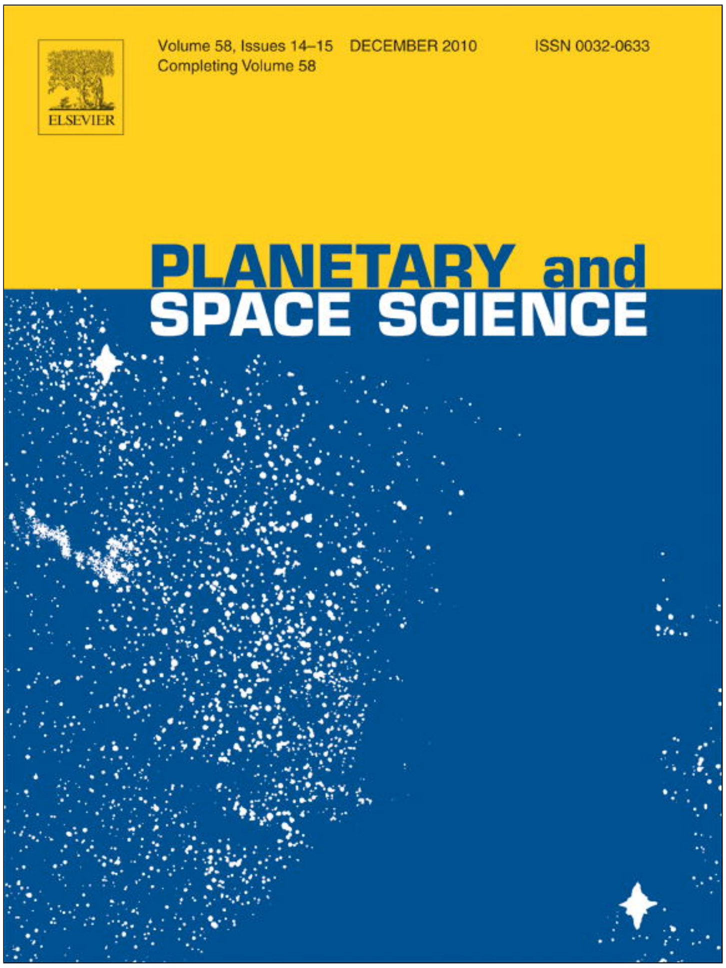
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Intermittent structures and magnetic discontinuities on small scales in MHD simulations and solar wind

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ABSTRACT

In this work we review some recent studies, in which properties of the magnetic field in high resolution simulations of MHD turbulence with spacecraft data are compared, focusing on methods used to identify classical discontinuities and intermittency statistics. Comparison of ACE solar wind data and simulations of MHD turbulence showed good agreement in waiting-time analysis of magnetic discontinuities, and in the related distribution of magnetic field increments. Further analyses showed that the magnetic discontinuities are not distributed without correlations, but rather that non-Poisson correlations, possibly in the form of burstiness or voids, are present in the data at least up to the typical correlation scale. The discontinuities or bursty coherent structures represent in this view the current sheets that form between magnetic flux tubes which may be a signature of intermittent, anisotropic, fully developed MHD turbulence.

1. Introduction

A well known feature of solar wind observations is the appearance of sudden changes in the magnetic field vector, defined as directional discontinuities (DDs), which are detected throughout the heliosphere (Burlaga, 1968; Tsurutani and Smith, 1979; Ness and Burlaga, 2001; Neugebauer, 2006). Many studies identify these as statistically advected tangential discontinuities (TDs), characterized by small components of the magnetic field normal to them, large variations of magnetic field intensity and density jumps across them, separating two different plasma regions, or propagating rotational discontinuities (RDs), which have large normal components of the magnetic field, but small variations of magnetic field intensity and of density (Hudson, 1970). There is still debate regarding the relative frequency (Neugebauer, 2006) and the origin of these structures (Vasquez et al., 2007). There are also ambiguities in identification of TDs and RDs, and differences in the defining criteria (Horbury et al., 2001; Knetter et al., 2004; ErdoS and Balogh, 2008).

Changes are often seen at time scales of 3–5 min, although similar discontinuities are seen at smaller time scales (Vasquez et al., 2007). A familiar interpretation is that these are classical ideal magnetohydrodynamic (MHD) discontinuities (Burlaga, 1968; Tsurutani and Smith, 1979; Ness and Burlaga, 2001). An alternative viewpoint is that both the fluctuations and discontinuities are facets of a nonlinear MHD cascade (Politano et al., 1998; Biskamp and Muller, 2000), and that these are interacting, not passive, and contribute to heating of the interplanetary plasma. In the former view the interplanetary medium evolves very little, and its features can be traced back to features in the lower corona, possibly even to the photosphere (Borovsky, 2008).

In this work, we are not examining the normal magnetic field which is, at least, required to distinguish between tangential and rotational discontinuities (Neugebauer, 2006). Our intention is to review observational and theoretical issues related to interplanetary discontinuities, making comparisons with MHD simulations, without regard to whether they are tangential or rotational.

It was found that methods for identifying classical discontinuities and for computing quantities related to intermittency are closely related (Greco et al., 2008). These approaches give very similar results when used as a basis for identifying "events" in either simulation data or in ACE solar wind magnetic field data (Greco et al., 2009a). In the simulations, we found that the typical events are connected with current sheets that form between adjacent magnetic flux tubes (Greco et al., 2008, 2009a). Indeed this is consistent with the fact that the solar wind exhibits many properties associated with intermittent turbulence (Burlaga, 1991; Marsch and Tu, 1994; Horbury et al., 1997; Sorriso-Valvo et al., 1999; Burlaga et al., 2006), but the question persists as to whether these properties arise locally or if they are remnants of coronal processes (e.g., Borovsky, 2008). The results show that coherent structures and discontinuities can arise rapidly, and

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therefore we suggest that at least some of the observed interplanetary discontinuities are formed locally. This conclusion is not in contrast with conclusions drawn in Bruno et al. (2004), where magnetic field directional changes were compared with increments computed from 1D parametric decay simulations. As in the present paper, the authors (Bruno et al., 2004) concluded that the distributions of increments provide important information concerning the turbulent cascade and the structure of interplanetary magnetic fluctuations.

2. Turbulence and discontinuities

The presence of discontinuities in the observed interplanetary magnetic field is suggestive of some kind of internal boundaries in the plasma. The main diagnostic we examined, describes properties of the magnetic field which is assumed to consist of a mean part $\mathbf{B}_0$ and a fluctuation $\mathbf{b}$, namely $\mathbf{B}=\mathbf{B_0}+\mathbf{b}$. The former may vary slowly in space while $\mathbf{b}$ is a complex turbulent field that varies in space and time. To describe rapid changes in the magnetic field, we looked at the increments $\Delta\mathbf{B}_s=\mathbf{B}(s+\Delta s)-\mathbf{B}(s)$ at points in space separated by Δs along some trajectory. When s is an inertial range separation, the increments have properties characteristic of the inertial range of turbulence (Monin and Yaglom, 1975). A slightly more economical description, and one that relates well to discontinuity analysis is obtained by looking at the time series and statistics of the magnitude of the vector increments,

$$|\Delta\mathbf{B}|=|\mathbf{B}(s+\Delta s)-\mathbf{B}(s)|, \quad (1)$$

once again Δs is the separation and we now suppress the argument s where convenient. Note that Eq. (1) takes into account both directional changes (such as TDs) as well as compressions due to magnetic field intensity fluctuations (Bruno et al., 2004).

In Fig. 1 we show two samples of time series of $|\Delta\mathbf{B}|$, one obtained from a 2D MHD simulation by sampling along box diagonals and another obtained from interplanetary magnetic field data measured by the ACE spacecraft (Greco et al., 2008, 2009a). We took a time interval from ACE data of 27 days (one Bartel rotation) in 2001, characterized by an average bulk velocity of 400 km/s (slow wind). The resolution is 16 s. To plot Fig. 1 we used a time separation of $\Delta t=32$ s and a separation length $\Delta s=2\Delta x$, where Δx is the spatial grid size of the simulation box. Simulation data are normalized to the correlation scale λ_c, which we chose to be equal $\frac{1}{15}$ of the box size, and ACE data are normalized to the correlation time $t_c=50$ min (Matthaeus et al., 2005). To compare MHD simulations with the solar wind dataset, the ratios $\Delta s/\lambda_c$ and $\Delta t/t_c$ are of the order of 0.08. It is apparent that both datasets are spiky, and the events that might be identified as discontinuities are evident. While discontinuities are sometimes picked out using more elaborate methods (e.g., Vasquez et al., 2007), the baseline property that there is a large sudden change of direction, can be associated with a simple cutoff or threshold applied to the datasets.

In the case of the simulation data it is possible to unambiguously identify what structures are associated with these discontinuity "events". This is particularly straightforward in two dimensions, as illustrated in Fig. 2. This illustrates field lines, associated magnetic islands and intensity of electric current density for a 2D incompressible MHD simulation of fully developed turbulence. It is a decaying turbulence run at kinetic and magnetic Reynolds numbers $R_v=R_m=1700$, carried out with a very accurate and well resolved 4096^2 Fourier pseudo-spectral code. The 2D approximation may be valid when a strong magnetic field is present (this may happen, in some circumstances, in the solar wind). Moreover, using 2D MHD can attain higher spatial resolution.

The picture shows the system when the mean square current density $\langle j^2\rangle$ is very near to its peak value. At this instant of time the peak of turbulent activity is achieved. When the turbulence is fully developed, coherent structures appear. They can be identified as magnetic islands that have different size and energy. At the regions between islands the perpendicular (out-of-plane) component of the current density j becomes very high (Matthaeus and Montgomery, 1980). As reported in Fig. 2, the out-of-plane component of the magnetic potential a shows a collection of

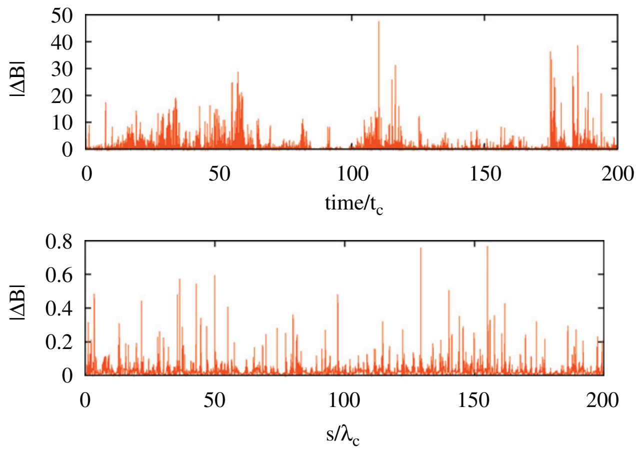

Fig. 1. Time/space series of the magnitude of magnetic vector increments computed from 2D MHD simulation (bottom) and solar wind ACE (top). In both cases data are acquired along a linear path (in solar wind using frozen-in flow) and normalized to the respective correlation scales. Here the scales are roughly comparable in terms of correlation scales, and the appearance of the datasets is similar, with spiky changes seen in both cases.

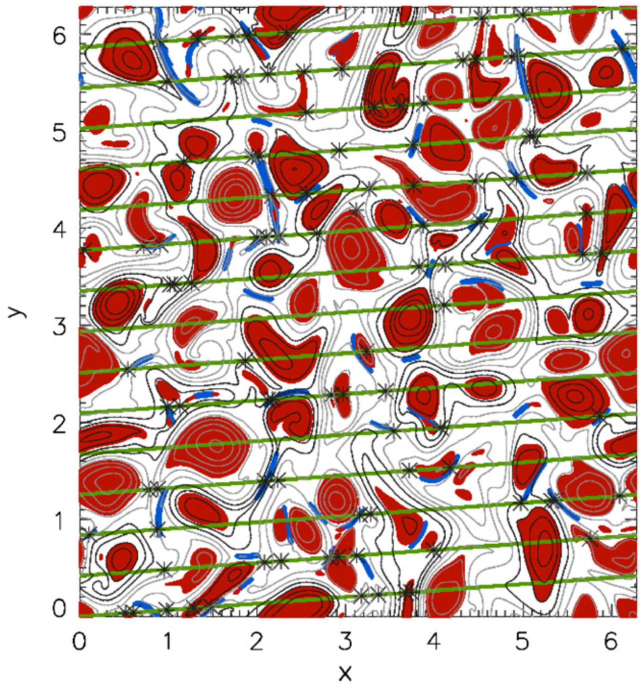

Fig. 2. A contour map of the out-of-plane vector potential (field lines) for $a>0$ (black contour lines) and $a<0$ (gray contour lines). The superposed colors represent magnetic islands (red) and strong current regions (blue) identified with a cellular automata technique (Servidio et al., 2009). Green lines represent a sample path through the simulation box. Gray stars are placed at the center of a discontinuity, selected in this case by the PVI method. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

magnetic islands having a wide distribution of sizes. In three dimensions these would correspond to flux tubes. Some of the magnetic islands are reconnecting with a nearest neighbor. Many of the islands are also bordered by strong sheets of electric current density. When these are sampled by crossing them, the result appears as a tangential discontinuity (TD) for inertial range increments. An example of TD is displayed in the lower part of Fig. 3. The figure shows that these events are characterized by a rotation of the magnetic field followed by a depression of the magnitude of **B** (panels c and d). None of these features were present at the initial time (not shown) in which the electric current is not concentrated but rather is randomly distributed by construction.

This observation of the generation of current sheets and their connection to TDs that can be "observed" in simulations leads naturally to the question of whether there might be a similar origin of discontinuities in solar wind turbulence. This question has been examined in some details (Greco et al., 2008) by looking at the distribution of waiting times between discontinuity events identified either by classical methods (designated "TS") or by a threshold on the value of $|\Delta \mathbf{B}|$ normalized to its own variance (designated partial variance of increments or "PVI"). The method for establishing the threshold is as follows. We choose a guess (low) threshold value of PVI, and all events above this threshold are excluded. Then, we calculate the kurtosis of the remaining signal. This is repeated with varying values of the threshold until the kurtosis of the remaining signal is equal to the value expected for the squared modulus of a random vector having independent, Gaussian distributed, components.

An example of the application of the latter method is shown in Fig. 2. A sample diagonal path through the simulation box is displayed. In the same figure we overlap the one-dimensional path to the two-dimensional structure of the magnetic field. As it can be seen, many of the detected discontinuities correspond (overlap) to those regions characterized by strong values of the current (reconnecting current sheets).

Greco et al. (2008) found that the two methods performed almost interchangeably and the waiting-time distributions were almost identical. Next Greco et al. (2009a) applied these methods to compare statistics of simulations and statistics of solar wind ACE magnetic field data. The upper part of Fig. 3 shows a directional change, selected by the PVI algorithm in the ACE magnetic field data. Panel a gives the components and panel b the magnitude of the magnetic field vector with 16 s time resolution. It is clear that the transition took place on a shorter time interval than the time resolution of the data from which the event was selected.

The normalized waiting-time distributions between events (like those shown in Fig. 3) (using either TS or PVI methods) were extremely similar in the solar wind and simulation datasets at (inertial range) separations shorter than the correlation scale. At larger separations the distributions, in the solar wind and in simulations, differ. Departures are not surprising since the large-scale structures in the solar wind are very different from those in the simulations. The simulation box contains only a certain number of correlation lengths much less than in the solar wind, and it has no large-scale features other than periodicity.

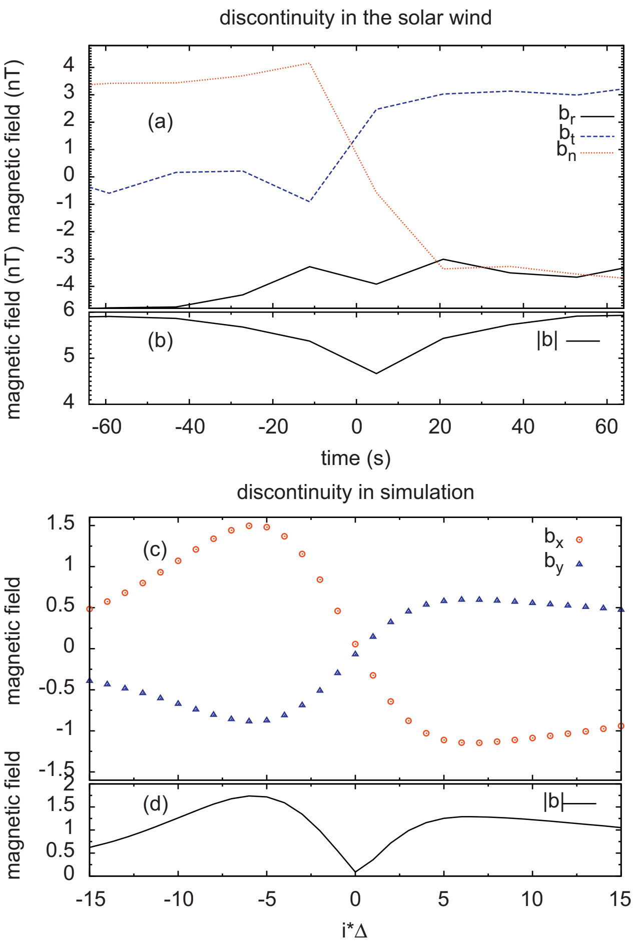

Fig. 3. Examples of discontinuities selected by the PVI method. Panel a: the three components of the magnetic field vector in solar wind data in the RTN reference frame; Panel b: magnitude of the magnetic field vector in solar wind data. The discontinuity, centered around zero, lasts few tens seconds; Panel c: the two components of the magnetic field vector in simulation data; and Panel d: magnitude of the magnetic field vector in simulation data. Δ is the resolution data.

3. Non-Gaussian statistics

The same study (Greco et al., 2009a) also found that probability distribution functions of the normalized increments of the magnetic field components also compare well. Fig. 4 illustrates PDFs of inertial range increments from high resolution

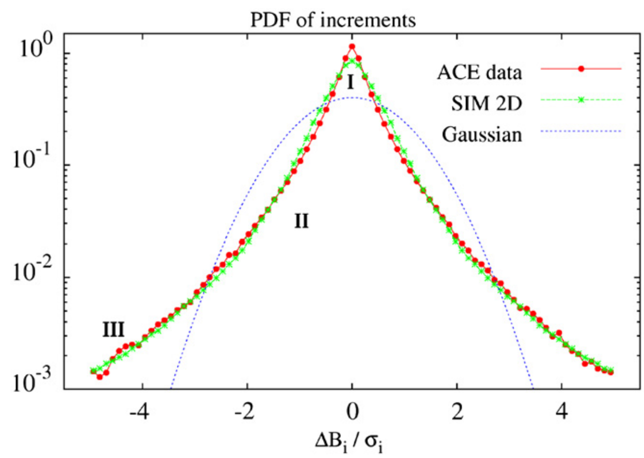

Fig. 4. The PDFs of normalized increments designated by $\Delta B_i/\sigma_i$, where for the ACE data this indicates $\Delta B_r/\sigma_r$, the radial magnetic field increment at 4 min separation and normalized by the standard deviation (red); for the 2D simulation this becomes the composite PDF of $\Delta B_x/\sigma_x$ and $\Delta B_y/\sigma_y$, the standard deviation-normalized increments of the two magnetic components perpendicular to the uniform applied magnetic field (green). For comparison a unit variance Gaussian is also shown (blue). Labels designate (I) super-Gaussian core, (II) sub-Gaussian wings and (III) super-Gaussian tails. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2D simulations, and ACE magnetic field data. A crucial point to realize is that the events that are identified as discontinuity events in either TS or PVI methods are associated with the tails of these distributions. The only technical difference is that PVI employs a threshold on the *magnitude* $|\Delta \mathbf{B}|$ while the PDFs shown here are those of a *Cartesian* component ΔB_i. The distributions from simulation and spacecraft data are remarkably similar, with cross-overs between sub- and super-Gaussian regions occurring at very nearly the same value of normalized separation.

In Fig. 4 we also suggest a classification into (I) super-Gaussian core, (II) sub-Gaussian wings at intermediate values, and (III) super-Gaussian tails. We can then ask the question: Where are the super-Gaussian tails coming from? In the case of the 2D simulations this is readily addressed by masking region III and visualizing the results. The comparison between Figs. 4 and 2 shows that Region III is due to the current sheets (blue regions) that form between the flux tubes. These current sheets represent the well-known small-scale coherent structures of MHD turbulence (Matthaeus and Lamkin, 1986; Carbone et al., 1990; Veltri, 1999; Servidio et al., 2008) that are linked to the magnetic field intermittency.

4. The local Poisson hypothesis for the discontinuities occurrence

In Greco et al. (2008), it was found through the analysis of Hall MHD simulations that distribution of waiting times (WT) between discontinuities for $s<\lambda_c$ is well described by a power law, while exponential functions work better for $s>\lambda_c$, where λ_c is the correlation length and s is the separation between suitably defined extreme events. Analysis of solar wind ACE data (Greco et al., 2009a) verified that power laws describe the distribution of WT for periods up to ~ 3000 s (50 min), which is on the order of the correlation length of magnetic fluctuations in the solar wind (Matthaeus et al., 2005).

The shape of the probability distribution function for waiting times can give useful information about the physics of the underlying process. The solar wind is characterized by the presence of strong small-scale magnetic fluctuations that behave as intermittent events (Burlaga, 1968; Tsurutani and Smith, 1979; Bruno et al., 2001; Vasquez et al., 2007). There is still debate regarding the relative frequency and the nature of these intermittent signals.

Some authors (Carbone et al., 2003) have shown that the waiting times (inter-arrival times) between these magnetic structures in the inner solar wind, measured by the Helios II satellite are distributed with a power law, extended over several decades. In fact, as turbulence may be viewed as a fragmentation process that carries energy from large to small scales, there exists a strong correlation between structures generated at different scales. This suggests that an underlying complex dynamics with long correlation times or "memory" is present (Bruno and Carbone, 2005).

In Greco et al. (2009b) the test for Poisson processes, introduced by Bi et al. (1989), was employed to further examine the distribution of waiting time for these events. The tests showed that the discontinuities are not distributed without correlations, but rather that non-Poisson correlations, possibly in the form of burstiness or voids, are present in the solar wind data at least up to the typical correlation scale. A similar conclusion emerges from Poisson analysis of the simulation dataset. On the contrary, a Poisson's random noise might well characterize the very large-scale solar wind fluctuations that at least in part are related to the presence of large structures of highly conducting plasma. Our tentative conclusion was that the discontinuities or bursty coherent structures represent in this view the current sheets that form between magnetic flux tubes (Veltri, 1999; Servidio et al., 2008; Borovsky, 2008) which may be a signature of intermittent, anisotropic, fully developed MHD turbulence.

5. Conclusions

We can draw a firm conclusion for the numerical experiments that the discontinuity events are formed spontaneously due to nonlinear couplings, cascade, and turbulence. They were not present in the initial data. The extension of this conclusion to the solar wind is tempting, but remains uncertain. Partial variance of increments and classical discontinuity measures identify the same events at better than 70% accuracy when applied to simulations and solar wind for scales comparable to and smaller than the correlation scale. Direct comparison of the probability distributions of increments at inertial range separations in the MHD simulations and in the solar wind magnetic field reveals a very similar three-part functional form. For the 2D case these are identified as super-Gaussian current sheets, sub-Gaussian flux tube cores, and weak sub-Gaussian currents between flux tubes. These results are consistent with the hypothesis that solar wind intermittency and many or most of its discontinuities are produced by MHD turbulence, even if we have not ruled out that some of these features originate in the lower corona. It is possible therefore that many or even most of the current structures in the solar wind, particularly inertial range structures that contribute to the tails of the PDFs of increments of **B**, are formed in situ by local rapid relaxation processes associated with turbulence (Servidio et al., 2008).

Finally, a magnetic discontinuity is a rapid change of the field across a very narrow part of the space, so that strong changes of the magnetic topology are necessarily involved. This implies the possibility that discontinuities and local magnetic reconnection events may be different faces of the same medal. Generally, magnetic reconnection has been often studied in simplified geometries and boundary conditions, but since it might occur in any region separating topologically distinct magnetic flux structures, it might be expected to be of importance in more general circumstances, including MHD turbulence. The latter possibility has been recently investigated, leading to the conclusion that in turbulence strong reconnection events locally occur (Servidio et al., 2009). Previous studies on discontinuities and theories of reconnection in turbulence could be combined in order to identify possible reconnection events between the intermittent events. Other informations like density, velocity outflows, and electric field may be crucial in order to develop numerical algorithms for the identification of reconnecting current sheets in the solar wind.

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