Violating the string winding number maximally in Anti-de Sitter space

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Abstract

We study n-string scattering amplitudes in three-dimensional Anti-de Sitter space (AdS_3). We focus our attention on the processes in which the winding number conservation is violated maximally; that is, those processes in which it is violated in $n-2$ units. A worldsheet conformal field theory calculation leads us to confirm a previous conjecture about the functional form of these observables.

String theory in AdS_3 has served as a model to study AdS/CFT correspondence beyond the supergravity approximation. What makes this possible is that in the three-dimensional case one has access to the worldsheet conformal field theory formulation in terms of the $SL(2,\mathbb{R})$ Wess-Zumino-Witten model (WZW) [1, 2], and thus several observables of the theory, like three-point functions, can be solved exactly. This, together with the existence of a non-renormalization argument [3], has led the authors of [4, 5, 6] to perform explicit checks of the matching between bulk and boundary correlators at the string theory level. The agreement found was exact, and this represents one of the few examples in which one sees the bulk-boundary correspondence to hold beyond the field theory approximation.

Besides its correlators being solvable, string theory in AdS_3 presents several interesting aspects. In particular, its spectrum is very rich and exhibits intriguing properties. As observed in [1, 2], in order to completely parameterize the spectrum of string theory in AdS_3, it is necessary to consider spectrally flowed sectors of the space of representation of the $sl(2)_k$ affine algebra. These sectors are labeled by an integer number ω, whose physical interpretation is that of specifying the winding number of the string states. This winding number is a dynamical degree of freedom associated to the presence of the B-field in the background. Not being a topological quantity, this winding number can in principle be violated if a string scattering process takes place. However, such a violation is not totally arbitrary, and it happens to be bounded from above following a curious pattern: In a scattering process that involves n strings, the total winding number $\Delta\omega = \sum_{i=1}^{n} \omega_i$ is restricted to obey the bound $|\Delta\omega| \leq n-2$; see [2]. In this paper, we will focus on the case where this bound is saturated; namely we will analyze n-string interaction processes satisfying $|\Delta\omega| = n-2$.

n-string amplitudes in AdS_3 space admit to be written in terms of Liouville theory correlation functions. This was proven in [7] for the case in which the winding number conservation is violated up to $n-3$ units (i.e. $|\Delta\omega| < n-2$.) A natural expression for the maximal case $|\Delta\omega| = n-2$ was also proposed in [7] following an educated guess; however, in such special case the proof in [7], based on the analysis of modular differential equations [8, 9], does not hold because the corresponding Liouville n-point correlators do not generically involve degenerate primaries. Therefore, when the winding number conservation is violated maximally, the formula in [7] that expresses n-string amplitudes in AdS_3 in terms of n-point Liouville correlators remains a conjecture. The aim of this paper is to show that a free field computation in the worldsheet conformal field theory actually confirms that formula. Proving so, amounts to review the free field formalism introduced in references [10, 11], which is particularly useful to compute worldsheet correlation functions that violate the winding number conservation. The formalism is based on the Dotsenko conjugate representations introduced for the $SU(2)$ case in references [12, 13] and extended to the $SL(2,\mathbb{R})$ case in references [10, 11, 14].

Let us begin by briefly reviewing string theory in AdS$_3$. The theory is described by the level-k $SL(2,\mathbb{R})$ WZW model, where k is given by $k = l^2/\alpha'$, being $-l^{-2}$ the curvature of AdS$_3$. The string spectrum is given by a subset of the direct sum of unitary $SL(2,\mathbb{R})$ representations [1]; while discrete representations correspond to short string states, the continuous series correspond to long string states, for which we do have an S-matrix interpretation. The string scattering amplitudes in AdS$_3$ are then given by integrating the $SL(2,\mathbb{R})$ WZW correlation functions over the worldsheet [2].

Correlation functions in the $SL(2,\mathbb{R})$ WZW model are defined by analytic continuation of correlation functions in the model formulated on $\mathbb{H}_+^3$, which corresponds to Euclidean AdS$_3$. It is convenient to start by discussing the model on $\mathbb{H}_+^3$ first. More specifically, we should actually start by considering the model on $\mathbb{H}_+^3/U(1)\times\mathbb{R}$, which, as suggested in [1] and shown in [10, 11, 14], is the adequate construction to describe winding string states. To describe the model on $\mathbb{H}_+^3/U(1)\times\mathbb{R}$, one may use Wakimoto free field representation [15] with the addition of extra fields: First, one adds a spacelike $U(1)$ free boson $X(z)$ to realize the coset $\mathbb{H}_+^3/U(1)$, as in [16, 17], and then adds an extra timelike free boson $T(z)$ to represent the $\mathbb{R}$ time direction. The piece $\mathbb{H}_+^3$ is realized by the standard Wakimoto representation, which consists of a β-γ ghost system and a boson ϕ with background charge.

Being described by the WZW model, the theory exhibits $SL(2,\mathbb{R})_k \times SL(2,\mathbb{R})_k$ affine Kac-Moody symmetry, whose holomorphic part can be expressed in terms of the following operator product expansion (OPE)

$$J^3(z)J^{\pm}(w) \simeq \pm\frac{J^{\pm}(w)}{(z-w)} + \mathcal{O}(1) \tag{1}$$

$$J^+(z)J^-(w) \simeq \frac{k}{(z-w)^2} + \frac{2\,J^3(w)}{(z-w)} + \mathcal{O}(1) \tag{2}$$

$$J^3(z)J^3(w) \simeq \frac{-k/2}{(z-w)^2} + \mathcal{O}(1) \tag{3}$$

where the $\mathcal{O}(1)$ stand for regular terms. Analogous OPE holds for the anti-holomorphic piece.

The double poles in the OPE above encode the contribution of the central element of the $sl(2)_k$ affine algebra.

Using the free field representation mentioned above, the $sl(2)_k$ currents may be realized as follows

$$
\begin{aligned}
J^+(z) &= \beta(z)\, e^{i\sqrt{\frac{2}{k}}(X(z)+T(z))}, & (4)\\
J^3(z) &= -\beta(z)\gamma(z) - \sqrt{\frac{k-2}{2}}\partial\phi(z) - i\sqrt{\frac{k}{2}}\partial X(z) - i\sqrt{\frac{k}{2}}\partial T(z), & (5)\\
J^-(z) &= (\beta(z)\gamma^2(z) + \sqrt{2k-4}\gamma(z)\partial\phi(z) + k\partial\gamma(z))\, e^{-i\sqrt{\frac{2}{k}}(X(z)+T(z))}. & (6)
\end{aligned}
$$

with the free field propagators

$$
\langle\phi(z)\phi(w)\rangle = \langle X(z)X(w)\rangle = -\langle T(z)T(w)\rangle = -\log(z-w), \quad \langle\beta(z)\gamma(w)\rangle = \frac{1}{(z-w)}; \qquad (7)
$$

and analogously for the anti-holomorphic contributions.

The states of the theory are labeled by indices j and m, as it is usual when classifying representations of $SL(2,\mathbb{R})$. It is also necessary to introduce an integer index ω to specify which spectral flow sector of the $sl(2)_k$ algebra the states are Kac-Moody primary with respect to. Then, we denote the states by kets $|j,m,\overline{m},\omega\rangle$.

The vertex operators that create these states are

$$
\Phi^{\omega}_{j,m,\overline{m}}(z) = c_0\, \gamma_{(z)}^{j-m} e^{\sqrt{\frac{2}{k-2}}j\phi(z)} e^{i\sqrt{\frac{2}{k}}mX(z)} e^{i\sqrt{\frac{2}{k}}(m+\frac{k}{2}\omega)T(z)} \times h.c. \qquad (8)
$$

where $h.c.$ stands for Hermitian conjugate, which is actually a misnomer as it involves the contributions that depend on $\overline{m}$. The factor c_0 is a normalization constant, independent of j and m.

Operators (8) create the *in*-states from the $SL(2,\mathbb{R})$ invariant vacuum $|0\rangle$; namely

$$
\lim_{z\to 0} \Phi^{\omega}_{j,m,\overline{m}}(z)\,|0\rangle = |j,m,\overline{m},\omega\rangle \qquad (9)
$$

as well as the *out*-states

$$
\langle j,m,\overline{m},\omega| = \lim_{z\to\infty}\; z^{2h^{\omega}_{j,m}}\overline{z}^{2h^{\omega}_{j,\overline{m}}}\;\langle 0|\,\Phi^{\omega}_{-1-j,m,\overline{m}}(z), \qquad (10)
$$

where $h^{\omega}_{j,m}$ is the conformal dimension of the operators,

$$
h^{\omega}_{j,m} = -\frac{j(j+1)}{k-2} - m\omega - \frac{k}{4}\omega^2. \qquad (11)
$$

It is worth noticing that the formula for the conformal dimension remains unchanged under the Weyl reflection $j \to -1-j$. That is, the states created by the operator $\Phi^{\pm\omega}_{-1-j,\mp m,\mp\overline{m}}$ have the same conformal dimension than those created by $\Phi^{\omega}_{j,m,\overline{m}}$.

Operators (8) have the following OPE with the $sl(2)_k$ Kac-Moody currents

$$
\begin{aligned}
J^3(z)\Phi^{\omega}_{j,m,\overline{m}}(w) &\simeq \frac{(m+k\omega/2)}{(z-w)}\Phi^{\omega}_{j,m,\overline{m}}(w)+\mathcal{O}(1) && (12)\\
J^{\pm}(z)\Phi^{\omega}_{j,m,\overline{m}}(w) &\simeq \frac{(\pm j-m)}{(z-w)^{1\pm\omega}}\Phi^{\omega}_{j,m\pm1,\overline{m}}(w)+\mathcal{O}((z-w)^{\mp\omega}) && (13)
\end{aligned}
$$

The theory also admits conjugate representations of the vertex operators. These are important ingredients in our discussion. Such conjugate representations are defined by operators

$$
\widetilde{\Phi}^{\omega}_{j,m,\overline{m}}(z)=\frac{1}{Z_{j,m}}\beta^{j+m}_{(z)}e^{-\sqrt{\frac{2}{k-2}}(j+\frac{k}{2})\phi(z)}e^{i\sqrt{\frac{2}{k}}(m-\frac{k}{2})X(z)}e^{i\sqrt{\frac{2}{k}}(m+\frac{k}{2}\omega)T(z)}\times h.c. \qquad (14)
$$

which create conjugate *in*-states

$$
\lim_{z\to 0}\widetilde{\Phi}^{\omega}_{j,m,\overline{m}}(z)\,|0\rangle=|j_n,m_n,\overline{m}_n,\omega_n)\,. \qquad (15)
$$

In (14) $Z^{-1}_{j,m}$ stands for a normalization factor, which gets fixed once one requires the two-point function between an operator (8) and its conjugate (14) to be normalized to one; namely $\langle 1,m,\overline{m},\omega\,|j,-m,-\overline{m},-\omega)\equiv 1$. This yields

$$
Z_{j,m}=(-1)^{j+m}c_0\,\Gamma(j+m+1). \qquad (16)
$$

Conjugate representation (14) was introduced in [14]. These operators can be thought of as a twisted version of the operators proposed in [16] to describe discrete states in the two-dimensional black hole background. Operators (14) create states in a conjugate representations $|j_n,m_n,\overline{m}_n,\omega_n)$. This is analogous to the $SU(2)$ case studied in [12].

It is easy to verify that operators $\Phi^{\omega}_{j,m,\overline{m}}$ and $\widetilde{\Phi}^{\omega}_{j,m,\overline{m}}$ create states with the same conformal dimension (11). Besides, one can also verify that (14) obeys the following OPE with the currents

$$
\begin{aligned}
J^3(z)\widetilde{\Phi}^{\omega}_{j,m,\overline{m}}(w) &\simeq \frac{(m+k\omega/2)}{(z-w)}\widetilde{\Phi}^{\omega}_{j,m,\overline{m}}(w)+\mathcal{O}(1) && (17)\\
J^{\pm}(z)\widetilde{\Phi}^{\omega}_{j,m,\overline{m}}(w) &\simeq \frac{(\mp1\mp j-m)}{(z-w)^{1\pm\omega}}\widetilde{\Phi}^{\omega}_{j,m\pm1,\overline{m}}(w)+\mathcal{O}((z-w)^{\mp\omega}); && (18)
\end{aligned}
$$

that is, conjugate operators $\widetilde{\Phi}^{\omega}_{j,m,\overline{m}}$ have exactly the same properties that the Weyl-reflected operator $\Phi^{\omega}_{-1-j,m,\overline{m}}$. In fact, as pointed out in [14], Weyl reflection can also be thought of as a conjugation operation associated to the zero-dimension field $\Phi^{0}_{-1,0,0}(z) = \gamma^{-1}_{(z)} e^{-\sqrt{\frac{2}{k-2}}\phi(z)} \times h.c.$

Important ingredients of the Coulomb gas realization that the free field approach leads to are the screening operators. These are given by

$$\widetilde{\Phi}^{-1}_{1-\frac{k}{2},\frac{k}{2},\frac{k}{2}}(z) = \beta(z)\ e^{-\sqrt{\frac{2}{k-2}}\phi(z)} \times h.c. \tag{19}$$

These operators have conformal dimension one and regular OPE with the Kac-Moody currents.

Another special case of operators (14) is the conjugate representation of the identity operator. This is given by the zero-dimension field

$$\widetilde{\Phi}^{0}_{0,0,0}(z) = \Phi^{1}_{-\frac{k}{2},-\frac{k}{2},-\frac{k}{2}}(z) = e^{-\sqrt{\frac{2}{k-2}}\frac{k}{2}\phi(z)} e^{-i\sqrt{\frac{k}{2}}X(z)} \times h.c. \tag{20}$$

Operator (20) was first introduced by Fateev and the brothers Zamolodchikov in their renowned FZZ unpublished paper [18], and in reference [2] it was dubbed *spectral flow operator*. Representation (20) is important to define the charge asymmetry conditions; see [12] for the details.

In references [10, 11, 14], conjugate representations were considered to describe string scattering amplitudes in AdS_3 in the case where the winding number is taken into account. Based on an adaptation of Dotsenko works [12, 13], a prescription was proposed to calculate the correlators on $\mathbb{H}^3_+/U(1) \times \mathbb{R}$. According to such prescription, the string scattering amplitudes of n-strings in AdS_3 are obtained by integrating over the worldsheet the following correlation function

$$X^{\Delta\omega}_{n} = \ \langle j_1, m_1, \overline{m}_1, \omega_1 | \prod_{t=2}^{p} \Phi^{\omega_t}_{j_t,m_t,\overline{m}_t}(z_t) \prod_{l=p+1}^{n-1} \widetilde{\Phi}^{\omega_l}_{j_l,m_l,\overline{m}_l}(z_l)\, | j_n, m_n, \overline{m}_n, \omega_n) \tag{21}$$

where $\Delta\omega = \sum_{i=1}^{n} \omega_i = p+1-n$ (notice that $p \geq 1$.) That is, the tree-level string amplitude is given by

$$\mathcal{A}^{\Delta\omega}_{\text{string}} = \int \prod_{l=3}^{n-1} d^2 z_l\ X^{\Delta\omega}_{n}, \tag{22}$$

integrating over $n-3$ vertex insertions on the sphere.

Here we are concerned with the amplitudes of processes in which the total winding number is violated in $n-2$ units; namely, we will consider correlation functions

$$X^{2-n}_{n} = \langle j_1, m_1, \overline{m}_1, \omega_1 | \prod_{l=2}^{n-1} \widetilde{\Phi}^{\omega_l}_{j_l,m_l,\overline{m}_l}(z_l)\, | j_n, m_n, \overline{m}_n, \omega_n) \,. \tag{23}$$

For this correlator not to vanish, it is necessary to insert a precise amount s of screening operators (19). s is determined by the charge asymmetry condition corresponding to the field $\phi(z)$, which yields $s = -1 - \sum_{i=1}^{n} j_i - (n-2)k/2$. On the other hand, the charge asymmetry conditions corresponding to the fields $X(z)$ and $T(z)$ demand $\sum_{i=1}^{n} m_i = \sum_{i=1}^{n} \overline{m}_i = (n-2)k/2$ and $\sum_{i=1}^{n} \omega_i = 2-n$.

For further purpose it will be necessary to renormalize the vertex operators $\widetilde{\Phi}^{\omega_i}_{j_i,m_i,\overline{m}_i}$ of the $n-2$ intermediate states, $i = 2,3,4,...n-1$. To do so, first we rewrite $Z_{j,\overline{m}}$ as follows

$$Z_{j,\overline{m}} = (-1)^{j+m} c_0\ \Gamma(j+\overline{m}+1) = \lim_{\varepsilon\to 0}\ c_0\ Z^{(\varepsilon)}_{j,\overline{m}} \qquad \text{with} \qquad Z^{(\varepsilon)}_{j,\overline{m}} = \frac{\Gamma(\varepsilon)}{\Gamma(\varepsilon - j - \overline{m})},$$

and then introduce a regularization factor to extract the divergence by renormalizing c_0; namely

$$\prod_{l=2}^{n-1} \frac{1}{Z_{j,m} Z_{j,\overline{m}}} = \lim_{\varepsilon\to 0} \varepsilon^{2-n}\ (c_0/\varepsilon)^{2-n} \prod_{l=2}^{n-1} \frac{1}{Z_{j,m} Z^{(\varepsilon)}_{j,\overline{m}}} = c^{2-n} \prod_{l=2}^{n-1} (-1)^{-j_l - m_l} \frac{\Gamma(-j_l - \overline{m}_l)}{\Gamma(1+j_l+m_l)}.$$

The amplitudes of a scattering process of n strings in which the winding number conservation is violated in $n-2$ units is then given by integrating the correlation function

$$\begin{aligned} X_n^{2-n} \quad = \quad & \frac{(-1)^{s-2j_n - m_n - \overline{m}_n}\Gamma(-s) c^{2-n}}{\Gamma(1+j_n+m_n)\Gamma(1+j_n+\overline{m}_n)} \prod_{l=2}^{n-1} \frac{(-1)^{-j_l - m_l}\Gamma(-j_l - \overline{m}_l)}{\Gamma(1+j_l+m_l)} \\ & \int \prod_{r=1}^{s} d^2 y_r \left\langle \gamma_{(z_1)}^{-1-j_1-m_1} \prod_{l=2}^{n} \beta_{(z_l)}^{j_l+m_l} \prod_{r=1}^{s} \beta_{(y_r)} \right\rangle \\ & \left\langle e^{-\sqrt{\frac{2}{k-2}}(j_1+1)\phi(z_1)} \prod_{l=2}^{n} e^{-\sqrt{\frac{2}{k-2}}(j_l+\frac{k}{2})\phi(z_l)} \prod_{r=1}^{s} e^{-\sqrt{\frac{2}{k-2}}\phi(y_r)} \right\rangle \\ & \left\langle e^{i\sqrt{\frac{2}{k}}m_1 X(z_1)} \prod_{l=2}^{n} e^{i\sqrt{\frac{2}{k}}(m_l - \frac{k}{2})X(z_l)} \right\rangle \left\langle e^{i\sqrt{\frac{2}{k}}(m_1+\frac{k}{2}\omega_1)T(z_1)} \prod_{l=2}^{n} e^{i\sqrt{\frac{2}{k}}(m_l+\frac{k}{2}\omega_l)T(z_l)} \right\rangle \times h.c. \end{aligned}$$

where the integrals over y_r come from the insertion of s screening operators, with $s = -1 - \sum_{i=1}^{n} j_i - (n-2)k/2$; and where we set $z_1 = \infty$, $z_2 = 1$, and $z_n = 0$.

Expanding the Wick contractions, and considering the free field propagators (7), one finds the integral expression

$$X_n^{2-n} = c^{2-n} \prod_{i=1}^{n} \frac{\Gamma(-j_i - \overline{m}_i)}{\Gamma(1+j_i+m_i)} \prod_{i<j}^{n-1,n} (z_i - z_j)^{\beta_{ij}} (\overline{z}_i - \overline{z}_j)^{\overline{\beta}_{ij}}$$

$$\prod_{i<j}^{n-1,n} |z_i - z_j|^{-2\alpha_i\alpha_j} \Gamma(-s) \int \prod_{r=1}^{s} d^2 y_r \prod_{i=1}^{n} \prod_{r=1}^{s} |z_i - y_r|^{-2\alpha_i b} \prod_{r<t}^{s-1,s} |y_r - y_t|^{-2b^2}, \qquad (24)$$

where we introduced the notation $\alpha_i = b(j_i + 1 + b^{-2}/2)$ with $b^{-2} = k-2$, and $\beta_{ij} = k/2 - m_i - m_j - k\omega_i\omega_j/2 - m_i\omega_j - m_j\omega_i$, and analogously for $\overline{\beta}_{ij}$ changing m_i and ω_i for $\overline{m}_i$ and $\overline{\omega}_i$ respectively. Notice that, in terms of α_i, we have $s = b^{-1}\sum_{i=1}^{n}\alpha_i + 1 + b^{-2}$.

In finding an expression like (24), the rapid way of dealing with the contraction of the β-γ system is that of first assuming the case of $j_1 - m_1$ being a positive integer and then extending the resulting expressions. Also, it was used in (24) that the product of the multiplicity factor coming from the Wick contraction of the β-γ contribution and the normalization of the n^{th} vertex can be rewritten as

$$\frac{\Gamma(-j_1-m_1)\Gamma(-j_1-\overline{m}_1)}{\Gamma(1+j_n+m_n)\Gamma(1+j_n+\overline{m}_n)} = \frac{\Gamma(-j_1-\overline{m}_1)}{\Gamma(1+j_1+m_1)}\frac{\Gamma(-j_n-\overline{m}_n)}{\Gamma(1+j_n+m_n)}(-1)^{j_n-j_1+\overline{m}_n-m_1}.$$

The z_i-dependent factor in the first line of (24) comes from the Wick contraction of the fields $X(z)$ and $T(z)$. In the second line of (24), on the other hand, one already sees the n-point Liouville correlation function appearing. In fact, Liouville correlation functions of primary operators $V_{\alpha_i}(z_i) = e^{\sqrt{2}\alpha_i\phi(z_i)}$ are given by

$$\langle \textstyle\prod_{i=1}^{n} V_{\alpha_i}(z_i)\rangle_{\text{L}} = \displaystyle\int \mathcal{D}\varphi \; e^{-\frac{1}{4\pi}\int d^2w((\partial\varphi)^2+(b+1/b)R\varphi+4\pi e^{2b\varphi})}\textstyle\prod_{i=1}^{n} e^{\sqrt{2}\alpha_p\varphi(z_i)} =$$

$$= \frac{\Gamma(-s)}{b}\textstyle\prod_{i<j}^{n-1,n}|z_i-z_j|^{-2\alpha_i\alpha_j}\displaystyle\int \textstyle\prod_{p=1}^{s} d^2y_p \prod_{i=1}^{n}\prod_{l=1}^{s}|z_i-y_l|^{-2\alpha_i b}\prod_{l<t}^{s-1,s}|y_l-y_t|^{-2b^2}$$

with $s = -b^{-1}\sum_{i=1}^{n}\alpha_i + 1 + b^{-2}$. This means that, after absorbing an irrelevant factor, we can write correlation functions (23) as follows

$$X_n^{2-n} = c^{2-n}\textstyle\prod_{i=1}^{n}\displaystyle\frac{\Gamma(-j_i-\overline{m}_i)}{\Gamma(1+j_i+m_i)}\textstyle\prod_{l<t}^{n-1,n}(z_l-z_t)^{\beta_{lt}}(\overline{z}_l-\overline{z}_t)^{\overline{\beta}_{lt}}\;\langle\prod_{i=1}^{n}V_{\alpha_i}(z_i)\rangle_{\text{L}}\,, \qquad (25)$$

recalling $\beta_{lt} = k/2 - m_l - m_t - k\omega_l\omega_t/2 - m_l\omega_t - m_t\omega_l$, $\alpha_i = b(j_i + 1 + b^{-2}/2)$, and $b^{-2} = k-2$.

(25) is exactly the expression conjectured in [7] for the case $|\Delta\omega| = n-2$, and this is what we wanted to prove. The worldsheet conformal field theory calculation in terms of free fields actually confirms that formula. It is worth pointing out that resorting to the prescription of [10, 11, 14] in terms of conjugate representations was crucial to find (25); a free field calculation in terms of the standard Wakimoto representation for the vertices (c.f. [17]) would never lead to such a direct computation, in particular because it is not clear in that case how to implement the winding number violation. Therefore, the result obtained here can be interpreted as a non-trivial test passed by the prescription of [10, 11, 14], which seems to be powerful enough to

yield an expression like (25) even in a case in which the modular differential equations are not at hand. Of course, even when convincing, a computation based on a free field realization can hardly be considered a rigorous proof; in particular, it strongly relies on analytic continuation of the integral formulas involved. However, it is still interesting that formula (25) is confirmed by these means. It has already been argued in [7] that free field computations in the FZZ dual theory done by Fateev in an unpublished paper [19] gave further evidence in favor of the validity of (25).

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