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Four-terminal resistance in a clean interacting quantum wire with invasive contacts

H. Aita [a,b,*], L. Arrachea [c], C. Naón [a,b]

[a] Departamento de Física, Facultad de Ciencias Exactas—UNLP, CC 67, La Plata 1900, Argentina
[b] IFLP—CONICET, Argentina
[c] Departamento de Física and IFIBA, Universidad de Buenos Aires, Pebellon I, Ciudad Universitaria, 1428 Buenos Aires, Argentina

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ABSTRACT

We investigate the behavior of the four-terminal resistance R_{4pt} in an interacting quantum wire described by a Luttinger liquid with an applied bias voltage V and coupled to two voltage probes. We extend previous results, obtained for very weakly coupled contacts, to the case in which the effects of the probes become non-trivially correlated.

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1. Introduction

Quantum transport in novel materials is one of the most active areas of present research in condensed matter physics [1]. The problems that arise are specially interesting in one-dimensional (1D) devices such as quantum wires and carbon nanotubes. In these cases the effect of electron–electron (e–e) interactions is crucial, leading to the so-called Luttinger liquid (LL) behavior [2], characterized by correlation functions which decay with interaction-dependent exponents [3]. On the other hand, one of the central issues in quantum transport is the actual nature of the resistance in a mesoscopic device. Improving the pioneering works that established the fundamental relation between the two-terminal conductance G and the universal quantum $G_0 = e^2/h$ [4], Büttiker has elaborated the concept of the multi-terminal resistance within scattering-matrix theory [5,6], for non-interacting electrons at low V and zero temperature. In the particular four-terminal case, the resistance R_{4pt} is expected to characterize the genuine resistance of the sample. One of the remarkable features of Büttiker's theory is the possibility of having $R_{4pt} < 0$ as a consequence of quantum interference. This effect has been experimentally observed [7,8].

While the consequences of elastic scattering due to impurities can be analyzed in terms of non-interacting electrons, the role of the e–e interaction in the behavior of R_{4pt} remains an open question. The proper evaluation of this quantity implies dealing with a multi-terminal setup as the one in Fig. 1, which is difficult to implement within theoretical approaches like those of Refs. [9–11]. Previous multi-terminal treatments in LL rely in the effective reduction to a non-interacting model by recourse to a Hartree–Fock decoupling of the interaction term [12] or focus in linear response in V [13]. More recently we presented an alternative approach, based on Keldysh non-equilibrium Green's functions, which allowed us to treat the e–e interaction exactly, while going beyond linear response [14]. In that work we considered the setup sketched in Fig. 1, which consists in a quantum wire described by a Tomonaga–Luttinger model under the influence of a voltage V. Two reservoirs are in contact with the wire at the points x_1,x_2 through very weak tunneling constants w_1 and w_2, their chemical potentials μ_1,μ_2 satisfy the condition of a vanishing current through the ensuing contacts. In this way, a continuous current I flows through the wire, and

$$\frac{R_{4pt}}{R_{2pt}} = \frac{\mu_1 - \mu_2}{V}. \tag{1}$$

Although the hypothesis of non-invasive contacts considerably simplifies the calculations, in real measurements this condition is very difficult to fulfill. Besides, within this approximation, potentially interesting quantum interference effects coming from the mutual influence of the reservoirs are ignored. The main purpose of this work is to start exploring the effect of invasive contacts on R_{4pt}.

2. Model and calculations

The full system is described by the action:

$$S = S_{wire} + S_{res} + S_{cont}, \tag{2}$$

where S_{res} describes the two reservoirs that constitute the voltage probes and

$$S_{wire} = \int dx\, dt \{\psi_r^\dagger [i(\partial_t + \partial_x) - \mu_r]\psi_r + \psi_l^\dagger [i(\partial_t - \partial_x) - \mu_l]\psi_l - g[\psi_r^\dagger \psi_r + \psi_l^\dagger \psi_l]^2\}, \tag{3}$$

where g is the e–e interaction in the forward channel, while $\mu_r = \mu + V/2$ and $\mu_l = \mu - V/2$. The term S_{cont} represents the tunneling

* Corresponding author at: Departamento de Física, FCE – UNLP, CC 67, La Plata 1900, Argentina.
E-mail address: lili@df.uba.ar (H. Aita).

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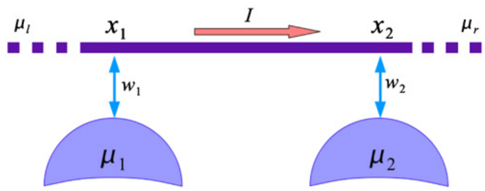

Fig. 1. Sketch of the setup: A voltage V is imposed on a Luttinger liquid, through the chemical potentials for the left and right movers: $\mu_{r,l} = \mu \pm V/2$. Two voltage probes are connected at the positions x_1,x_2. The corresponding chemical potentials $\mu_{1,2}$ are fixed by the condition of zero current through the contacts.

between the reservoirs and the wire,

$$S_{cont} = \sum_{j=1,2} \sum_{\alpha,\beta=r,l} \int dx\, w_j \delta(x - x_j) \times [e^{\mp i(k_F + k'_F)x} \psi^\dagger_\alpha \chi_{\beta,j} + h.c.], \tag{4}$$

where the fields $\chi^\dagger_{\beta,j}$, with $\alpha,\beta = l,r$ and $j = 1,2$ are the degrees of freedom of the reservoirs. The upper and lower sign corresponds to l and r, respectively. We carry out the gauge transformation $\psi^\dagger_{l,r}(x) \to e^{\pm ik_F x} \psi^\dagger_{l,r}(x)$.

The tunneling current from the probes to the wire casts

$$I_j \doteq iw_j \sum_{\alpha,\beta} \langle \chi^\dagger_{\alpha,j}(t) \psi_\beta(x_j,t) - \psi^\dagger_\beta(x_j,t) \chi_{\alpha,j}(t) \rangle. \tag{5}$$

In the following, we evaluate the currents I_j using perturbation theory with respect to the coupling between the wire and the probes. In Ref. [14] we considered "non-invasive" probes, which correspond to completely uncorrelated probes. Thus, the currents were evaluated up to second order in the tunneling coupling w, which corresponds to the lowest order in this parameter. In the present work we compute up to order w^4. As we shall see, this correction allows us to capture inelastic scattering effects induced by the coupling to the probe, as well as new interference effects coming from contributions proportional to $w_1^2 w_2^2$, which are due to correlations between the probes.

The tunneling current can be written in terms of Green's functions of the wire (G) and the reservoirs (g_j):

$$I_j = -2 \sum_{\alpha\beta,j} \Re \left\{ w_j^2 \int \frac{d\omega}{2\pi} [G^<_{\alpha\alpha}(x_j,x_j;\omega) g^A_{\beta_j}(\omega) + G^R_{\alpha\alpha}(x_j,x_j;\omega) g^<_{\beta_j}(\omega)] \right\}. \tag{6}$$

The above expression for the current is exact. The approximations are made in evaluating the Green functions $G^R_{\alpha\alpha}(x,x';\omega)$ and $G^<_{\alpha\alpha}(x,x';\omega)$. In doing so, we have assumed that the self-energy of the Luttinger liquid is the same that the self-energy of the LL coupled to the probes. The expansion of the expression for the current in different orders of the parameters w_j gives $I_j = I_j^1 + I_j^2$, where I_j^1 is $O(w_j^2)$ and corresponds to the result derived in Ref. [14] for non-invasive probes, while I_j^2 is $O(w_j^4)$ and this is precisely the contribution we aim to analyze in the present work. Concretely:

$$I_j^{(2)} = w_j^2 \sum_{i=1,2} \sum_{\alpha,\beta=l,r} w_i^2 \int \frac{d\omega}{2\pi} \tag{7}$$

$$\times \{[G^<_\alpha(x_i,x_j;\omega) g^>_j(\omega) - G^>_\alpha(x_i,x_j;\omega) g^<_j(\omega)] \times G^R(x_j,x_i;\omega) g^R_i(\omega)$$
$$+ [G^<_\alpha(x_j,x_i;\omega) g^>_j(\omega) - G^>_\alpha(x_j,x_i;\omega) g^<_j(\omega)] \times G^A(x_i,x_j;\omega) g^A_i(\omega)$$
$$+ [g^>_j(\omega) g^<_i(\omega) - g^<_j(\omega) g^>_i(\omega)] \times G^A_\alpha(x_i,x_j;\omega) G^R_\beta(x_j,x_i;\omega)\}. \tag{8}$$

The chemical potentials of the probes μ_j are determined from the condition of local equilibrium of the probe in relation to the driven LL. This corresponds to $I_j = 0$ through each of the contacts between the LL and the probes. Thus, we have a system of two transcendental equations which we have solved numerically in order to find μ_1 and μ_2. Replacing these values in (1) we find R_{4pt}/R_{2pt} in terms of the parameters of the system, namely, K, V, x_1 and x_2. Our main results are displayed in Figs. 2 and 3.

3. Discussion

Let us now gather our main results and conclusions. First of all we recall that in the case of non-invasive contacts, a non-vanishing R_{4pt}/R_{2pt} is possible in the present setup provided that the wire contains some source of backscattering, like a barrier or an impurity [14] (see also Ref. [6]). Instead, when we take into account the higher order terms in the coupling w, there is a non-vanishing four-terminal resistance, even in the absence of impurities. This is because within the $O(w^4)$ approach we are able to capture processes where a given probe senses the effect introduced by the other one. In addition, coupling a probe to the wire beyond the non-invasive condition, introduces inelastic scattering processes and decoherence. This manifests itself in the fact that the four terminal resistance is always positive, while in the case of an elastic impurity sensed by non-invasive probes it may also be negative, due to the coherence nature of the electronic transport through the wire.

In Fig. 2 we plot R_{4pt}/R_{2pt} as a function of the bias voltage V, for different values of the e–e interaction K. This suggests that the many-body interactions tend to screen the resistive behavior introduced by the coupling between the probes and the wire. We see that R_{4pt}/R_{2pt}, after an initial monotonous growth, presents an oscillatory behavior. The period of these oscillations is given by $2\pi/K(x_2 - x_1)$. Moreover, the magnitude of the resistance decreases with increasing e–e interaction. Interestingly, for values of V larger than the first maximum the resistance can be very well fit as $R_{4pt}/R_{2pt} \approx A + B \sin(KV(x_2 - x_1))/V^{2\gamma+1}$, where $\gamma = (K + K^{-1} - 2)/4$ and A and B are constants which are proportional to the product $w_1 w_2$ (remember that we have set $w_1 = w_2 = w$ in this study). Thus, for non-invasive probes, for which $w^2 \to 0$, the whole effect disappears and one has $R_{4pt}/R_{2pt} = 0$ unless an impurity is present in the wire.

The physical interpretation of these findings is the following. For low enough V the ratio R_{4pt}/R_{2pt} displays a non-ohmic behavior as a function of V, given by a power law with an interaction dependent exponent. For higher values of V, a classical ohmic-like resistive behavior is established, related to inelastic backscattering processes

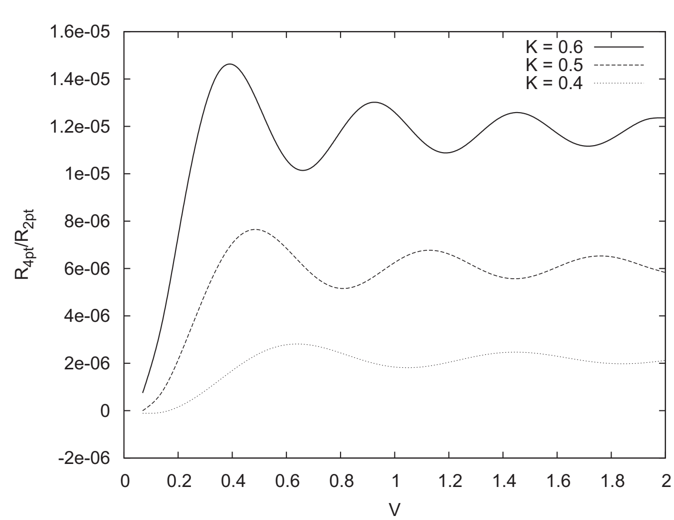

Fig. 2. R_{4pt}/R_{2pt} as a function of the voltage V, for different values of e–e interaction strength K. The positions of the probes are $x_1 = -10$ and $x_2 = 10$, and the strength of the couplings are $w_1 = w_2 = 0.1$.

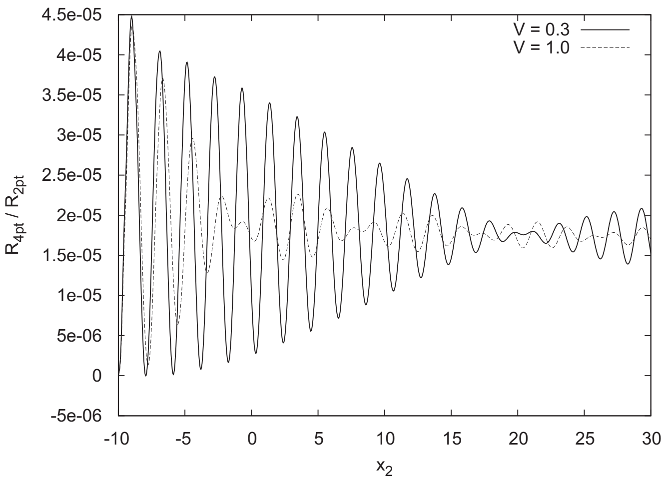

Fig. 3. R_{4pt}/R_{2pt} as a function of the position of the second probe x_2, given the first probe fixed at $x_1=-10$. The strength of the e–e interaction is $K=0.7$, and the couplings $w_1=w_2=0.1$.

taking place at the probes. This behavior implies a constant value of R_{4pt}/R_{2pt}, which is given by the constant A introduced above.

Fig. 3 shows the behavior of R_{4pt}/R_{2pt} as a function of the distance between the reservoirs, for different values of V. We observe the occurrence of $2k_F$ Friedel-like oscillations. This feature has also been shown in non-interacting [6], disordered [15] as well as in interacting [14] and time-dependent pumped [16] systems with non-invasive probes. In such cases the origin is the interference effects generated in the coherent transport due to back-scattering events at a static or dynamical impurity. In our case, these oscillations are due to interference effects between the two voltage probes. Interestingly, in the present case they also display a modulation with period $2\pi/KV$, while for a given K their amplitudes decrease with increasing V.

Four-terminal configurations have been also considered in transport experiments in quantum dots embedded in Aharonov–Bohm rings [17]. In such geometries, the interference effects have been exploited to infer the phase of the transmission function. In quantum dots, the behavior of the four-terminal resistance has been experimentally studied and universal fluctuations similar to the one observed in the conductance of these devices have been observed [18]. In agreement with our results, four-terminal resistance fluctuations are found to increase, as the coupling to the probes is increased. A closer comparison with such devices, however, implies including elastic scattering processes and disorder induced by impurities, in our treatment. A step in this direction has been done in Ref. [19], where the effect of a single impurity in the behavior of R_{4pt}/R_{2pt} of a biased Luttinger wire has been analyzed.

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