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Electronic transitions in quantum dots and rings induced by inhomogeneous off-centered light beams

G F Quinteiro, A O Lucero and P I Tamborenea

Departamento de Física and IFIBA, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón I, 1428 Ciudad de Buenos Aires, Argentina

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Abstract

We theoretically investigate the effect of inhomogeneous light beams with (twisted light) and without (plane-wave light) orbital angular momentum on semiconductor-based nanostructures, when the symmetry axes of the beam and the nanostructure are displaced parallel to each other. Exact analytical results are obtained by expanding the off-centered light field in terms of the appropriate light modes centered around the nanostructure. We demonstrate how electronic transitions involving the transfer of different amounts of orbital angular momentum are switched on and off as a function of the separation between the axes of the beam and the system. In particular, we show that even off-centered plane-wave beams induce transitions such that the angular momenta of the initial and final states are different.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Over the last few decades, the design and subsequent study of man-made nanostructures has occupied a crucial place in pure and applied condensed matter research. The importance of these systems stems from the fact that, on the one hand, they are excellent tools to probe the laws of physics at different length scales and, on the other hand, their technological applications are vast, spanning electronics, optical devices, and spintronics. Quantum dots (QDs) are a paradigmatic example that illustrates the general trend [1, 2]. QDs mimic atoms at a larger length scale (they are sometimes called artificial atoms) and allow a flexible control of the 'atomic' properties, like the strength of the Coulomb interaction via the dielectric constant, the number of electrons, the shape of the confining potential, the spin–orbit interaction, etc. Quantum rings (QRs) [3, 4] are another important example of nanostructured systems where interesting physical effects can be explored—the Aharonov–Bohm effect, persistent currents, quantum interference, etc.

Much research is being carried out on semiconductor nanostructures, which present, among other interesting features, a strong coupling with optical (or near-optical) electromagnetic fields. Naturally, the electronic states of these structures can be probed and manipulated using a variety of light pulses. For instance, the use of femtosecond light pulses is usually claimed to outperform other methods when an ultrafast control is sought. However, there has been very little exploration of what can be done on semiconductor nanostructures by taking advantage of the inhomogeneous nature of light beams. In particular, the interaction of these structures with twisted light (TL) [5]—i.e. light carrying orbital angular momentum (OAM)—is only beginning to be studied, although much effort is being devoted to the study of TL in other areas of physics [6]. We have recently initiated studies in the topic of TL–semiconductor interaction, both for QDs [7] and QRs [8], and also for bulk and quasi-two-dimensional systems [9, 10].

In particular, our previous article in QDs explored the interaction of disc-shaped QDs with twisted light, in the case where the beam axis coincides with the symmetry axis of the QD. We predicted that the transitions are not vertical and may connect states having different values of the orbital angular momentum of the electron, and from an applied standpoint, that the use of light carrying OAM would facilitate the ultrafast manipulation of the electronic states in the system. It is noticeable that the interaction of twisted light with QRs and QDs is richer, in a sense, than the interaction with molecules, whose internal electronic states cannot be modified as easily by the light, in view of their small size and the applicability of the dipole approximation [11].

This paper extends our previous work on QDs and QRs to the general case of the interaction of inhomogeneous fields—with special emphasis on TL beams—with QDs or QRs when the symmetry axis of the nanostructure and the beam axis do not coincide. The problem we study is of interest, for it addresses realistic experimental situations, like the irradiation of an ensemble of nanostructures, where each nanostructure will see a displaced light beam. Also, when a single nanostructure is meant to be addressed, our theory shows how an imprecise centering of the beam can alter significantly the optical transitions. Finally, from a constructive point of view, our calculations display the wealth of opportunities for optical control of electronic states which can be achieved by simply changing the positioning, waist and orbital angular momentum of the light beam.

The differences between the centered and the off-centered geometries of the beam with respect to the QR or QD can be simply appreciated when the light beam does not carry OAM, that is, for the usual plane-wave beams. In the centered geometry, these beams can only induce vertical optical transitions, that is, transitions between states whose envelope wavefunctions have the same angular momentum (there is a separate angular momentum conservation involving the spin angular momentum of the light and the angular momentum associated with the microscopic part of the electronic wavefunctions, which follows the traditional principles of optical transitions in solids). For these plane-wave beams, we show here how the off-centered geometry induces non-vertical transitions which are not allowed in the centered one. This arises mathematically from the decomposition of the off-centered laser-beam field in field modes centered in the nanostructure. In this expansion multiple components with non-zero OAM appear, which activate the non-vertical transitions like twisted light does in a centered geometry. From this simple case one can envision a similar phenomenon even if the applied off-centered beam has non-zero OAM. This is, in a nutshell, the essence of the phenomena explored here.

This paper is organized as follows. Section 2 briefly covers the already studied problem of the interaction of TL with electrons in nanostructures (QDs and QRs) when the beam and system symmetry axes coincide. Section 3 develops the theory used to solve the general problem of a TL beam displaced from the symmetry axis of the nanostructure and presents in detail the differences between the centered and off-centered geometries. Conclusions are given in section 4.

2. Background: centered beam

In this section we treat the problem of the interaction of TL with QDs and QRs, in the case where both symmetry axes coincide. The model and results are based on previous studies [7, 8] and serve as an introduction to the more general problem of off-centered beams.

Succinctly stated, twisted light is light carrying orbital angular momentum $\hbar l$ per photon, characterized by inhomogeneous fields having a phase $\mathrm{e}^{il\theta}$ and radial dependence of the Laguerre [12] or Bessel [13] form. In the Coulomb gauge, the inhomogeneous vector potential has both transverse and longitudinal components [9]. Under typical experimental conditions, in which $q_r/q_z < 1$, the transverse component is the dominant one, and in cylindrical coordinates is [9]

$$\begin{aligned}\mathbf{A}_l(\mathbf{r},t) &= \boldsymbol{\epsilon}_\sigma F_l(\mathbf{r})\mathrm{e}^{\mathrm{i}(q_z z-\omega t)} + \text{c.c.}\\ &= \mathbf{A}_l^{(+)}(\mathbf{r},t) + \mathbf{A}_l^{(-)}(\mathbf{r},t),\end{aligned} \tag{1}$$

where $\boldsymbol{\epsilon}_\pm = \hat{x} \pm \mathrm{i}\hat{y}$ is the transverse circular-polarization vector. We choose to work with Bessel modes for the radial dependence, since these will allow for analytical results in section 3. Thus, we take

$$F_l(\mathbf{r}) = J_l(q_r r)\mathrm{e}^{\mathrm{i}l\theta}. \tag{2}$$

Quantum dots and rings based on semiconductor materials can be made to confine both electrons and holes. Electronic states in these structures can be mathematically described by the product of envelope $R(r)\mathrm{e}^{\mathrm{i}m\theta}Z(z)$, microscopic $u_b(\mathbf{r})$ (here taken at zero crystal momentum) and spin ξ functions:

$$\Psi_b(\mathbf{r}) = [R(r)\mathrm{e}^{\mathrm{i}m\theta}Z(z)]u_b(\mathbf{r})\,\xi. \tag{3}$$

In semiconductor systems, strain lifts the degeneracy of the heavy-hole and light-hole bands. Therefore, a two-band model in the effective-mass approximation with one conduction ($b = c$) and one heavy-hole valence ($b = v$) band is sufficient to treat our problem. In addition, we have assumed that the envelope function is separable. Furthermore, since the confinement in the z direction is much stronger than that in the x–y plane, it is a good approximation to assume that the electron remains in the lowest-energy z eigenstate.

The coupling between the semiconductor structure and the TL beam is modeled using the minimal-coupling Hamiltonian, which to lowest order in the vector potential is given by

$$h_l = -\frac{q}{m_\mathrm{e}}\mathbf{A}_l(\mathbf{r},t)\cdot\mathbf{p}, \tag{4}$$

where $q = -e$, m_e and $\mathbf{p}$ are the charge, mass and momentum of the electron, respectively. This coupling causes electronic transitions between the conduction and valence bands. The physics of the interaction can be derived—e.g. using Fermi's golden rule, master equation or Heisenberg equations—from the matrix elements of h_l. Since we are interested in interband transitions, the rotating-wave approximation is applied and we obtain for the matrix elements of the light–matter interaction

$$\begin{aligned}\langle c\alpha'|\,h_l^{(+)}\,|v\alpha\rangle &= -\frac{2\pi q}{m_\mathrm{e}}\mathrm{e}^{-\mathrm{i}\omega t}(\boldsymbol{\epsilon}\cdot\mathbf{p}_{cv})\delta_{l,(m'-m)}\\ &\quad\times\delta_{\xi',\xi}\int_0^\infty \mathrm{d}r\, r J_l(q_r r)R^*_{c\alpha'}(r)R_{v\alpha}(r),\end{aligned} \tag{5}$$

$$\begin{aligned}\langle v\alpha'|\,h_l^{(-)}\,|c\alpha\rangle &= -\frac{2\pi q}{m_\mathrm{e}}\mathrm{e}^{\mathrm{i}\omega t}(\boldsymbol{\epsilon}^*\cdot\mathbf{p}_{vc})\delta_{l,(m-m')}\\ &\quad\times\delta_{\xi',\xi}\int_0^\infty \mathrm{d}r\, r J_l(q_r)R^*_{v\alpha'}(r)R_{c\alpha}(r),\end{aligned} \tag{6}$$

where α is a collective index that gathers all quantum numbers relevant for QDs (radial and angular) or QRs (angular) and contains in particular the angular quantum number m that appears in equation (3). We used the usual matrix element $\mathbf{p}_{bb'} = \frac{1}{a^3}\int_{a^3}\mathrm{d}^3r\, u_b(\mathbf{r})^*\,(-\mathrm{i}\hbar\nabla)u_{b'}(\mathbf{r})$. The expressions given above represent absorption ($v \to c$) and emission ($c \to v$) of light, respectively (for a detailed derivation, see [7] and [8]).

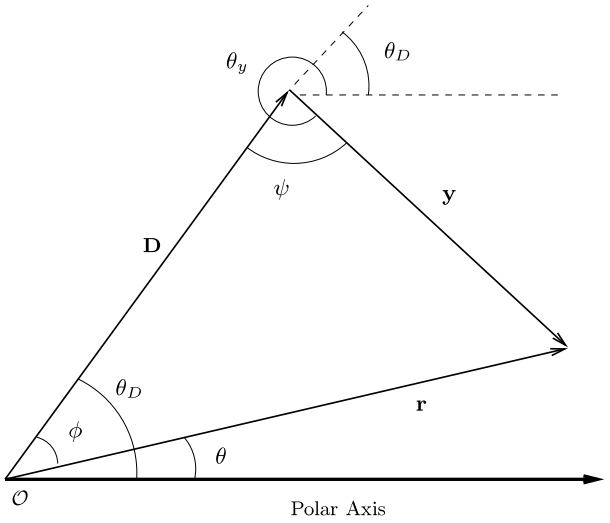

Figure 1. Relative positions of the nanoparticle and the light beam. The QD/QR is placed at the origin $\mathcal{O}$ and the TL beam axis passes through **D**. The z axis is normal to the plane of the drawing.

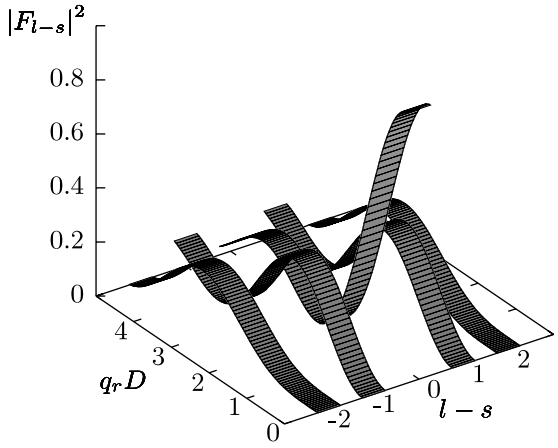

Figure 2. Weights corresponding to the decomposition of the off-centered TL beam in terms of TL beams centered at the origin.

3. Off-centered beam

We now consider the case of light beams whose center is displaced with respect to that of the nanostructure. We formulate the theory for TL beams with arbitrary OAM, given by l; the case of plane waves is then easily derived by setting $l = 0$. The geometry of the problem is shown in figure 1. The nanostructure is centered at the origin of coordinates, with symmetry axis along z, and a beam of TL propagates in the z direction, displaced from the origin by a distance D. Hereafter, we shall refer to this situation as having an 'off-centered' TL beam (with respect to the nanostructure). We first formulate the theory and then analyze the main aspects of its predictions.

3.1. Theory

Let us use equation (1) to write the vector potential centered at **D** (herein, we indicate displaced vector potentials and Hamiltonians by a tilde):

$$\tilde{\mathbf{A}}_l(\mathbf{y}, t) = \boldsymbol{\epsilon}_\sigma F_l(\mathbf{y}) \mathrm{e}^{\mathrm{i}(q_z z - \omega t)} + \text{c.c.}, \tag{7}$$

where

$$\begin{aligned} F_l(\mathbf{y}) &= J_l(q_r y)\mathrm{e}^{\mathrm{i}l\theta_y} \\ &= J_l(q_r y)(-1)^l \mathrm{e}^{\mathrm{i}l\psi}\mathrm{e}^{\mathrm{i}l\theta_D} \end{aligned}$$

since $\theta_y = \pi + \psi + \theta_D$. The relation [14]

$$J_l(q_r y)\mathrm{e}^{\mathrm{i}l\psi} = \sum_{s=-\infty}^{\infty} J_{l+s}(q_r D) J_s(q_r r)\mathrm{e}^{\mathrm{i}s\phi} \tag{8}$$

allows us to express $F_l(\mathbf{y})$ in terms of centered coordinates as

$$\begin{aligned} F_l(\mathbf{y}) &= (-1)^l \sum_{s=-\infty}^{\infty} J_{l+s}(q_r D) J_s(q_r r)\mathrm{e}^{\mathrm{i}s(\theta_D-\theta)}\mathrm{e}^{\mathrm{i}l\theta_D} \\ &= \sum_{s=-\infty}^{\infty} (-1)^{l-s} F_{l-s}(\mathbf{D}) F_s(\mathbf{r}), \end{aligned}$$

where the last equation follows by changing s to $-s$. From equation (7), the final form of the vector potential centered at position **D**:

$$\begin{aligned} \tilde{\mathbf{A}}_l(\mathbf{y}, t) = &\sum_{s=-\infty}^{\infty} (-1)^{l-s} F_{l-s}(\mathbf{D})\, \mathbf{A}_s^{(+)}(\mathbf{r}, t) \\ &+ \sum_{s=-\infty}^{\infty} (-1)^{l-s} F^*_{l-s}(\mathbf{D})\, \mathbf{A}_s^{(-)}(\mathbf{r}, t) \end{aligned} \tag{9}$$

is given in terms of a superposition of TL vector potentials centered at the origin, each having different OAM, and weighted by $F_{l-s}(\mathbf{D})$. This expansion is easily understood by analogy with the transformation of angular momentum in classical mechanics, when the proper axis—from which the motion is seen as a pure rotation—is changed to an arbitrary axis. Although our transformation makes the representation of $\tilde{\mathbf{A}}_l(\mathbf{y}, t)$ more complex, it is justified by the need to refer both the TL beam and the electronic states to the same axis. Figure 2 shows the weights squared $|F_{l-s}(\mathbf{D})|^2 = J_{l-s}(q_r D)^2$ of the decomposition of equation (9).

It is now a simple algebraic matter to write down the interaction Hamiltonian $\tilde{h}_l = (-q/m_\mathrm{e})\tilde{\mathbf{A}}_l(\mathbf{y}, t) \cdot \mathbf{p}$, which becomes

$$\begin{aligned} \tilde{h}_l = &\sum_{s=-\infty}^{\infty} (-1)^{l-s} F_{l-s}(\mathbf{D}) \left[-\frac{q}{m_\mathrm{e}} \mathbf{A}_s^{(+)}(\mathbf{r}, t) \cdot \mathbf{p} \right] \\ &+ \sum_{s=-\infty}^{\infty} (-1)^{l-s} F^*_{l-s}(\mathbf{D}) \left[-\frac{q}{m_\mathrm{e}} \mathbf{A}_s^{(-)}(\mathbf{r}, t) \cdot \mathbf{p} \right], \end{aligned}$$

or, using the Hamiltonian at the origin h_l

$$\begin{aligned} \tilde{h}_l &= \sum_{s=-\infty}^{\infty} (-1)^{l-s} F_{l-s}(\mathbf{D}) h_s^{(+)} + \sum_{s=-\infty}^{\infty} (-1)^{l-s} F^*_{l-s}(\mathbf{D}) h_s^{(-)} \\ &= \tilde{h}_l^{(+)} + \tilde{h}_l^{(-)}. \end{aligned} \tag{10}$$

The matrix elements of the Hamiltonian, equation (10), are determined using the matrix elements for the centered case, equations (5) and (6), and give

$$\langle c\alpha' | \tilde{h}_l^{(+)} | v\alpha \rangle = \sum_{s=-\infty}^{\infty} (-1)^{l-s} F_{l-s}(\mathbf{D}) \langle c\alpha' | h_s^{(+)} | v\alpha \rangle$$

$$\langle v\alpha'|\,\tilde{h}_l^{(-)}\,|c\alpha\rangle = \sum_{s=-\infty}^{\infty}(-1)^{l-s}F^*_{l-s}(\mathbf{D})\langle v\alpha'|\,h_s^{(-)}\,|c\alpha\rangle.$$

The sums are solved thanks to the delta function $\delta_{s,\pm(m'-m)}$ that appear in the matrix elements of $h_s^{(\pm)}$. Then

$$\langle c\alpha'|\,\tilde{h}_l^{(+)}\,|v\alpha\rangle = (-1)^{l-(m'-m)}F_{l-(m'-m)}(\mathbf{D}) \times \langle v\alpha'|\,h^{(+)}_{m'-m}\,|c\alpha\rangle \quad (11)$$

$$\langle v\alpha'|\,\tilde{h}_l^{(-)}\,|c\alpha\rangle = (-1)^{l-(m-m')}F^*_{l-(m-m')}(\mathbf{D}) \times \langle v\alpha'|\,h^{(-)}_{m-m'}\,|c\alpha\rangle. \quad (12)$$

The final expressions are

$$\langle c\alpha'|\,\tilde{h}_l^{(+)}\,|v\alpha\rangle = -\frac{2\pi q}{m_e}e^{-i\omega t}(\boldsymbol{\epsilon}_\sigma\cdot\mathbf{p}_{cv})F_{l-(m'-m)}(\mathbf{D}) \times (-1)^{l-(m'-m)}\int_0^\infty dr\, r\, J_{m'-m}(qr)R^*_{c\alpha'}(r)R_{v\alpha}(r), \quad (13)$$

$$\langle v\alpha'|\,\tilde{h}_l^{(-)}\,|c\alpha\rangle = -\frac{2\pi q}{m_e}e^{i\omega t}(\boldsymbol{\epsilon}^*_\sigma\cdot\mathbf{p}_{vc})F^*_{l-(m-m')}(\mathbf{D}) \times (-1)^{l-(m-m')}\int_0^\infty dr\, r\, J_{m-m'}(qr)R^*_{v\alpha'}(r)R_{c\alpha}(r), \quad (14)$$

where, to ease the notation, we eliminated the delta functions for the spin indices and assumed that the bands are the correct ones which yield non-vanishing matrix elements.

3.2. Results and analysis

We would like to draw first a general picture of what happens to the electronic transitions when the TL beam is displaced. To this end, let us consider the main ingredient in the calculation of the transition probability using Fermi's golden rule, namely

$$|\langle v\alpha'|\,\tilde{h}_l^{(+)}\,|c\alpha\rangle|^2 = \kappa|F_n(\mathbf{D})|^2 = \kappa J_n(q_r D)^2, \quad (15)$$

where $n = l-(m'-m)$. Note that κ contains relevant information that will be unfolded later in this section, when precise numerical calculations are presented.

For concreteness, let us assume that the TL beam possesses $l = 1$; it is worth stressing that l is the OAM of the beam as seen from its proper axis. When the beam is centered, $J_n(q_r D = 0)^2 \neq 0$ only for $n = 0$. This implies that the only allowed transition is that connecting states that differ by one unit of angular momentum: $m'-m = 1$, see figure 3(a). As the beam is displaced off the nanostructure symmetry axis, other transitions between states having $m'-m = \pm 2, \pm 3, \ldots$ become allowed, see figure 3(b). To understand this apparent violation of conservation of OAM, we recall that, according to equation (9), the off-centered beam is a superposition of centered beams with varying values of OAM. Due to the n dependence of the zeros of the Bessel function, there are specific values of $q_r D$ such that, for instance, the transition matrix element between states differing by one unit of angular momentum is zero, while the probability for other transitions remains finite, as illustrated in figure 3(c).

In order to perform a numerical analysis, we shall now consider the whole expressions in equations (13) and (14). Thus, we need to consider specific radial functions. For the case of a QR, employing for the sake of simplicity a hard-wall model:

$$R(r) = \sqrt{\frac{2}{r_0 d}}\sin\left[\frac{\pi}{d}(r - r_0 + d/2)\right], \quad (16)$$

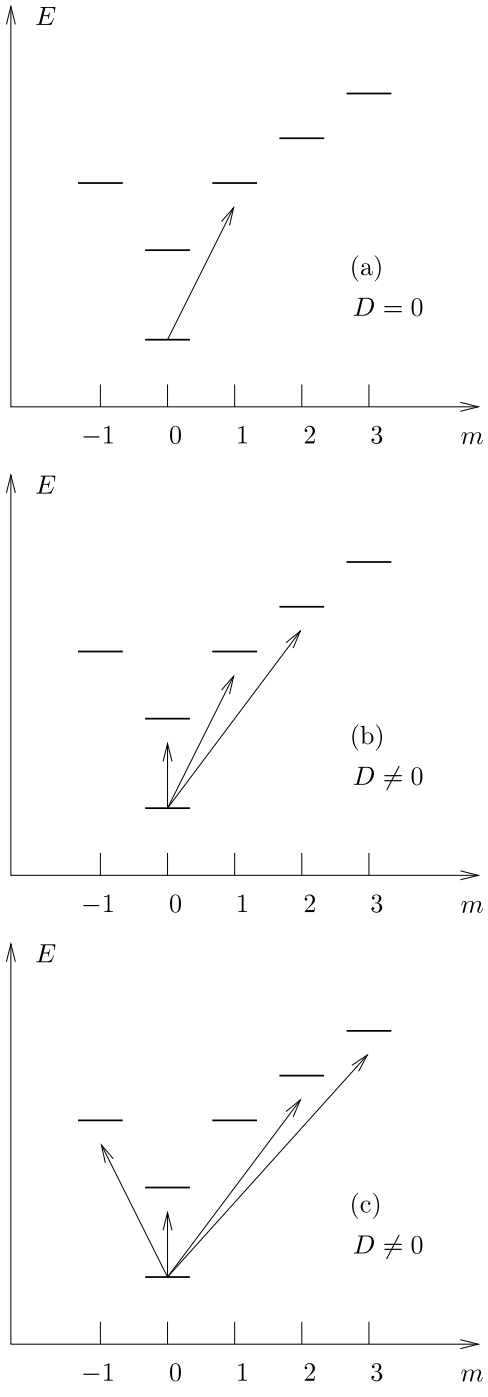

Figure 3. Pictorial representation of the possible transitions in a QR, induced by TL having OAM $l = 1$. (a) A centered beam produces a non-vertical transition between a valence-band and a conduction-band state differing by one unit of angular momentum m. (b) An off-centered beam produces several non-vertical transitions. (c) For a particular choice of $q_r D$ the transition from the upper valence-band state to the $m = 1$ conduction-band state becomes negligible.

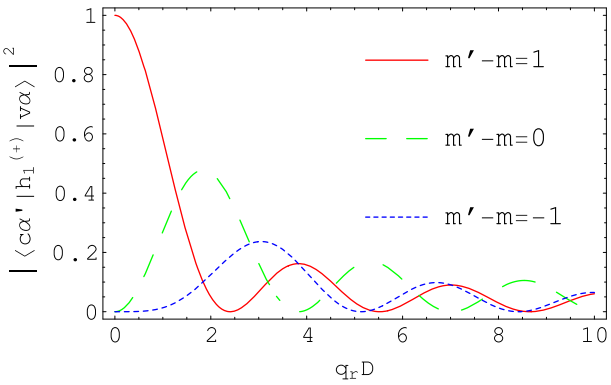

Figure 4. Normalized transition matrix element for a TL beam with $l = 1$ and for different values of the initial and final angular momentum quantum number.

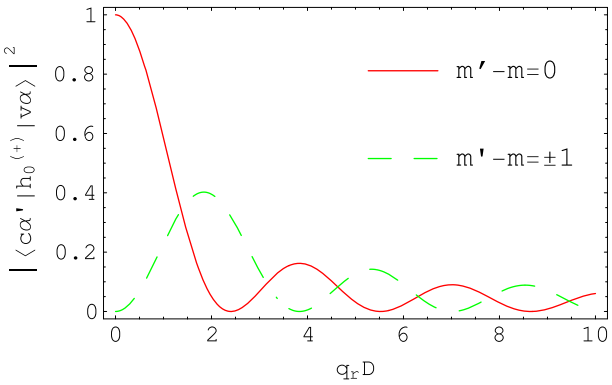

Figure 5. Normalized transition matrix element for plane-wave light ($l = 0$) and for different values of the initial and final angular momentum quantum number.

with r_0 and d the radius and width of the ring; $R(r) = 0$ outside the ring. We assume $r_0 \gg d$, which allows us to restrict the analysis to one subband. For a QD, considering a parabolic confinement, which is the most widely used model for quasi-two-dimensional dots, the radial wavefunction is

$$R_{sm}(r) = \frac{(-1)^s}{\sqrt{2\pi}\,\ell}\sqrt{\frac{s!}{(s+|m|)!}}\mathrm{e}^{-r^2/(4\ell^2)} \times \left(\frac{r}{\sqrt{2}\,\ell}\right)^{|m|} L_s^{|m|}(r^2/(2\ell^2)), \tag{17}$$

where ℓ is a characteristic length for the confinement of electrons, s and m are the radial and angular momentum quantum numbers, and $L_s^{|m|}(x)$ is a Laguerre polynomial.

For the QR, the integral is (the quantum number indices in $R(r)$ are unnecessary)

$$\int_0^\infty \mathrm{d}r\, r J_{m'-m}(qr)|R(r)|^2 = J_{m'-m}(q_r r_0). \tag{18}$$

For the case of a QD, a redefinition of the radial coordinate to $x = r^2/(2\ell^2)$ allows us to simplify the expression [7]. In the following, we exemplify this by using the case of a QR. The matrix element for the absorption of light becomes

$$\langle c\alpha'|\,\tilde{h}_l^{(+)}\,|v\alpha\rangle = -(-1)^n\frac{2\pi q}{m_\mathrm{e}}(\boldsymbol{\epsilon}_\sigma\cdot\mathbf{p}_{cv}) \times F_n(\mathbf{D})J_{m'-m}(q_r r_0)\mathrm{e}^{-\mathrm{i}\omega t}. \tag{19}$$

We are now in a position to precisely determine the transition probability. We see at once that the probability for the transition $\{v\alpha\} \to \{c\alpha'\}$ is determined by the displacement $\mathbf{D}$, the difference $m' - m$ between the angular momentum quantum numbers and the relative sizes of QR (r_0) to beam waist ($\simeq q_r^{-1}$).

For a general displacement $D \neq 0$, several transition matrix elements have finite values, as explained before. The relative strength of each transition is determined by $[J_{l-(m'-m)}(q_r D)J_{m'-m}(q_r r_0)]^2$.

Let us exemplify this for a TL beam with $l = 1$; then, we have $n = 1 - (m' - m)$. Figure 4 shows the quantity $[J_{1-(m'-m)}(q_r D)J_{m'-m}(q_r r_0)]^2$ for different values of D and $m'-m$. We observe what was said previously from a qualitative point of view. At zero displacement, the only contribution comes from transitions differing by one unit in their angular momentum. When the beam is displaced, other transitions become allowed. An extinction occurs when the value $q_r D$ is a zero of the Bessel function. It is clear what happens for an arbitrary value of l: the main contribution close to $R = 0$ arises from the states differing by l in their orbital angular momentum.

Another interesting situation arises when the beam carries no OAM, i.e. $l = 0$ in equation (19)—we refer to this as the plane-wave situation. In figure 5 we analyze $[J_{m'-m}(q_r D)J_{m'-m}(q_r r_0)]^2$. As expected, when the beam is centered on the nanoparticle, the only allowed transitions are vertical, i.e. $\{v\alpha\} \to \{c\alpha\}$; however, when the beam axis is displaced, non-vertical transitions become allowed. An important difference arises with respect to the $l \neq 0$ case; since the function $[J_{m'-m}(q_r D)J_{m'-m}(q_r r_0)]^2$ is symmetric on $m' - m$, the probability for transitions in a pair $m' - m = \pm\delta$, with δ an natural number, is the same, and in most situations there will be no net transfer of orbital angular momentum. However, this symmetry can be broken by the application of a magnetic field, which detunes one of the possible final states, e.g. the $m' - m = \delta$ compared to the $m' - m = -\delta$. Also, this symmetry is broken (even without an applied magnetic field) if the initial population of valence-band states is asymmetric with respect to the quantum number m. The peculiar effects that we predict are the result of the finite waist of the beam and the fact that, to treat it adequately, we have gone beyond the dipole-moment approximation.

The relative size of QR to beam waist impacts on the response of QRs differing in their radii r_0, but lying at the same distance D from the beam axis—this situation may arise, for example, due to the statistical nature of the fabrication process of an ensemble of nanoparticles. As a consequence of the factor $J_{m'-m}(q_r r_0)$ in equation (19), the various electronic transitions are enhanced and inhibited to some degree, and this results in a different evolution of the electronic states in each QR.

We now make some final comments regarding the general characteristics of the interaction of twisted light with solids, which apply also to the situations analyzed here. Interband

optical transitions induced by twisted light in semiconductors are affected in the usual way by whether the energy gap of the material is direct or indirect. A direct gap, like that of GaAs, is of course more desirable than an indirect one, like that of Si, since the latter presents a reduced joint density of states which severely limits the optical-transition probabilities. The finite value of the lateral momentum of the twisted light (the parameter q_r in equation (2)) does not modify this general feature of interband optical transitions significantly since its magnitude is negligible compared to the momentum difference between the extrema of the bands involved in the transition.

Another issue worth remarking on is the fact that we employ a semiclassical treatment in which the electrons are treated quantum mechanically and the light is treated using classical electromagnetism. This approach is completely justified in the usual optical experiments not involving electromagnetic cavities. If the quantum rings or dots were enclosed in a cavity it would be essential to adopt a fully quantum electrodynamical (QED) approach to treat the interplay between electrons and cavity photons. QED studies of twisted light have already been reported in the literature [12, 15, 16].

4. Conclusions

We have theoretically investigated the effect that an inhomogeneous light beam, such as plane waves ($l = 0$) with a finite waist or twisted light beams ($l \neq 0$), has on semiconductor-based nanostructures when the symmetry axes of the beam and the nanoparticle do not coincide.

The problem was analytically solved for the general case of twisted light with $l = 0, 1, 2, \ldots$ by writing the off-centered beam as a superposition of twisted light beams centered at the position of the nanoparticle. This decomposition allowed us to study the possible electronic transitions in terms of the problem of a centered beam illuminating a nanoparticle, which we have already investigated in previous works.

We showed that different transitions between states in the nanoparticle are switched on and off as a function of the distance from the beam to the nanoparticle axes. In addition, the strength of the transition is determined by the relative size of the nanostructure to the beam waist. Our results also predict that a plane-wave beam with a finite waist will induce both vertical and non-vertical transitions, depending on the displacement of the axes.

Our study indicates that, under several experimental conditions, care must be taken when interpreting the results of light–nanoparticle interaction. For example, if an ensemble of nanostructures is illuminated in normal incidence using a beam whose waist is roughly of the same size or smaller than the ensemble, each nanoparticle will respond in a different way, irrespective of the beam being a plane wave or twisted light. In a situation where the experiment is carried out on a single nanostructure, we saw that the precise positioning of the TL axis is crucial to know what transitions are being excited.

The issue of experimental detection of the transfer of orbital angular momentum from TL beams to semiconductors and semiconductor nanostructures is a challenging one. Detection of the effects mentioned in this paper could possibly be achieved through measurement of the ultrafast magnetic fields generated when OAM is transferred [9] to a given quantum dot, but the effect is small and would require a very sensitive and highly positionable magnetic-field probe. The ultrafast aspect of the phenomenon (momentum relaxation washes out the electronic currents generated by the photo-excitation) makes time-resolved approaches more desirable, and time-resolved Faraday rotation is a possible candidate in this sense. Stimulated emission of the off-centered quantum dots and rings [17] could also be used as a probe of the transferred angular momentum, which would be different from that of the photons of the beam.

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