Astrophys Space Sci
DOI 10.1007/s10509-010-0592-7

ORIGINAL ARTICLE

Crossing the phantom divide with a classical Dirac field

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Received: 15 December 2010 / Accepted: 31 December 2010

Abstract In this paper we consider a spatially flat Friedmann-Robertson-Walker (FRW) cosmological model with cosmological constant, containing a stiff fluid and a classical Dirac field. The proposed cosmological scenario describes the evolution of effective dark matter and dark energy components reproducing, with the help of that effective multifluid configuration, the quintessential behavior. We find the value of the scale factor where the effective dark energy component crosses the phantom divide. The model we introduce, which can be considered as a modified ΛCDM one, is characterized by a set of parameters which may be constrained by the astrophysical observations available up to date.

Keywords Classical Dirac fields · Phantom divide

1 Introduction

According to the standard cosmology the total energy density of the Universe is dominated today by both dark matter and dark energy densities. The dark matter, which includes all components with nearly vanishing pressure, has an attractive gravitational effect like usual pressureless matter and neither absorbs nor emits radiation. The dark energy component in general is considered as a kind of vacuum energy with negative pressure and is homogeneously distributed and, unlike dark matter, is not concentrated in the galactic halos nor in the clusters of galaxies. The observational data provide compelling evidence for the existence of dark energy which dominates the present-day Universe and accelerates its expansion.

In principle, any matter content which violates the strong energy condition and possesses a positive energy density and negative pressure, may cause the dark energy effect of repulsive gravitation. So the main problem of modern cosmology is to identify the form of dark energy that dominates the Universe today.

In the literature the most popular candidates are cosmological constant Λ, quintessence and phantom matter. Their equation of state is given by $w = p/\rho$, where $w = -1$, $w > -1$ and $w < -1$ respectively. Dark energy composed of just a cosmological term Λ is fully consistent with existing observational data. However, these data do not exclude the possibility of explaining the observed acceleration with the help of phantom matter. The cosmological constant can be associated with a time independent dark energy density; the energy density of quintessence scales down with the cosmic expansion, and the energy density of phantom matter increases with the expansion of the Universe.

Mostly, the attention has been paid to dark energy as high energy scalar fields, characterized by a time varying equation of state, for which the potential of the scalar field plays an important role. Among scalar field models we can enumerate quintessence models (Ratra and Peebles 1988; Chiba et al. 1997; Carroll 1998), Chameleon fields (Khoury and Weltman 2004; Brax et al. 2004), K-essence (Myrzakulov 2011; Chiba et al. 2000; Armendáriz-Picón et al. 2000, 2001; Aguirregabiria et al. 2004, 2005), G-essence (Yerzhanov et al. 2010; Kulnazarov et al. 2010), Chaply-

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Published online: 13 January 2011

gin gases (Kamenshchik et al. 2001; Bento et al. 2002; Chimento 2006), tachyons (Padmanabhan 2002; Padmanabhan and Choudhury 2002; Chimento 2004), phantom dark energy (Caldwell 2002), cosmon models (Bauer et al. 2010; Grande et al. 2006, 2007), etc.

In general the crossing of the phantom divide cannot be achieved with a unique scalar field (Melchiorri et al. 2003; Vikman 2005; Sen 2005). This fact has motivated a lot of activity oriented towards different ways to realize such a crossing (Caldwell and Doran 2006; Cai et al. 2005, 2006, 2010; Sahni 2005; Sahni and Shtanov 2003; Aref'eva et al. 2005; McInnes 2005; Hu 2005; Wei and Zhang 2003; Wei and Tian 2004; Wei 2005; Wei et al. 2005, 2006; Wei and Cai 2005; Feng et al. 2005, 2006; Xia et al. 2005; Wu and Yu 2005; Zhang 2005; Elizalde et al. 2004; Perivolaropoulos 2005; Zhao et al. 2005; Li et al. 2005; Huang and Guo 2005; Zhang et al. 2006a, 2006b; Guo et al. 2005; Anisimov and Babichev 2005; Lazkoz and Leon 2006; Deffayet et al. 2010; Lim et al. 2010). For instance, in Ref. (Chimento and Lazkoz 2006) was explored the so called kinetic k-essence models (Chimento and Feinstein 2004; Chimento 2004; Scherrer 2004; Chimento et al. 2005; Chimento and Forte 2006; Aguirregabiria et al. 2004), i.e. cosmological models with several k-fields in which the Lagrangian does not depend on the fields themselves but only on their derivatives. It was shown that the dark energy equation of state transits from a conventional to a phantom type matter. Note that formally, one can get the phantom matter with the help of a scalar field by switching the sign of kinetic energy of the standard scalar field Lagrangian (Caldwell 2002). So that the energy density $\rho_{ph} = -(1/2)\Phi^2 + V(\Phi)$ and the pressure $p_{ph} = -(1/2)\Phi^2 - V(\Phi)$ of the phantom field leads to $\rho_{ph} + p_{ph} = -\Phi^2 < 0$, violating the weak energy condition.

In the Universe nearly 70% of the energy is in the form of dark energy. Baryonic matter amounts to only 3–4%, while the rest of the matter (27%) is believed to be in the form of a non-luminous component of non-baryonic nature with a dust-like equation of state ($w = 0$) known as cold dark matter (CDM). In this case, if the dark energy is composed just by a cosmological constant, then this scenario is called ΛCDM model.

Below, we analyze a FRW universe having cosmological constant and filled with a stiff fluid and a classical Dirac field (CDF). With this matter configuration, we will see that the FRW universe evolves from a non-accelerated stage at early times to an accelerated scenario at late times recovering the standard ΛCDM cosmology. The CDF may be justified by an important property: in a spatially flat homogeneous and isotropic FRW spacetime it behaves as a "perfect fluid" with an energy density, not necessarily positive definite. This pressureless "perfect fluid" can be seen as a kind of "dust". In particular, motivated by the fact that the dark matter is generally modeled as a system of collisionless particles (Wang et al. 2005; Müller 2005; Arbey 2005; Copeland et al. 2006), we have the possibility of giving to cold dark matter content an origin based on the nature of the CDF. On the other hand, the stiff fluid is an important component because, at early times, it could describe the shear dominated phase of a possible initial anisotropic scenario, dominating the remaining components of the model.

The organization of the paper is as follows: In Sect. 2 we present the dynamical field equations for a FRW cosmological model with a matter source composed of a stiff fluid and a CDF. In Sect. 3 the behavior of the dark energy component is studied. In Sect. 4 we conclude with some remarks.

2 Dynamical field equations

We shall adopt a spatially flat, homogeneous and isotropic spacetime described by the FRW metric

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \tag{1}$$

where $a(t)$ is the scale factor. The spacetime contains a cosmic fluid composed by (i) a stiff fluid $\rho_s = \rho_{s0}/a^6$ and (ii) a homogeneous classical Dirac field ψ. The Einstein-Dirac equations are

$$3H^2 - \Lambda = \frac{\rho_{s0}}{a^6} + \rho_D, \tag{2}$$

$$(\Gamma^i \nabla_i - \alpha)\psi = 0, \tag{3}$$

where $H = \dot{a}/a$ is the Hubble expansion rate, α is a constant and the dot denote differentiation with respect to the cosmological time. Here ρ_D represents the energy density of the CDF.

The dynamical equation for the CDF in curved spacetime can be obtained using the vierbein formalism. So, Γ^i are the generalized Dirac matrices, which satisfy the anticommutation relations

$$\{\Gamma^i, \Gamma^k\} = -2g^{ik} I, \tag{4}$$

with the metric tensor g^{ik} and I the identity 4×4 matrix. They can be defined in terms of the usual representation of the flat space-time constant Dirac matrices γ^i as

$$\Gamma^0 = \gamma^0, \qquad \Gamma^\beta = \frac{\gamma^\beta}{a}, \tag{5}$$

where the Dirac matrices γ^i can be written with the Pauli matrices σ^β as

$$\gamma^0 = i\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \gamma^\beta = i\begin{pmatrix} 0 & -\sigma^\beta \\ \sigma^\beta & 0 \end{pmatrix} \tag{6}$$

and

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{7}$$

with I the identity 2×2 matrix. The symbol $\nabla_i = \partial_i + \Sigma_i$ denotes the spinorial covariant derivatives, being the spinorial connection Σ_i defined by $\nabla_i \Gamma_k = 0$. Then it leads to

$$\Sigma_0 = 0, \qquad \Sigma_\beta = \frac{1}{2} H \Gamma^0 \Gamma_\beta. \tag{8}$$

Coming back to the Dirac equation (3), it takes the form

$$\left[\Gamma^0 \left(\partial_t + \frac{3}{2} H\right) - \alpha\right] \psi(t) = 0. \tag{9}$$

Restricting ourselves to the metric (1), the general solution of the latter equation consistent with (2) is given by

$$\psi(t) = \frac{1}{a^{3/2}} \begin{pmatrix} b_1 e^{-i\alpha t} \\ b_2 e^{-i\alpha t} \\ d_1^* e^{i\alpha t} \\ d_2^* e^{i\alpha t} \end{pmatrix} \tag{10}$$

with arbitrary complex coefficients b_1, b_2, d_1 and d_2. The only nonvanishing component of the energy-momentum tensor for the CDF is

$$T^{\mathrm{D}}_{00} = \frac{\alpha}{a^3} (|b_1|^2 + |b_2|^2 - |d_1|^2 - |d_2|^2) \equiv \frac{\rho_{D0}}{a^3}, \tag{11}$$

where $\rho_{D0} = \alpha(b^2 - d^2)$, $b^2 = |b_1|^2 + |b_2|^2$ and $d^2 = |d_1|^2 + |d_2|^2$. For positive values of ρ_{D0} this source formally behaves as a perfect fluid representing a classical dust. However, the CDF will allow us to extend the analysis for negative values of the energy density. Here, we restrict ourselves to the physical sector $d^2 < b^2$, this means that $\rho_D \geq 0$.

From (11) we see that the energy density of the CDF is given by $\rho_D = \rho_{D0}/a^3$. Thus we shall rewrite the Friedmann equation (2) in the following form:

$$3H^2 = \rho_m + \rho_x, \tag{12}$$

where

$$\rho_m = \frac{\alpha b^2}{a^3}, \tag{13}$$

$$\rho_x = \frac{\rho_{s0}}{a^6} - \frac{\alpha d^2}{a^3} + \Lambda. \tag{14}$$

Here α, b^2 and d^2 have dimensions of time^{-1}.

Notice that there is some arbitrariness in the distribution of the four components of (11) between dark matter and dark energy, since it is not clear that we can represent each dark component with a mix of positive and negative energy components. To avoid this arbitrariness we have included both positive energy components of CDF into dark matter and both negative energy components of CDF into dark energy. This assignment has an invariant meaning because it is preserved under linear transformations of CDF.

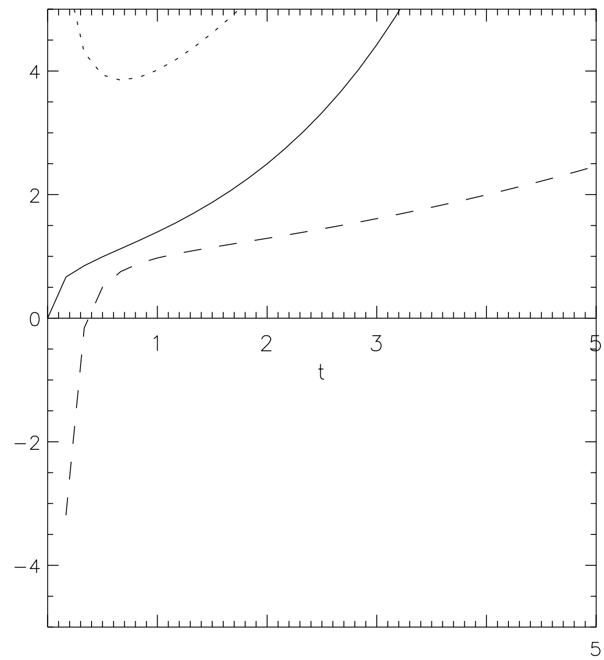

Fig. 1 We show the behavior of the scale factor (*solid line*) and its derivatives $\dot{a}$ (*dotted line*) and $\ddot{a}$ (*dashed line*). From this it is clear that this cosmological scenario exhibits an accelerated expansion since there is a stage where $\ddot{a} > 0$

This distribution allows us to interpret the positive part of the Dirac energy–momentum tensor as a pressureless matter (giving rise to the total ("true") observable matter ρ_m); while its negative part, together with the stiff fluid and the cosmological constant, we interpret as the effective dark energy component ρ_x. It must be noted that this is equivalent to assume that we have a dust component $\rho_M = \rho_{M0} a^{-3}$ and a CDF $\rho_D = \rho_{D0} a^{-3}$ associated with a negative energy density (i.e. with $d^2 > b^2$) in the dark energy sector.

The general solution of the Einstein equation (12) with sources (13) and (14) takes the form

$$a^3(t) = \frac{\alpha(b^2 - d^2)}{2\Lambda} \left[-1 + \cosh \sqrt{3\Lambda} t\right] + \sqrt{\frac{\rho_{s0}}{\Lambda}} \sinh \sqrt{3\Lambda} t, \tag{15}$$

where we have set the initial singularity at $t = 0$. In Fig. 1 is shown the behavior of the scale factor and its derivatives.

3 Dark energy evolution

It is well known that the present observational data do not exclude the existence of the phantom dark energy; thus it is interesting to consider the possibility of crossing the phantom divide. We shall see that the Dirac field component can

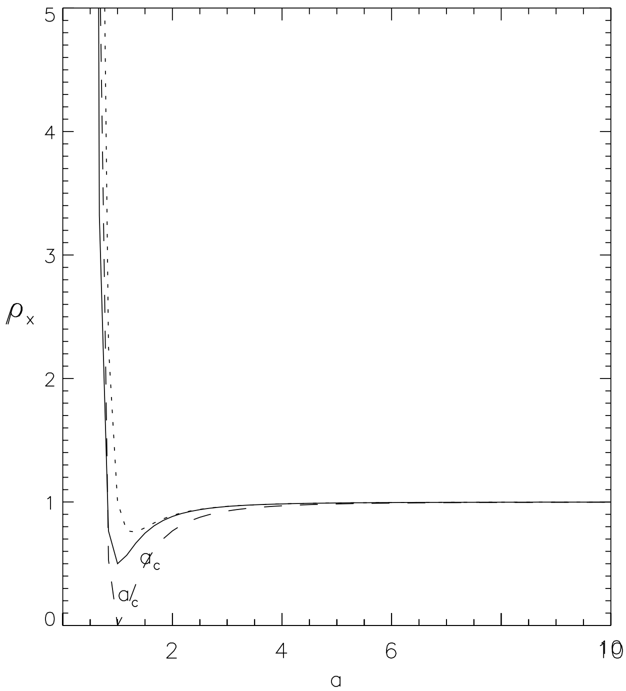

Fig. 2 We show the behavior of ρ_x as a function of time. The *dashed line* represents the limit case $a_c = a_m$, i.e. $\alpha^2 d^4 = 4\Lambda\rho_{s0}$, while *solid* and *dotted lines* represent a typical case satisfying the condition $a_c > a_m$ since $\alpha^2 d^4 < 4\Lambda\rho_{s0}$

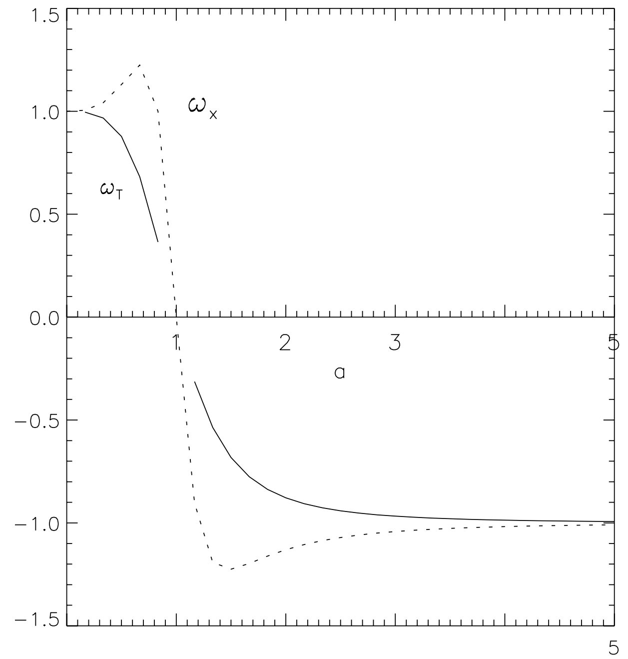

Fig. 3 We show the behavior of the dark energy state parameter w_x (*dashed line*) and the state parameter for the full source content w_T (*solid line*). It is clear that the dark energy violates the dominant energy condition while the full source content does not violate it. Note also that the dark energy component remains in the phantom region as it enters into it after crossing the phantom divide

produce a strong variation of the dark energy density ρ_x, allowing one to have such a crossing at early times.

In order to have a positive ρ_x, we choose the parameters of the models according to the following restriction:

$$\alpha^2 d^4 < 4\Lambda\rho_{s0}. \tag{16}$$

Since $a(t)$ is an increasing function of time, this effective dark energy component decreases until it reaches a minimum value $\rho_{xc} = \Lambda - \alpha^2 d^4/4\rho_{s0}$ at $a_c = (2\rho_{s0}/\alpha d^2)^{1/3}$ where the dark component crosses the phantom divide and begins to increase with time (see Fig. 2). Fundamentally, the effective dark energy component crosses the phantom divide due to the presence of the d-parameter. In fact, the cosmological evolution of dark energy depends on the negative term $-\alpha d^2/a^3$, see (14), because it produces a minimum at $\dot{\rho}_x(a_c) = 0$, showing the importance of considering the CDF as a source of the Einstein equation.

Assuming that total matter and dark energy are coupled only gravitationally, then they are conserved separately, so we have that

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{17}$$

$$\dot{\rho}_x + 3H(1 + w_x)\rho_x = 0, \tag{18}$$

where we have assumed the equation of state $p_x = w_x\rho_x$ for the dark energy component. Taking into account that the dark energy component has a variable state parameter $w_x = w_x(a)$, we define as $a_m = (\rho_{s0}/\Lambda)^{1/6}$ the value of the scale factor where this equation of state coincides with the matter one, that is $w_x = 0$. So that, the above restriction $\alpha^2 d^4 < 4\Lambda\rho_{s0}$ now becomes $a_m < a_c$. In terms of the above parameters, a_c and a_m, the dark energy state parameter w_x can be written as

$$w_x = \frac{1 - (a/a_m)^6}{1 - 2(a/a_c)^3 + (a/a_m)^6}. \tag{19}$$

On the other hand we can consider the state equation for the full source content. For this 2-component system we define the total pressure $p_T = w_T\rho_T$, where $p_T = p_x$ and the total energy density $\rho_T = \rho_m + \rho_x$. This implies that

$$w_T = \frac{w_x}{1 + r}, \tag{20}$$

where $r = \rho_m/\rho_x$. In Fig. 3 we compare the behaviors of the state parameters of the dark energy and the full source content.

Let us consider in more detail the ratio of energy densities of matter and dark energy. The defined above ratio takes the form

$$r = \frac{\alpha b^2 a^3}{\Lambda a^6 - \alpha d^2 a^3 + \rho_{s0}}. \tag{21}$$

It is easy to show that this ratio has a maximum at $a = a_c = (\rho_{s0}/\Lambda)^{1/6}$ (at this point the $d^2r/da^2 < 0$). This maximum value is given by

$$r_{max} = \frac{\alpha b^2\sqrt{\rho_{s0}/\Lambda}}{2\rho_{s0} - \alpha d^2\sqrt{\rho_{s0}/\Lambda}}. \tag{22}$$

It is clear that for cosmological scenarios where $r_{max} < 1$ the dark energy component always dominates over the dark matter during all cosmological evolution. Thus, in order to have stages where dark matter dominates over dark energy, we have to require that $r_{max} > 1$. So in this case we would have two values for the scale factor where the energy density of dark matter equals the energy density of the dark energy:

$$a_\pm = \left[\frac{\alpha(b^2+d^2) \pm \sqrt{\alpha^2(b^2+d^2)^2 - 4\Lambda\rho_{s0}}}{2\Lambda}\right]^{1/3}. \tag{23}$$

Thus, for cosmological scenarios where $r_{max} > 1$, at the beginning the dark energy dominates over the dark matter until the scale factor reaches the value $a = a_-$ where the dark matter energy density equals the energy density of dark energy and then it begins to dominate. This stage of domination of the dark matter is prolonged until the moment when the scale factor reaches the value $a = a_+$ and the dark energy again starts to dominate over the dark matter (see Fig. 4).

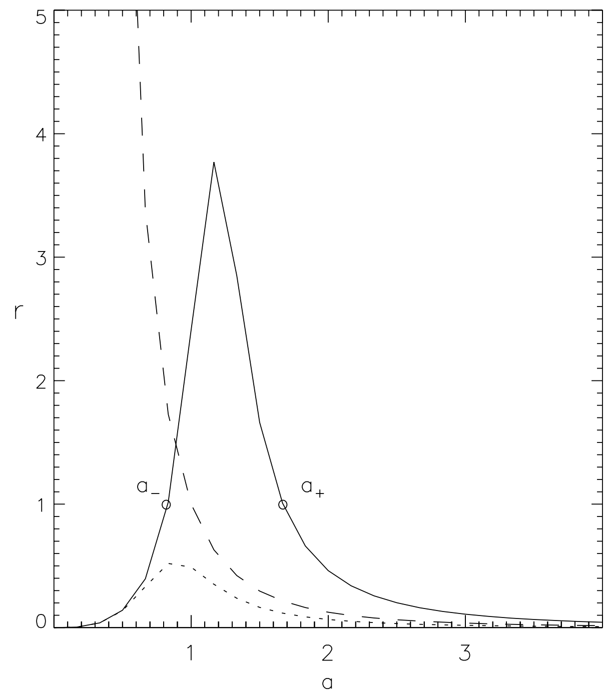

Fig. 4 We show the behavior of the ratio $r = \rho_m/\rho_x$ as a function of the scale factor a. The *dashed line* represents the ΛCDM cosmology, where $r \propto 1/a^3$. The *solid line* represents the case where $r > 1$ and so at the beginning dark energy dominates, then at the range $a \in (a_-, a_+)$ the dark matter component dominates over dark energy and, for $a > a_+$, the dark energy component dominates again. The *dotted line* represents a scenario where the dark energy always dominates over the dark matter, since $r_{max} < 1$

3.1 Constraints on cosmological parameters

In order to confront our model and the cosmological observations we shall use the constraints on cosmological parameters obtained from the analysis of observational data assuming the Λ-CDM model. This choice is justified by the fact that our model differs from Λ-CDM only at early times. It must be noted that in general such a procedure of using constraints derived from a fit of one specific model can give at best rough estimations for the parameters of a different cosmological model.

The proposed scenario is characterized by four parameters which may be constrained by the astrophysical observations available up to date. Since we have considered flat FRW cosmological scenarios, the dimensionless density parameters are constrained today by

$$\Omega_{m,0} + \Omega_{x,0} = 1. \tag{24}$$

From (12)–(14) we have that

$$3H^2 = \frac{\alpha b^2}{a^3} + \frac{\rho_{s0}}{a^6} - \frac{\alpha d^2}{a^3} + \Lambda. \tag{25}$$

Evaluating it today (where we set $a = 1$) we have that

$$\rho_{crit} = 3H_0^2 = \alpha b^2 + \rho_{s0} - \alpha d^2 + \Lambda, \tag{26}$$

so the two dimensionless density parameters are given by

$$\Omega_{m,0} = \frac{\rho_m(a=1)}{\rho_{crit}} = \frac{\alpha b^2}{\alpha b^2 + \rho_{s0} - \alpha d^2 + \Lambda}, \tag{27}$$

$$\Omega_{x,0} = \frac{\rho_x(a=1)}{\rho_{crit}} = \frac{\rho_{s0} - \alpha d^2 + \Lambda}{\alpha b^2 + \rho_{s0} - \alpha d^2 + \Lambda}. \tag{28}$$

Now it may be shown that, in general, for a flat FRW cosmology the deceleration parameter q is given by

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2} + \frac{3}{2}\frac{p_T}{\rho_T}. \tag{29}$$

Taking into account that we have $p_T = p_x = \omega_x\rho_x$ and $\rho_T = \rho_m + \rho_x$ the deceleration parameter is given by

$$q = \frac{1}{2} + \frac{3}{2}\omega_x(1 - \Omega_m), \tag{30}$$

and evaluating it today (i.e. $a = 1$) and using (19) we obtain

$$\alpha(b^2 - d^2)(1 - 2q_0) + 2\rho_{s0}(2 - q_0) = 2\Lambda(1 + q_0). \tag{31}$$

Thus, for accelerated scenarios, $q < 0$, we require a positive cosmological constant. Another constraint may be introduced by taking into account the moment when the Universe has started to accelerate again. In other words, this is related to the moment when the Universe starts violating the strong energy condition ($\rho + p \geq 0$ and $\rho + 3p \geq 0$). So we must require the inequality $\rho + 3p < 0$. Now from the condition $\ddot{a} = 0$ and the equivalent Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p), \tag{32}$$

we conclude that $\rho + 3p = 0$, which implies that $\rho_m + \rho_x + 3\omega_x \rho_x = 0$, obtaining the condition

$$\alpha(b^2 - d^2)(1 + z_{acc})^3 + 4\rho_{s0}(1 + z_{acc})^6 = 2\Lambda, \tag{33}$$

where the equation $1/a = (1 + z)$ was used. Here z_{acc} is the value of the redshift when the Universe starts to accelerate again.

In conclusion we have the four conditions (27), (28), (31) and (33) for the four parameters αb^2, ρ_{s0}, αd^2 and Λ of our model.

Note that from (31) and (33) we have that

$$\rho_{D0} = \alpha(b^2 - d^2) = K\rho_{s0}, \tag{34}$$

where

$$K = \frac{4(1 + q_0)\bar{z}^2 - 2(2 - q_0)}{(1 - 2q_0) - (1 + q_0)\bar{z}}, \quad \bar{z} = (1 + z_{acc})^3. \tag{35}$$

Since ρ_{D0} is related to the energy density of the CDF, we must require that $K > 0$. So from (27), (28), (31) and (33) we have that

$$\alpha b^2 = 3H_0^2 \Omega_{m,0}, \tag{36}$$

$$\rho_{s0} = \frac{6H_0^2}{K(2 + \bar{z}) + 2(1 + 2\bar{z}^2)}, \tag{37}$$

$$d^2 = b^2 - \frac{6KH_0^2}{\alpha[K(2 + \bar{z}) + 2(1 + 2\bar{z}^2)]}, \tag{38}$$

$$\Lambda = \frac{3H_0^2 \bar{z}(K + 4\bar{z})}{K(2 + \bar{z}) + 2(1 + 2\bar{z}^2)}, \tag{39}$$

where we have used (24) and (34).

Now the four model parameters need to be constrained. We do this by using (36)–(39) and by considering the increasing bulk of observational data that have been accumulated during the past decade. The present expansion rate of the Universe is measured by the Hubble constant. From the final results of the Hubble Space Telescope Key Project (Freedman et al. 2001) to measure the Hubble constant we know that its present value is constrained to be $H_0 = 68 \pm 7$ km s^{-1} Mpc^{-1} (Chen et al. 2003). Equivalently we can write $H_0^{-1} = 9.776h^{-1}$ Gyr, where h is a dimensionless quantity and varies in the range $0.61 < h < 0.75$.

Now assuming a flat Universe, i.e. (24) is valid, Perlmutter et al. (1999) found that the dimensionless density parameter $\Omega_{m,0}$ may be constrained to be ~ 0.3, implying from (24) that $\Omega_{x,0} \sim 0.7$ (Copeland et al. 2006; Capozziello 2007; Cardone et al. 2004), and the present day deceleration parameter q_0 may be constrained to be $-1 < q_0 < -0.64$ (Copeland et al. 2006; Capozziello 2007; Cardone et al. 2004; Jackson 2004).

For consistency we also need to compare the age of the Universe determined from our model with the age of the oldest stellar populations, requiring that the Universe be older than these stellar populations. Specifically, the age of the Universe t_0 is constrained to be $t_0 > 11$–14 Gyr (Copeland et al. 2006; Krauss and Chaboyer 2003; Grattonet et al. 2003; Imbriani et al. 2004; Hansen et al. 2002, 2004; Jimenez et al. 1996; Richer et al. 2002).

So let us calculate the age of the Universe from the Friedmann equation (25). This equation may be written as

$$H^2 = \frac{H_0^2}{a^6}\left(\frac{\rho_{s0} + \alpha a^3(b^2 - d^2) + a^6\Lambda}{\rho_{s0} + \alpha(b^2 - d^2) + \Lambda}\right), \tag{40}$$

obtaining (24) when the expression (40) is evaluated today. Then the age of the Universe may be written as

$$\begin{aligned} t_0 &= \int_0^\infty \frac{dz}{H(1+z)} \\ &= \frac{1}{H_0}\int_0^\infty \sqrt{\frac{\alpha(b^2 - d^2) + \rho_{s0} + \Lambda}{\alpha(b^2 - d^2)(1+z)^3 + \rho_{s0}(1+z)^6 + \Lambda}} \\ &\quad \times \frac{dz}{(1+z)}. \end{aligned} \tag{41}$$

In Table 1 we include some values obtained from (36)–(39) for the parameters αb^2, αd^2, ρ_{s0}, Λ and K corresponding to some given values of the parameters H_0, q_0 and z_{acc} (with $c = 1$ and $G = 1$). For the Hubble parameter H_0 is considered both possible values H_{0-} for $h = 0.61$ and H_{0+} for $h = 0.75$, and for the matter dimensionless density parameter we have taken $\Omega_{m,0} = 0.3$. The last two columns represent the age of the Universe determined from the model parameters for $h = 0.75$ and $h = 0.61$ respectively. Clearly in the proposed model there are configurations which are allowed, being their ages larger than the oldest known stellar ages, since there exist combinations of the parameters αb^2, αd^2, ρ_{s0}, Λ which give $t_0 > 11$–14 Gyr satisfying the stellar population constraints.

3.2 The effect of the d-parameter

Now it is interesting to get some insights concerning the nature of the d-parameter for studying the effect on the cosmological evolution of the negative term $-\alpha d^2/a^3$ in (14) for the dark energy component ρ_x. It can be shown that in the

Table 1 In this table we show some values of the model parameters obtained for given H_0, q_0 and z_{acc} ($\Omega_{m,0} = 0.3$, $c = 1$, $G = 1$)

H_0 (s^{-1})	q_0	Z_{acc}	αb^2 (s^{-2})	αd^2 (s^{-2})	ρ_{s0} (s^{-2})	Λ (s^{-2})	K	H_{0+}, t_0	H_{0-}, t_0
H_{0+}	−0.68	0.587	5.315×10^{-36}	2.009×10^{-36}	2.368×10^{-37}	2.655×10^{-35}	14	12 (Gyr)	15 (Gyr)
H_{0-}	−0.68	0.587	3.516×10^{-36}	1.329×10^{-36}	1.566×10^{-37}	1.756×10^{-35}	14	12 (Gyr)	15 (Gyr)
H_{0+}	−0.68	0.94	5.315×10^{-36}	1.538×10^{-36}	1.415×10^{-39}	1.210×10^{-34}	2619	11 (Gyr)	13 (Gyr)
H_{0-}	−0.68	0.94	3.516×10^{-36}	1.017×10^{-36}	9.361×10^{-40}	8.006×10^{-35}	2619	21 (Gyr)	25 (Gyr)
H_{0+}	−0.9	0.6	5.315×10^{-36}	5.126×10^{-36}	4.961×10^{-37}	1.751×10^{-35}	0.38	11 (Gyr)	13 (Gyr)
H_{0-}	−0.9	0.6	3.516×10^{-36}	3.391×10^{-36}	3.281×10^{-37}	1.158×10^{-35}	0.38	11 (Gyr)	13 (Gyr)
H_{0+}	−0.9	1	5.315×10^{-36}	4.333×10^{-36}	9.926×10^{-38}	2.217×10^{-35}	9.9	14 (Gyr)	17 (Gyr)
H_{0-}	−0.9	1	3.516×10^{-36}	2.866×10^{-36}	6.566×10^{-38}	1.466×10^{-35}	9.9	14 (Gyr)	17 (Gyr)

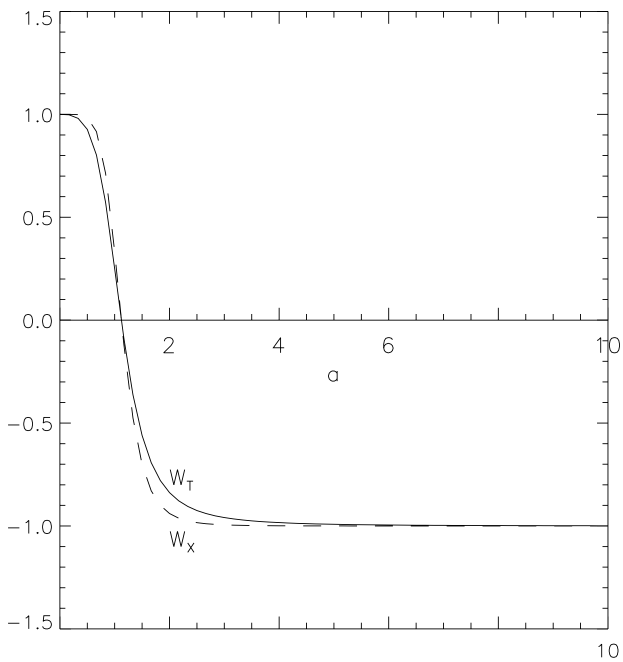

Fig. 5 We show the behavior of w_x and of w_T as a functions of the scale factor. The *dashed line* represents the behavior of the dark energy state parameter, while the *solid line* represents the behavior of the state parameter of the full source content. Both satisfy the DEC since in this case $d = 0$

proposed cosmological scenario the dominant energy condition (DEC) is violated thanks to the presence of this parameter. Effectively, if $d = 0$ the dark energy state parameter w_x and the state parameter of the full source content w_T are given by

$$w_x = \frac{\rho_{s0} - \Lambda a^6}{\rho_{s0} + \Lambda a^6}, \qquad w_T = \frac{\rho_{s0} - \Lambda a^6}{\rho_{s0} + \Lambda a^6 + \alpha b^2 a^3}, \tag{42}$$

respectively. From these expressions we see that always $-1 < w_x < 1$ which implies that now the dark energy component satisfies DEC, as well as w_T. Their general behavior is shown in Fig. 5 (compare with Fig. 3).

However, the fulfillment of the DEC does not imply that the Universe has a decelerated expansion. From (32) we may write that

$$\frac{6\ddot{a}}{a} = -(\rho_T + 3p_T) = -\left(\frac{4\rho_{s0}}{a^6} + \frac{\alpha(b^2 - d^2)}{a^3} - 2\Lambda\right), \tag{43}$$

and putting $d = 0$ we see that the accelerated expansion is realized if $a > a_{acc} = ((\alpha b^2 + \frac{\sqrt{\alpha^2 b^4 + 32\Lambda\rho_{s0})}}{4\Lambda})^{1/3}$. Another property of the d-parameter to be considered is its effect on the deceleration parameter q_0. From (38), which is independent of the Hubble parameter H_0 and, using (35) we can express the deceleration parameter q_0 as a function of the z_{acc} obtaining

$$q_0(z_{acc}) = \frac{(\bar{z} - 1)((2\bar{z} + 1)(b^2 - d^2)\alpha - 4H_0^2(\bar{z} + 1))}{2H_0^2(2\bar{z}^2 + 1)}, \tag{44}$$

where, as before, $\bar{z} = (1 + z_{acc})^3$. We can see that in this case $q_0(z_{acc})$ rapidly tends to the value

$$q_{0,\infty} = -1 + \frac{\alpha}{2H_0^2}(b^2 - d^2) = -1 + \frac{3\Omega_{m,0}}{2}\left(1 - \frac{d^2}{b^2}\right), \tag{45}$$

obtaining, for $d = 0$ and $\Omega_{m,0} = 0.3$, the value $q_{0,\infty} = -0.55$ and, from this value the deceleration parameter reaches the value $q_{0,\infty} = -1$ for $d \approx b$ (see Fig. 6). So the parameter d directly affects the range of validity of the deceleration parameter which is constrained to be in the range $-1 < q_0 < 0$. Note that the value $q_{0,\infty} = -1$ for $d = b$ is independent of the value of the dimensionless density parameter $\Omega_{m,0}$.

4 Concluding remarks

Since today the observations constrain the value of ω to be close to $\omega = -1$, we have considered broader cosmological scenarios in which the equation of state of dark energy changes with time. The two principal ingredients of the

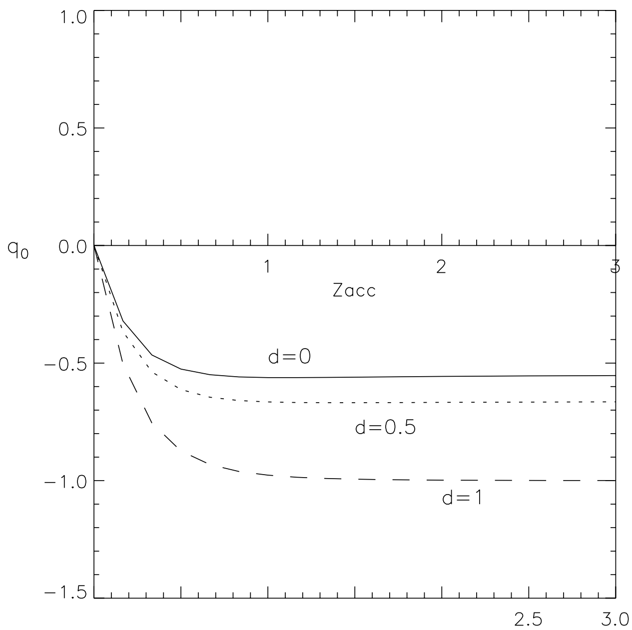

Fig. 6 We show the behavior of the deceleration parameter q_0 as a function of z_{acc} for three values of the parameter d $(0, 1/2, 1)$ with $\Omega_{m,0} = 0.3$ and $b = 1$. We can see that the deceleration parameter rapidly tends to the values $(-0.55, -0.6625, -1)$ respectively

model are a stiff fluid which dominates at early time (Dutta and Scherrer 2010) and a CDF. The positive part of the latter was associated with a dark matter component while its negative part was considered as a part of the dark energy component and was responsible for the effective dark energy density crossing the phantom divide (see and compare Figs. 3 and 5). At the end, this cosmological model becomes accelerated recovering the standard ΛCDM cosmology.

In general, the model may be seen as a continuation of the inflation era. In the inflationary paradigm the scalar field Φ, driven by the potential $V(\Phi)$, generates the inflationary stage. In the slow roll limit $\dot{\Phi}^2 \ll V(\Phi)$, with $\omega_\Phi \approx -1$, we have a superluminal expansion while in the kinetic-energy dominated limit $\dot{\Phi}^2 \gg V(\Phi)$, with $\omega_\Phi \approx 1$, we have a stiff matter scenario characterized by a subluminal expansion. Taking into account that our model has a variable equation of state, we can think of it as a transient model which interpolates smoothly between different barotropic eras as, for instance, radiation dominated era, matter dominated era an so on. In other words, from (19)–(21) we see that $w_x \to 1$ (and $w_T \to 1$) for $a \to 0$ implying that the energy density ρ_x behaves like $1/a^6$ and matching, after inflation, with the kinetic-energy mode of the scalar field $\rho_\Phi \propto 1/a^6$. Now from (19) we see that ρ_x passes through a radiation dominated stage (i.e. $\omega_x = 1/3$) for

$$a_{rad} = \frac{a_m}{2^{2/3}} \left[\left(\frac{a_m}{a_c} \right)^3 + \sqrt{8 + \left(\frac{a_m}{a_c} \right)^6} \right], \quad (46)$$

behaving like $\rho_x \approx 1/a_{rad}^4$ and dominating over ρ_m. After that, at $a = a_m$, we have $w_x = 0$ and the effective dark component behaves as a pressureless source thus obtaining a matter-dominated stage. Finally the model evolves from this state to a vacuum-energy dominated scenario. It is interesting to note that in the a matter-dominated stage, if the condition (16) is fulfilled, then the total energy density is given by $\rho_T = \alpha(b^2 - d^2)/a_m^3 + 2\Lambda$ and, if $a_m = a_c$ then $\rho_T = \alpha b^2/a_m^3$, being $\rho_x = 0$, implying that we have at this stage only the dark matter component.

The above results indicate that a cosmological scenario based on a CDF component and the effective multifluid configuration ρ_x can, in certain cases, reproduce the quintessential behavior (see Figs. 3 and 5). In fact, the state parameter of the total matter content $-1 < w_T < 1$ is constrained the same as is the state parameter of the scalar field in quintessence models. In this manner we avoid the use of scalar fields and particular classes of potentials for describing the dark energy component.

Finally, all the parameters of the model have been expressed in terms of the observable quantities which may be constrained by the astrophysical observational data. In effect, in Table 1 some values of the model parameters αb^2, αd^2, ρ_{s0} and Λ were included, which correspond to some given values of the parameters H_0, q_0, z_{acc} and $\Omega_{m,0}$ constrained by astrophysical observations.

Acknowledgements LPC acknowledges the hospitality of the Physics Department of Universidad del Bío-Bío where a part of this work was done. The authors thank Paul Minning for carefully reading this manuscript. This work was partially supported by CONICYT through grants FONDECYT N^0 1080530 (MC), MECESUP USA0108 (MC and LPC) and Project X224 by the University of Buenos Aires along with Project 5169 by the CONICET (LPC). It was also supported by the Dirección de Investigación de la Universidad del Bío–Bío (MC) and Consejo Nacional de Investigaciones Científicas y Técnicas (LPC).

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