this limit was stablished after the explicit integration of the internal group-valuated variables that is not our case here.

(6) In similar form that the Eddington conjecture: $R_{(\alpha\beta)}g_{\alpha\beta}$, we have a condition over the ratios as follows:

$$\frac{R_{(\alpha\beta)}}{R_s} \propto \frac{g_{\alpha\beta}}{D} \tag{14}$$

that seems to be universal.

(7) The equations are the simplest ones when $b^{-2} = 0\,(\beta = 0)$, taking the exact the following form

$$R_{(\alpha\beta)} - \frac{g_{\alpha\beta}}{4}R_s \tag{15}$$
$$= \underbrace{\frac{R_s}{2\alpha^2}\left[F_{\alpha\lambda}F_\beta^\lambda - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right]}_{\text{Maxwell-like}} + 2\lambda\underbrace{\left[g_{\alpha\beta} + \frac{1}{W}F_{\alpha\lambda}F_\beta^\lambda\right]}_{\tilde{g}_{\text{eff}}},$$

notice that the pseudoscalar part dissapear. This particular case (e.g. projective invariant) will be used through this work.

2.2. $\delta_f L_G$

Let us to take as starting point the geometrical lagrangian (1)

$$\mathcal{L}_{gs} = \sqrt{\det\left[\lambda g_{\alpha\beta}\left(1 + \frac{R_s}{4\lambda}\right) + \lambda F_{\alpha\beta}\left(1 + \frac{R_A}{4\lambda}\right)\right]} \tag{16}$$

$$= \sqrt{|g|}\lambda^2\alpha^2\left(\sqrt{1 + \frac{1}{2}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{16}\left(\mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu}\right)^2}\right) \tag{17}$$

then, having into account that: $R_A = f^{\mu\nu}R_{\mu\nu}$ and $\frac{\partial \ln\left(\det F_{\mu\nu}\right)}{\partial F_{\alpha\beta}} = f^{\alpha\beta}$ we obtain

$$\frac{\delta L_G}{\delta F_{\sigma\omega}} = 0 \rightarrow \left(\frac{\sqrt{|g|}\lambda\beta}{2\mathbb{R}b}\right)\left[\mathbb{F}^{\sigma\omega}\beta - \frac{\mathbb{F}}{4\lambda}R_{[\mu\nu]}\chi^{\mu\nu\sigma\omega}\right] = 0 \tag{18}$$

where: $\mathbb{F} \equiv \left[F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}b^{-2}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right]$, $\mathbb{F}^{\sigma\alpha} \equiv \left[F^{\sigma\alpha} - \frac{1}{4}b^{-2}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)\tilde{F}^{\sigma\alpha}\right]$ and $\chi^{\mu\nu\sigma\omega} \equiv f^{\mu\omega}f^{\sigma\nu} - f^{\mu\sigma}f^{\omega\nu}$. Notice that the quantity b also was taking into account in the expression (18).

Contracting (18) with $F_{\alpha\beta}$, a condition over the curvature and the electromagnetic field invariants is obtained as

$$\left(\frac{\sqrt{|g|}\lambda\beta}{\mathbb{R}b}\right)\mathbb{F}\left[\beta - \frac{R_A}{2\lambda}\right] = 0.$$

This condition is satisfield for $R_A = -4\lambda$ is the exact projective invariant case (that correspond with $\beta = 0$), and for $R_A = 2\lambda$.

3. GEOMETRY INDUCES PHYSICS

3.1. Emergent Trace Free Gravitational Equations: the Meaning of Λ

Starting from the trace free equation (13)

$$\underbrace{\overset{\circ}{R}_{\alpha\beta}-\frac{g_{\alpha\beta}}{2}\overset{\circ}{R}}_{\equiv G_{\alpha\beta}} = \underbrace{6\left(-h_\alpha h_\beta + \frac{g_{\alpha\beta}}{2}h_\gamma h^\gamma\right)}_{\equiv T^h_{\alpha\beta}} + \frac{g_{\alpha\beta}}{2}R_s + T^F_{\alpha\beta} + 2\lambda\rho_{\alpha\beta}, \tag{19}$$

$$\rho_{\alpha\beta} \equiv g_{\alpha\beta} + \frac{1}{\mathcal{W}}\left(F_{\alpha\lambda}F^\lambda_\beta + \frac{F_{\mu\nu}\tilde{F}^{\mu\nu}}{2b^2}\left(\frac{F_{\eta\rho}\tilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\tilde{F}^\lambda_\beta\right)\right), \tag{20}$$

$$T^F_{\alpha\beta} \equiv \frac{R_s}{2\mathcal{W}}\left\{\left(F_{\alpha\lambda}F^\lambda_\beta - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right) + \frac{1}{2b^2}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\left(\frac{F_{\eta\rho}\tilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\tilde{F}^\lambda_\beta\right) + \frac{\left(F_{\eta\rho}\tilde{F}^{\eta\rho}\right)^2}{2}\frac{R_{(\alpha\beta)}}{R_s}\right]\right\}. \tag{21}$$

(Quantities and operators with a circle above are those corresponding to general relativity.)

Then, as $\overset{\circ}{\nabla}{}^\alpha G_{\alpha\beta} = \overset{\circ}{\nabla}{}^\alpha\left(T^h_{\alpha\beta} + T^F_{\alpha\beta}\right) = 0$ consequently:

$$\begin{aligned}&\nabla^\alpha\left(\frac{g_{\alpha\beta}}{2}R_s + 2\lambda\rho_{\alpha\beta}\right) = 0\\ \Rightarrow &\left(\frac{g_{\alpha\beta}}{2}R_s + 2\lambda\rho_{\alpha\beta}\right) = \Lambda g_{\alpha\beta} \to R_s = 2\Lambda\end{aligned} \tag{22}$$

come back to the original trace free expressions we have the expected formula

$$\begin{aligned}&\underbrace{\overset{\circ}{R}_{\alpha\beta}-\frac{g_{\alpha\beta}}{2}\overset{\circ}{R}}_{\equiv G_{\alpha\beta}}\\ &= \underbrace{6\left(-h_\alpha h_\beta + \frac{g_{\alpha\beta}}{2}h_\gamma h^\gamma\right)}_{\equiv T^h_{\alpha\beta}} + T^F_{\alpha\beta} + \Lambda g_{\alpha\beta}.\end{aligned} \tag{23}$$

Remark 3. Tracing the first expression in (22) we have $R_s = 2\Lambda = \overset{\circ}{R} + 6h_\mu h^\mu$ linking the value of the curvature and the norm of the torsion vector field. Consequently, if the dual of the torsion field have the role of the energy-matter carrier, the meaning of lambda as the vacuum energy is immediately stablished.

3.2. The Constancy of G

At this level, no assertion can state with respect to G or even with respect to c. The link with the general relativistic case is given by the identification of electromagnetic energy-momentum tensor with the term analogous $T^F_{\alpha\beta}$ in our metric variational equations:

$$\begin{aligned}&\frac{8\pi G}{c^4}\left(F_{\alpha\lambda}F^\lambda_\beta - F_{\mu\nu}F^{\mu\nu}\frac{g_{\alpha\beta}}{4}\right)\\ \to &\frac{R_s}{2\mathcal{W}}\left(F_{\alpha\lambda}F^\lambda_\beta - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right).\end{aligned}$$

Consequently we have:

$$\kappa = \frac{8\pi G}{c^4} \to \frac{R_s}{2\mathbb{R}^2\alpha^2} \text{ and } \frac{g_{\alpha\beta}}{4} = \frac{R_{(\alpha\beta)}}{R_s} \tag{24}$$

the above expression only said that the ratio must remains constant due the Noether symmetries and conservation laws of the field equations. Notice that (as in the case of b) there exist a limit for all the physical fields coming from the *geometrical invariants quantities.*

3.3. Spacetime 3 + 1 Splitting and Electrodynamics

the starting point will be the line element in 3 + 1 splitting [6, 7] (Appendix III): the 4-dimensional spacetime is split into 3-dimensional space and 1-dimensional time to form a foliation of 3-dimensional spacelike hypersurfaces. The metric of the spacetime is consequently, given by $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$ where γ_{ij} is the metric of the 3-dimensional hypersurface, α is the lapse function, and β^i is the shift function (see Appendix for details) . For any nonlinear Lagrangian, in sharp contrast with the Einstein–Maxwell case, the field equations $d * \mathbb{F} = *\mathbb{J}$ and the Bianchi-geometrical condition $dF = 0$ (where we have defined the Hodge dual $*$ and $\mathbb{F} = \frac{\partial\mathcal{L}}{\partial F}$) are expressed by the vector fields

$$E, B, \mathbb{E} = \frac{\partial\mathcal{L}}{\partial E}, \mathbb{B} = \frac{\partial\mathcal{L}}{\partial B} \tag{25}$$

that live into the slice. Notice the important fact that due $\mathbb{F} = \frac{\partial\mathcal{L}}{\partial F}$ the dynamical equations are highly nonlinear as in the Born-Infeld case. In our case given by the geometrical Lagrangian $\mathcal{L}_g$ ($\mathcal{L}$ here not be confused with the Lie derivative $\mathcal{L}_\beta$!)

$$\nabla\cdot\mathbb{E} = -\bar{h}\cdot\mathbb{B} + 4\pi\rho_e \tag{26}$$

$$\nabla\cdot B = 0 \tag{27}$$

$$\begin{aligned}\nabla\times(\alpha E) &= -(\partial_t - \mathcal{L}_\beta)B\\ &= -\partial_0 B + (\beta\cdot\nabla)B - (B\cdot\nabla)\beta\end{aligned} \tag{28}$$

$$\begin{aligned}\nabla\times(\alpha\mathbb{B}) + h_0\mathbb{B} - \bar{h}\times\mathbb{E} &= -(\partial_t - \mathcal{L}_\beta)\mathbb{E} + 4\pi\alpha j\\ &= \partial_0\mathbb{E} - (\beta\cdot\nabla)\mathbb{E} + (\mathbb{E}\cdot\nabla)\beta + 4\pi\alpha j\end{aligned} \tag{29}$$

where h^{μ} is the dual torsion vector and external currents (ρ_e, j)were introduced. Notice that, as we will see later, "external" currents (ρ_e, j) have a geometrical character and they are geometrically implemented due the torsion vector h^{μ}.

4. DYNAMO EFFECT AND GEOMETRICAL ORIGIN OF $\alpha\Omega$ TERM

In the case of weak field approximation and in 3 + 1 representation $\left(F^{01} \to E^i, F^{jk} \to B^i\right)$ the electromagnetic dymnamical equations take the form

$$\nabla_\nu F^{\nu\mu} = T^{\mu\nu\rho} F_{\nu\rho} = \varepsilon^{\mu\nu\rho}{}_\delta h^\delta F_{\nu\rho} \quad (d{*}F = {*}J), \tag{30}$$

$$\overline{\nabla} \cdot \overline{E} = -\overline{h} \cdot \overline{B}, \tag{31}$$

$$\partial_t \overline{E} - \overline{\nabla} \times \overline{B} = h^0 \overline{B} - \overline{h} \times \overline{E}, \tag{32}$$

(where the upper bar represents a spacial vector) and

$$\nabla_\nu {*}F^{\nu\mu} = 0 \ (dF = 0), \tag{33}$$

$$\overline{\nabla} \cdot \overline{B} = 0, \tag{34}$$

$$\partial_t \overline{B} = -\overline{\nabla} \times \overline{E}. \tag{35}$$

Putting all toghether, the set of equations takes a familiar form

$$\overline{\nabla} \cdot \overline{E} + \overline{h} \cdot \overline{B} = \rho_{\text{ext}}, \tag{36}$$

$$\begin{aligned} &\partial_t \overline{E} - \overline{\nabla} \times \overline{B} \\ &= h^0 \overline{B} - \overline{h} \times \overline{E} - \sigma_{\text{ext}} \left[\overline{E} + \overline{v} \times \overline{B}\right], \end{aligned} \tag{37}$$

$$\overline{\nabla} \cdot \overline{B} = 0, \tag{38}$$

$$\partial_t \overline{B} = -\overline{\nabla} \times \overline{E}. \tag{39}$$

where we have introduced again external charge density and current. However as we pointed out before, all external currents can be geometrically introduced, via a geometrically induced Lorentz-like force. Following the standard procedure we take the rotational to the second equation above obtaining straigforwardly the modified dynamo equation

$$\begin{aligned} &\overline{\nabla} \times \partial_t \overline{E} + \overline{\nabla}^2 \overline{B} = \overline{\nabla} \times \left(h^0 \overline{B}\right) \\ &+ \left(\overline{h} \cdot \overline{B} - \rho_{\text{ext}}\right) \overline{h} + \left(\overline{\nabla} \cdot \overline{h}\right) \overline{E}, \\ &- \sigma_{\text{ext}} \left[\partial_t \overline{B} + \left(\overline{\nabla} \cdot \overline{v}\right) \overline{B}\right] \end{aligned} \tag{40}$$

where the standard identities of the vector calculus plus the first, the third and the fourth equations above have been introduced. Notice that in the case of the standard approximation and (in the spirit of this research) without any external or additional ingredients, we have

$$\begin{aligned} \overline{\nabla}^2 \overline{B} &= h^0 \left(\overline{\nabla} \times \overline{B}\right) + \overline{\nabla} h^0 \times \overline{B} \\ &+ \left(\overline{h} \cdot \overline{B}\right) \overline{h} + \left(\overline{\nabla} \cdot \overline{h}\right) \overline{E}. \end{aligned} \tag{41}$$

Here we can see that there exist and α-term with a pure geometrical origin, and not only a turbulent one that is given by h^0 (the zero component of the dual of the torsion tensor).

4.1. Comparison with the Mean Field Formalism

Now we compare the above linerized equations with respect to the mean field formalism [25]. Starting from expresions (30)–(35) as before, we have:

$$\begin{aligned} &\eta \overline{\nabla}^2 \overline{B} + \overline{\nabla} \times \left(\overline{v} \times \overline{B}\right) \\ &- \partial_t \overline{B} \underbrace{+ \ \eta \overline{\nabla} \times \left(-h^0 \overline{B} + \overline{h} \times \overline{E}\right)}_{\mathscr{E}_{\text{Geom}}} = 0, \end{aligned} \tag{42}$$

$\mathscr{E}_{\text{Geom}}$ takes the place of electromotive force due the torsion field with full analogy as $\mathscr{E} = \langle u \times b \rangle$ is the mean electromotive force due to fluctuations. Also as in the mean field case that there are the splitting

$$\mathscr{E} = \mathscr{E}^{\langle 0 \rangle} + \mathscr{E}^{\langle \overline{B} \rangle}, \tag{43}$$

with $\mathscr{E}^{\langle 0 \rangle}$ independent of $\langle \overline{B} \rangle$ and $\mathscr{E}^{\langle \overline{B} \rangle}$ linear and homogeneous in $\overline{B}$, we have in the torsion case the following correspondence

$$-h^0 \overline{B} \leftrightarrow \mathscr{E}^{\langle \overline{B} \rangle}$$
$$\overline{h} \times \overline{E} \leftrightarrow \mathscr{E}^{\langle 0 \rangle}$$
$$\text{geometrical} \leftrightarrow \text{turbulent}.$$

Consequently, the problems of mean-field dynamo theory that are concerned with the generation of a mean EMF by turbulence, have in this model a pure geometric counterpart. In the past years, attention has shifted from kinematic calculations, akin to those familiar from quasilinear theory for plasmas, to self-consistent theories which account for the effects of small scale magnetic fields (including their back-reaction on the dynamics) and for the constraints imposed by the topological conservation laws, such as that for magnetic helicity. Here the torsion vector generalize (as we can see from above set of equations) the concept of helicity. The consequence of this role of the dual torsion field is that the traditionally invoked mean-field dynamo mechanism (i.e. the so-called alpha effect) may be severely quenched or increased at modest fields and magnetic Reynolds numbers, and that spatial transport of this generalized magnetic helicity is crucial to mitigating this quench. Thus, the dynamo problem is seen in our model as one of generalized helicity transport, and so may be tackled like other problems in turbulent transport. A key element in this approach is to understand the evolution of the torsion vector field besides of the turbulence energy and the generalized helicity profiles in space-time. This forces us to confront the problem of spreading of strong MHD turbulence, and a spatial variant or analogue of the selective decay problem with the dynamics of the torsion field.

4.2. The Generalized Lorentz Force

An important point in any theory beyond relativity is the concept of force. As is known, general relativity has deficiencies at this point. Now we are going to show that it is possible to derive from our proposal the Lorentz force as follows. From expression (32) the geometrical induced current is recognized

$$\partial_t \overline{E} - \overline{\nabla} \times \overline{B} = h^0 \overline{B} - \overline{h} \times \overline{E} \equiv \overline{J}, \tag{44}$$

$$\overline{J} \times \overline{B} = \left(h^0 \overline{B} - \overline{h} \times \overline{E} - \overline{j}_{\text{ext}}\right) \times \overline{B} \tag{45}$$

$$= -\left[\left(\overline{h} \cdot \overline{B}\right)\overline{E} - \left(\overline{E} \cdot \overline{B}\right)\overline{h}\right] - \overline{j}_{\text{ext}} \times \overline{B}, \tag{46}$$

we assume j_{ext} proportional to the velocity and other contributions. Consequently reordering terms from above, a geometrically induced Lorentz-like force arises

$$\left(\overline{J} + \overline{j}_{\text{ext}}\right) \times \overline{B} = -\left[\underbrace{\left(\overline{h} \cdot \overline{B}\right)}_{\rho_{\text{geom}}}\overline{E} - \left(\overline{E} \cdot \overline{B}\right)\overline{h}\right] \tag{47}$$

$$\begin{aligned}&\rightarrow \underbrace{\left(\overline{h} \cdot \overline{B}\right)}_{\rho_{\text{geom}}}\overline{E} + \underbrace{\left(\overline{J} + \overline{j}_{\text{ext}}\right)}_{j_{\text{gen}}} \times \overline{B} \\ &= \left(\overline{E} \cdot \overline{B}\right)\overline{h} \rightarrow \text{Lorentz induced force},\end{aligned} \tag{48}$$

being the responsible of the induced force, the torsion vector itself. Notice from the above equation the following issues:

(1) It is not necessary to introduce the external current because it can be absorbed in $\overline{J}$.

(2) We can eliminate the electric field as follows

$$\overline{E} = \frac{\left(\overline{E} \cdot \overline{B}\right)\overline{h} - \overline{j}_{\text{gen}} \times \overline{B}}{\left(\overline{h} \cdot \overline{B}\right)}, \tag{49}$$

being the above expression very important in order to replace the electric field into the dynamo equations, being the natural and correct form to introduce the external current in the unified theory.

4.3. The Vector h_μ and the Energy-Matter Interpretation

One of the characteristics that more attract the attention in unified field theoretical models is the possibility to introduce the energy and matter through its geometrical structure. In our case the torsion field takes the role of RHS of the standard GR gravity equation by mean its dual, namely h_μ.

Consequently, in order to explain the physical role of h_μ we know (due the Hodge-de Rham decomposition [Appendix II]) that it can be decomposed as:

$$\begin{aligned}h_\alpha &= \nabla_\alpha \Omega + \varepsilon_\alpha^{\beta\gamma\delta} \nabla_\beta A_{\gamma\delta} \\ &+ \gamma_1 \overbrace{\varepsilon_\alpha^{\beta\gamma\delta} M_{\beta\gamma\delta}}^{\text{axial vector}} + \gamma_2 \overbrace{P_\alpha}^{\text{polar vector}},\end{aligned} \tag{50}$$

where γ_1 and γ_2 can be phenomenologically related to physical constants (e.g: $\gamma_1 = \gamma_2 = \gamma = \frac{8\pi}{c}\sqrt{G}$ is a physical constant related to the Blackett formula [20]). The arguments in favour of this type of theories and from the decomposition (50) can be resumed as follows:

(i) the existence of an angular momentum Helmholtz theorem [23, 24]: the theorem in analysis is exacly as in E_3 but, in the four dimensional case M_4 there exists an additional *axial vector*;

(iii) the concept of *chirality* is achieved in the model by the existency of polar and axial vectors in expression (50).

(iv) if Ω, $A_{\gamma\delta}$ are the wave tensors and $\varepsilon_\alpha^{\beta\gamma\delta} M_{\beta\gamma\delta}, P_\alpha$ the particle vectors (vector and axial part), the concept of an inertial-wave vector s introduced in the equation (50).

Consequently, from the Eq. motion for the torsion namely: $\nabla_\alpha T^{\alpha\beta\gamma} = -\lambda F^{\beta\gamma}$ and back to Eq. (50) we obtain the following important equation

$$\overset{\circ}{\square} A_{\gamma\delta} - \gamma\left[\nabla_\alpha M^{\alpha}_{\ \gamma\delta} + \left(\nabla_\gamma P_\delta - \nabla_\delta P_\gamma\right)\right] = -\lambda F_{\gamma\delta}. \tag{51}$$

Consider in particular as simplest example, the case when $\lambda F_{\gamma\delta} \rightarrow 0$:

$$\overset{\circ}{\square} A_{\gamma\delta} = \gamma\left[\nabla_\alpha M^{\alpha}_{\ \gamma\delta} + \left(\nabla_\gamma P_\delta - \nabla_\delta P_\gamma\right)\right]. \tag{52}$$

We can immediately see that, if $M^{\alpha}_{\ \gamma\delta}$ is identified with the intrinsec spin angular momentum of the ponderable matter, P_δ is its lineal momentum vector and $A_{\gamma\delta}$ is the gravitational radiation tensor, then Eq. (52) states that the sum of the intrinsec spin angular momentum and the orbital angular momentum of ponderable matter is conserved if the gravitational radiation is absent [26–28].

4.4. Killing-Yano Systems and the Vector h_μ

Without enter in many details (these will be treated somewhere) the antisymmetric tensor $A_{\gamma\delta}$ in the h_β composition is related with the Killing [33] and Killing–Yano [34] systems and physically speaking with the notoph field [35]. Consequently we can introduce two types of couplings into the $A_{\gamma\delta}$ divergence : it correspond with the generalized current interpretation that also has h_β.

(i) Defining

$$A_{\gamma\delta} \equiv A_{[\gamma;\delta]}, \tag{53}$$

such that

$$\overset{\circ}{\nabla}_\rho A_{[\gamma;\delta]} = \frac{4\pi}{3}\left(j_{[\gamma}g_{\delta]\rho}\right), \tag{54}$$

then, in this case we can identify $A_{\gamma\delta} = 2F_{\gamma\delta}$ because $F^{\delta}_{\gamma;\delta} = 4j_\gamma$ and $A_{[\gamma\delta;\rho]} = F_{[\gamma\delta;\rho]} = 0$.

In this case the contribution of $A_{\gamma\delta}$ to h_β dissapear.

(ii) Let us consider

$$A_{[\gamma;\delta];\rho} = \frac{4\pi}{3} j_{[\gamma}F_{\delta]\rho}, \tag{55}$$

having into account the vorticity vector also

$$\omega_\mu \equiv u^\lambda \varepsilon_{\lambda\mu\nu\rho}\nabla^\nu u^\rho, \tag{56}$$

and considering a plasma with electrons, protons etc.

$$j^\gamma \sim A^\gamma + q_s n_s u_s^\gamma, \tag{57}$$

where A_μ is the vector potential and q_s is the particle charge, n_s is the number density (in the rest frame) and the four-velocity of species s is u_s^γ. In this case h_α takes the form

$$h_\alpha = \nabla_\alpha\Omega + \varepsilon_\alpha^{\beta\gamma\delta}\nabla_\beta A_{\gamma\delta} + \gamma_1\varepsilon_\alpha^{\beta\gamma\delta}M_{\beta\gamma\delta} + \gamma_2 P_\alpha \tag{58}$$

$$\begin{aligned}\rightarrow h_\alpha &= \nabla_\alpha\Omega + \varepsilon_\alpha^{\gamma\delta\rho}\frac{4\pi}{3} j_{[\gamma}F_{\delta]\rho} \\ &- \gamma_1 u^\lambda\varepsilon_{\lambda\alpha\nu\rho}\nabla^\nu u^\rho + \gamma_2 P_\alpha,\end{aligned} \tag{59}$$

$$\begin{aligned}h_\alpha &= \nabla_\alpha\Omega + \varepsilon_\alpha^{\gamma\delta\rho}\frac{4\pi}{3}\left[A + q_s n_s u_s\right]_{[\gamma}F_{\delta]\rho} \\ &- \gamma_1 u^\lambda\varepsilon_{\lambda\alpha\nu\rho}\nabla^\nu u^\rho + \gamma_2 P_\alpha.\end{aligned} \tag{60}$$

Consequently in 3 + 1 decomposition we have

$$h_0 = \nabla_0\Omega + \varepsilon_\alpha^{\gamma\delta\rho}\frac{4\pi}{3}\bar{j}\cdot\bar{B} + \gamma_1\bar{u}\cdot\left(\bar{\nabla}\times\bar{u}\right) + \gamma_2 P_0, \tag{61}$$

$$\begin{aligned}h_0 &= \nabla_0\Omega + \varepsilon_\alpha^{\gamma\delta\rho}\frac{4\pi}{3}\left[\bar{A}\cdot\left(\bar{\nabla}\times\bar{A}\right) + q_s n_s \bar{u}_s\cdot\bar{B}\right] \\ &+ \gamma_1\bar{u}\cdot\left(\bar{\nabla}\times\bar{u}\right) + \gamma_2 P_0,\end{aligned} \tag{62}$$

and

$$\begin{aligned}h_i &= \nabla_i\Omega + \frac{4\pi}{3}\left[-\left(\bar{j}\times\bar{E}\right)_i + j_0\bar{B}_i\right] \\ &+ \gamma_1\left[u_0\left(\bar{\nabla}\times\bar{u}\right) + \left(\bar{u}\times\bar{\nabla}u_0\right) + \left(\bar{u}\times\dot{\bar{u}}\right)\right]_i + \gamma_2 P_i,\end{aligned} \tag{63}$$

$$\begin{aligned}h_i &= \nabla_i\Omega + \frac{4\pi}{3}\left[-\left((\bar{A} + q_s n_s\bar{u}_s)\times\bar{E}\right)_i \right. \\ &\left. + \left(\Phi + q_s n_s u_{0s}\right)\bar{B}_i\right] \\ &+ \gamma_1\left[u_0\left(\bar{\nabla}\times\bar{u}\right) + \left(\bar{u}\times\bar{\nabla}u_0\right) + \left(\bar{u}\times\dot{\bar{u}}\right)\right]_i + \gamma_2 P_i.\end{aligned} \tag{64}$$

Notice that in h_0 we can recognize the magnetic and vortical helicities

$$\begin{aligned}h_0 &= \nabla_0\Omega \\ &+ \varepsilon_\alpha^{\gamma\delta\rho}\frac{4\pi}{3}\left[h_M + q_s n_s\bar{u}_s\cdot\bar{B}\right] + \gamma_1 h_V + \gamma_2 P_0,\end{aligned} \tag{65}$$

which will be important in what follows.

5. PROTOGALACTIC ORIGIN OF THE MAGNETIC FIELD

5.1. General Issues

As is more or less understood, some authors demonstrated that strong magnetic fields are produced from a zero initial magnetic field during the pregalactic era, when the galaxy is first forming. Here we will apply the methematical formulation developed in (21, 22) to the protogalactic escenario. Their development in such approaches proceeds in three phases. In the first phase, weak magnetic fields are created by the Biermann battery mechanism. During the second phase, results only from a numerical simulation make it appear likely that homogenous isotropic Kolmogorov turbulence develops that is associated with gravitational structure formation of galaxies. Assuming that this turbulence is real, then these weak magnetic fields will be amplified to strong magnetic fields by this Kolmogorov turbulence. During this second phase, the magnetic fields reach saturation with the turbulent power, but they are coherent only on the scale of the smallest eddy. During the third phase, which follows this saturation, it is expected that the magnetic field strength will increase to equipartition with the turbulent energy and that the coherence length of the magnetic fields will increase to the scale of the largest turbulent eddy, comparable to the scale of the entire galaxy. The resulting magnetic field represents a galactic magnetic field of primordial origin. No further dynamo action after the galaxy forms is necessary to explain the origin of magnetic fields. However, the magnetic field will certainly be altered by dynamo action once the galaxy and the galactic disk have formed. However that mechanism brings many doubts, because many things come into play to justify the generation and maintenance of the magnetic field. We will now see the role of torsion in magnetogenesis and the dynamics of the galactic magnetic field. We also will see how the magnetic and vortex helicities are included in the torsion vector, and the necessity to introduce the vorticity in the fundamental 2-form together with the magnetic field. From the formula of the induction, namely

$$\bar{\nabla}^2\bar{B} + \underbrace{\bar{\nabla}\times\left(-h^0\bar{B} + \bar{h}\times\bar{E}\right)}_{\mathscr{E}_{\mathrm{Geom}}} = 0, \tag{66}$$

and using the Eq. (49) to eliminate the electric field as function of the torsion, the generalized current and the magnetic field:

$$\overline{h}\times\overline{E}=\frac{-\overline{h}\times\left(\overline{j}_{\text{gen}}\times\overline{B}\right)}{\left(\overline{h}\cdot\overline{B}\right)}=-\frac{\left(\overline{h}\cdot\overline{B}\right)\overline{j}_{\text{gen}}-\left(\overline{h}\cdot\overline{j}_{\text{gen}}\right)\overline{B}}{\left(\overline{h}\cdot\overline{B}\right)},\tag{67}$$

$$\overline{h}\times\overline{E}=-\overline{j}_{\text{gen}}+\frac{\left(\overline{h}\cdot\overline{j}_{\text{gen}}\right)}{\left(\overline{h}\cdot\overline{B}\right)}\overline{B}=\left|\overline{j}_{\text{gen}}\right|\left(-n_{\overline{j}_{\text{gen}}}+\frac{\cos\alpha}{\cos\beta}n_B\right),\tag{68}$$

being α the angle between the vector torsion $\overline{h}$ and the generalised current $\overline{j}_{\text{gen}}$ and β the angle between $\overline{h}$ and the magnetic field $\overline{B}$. Above n_B and $n_{\overline{j}_{\text{gen}}}$ are unitary vectors in the direction of $\overline{B}$ and $\overline{j}_{\text{gen}}$ respectively. Notice the important fact that the RHS of Eq. (68) is independent of the torsion and the magnetic field. Consequently we obtain

$$\overline{\nabla}^2\overline{B}+\underbrace{\overline{\nabla}\times\left[-\overline{j}_{\text{gen}}+\left(\frac{\left(\overline{h}\cdot\overline{j}_{\text{gen}}\right)}{\left(\overline{h}\cdot\overline{B}\right)}-h^0\right)\overline{B}\right]}_{\mathscr{E}_{\text{Geom}}}=0.\tag{69}$$

We introduce the explicitly the physical situation via the generalised current $\overline{j}_{\text{gen}}$

$$-\overline{j}_{\text{gen}}\sim\sigma_{\text{ext}}\left[\overline{E}+\overline{v}\times\overline{B}\right]+\left(\frac{c}{e}\frac{\overline{\nabla}p}{n_e}\right),\tag{70}$$

then

$$\overline{\nabla}^2\overline{B}+\underbrace{\overline{\nabla}\times\left[\sigma_{\text{ext}}\left[\overline{E}+\overline{v}\times\overline{B}\right]+\left(\frac{c}{e}\frac{\overline{\nabla}p}{n_e}\right)+\left(\frac{\left(\overline{h}\cdot\overline{j}_{\text{gen}}\right)}{\left(\overline{h}\cdot\overline{B}\right)}-h^0\right)\overline{B}\right]}_{\mathscr{E}_{\text{Geom}}}=0,\tag{71}$$

$$\overline{\nabla}^2\overline{B}+\underbrace{\sigma_{\text{ext}}\left[\left(-\partial_t\overline{B}\right)+\overline{\nabla}\times\left(\overline{v}\times\overline{B}\right)\right]+\overline{\nabla}\times\left(\frac{c}{e}\frac{\overline{\nabla}p}{n_e}\right)+\overline{\nabla}\times\left(\frac{\left(\overline{h}\cdot\overline{j}_{\text{gen}}\right)}{\left(\overline{h}\cdot\overline{B}\right)}-h^0\right)\overline{B}}_{\mathscr{E}_{\text{Geom}}}=0,\tag{72}$$

finally the expected geometrically induced expression is obtained:

$$\partial_t\overline{B}=\eta\overline{\nabla}^2\overline{B}+\overline{\nabla}\times\left(\overline{v}\times\overline{B}\right)+\eta\overline{\nabla}\times\left[\left(\frac{c}{e}\frac{\overline{\nabla}p}{n_e}\right)+\alpha\overline{B}\right]=\partial_t\overline{B},\tag{73}$$

$$\to\eta\underbrace{\overline{\nabla}^2\overline{B}}_{\text{diffusive}}+\underbrace{\overline{\nabla}\times\left(\overline{v}\times\overline{B}\right)}_{\text{advective}}+\eta\underbrace{\overline{\nabla}\times\left(\alpha\overline{B}\right)}_{\alpha\text{-term}}+\underbrace{\frac{c}{e}\frac{\overline{\nabla}p\times\overline{\nabla}n_e}{n_e^2}}_{\text{Biermann battery}}=-\partial_t\overline{B},\tag{74}$$

where $\eta\equiv\frac{1}{\sigma_{\text{ext}}}$ as usual and we also define

$$\alpha\equiv\left(\frac{\left(\overline{h}\cdot\overline{j}_{\text{gen}}\right)}{\left(\overline{h}\cdot\overline{B}\right)}-h^0\right)=\left(\frac{\cos\alpha\left|\overline{j}_{\text{gen}}\right|}{\cos\beta\left|\overline{B}\right|}-h^0\right).\tag{75}$$

5.2. Seed Magnetic Field

Notice from the last expression that $\alpha\overline{B}$ is explicitly

$$\alpha\overline{B}=\frac{\cos\alpha\left|\overline{j}_{\text{gen}}\right|}{\cos\beta}n_B-h^0\overline{B},\tag{76}$$

or (eliminating the unitary vector)

$$\alpha\left|\overline{B}\right|=\frac{\cos\alpha\left|\overline{j}_{\text{gen}}\right|}{\cos\beta}-h^0\left|\overline{B}\right|,\tag{77}$$

we see the term independent of the magnetic field. Considering only the terms of interest without the diffusive and advective term (only temporary dependence) in the induction equation namely

$$\eta\underbrace{\overline{\nabla}\times\left(\alpha\overline{B}\right)}_{\alpha\text{-term}}=-\partial_t\overline{B},\tag{78}$$

$$\eta\overline{\nabla}\left(\frac{\cos\alpha\left|\overline{j}_{\text{gen}}\right|}{\cos\beta}\right)=-\partial_t\left|\overline{B}\right|,\tag{79}$$

we see that the currents given by the fields (related to the geomtry via h_α) originate the magnetic field. If we consider as in the next section the currents of the fields of theory (fermions, etc) the seed would be the currents itselves. The other missing point is, from the same unified formulation, to derive the fluid equations (which as is known does not have a definite

Lagrangian formulation) that have analogous formulas for vorticity than for the magnetic field B. This would mean that the 2-form of vorticity must also be included in the fundamental antisymmetric tensor together with the electromagnetic field.

6. TORSION, AXION ELECTRODYNAMICS OR CHERN SIMONS THEORY?

Let us review briefly the electromagnetic sector of the theory QCD based in a gauge symmetry $SU(3)\times U(1)$

$$\begin{aligned} &L_{QCD/QED} \\ &= +\sum \overline{\psi}_f\left[\gamma^\mu\left(\partial_\mu - ig_f t^\alpha A_\mu^\alpha - iq_f A_\mu\right) - m_f\right]\psi_f \\ &- \frac{G_{\mu\nu}^\alpha G^{\alpha\mu\nu}}{4} - \frac{F_{\mu\nu}F^{\mu\nu}}{4} - \frac{g^2\theta G_{\mu\nu}^\alpha \tilde{G}^{\alpha\mu\nu}}{32\pi^2} - \frac{g^2\theta F_{\mu\nu}\tilde{F}^{\mu\nu}}{32\pi^2}. \end{aligned} \tag{80}$$

As is well know, electromagnetic fields will couple to the electromagnetic currents, namely: $J_\mu = \sum_f q_f \overline{\psi}_f \gamma_\mu \psi_f$ consequently, there appear term will induce through the quark loop the coupling of $F_{\mu\nu}\tilde{F}^{\mu\nu}$ (the anomaly) to the QCD topological charge. The effective Lagrangian can be written as

$$L_{MCS} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} - A_\mu J^\mu - \frac{c}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{81}$$

where a pseudo-scalar field $\theta = \theta(\overline{x}, t)$ (playing the role of the axion field) is introduced and $c = \sum_f \frac{(q_f e)^2}{2\pi^2}$. This is the Chern–Simons Lagrangian where, if θ is constant, the last term is a total divergence: $F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu J^\mu_{CS}$. The question appear if θ is not a constant $\theta F_{\mu\nu}\tilde{F}^{\mu\nu} = \theta\partial_\mu J^\mu_{CS} = \partial_\mu\left(\theta J^\mu_{CS}\right) - J^\mu_{CS}\partial_\mu\theta$.

Now we can see from the previous section that if, from the general decomposition of the four dimensional dual of the torsion field via the Hodge de Rham theorem we retain b_α as gradient of a pseudoscalar (e.g: axion) these equations coincide in form with the respective equation for MCS theory. Precisely because under this condition $h_\alpha = \nabla_\alpha\theta$, in flat space (curvature = 0 but torsion ≠ 0) the equations become the same as in [8] namely

$$\overline{\nabla}\cdot\overline{E} - c\overline{P}\cdot\overline{B} = \rho_{\text{ext}}, \tag{82}$$

$$\begin{aligned} &\partial_t\overline{E} - \overline{\nabla}\times\overline{B} \\ &= -c\dot{\theta}\overline{B} + c\overline{P}\times\overline{E} - \sigma_{\text{ext}}\left[\overline{E} + \overline{v}\times\overline{B}\right], \end{aligned} \tag{83}$$

$$\overline{\nabla}\cdot\overline{B} = 0, \tag{84}$$

$$\partial_t\overline{B} = -\overline{\nabla}\times\overline{E}. \tag{85}$$

provided:

$$h^0 \to -c\dot{\theta}, \tag{86}$$

$$\overline{h} \to -c\overline{P}. \tag{87}$$

where from QCD the constant c is determined as $c = \frac{e^2}{2\pi}$ and the $\partial_\mu\theta = \left(\dot{\theta}, \overline{P}\right)$ in the [8] notation.The main difference is that while in the case o photons in axion ED was given by [9] the Lagrangian where that above equations are derived is

$$\begin{aligned} L_{MCS} &= -\frac{F_{\mu\nu}F^{\mu\nu}}{4} - A_\mu J^\mu + \frac{c}{4}P_\mu J^\mu_{CS}, \\ J^\mu_{CS} &\equiv \varepsilon^{\mu\sigma\rho\nu}A_\sigma F_{\rho\nu} \end{aligned} \tag{88}$$

in our case is the dual of the torsion field(that we take as the gradient of a pseudoscalar) responsible for the structure of the set of equations.

7. DISCUSSION AND PERSPECTIVES

From the functional action proposed, that is as square root or measure (Nambu–Goto/Born–Infeld type), the dynamic fundamental equations were derived: an equation analogous to trace free Einstein equations TFE and a dynamic equation for the torsion (which was taken totally antisymmetric). Although the aim of this paper was not to introduce a full theoretical basis of the model (that is given in a separated article), from this starting point we bring some results and possible explanations about a few problems of the current research. The most remarkable are

(i) The cosmological term appear as integration constant of a natural manner and is linked with the curvature and fundamental fields.

(ii) The fundamental constants (as G) are really functions of the spacetime coordinates geometrically induced and linked between them.

(iii) There are a geometrical origin (not turbulent) of the α-term and the dynamo effect given by the torsion vector field.

(v) we show that primordial protogalagtic magnetic fields can be originated by the dual torsion field h_μ.

(vi) the relation between the effective QCD with the anomalous sector(Maxwell–Chern–Simmons) and the unified model proposed here was explicity given and clarified. The conditions where the torsion can play the same role that the gradient of θ-factor were pointed out.

8. APPENDIX I

A. Basis of the Metrical-Affine Geometry

The starting point is a hypercomplex construction of the (metric compatible) spacetime manifold [17, 18]

$$M, g_{\mu\nu} \equiv e_\mu \cdot e_\nu \tag{A.1}$$

where for each point $p \in M$ there exists a local affine space A. The connection over A, $\tilde{\Gamma}$, define a generalized affine connection Γ on M, specified by (∇, K), where K is an invertible (1,1) tensor over M. We will demand for the connection to be compatible and rectilinear, that is

$$\nabla K = KT, \quad \nabla g = 0 \tag{A.2}$$

where T is the torsion, and g the spacetime metric (used to raise and lower the indices and determining the geodesics), that is preserved under parallel transport. This generalized compatibility condition ensures that the generalized affine connection Γ maps autoparallels of Γ on M into straight lines over the affine space A (locally). The first equation above is equal to the condition determining the connection in terms of the fundamental field in the UFT non-symmetric. Hence, K can be identified with the fundamental tensor in the non-symmetric fundamental theory. This fact gives us the possibility to restrict the connection to a (anti-) Hermitian theory.

The covariant derivative of a vector with respect to the generalized affine connection is given by

$$A^{\mu}{}_{;\nu} \equiv A^{\mu}{}_{,\nu} + \Gamma^{\mu}_{\alpha\nu}A^{\alpha}, \tag{A.3}$$

$$A_{\mu;\nu} \equiv A_{\mu,\nu} - \Gamma^{\alpha}_{\mu\nu}A_{\alpha}. \tag{A.4}$$

The generalized compatibility condition (A.2) determines the 64 components of the connection by the 64 equations

$$K_{\mu\nu;\alpha} = K_{\mu\rho}T^{\rho}{}_{\nu\alpha} \text{ where } T^{\rho}_{\nu\alpha} \equiv 2\Gamma^{\rho}_{[\alpha\nu]}. \tag{A.5}$$

Notice that by contracting indices ν and α in the first equation above, an additional condition over this hypothetic fundamental (nonsymmetric) tensor K is obtained

$$K^{\alpha}_{\mu\alpha;} = 0,$$

that, geometrically speaking, reads

$$d^{*}K = 0.$$

This is a current-free condition over the tensor K that can be exemplified in the simplest case with the prototype of non-symmetric fundamental tensor: $K_{\mu\nu} = g_{\mu\nu} + f_{\mu\nu}$

$$d^{*}K = d^{*}g + d^{*}f \Rightarrow d^{*}f = 0 \text{ (current free e.o.m.)},$$

where *usually* $g_{\mu\nu}$ plays the role of the spacetime metric and $f_{\mu\nu}$ the role of electromagnetic field.

The metric is uniquely determined by the metricity condition, which puts 40 restrictions on the derivatives of the metric

$$g_{\mu\nu,\rho} = 2\Gamma_{(\mu\nu)\rho}. \tag{A.6}$$

The spacetime curvature tensor, that is defined in the usual way, has two possible contractions: the Ricci tensor $R^{\lambda}_{\mu\lambda\nu} = R_{\mu\nu}$, and the second contraction $R^{\lambda}_{\lambda\mu\nu} = 2\Gamma^{\lambda}{}_{\lambda[\nu,\mu]}$, which is identically zero due to the metricity condition (A.2).

In order to find a symmetry of the torsion tensor, let us denote the inverse of K by $\hat{K}$. Therefore, $\hat{K}$ is uniquely specified by condition $\hat{K}^{\alpha\rho}K_{\alpha\sigma} = K^{\alpha\rho}\hat{K}_{\alpha\sigma} = \delta^{\rho}_{\sigma}$.

As it was pointed out in [12–16], inserting explicitly the torsion tensor as the antisymmetric part of the connection in (A.6), and multiplying by $\frac{1}{2}\hat{K}^{\alpha\nu}$, results, after straight forward computations, in

$$\left(Ln\sqrt{-K}\right)_{,\mu} - \Gamma^{\nu}_{(\mu\nu)} = 0, \tag{A.7}$$

where $K = \det\left(K_{\mu\rho}\right)$. Notice that from expression (A.6) we arrive to the relation between the determinants K and g:

$$\frac{K}{g} = \text{const},$$

(strictly a constant scalar function of the coordinates). Now we can write

$$\Gamma^{\nu}{}_{\alpha\nu,\beta} - \Gamma^{\nu}{}_{\beta\nu,\alpha} = \Gamma^{\nu}{}_{\nu\beta,\alpha} - \Gamma^{\nu}{}_{\nu\alpha,\beta}, \tag{A.8}$$

as the first term of (A.8) is the derivative of a scalar. Then, the torsion tensor has the symmetry

$$T^{\nu}{}_{\nu[\beta,\alpha]} = T^{\nu}{}_{\nu[\alpha,\beta]} = 0. \tag{A.9}$$

This implies that the trace of the torsion tensor, defined as $T^{\nu}{}_{\nu\alpha}$, is the gradient of a scalar field

$$T_{\alpha} = \nabla_{\alpha}\phi. \tag{A.10}$$

In reference [18] an interesting geometrical analysis is presented of non-symmetric field structures. There, expressions precisely as (A.1) and (A.2) ensure that the basic non-symmetric field structures (i.e. K) take on a definite geometrical meaning when interpreted in terms of affine geometry.

9. APPENDIX II

A. Electrodynamical Equations in 3 + 1

The starting point will be the line element in $3+1$ splitting [6, 7]: the 4-dimensional spacetime is split into 3-dimensional space and 1-dimensional time to form a foliation of 3-dimensional spacelike hypersurfaces. The metric of the spacetime is consequently, given by

$$ds^{2} = -\alpha^{2}dt^{2} + \gamma_{ij}\left(dx^{i} + \beta^{i}dt\right)\left(dx^{j} + \beta^{j}dt\right),$$

where γ_{ij} is the metric of the 3-dimensional hypersurface, α is the lapse function, and β^i is the shift function. At every spacetime point, a fiducial observer (FIDO) is introduced in such a way that his corresponding world-line is perpendicular to the hypersurface where he is stationary.

His FIDO 4-vector velocity is then given by

$$U^{\mu} = \frac{1}{\alpha}\left(1, -\beta^i\right), \quad U_{\mu} = (-\alpha, 0, 0, 0),$$

one deals with the physical quantities defined on the 3-dimensional hypersurface as measured by the FIDO. For example, the electric field and the magnetic field are defined with the help of the U^{μ} respectively, by

$$E^{\mu} = F^{\mu\nu}U_{\nu}$$
$$B^{\mu} = -\frac{1}{2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}U_{\nu}F_{\mu\nu}$$

notice that the zero components are null: $E^0 = B^0 = 0$. Also, the 4-current J^{μ} can be similarly decomposed as

$$J^{\mu} = \rho_e U^{\mu} + j^{\mu},$$

where we defined

$$\rho_e = -J^{\mu}U_{\mu}$$
$$j^{\mu} = J^{\mu} + J^{\nu}U_{\nu}U^{\mu},$$

then $j^0 = 0$. So that j, E and B can be treated as 3-vectors in spacelike hypersurfaces. In terms of these 3-vectors the Maxwell Eqs. can be written as

$$\begin{aligned}
\nabla \cdot E &= 4\pi\rho_e \\
\nabla \cdot B &= 0 \\
\nabla \times (\alpha E) &= -(\partial_t - \mathcal{L}_{\beta})B \\
&= -\partial_0 B + (\beta \cdot \nabla)B - (B \cdot \nabla)\beta \\
\nabla \times (\alpha B) &= -(\partial_t - \mathcal{L}_{\beta})E + 4\pi\alpha j \\
&= \partial_0 E - (\beta \cdot \nabla)E + (E \cdot \nabla)\beta + 4\pi\alpha j.
\end{aligned}$$

The derivatives in these equations are covariant derivatives with respect to the metric of the absolute space γ_{ij} being $\mathcal{L}_{\beta}$ the Lie derivative operator geometrically defined as: $\mathcal{L}_{\beta}V = d\left(i_{\beta} \cdot V\right)$ with V a vector field.

ZAMOs observers

$$U = \frac{1}{\alpha}\left(\partial_t - \beta^i e_i\right),$$

in the Boyer–Lindquist coordinates we have e_r, e_{θ} and $e_{\varphi} = \frac{1}{\sqrt{g_{\varphi\varphi}}}\partial_{\varphi}$. The plasma 4—velocity (medium) u can be expressed as $u = \gamma\left(U + \bar{v}\right)$ where $\bar{v}$ is the plasma 3—velocity with respect to the ZAMOs.

10. ACKNOWLEGMENTS

I am very grateful to the JINR Directorate and the BLTP for his hospitality and CONICET-ARGENTINA for financial support. Many thanks are given to Max Camenzind for his comments about gauge theories and gravitation, Brett Mc Innes that bring me important references and to Andrej Arbuzov for his useful insigts and comments.

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