A dipolar self-induced Josephson junction

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We propose a new scheme for observing Josephson oscillations and macroscopic quantum self-trapping in a toroidally confined Bose-Einstein condensate: a dipolar self-induced Josephson junction. Polarizing the atoms perpendicularly to the trap symmetry axis, an effective ring-shaped, double-well potential is achieved which is induced by the dipolar interaction. By numerically solving the three-dimensional time-dependent Gross-Pitaevskii equation we show that coherent tunneling phenomena such as Josephson oscillations and quantum self-trapping can take place. The dynamics in the self-induced junction can be qualitatively described by a two-mode model taking into account both s-wave and dipolar interactions.

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Josephson effects are a signature of quantum coherence in macroscopic many-body systems. Firstly predicted and observed when two superconductors were connected through a weak link [1], Josephson effects have also been experimentally observed in a variety of systems: in superfluid helium flowing through a submicrometer aperture [2] and an array of nano-apertures [3]; in contact-interacting Bose-Einstein condensates confined in a double-well trap [4, 5] and in an optical lattice [6]; and very recently in exciton-polariton systems in semiconductors [7]. Josephson phenomena can also appear in multicomponent systems [8], either in condensate mixtures or in spinor condensates. When the tunneling occurs between different hyperfine states, the process is referred to as internal Josephson effect [9]. All these systems are realizations of a Josephson junction (JJ). In Bose-Einstein condensates (BECs), due to the nonlinearity introduced by the s-wave contact interaction, there appears a new phenomenon called macroscopic quantum self-trapping [4, 10], characterized by the locking of most of the atoms in one of the two wells.

Dipolar Bose-Einstein condensates (dBECs) [11] offer a new playground for the study of Josephson effects due to the anisotropic and long-range character of their interaction [12]. Here we present a novel scenario for investigating the coherence properties of quantum transport: a dipolar self-induced Josephson junction (SIJJ). Such a junction is not directly created by an external potential, but rather based on the appearance of a ring-shaped double-well potential in a toroidally confined dipolar condensate due to the anisotropy of the interaction. Josephson dynamics in dipolar condensates has been addressed in the literature for external double well potentials [13] and in spinor dipolar condensates [14], but this is the first time to our knowledge that a SIJJ is investigated.

We consider $N = 5 \times 10^4$ atoms of ^{52}Cr with magnetic dipole moment $\mu = 6\ \mu_B$ (with μ_B the Bohr magneton) confined in a pancake-shaped toroidal trap [15]

$$V_{\rm t}(\mathbf{r}) = \frac{m}{2}(\omega_\perp r_\perp^2 + \omega_z z^2) + V_0\, \exp(-2\, r_\perp^2 /\, w_0^2)\,, \quad (1)$$

where m is the atomic mass, $r_\perp = \sqrt{x^2+y^2}$ is the distance to the trap symmetry axis, and $\omega_\perp = 8.4 \times 2\pi$ s^{-1} and $\omega_z = 92.5 \times 2\pi$ s^{-1} are the radial and axial harmonic trap frequencies. $V_0 = 30\ \hbar\omega_\perp$ and $w_0 = 10\ \mu$m are the strength and waist of the Gaussian beam that creates a hole in the condensate along the z axis. We consider the dipoles oriented perpendicularly to the trap axis (say y axis), and a small and repulsive s-wave scattering length, $a = 14\ a_B$ (with a_B the Bohr radius), to ensure that the dBEC is stable but dominated by the dipolar interaction. In this configuration the axial symmetry imposed by the external confinement is broken, and the anisotropic character of the dipolar interaction is enhanced [15].

The combination of the external toroidal trapping and the mean-field dipolar potential, $V_{\rm d}(\mathbf{r})$, defines an effective potential, $V_{\rm eff}(\mathbf{r}) = V_{\rm t}(\mathbf{r}) + V_{\rm d}(\mathbf{r})$, which has the shape of a ring-shaped double well. This structure is shown in the top left panel of Fig. 1 for the $z = 0$ plane, with the two potential wells in the direction perpendicular both to the trap symmetry axis and the polarization axis. Since the centered Gaussian potential introduces a strong repulsive barrier at $r_\perp = 0$ that prevents the atoms from tunneling through it, the double well structure arises in the azimuthal direction. The bottom left panel of Fig. 1 shows the minimum effective potential along the azimuthal coordinate, φ, with the two minima at $\varphi = 0$ and $\varphi = \pi$ and the two barriers at $\varphi = \pi/2$ and $\varphi = 3\pi/2$. According to this effective ring-shaped double well, the atoms localize mostly in the attractive regions inside the wells, producing an azimuthal density dependence. This is seen in the right panels of Fig. 1,

which show the density (top) and the maximum density along φ (bottom) on the $z = 0$ plane.

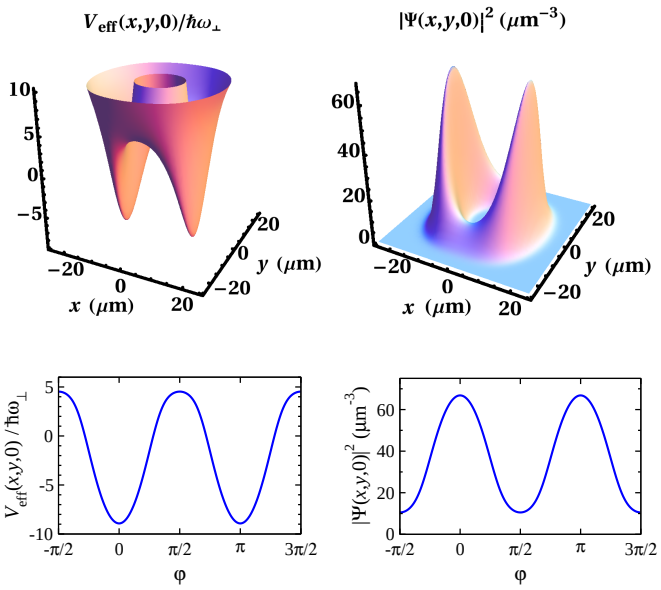

FIG. 1: Top: effective potential (left) and corresponding ground state density (right) in the $z = 0$. Bottom: minimum effective potential (left) and maximum density (right) in the $z = 0$ plane as a function of the azimuthal angle φ.

The system resembles two dipolar condensates in a ring-shaped geometry that are coupled via two links. If the barrier height, $\Delta V_{\rm eff}$, is large compared to the chemical potential, $\tilde{\mu}$, the system behaves as two weakly linked condensates. The weak link condition can be reached by tuning the s-wave scattering length to small values, which should still be large enough to prevent spatial spontaneous symmetry breaking [15]. We find that the ground state verifies $\Delta V_{\rm eff}/\tilde{\mu} = 1.1$, which is at the limit of the weak link condition. The system can thus be thought of as a self-induced Josephson junction. In the remaining of this letter we will show that it can behave as a bosonic JJ, presenting Josephson as well as self-trapping dynamics. Note here that this junction really consists of two coupled SIJJs, which in this scenario behave in phase in much the same way as the array of nano-apertures does in the experiments with helium [3].

We study the dynamics of this dipolar SIJJ within the mean-field framework by solving the full-3D time-dependent Gross-Pitaevskii (TDGP) equation:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm t}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2 + V_{\rm d}(\mathbf{r},t)\right]\Psi(\mathbf{r},t) = \\ = i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)\,, \qquad (2)$$

where $g = 4\pi\hbar^2 a/m$ is the coupling constant of the contact interaction. Note that the mean field dipolar potential, $V_{\rm d}(\mathbf{r},t)$, depends on time through the wave function $\Psi(\mathbf{r},t)$. At each time, it can be written in terms of the microscopic dipole-dipole interaction as

$$V_{\rm d}(\mathbf{r},t) = \frac{\mu_0\mu^2}{4\pi}\int d\mathbf{r}'\frac{1-3\cos^2\theta}{|\mathbf{r}-\mathbf{r}'|^3}|\Psi(\mathbf{r}',t)|^2\,, \qquad (3)$$

where $\mathbf{r}$ and $\mathbf{r}'$ are the positions of two aligned dipoles and θ is the angle between their relative position and the polarization axis. The TDGP equation with the dipolar term becomes an integro-differential equation. To solve the time evolution we have used a Hammings algorithm (predictor, corrector, modifier) initialized by a fourth-order Runge-Kutta method. The dipolar term has been treated using Fourier transform techniques (see Ref. [16] and references therein). The spatial grid used in the computation contains $128 \times 128 \times 64$ points with a spacing of 0.6, 0.6 and 0.4 μm, respectively for the directions x, y and z. The time step used is 3×10^{-6} s.

Josephson effects in BECs are characterized in terms of two conjugate variables: the population imbalance and the phase difference between the two wells. We define the left and right wells as the regions where $x < 0$ and $x > 0$, respectively. The population imbalance and phase difference are given by $Z(t) = (N_L(t) - N_R(t))/N$ and $\phi(t) = \phi_R(t) - \phi_L(t)$, respectively, where $N_{L(R)}(t)$ corresponds to the number of atoms on the left (right) well, and $\phi_{L(R)}(t)$ the phase of the dipolar condensate averaged on the left (right) well. To prepare the system with an initial population imbalance $Z(0) \neq 0$, we solve the stationary Gross-Pitaevskii equation in imaginary time [15] with a small tilting in the external potential, ensuring thus the desired asymmetry in the population of the two wells. We then let the system evolve in time without any tilting, according to Eq. (2).

For small values of the initial imbalance the system exhibits Josephson oscillations between the two self-induced wells. Figure 2 shows the dynamic evolution of the population imbalance and the phase difference corresponding to initial conditions $Z(0) = 0.1$ and $\phi(0) = 0$ (solid lines). $Z(t)$ and $\phi(t)$ present sinusoidal oscillations shifted by $\pi/2$, and the time average of the population imbalance is zero. In this regime, the atoms tunnel periodically from the left to the right well and back. The wave function of the dBEC at each well remains coherent during this process, that is with a uniform phase. This is translated into a phase difference that oscillates in time at the same frequency as the imbalance.

For a large initial population imbalance, the SIJJ enters the regime of self-trapping oscillations. This can be clearly seen in Fig. 3, where the imbalance and the phase difference are plotted as a function of time for initial conditions $Z(0) = 0.65$ and $\phi(0) = 0$ (solid lines). In this situation, the time average of the imbalance remains around 0.5 and the phase difference is unbounded (running phase mode). Although in this regime the atoms remain locked in one of the wells, there is still some tunneling of particles at a frequency higher than in the

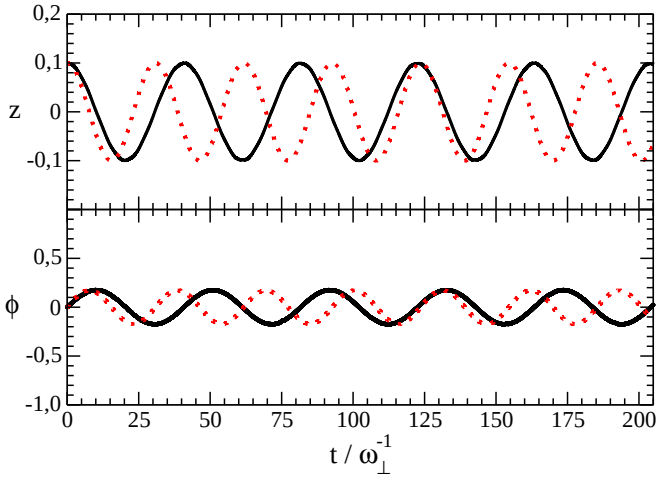

FIG. 2: Time evolution of the population imbalance (top panel) and phase difference (bottom panel) with initial conditions $Z(0) = 0.1$ and $\phi(0) = 0$. Solid lines are the TDGP results, and dashed lines correspond to the TMM. The phase difference is expressed in units of π.

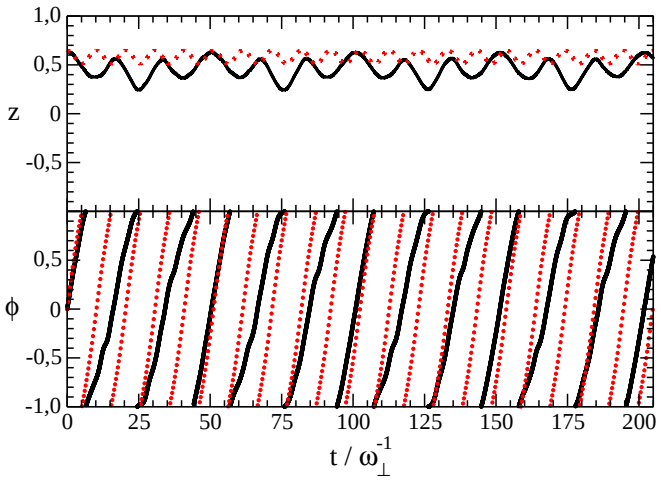

FIG. 3: Time evolution of the population imbalance (top panel) and phase difference (bottom panel) with initial conditions $Z(0) = 0.65$ and $\phi(0) = 0$. Solid lines are the TDGP results, and dashed lines correspond to the TMM. The phase difference is expressed in units of π.

Josephson regime. These imbalance oscillations are non-sinusoidal and have a complicated structure, which hints at a richer self-trapping dynamics. However, this point is beyond the purpose of the present letter, and will be addressed in a future work.

Figures 2 and 3 show that a toroidally confined dBEC behaves as a SIJJ, and that such a structure can sustain both Josephson and self-trapping dynamics. In this sense, the SIJJ is robust, since the dipolar interaction is strong enough to keep the double-well shape of the effective potential in time. Even when the initial imbalance is large, the double well structure is well defined, although in this case the double well is not symmetric. We have therefore shown that a new class of systems exist where the double-well potential structure is self-induced by an anisotropic interaction (in this case the dipolar interaction), and that for these systems Josephson and self-trapping oscillations are predicted to occur depending on the initial population imbalance between the two wells.

To gain insight into the tunneling dynamics obtained by evolving the TDGP equation, we have performed a two-mode analysis of the SIJJ, taking into account both s-wave and dipolar interactions. The two-mode model (TMM) lies on the assumption that the dynamical behavior of a JJ can be fully captured by analyzing the coherent dynamics between two spatially localized modes: the left and right modes, respectively $\Phi_L(\mathbf{r})$ and $\Phi_R(\mathbf{r})$. In this approximation, the condensate order parameter is written using the ansatz [10]

$$\Psi(\mathbf{r},t) = \psi_L(t)\Phi_L(\mathbf{r}) + \psi_R(t)\Phi_R(\mathbf{r})\,, \tag{4}$$

with $\langle\Phi_i|\Phi_j\rangle = \delta_{ij}$, and the coefficients fulfill $\psi_j(t) = \sqrt{N_j(t)}e^{i\phi_j(t)}$, $i,j = L,R$. Note that in this ansatz the time evolution is contained only in the coefficients $\psi_j(t)$.

By substituting ansatz (4) into Eq. (2) and performing some algebra [17], one obtains the two-mode equations for a symmetric dipolar SIJJ,

$$\dot{Z} = (-1+\alpha)\sqrt{1-Z^2}\sin\phi + \varepsilon(1-Z^2)\sin 2\phi \tag{5}$$

$$\dot{\phi} = \Lambda Z - (-1+\alpha)\frac{Z}{\sqrt{1-Z^2}}\cos\phi - \varepsilon Z\cos 2\phi\,, \tag{6}$$

with

$$\Lambda = \frac{U}{2K} - \frac{B+2I_1+D_1}{2K} \tag{7}$$

$$\alpha = \frac{I_2+D_3}{K}\,N \tag{8}$$

$$\varepsilon = \frac{I_1+D_1}{2K}\,N\,. \tag{9}$$

The different parameters appearing in (7)–(9) are given by the integrals

$$K = -\int\left(-\frac{\hbar^2}{2m}\nabla\Phi_L\nabla\Phi_R + \Phi_L\Phi_R V_{\rm t}\right)d\mathbf{r} \tag{10}$$

$$U = \int d\mathbf{r}\left[g|\Phi_L|^4 + \int v_{\rm d}|\Phi_L'|^2 d\mathbf{r}'|\Phi_L|^2\right]d\mathbf{r} \tag{11}$$

$$B = \int d\mathbf{r}|\Phi_L|^2\int v_{\rm d}|\Phi_R'|^2 d\mathbf{r}' \tag{12}$$

$$I_1 = g\int|\Phi_L|^2|\Phi_R|^2 d\mathbf{r}\,,\quad I_2 = g\int|\Phi_L|^2\Phi_L\Phi_R d\mathbf{r} \tag{13}$$

$$D_1 = \int d\mathbf{r}\,\Phi_L\Phi_R\int d\mathbf{r}' v_{\rm d}\Phi_L'\Phi_R' \tag{14}$$

$$D_3 = \int d\mathbf{r}\,\Phi_L\Phi_R\int d\mathbf{r}' v_{\rm d}|\Phi_L'|^2\,, \tag{15}$$

where $v_{\rm d} = \mu_0\mu^2(1-3\cos^2\theta)/(4\pi|\mathbf{r}-\mathbf{r}'|^3)$ is the microscopic dipole-dipole interaction and $\Phi'_{L(R)} = \Phi_{L(R)}(\mathbf{r}')$. Note that the only parameters that appear in Eqs. (5) and (6) are Λ, α and ε, which here depend on both dipolar and s-wave interactions, in contrast to the TMM derivations found in the literature [10, 13, 17]. To compute the integrals (10)–(15) and therefore to determine the parameters (7)–(9), we have used the modes defined as $\Phi_{L(R)}(\mathbf{r}) = (\Phi_{\rm s}(\mathbf{r}) \pm \Phi_{\rm as}(\mathbf{r}))/\sqrt{2}$, where $\Phi_{\rm s}(\mathbf{r})$ and $\Phi_{\rm as}(\mathbf{r})$ are the symmetric (ground state) and antisymmetric (first excited state) wave functions of the double well potential of Fig. 1. For the case analyzed here, $\Lambda = 17.84$, $\alpha = 0.1307$ and $\varepsilon = 0.1156$.

We have obtained the dynamics within the TMM by solving Eqs. (5) and (6). The results are given as dotted lines in Figs. 2 and 3, for the same initial conditions. We see from the figures that the TMM is a good qualitative approximation to the full dynamics given by the TDGP Eq. (2): the order of magnitude of the frequency and amplitude of the oscillations, as well as the dynamical regime imposed by the initial conditions, are well predicted.

The discrepancy between the TDGP dynamics and the TMM can be attributed to a combination of different factors. Firstly, it has been recently shown that the fact that the system does not lie in the deep weak-link limit gives rise to a frequency in the two-mode approximation larger than the experimental one [18], which lies closer to the TDGP result. Secondly, the self-induced nature of the double well means that it depends on time, which is not taken into account in a TMM with time-independent parameters. Lastly, the SIJJ presented here is clearly two-dimensional, whereas the TMM mimics it as one-dimensional. Dynamics in other directions different than x might affect the behavior of the imbalance and the phase difference. This opens the possibility of new physical aspects of a SIJJ which are not present in a usual JJ.

The whole system is scalable in terms of the dimensionless constants $\lambda = \omega_z/\omega_\perp$, $\tilde{w}_0 = w_0/a_\perp$, $\tilde{C} = 4\pi N a/a_\perp$, $D = N\mu_0\mu^2 m/(4\pi\hbar^2 a_\perp)$ and $\tilde{V}_0 = V_0/(\hbar\omega_\perp)$, which are the coefficients of the different terms of the dimensionless TDGP equation. The oscillator length is defined as usual as $a_\perp = \sqrt{\hbar/(m\omega_\perp)}$. For the case considered here $\lambda = 11$, $\tilde{w}_0 = 2.08$, $\tilde{C} = 96.77$, $D = 24.99$ and $\tilde{V}_0 = 30$. With these dimensionless constants it would be easy to export the same physics to another set of parameters experimentally accessible in ^{52}Cr [11], or even to condensates of alkali gases such as ^{39}K or ^{7}Li, where dipolar effects have been recently observed in Refs. [19] and [20], respectively, making use of a soft zero crossing of the scattering length near a Feshbach resonance. Moreover, this scenario can be extended to other systems, such as exciton-polaritons in semiconductors, which can be polarized and where the ring-shaped geometry is experimentally easier to implement than an external double well [21].

In conclusion, we propose a dipolar self-induced Josephson junction. This junction is created by the anisotropic character of the dipolar interaction modulated by a toroidal trap, which gives rise to a ring-shaped, double-well effective potential. The time-dependent Gross-Pitaevskii equation predicts Josephson oscillations as well as a self-trapping regime in this system, depending on the initial population imbalance. Moreover, the self-induced Josephson junction can be analized in a two-mode picture, giving qualitative agreement with the Gross-Pitaevskii results. The system formed by the coupled self-induced Josephson junctions can serve as a starting point for the realization of the bosonic analog of the superconducting quantum interference device (SQUID) [1], due to the close analogy between the two systems.

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