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Spin–spin indirect interaction at low-energy excitation in zero-dimensional cavities

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Abstract

We solve the low-energy part of the spectrum of a model that describes a circularly polarized cavity mode strongly coupled to two exciton modes, each of which is coupled to a localized spin of arbitrary magnitude. In the regime in which the excitons and the cavity modes are strongly coupled, forming polaritons, the low-energy part of the spectrum can be described by an effective spin model, which contains a magnetic field, an axial anisotropy, and an Ising interaction between the localized spins. For detunings such that the low-energy states are dominated by nearly degenerate excitonic modes, the description of the low-energy states by a simple effective Hamiltonian ceases to be valid and the effective interaction tends to vanish. Finally, we discuss a possible application to two-qubit quantum computing operations in a system of transition-metal impurities embedded in quantum dots inside a micropillar.

1. Introduction

In recent years there has been great interest in the field of cavity quantum electrodynamics. In particular, systems with strong coupling between single quantum dots (QDs) and high quality microcavities have been studied for different reasons, including to gain insight into different quantum optics effects [1–9], such as quantum decoherence, entanglement and possible applications in quantum information processing [1–4, 8]. For example, some of these systems were proposed as a single-photon source [3, 7] for realization of all-optical quantum computing [8]. The strong coupling (SC) regime takes place when the coupling between a quantum emitter and the cavity mode is strong compared to their decay rates. In this case, the emitter and cavity coherently exchange energy back and forth, leading to Rabi oscillations. The SC between single (In, Ga)As QD and micropillar cavity modes [1] has become apparent in photoluminescence data which displayed anti-crossings between the QD exciton and cavity-mode dispersion relations [1, 2, 4]. The SC regime has also become apparent in resonant Raman scattering due to optical phonons in planar II–VI-type semiconductor microcavities [10, 11].

Essential to quantum computation is the capability to produce one- and two-qubit operations. In 1999, Imamoglu *et al* [8] showed the possibility of inducing an effective interaction between spins in a QD mediated by photons. More recently Quinteiro *et al* studied the optically induced coupling between 1/2 spins in a two-dimensional (2D) microcavity [12]. This work was later extended to zero-dimensional (0D) cavities [13], where it was shown that, due to selection rules, the light-mediated effective spin–spin interaction is of the Ising type part of the spin. It has been shown that the Ising part of the spin–spin interaction is sufficient to perform two-qubit operations [14].

The construction of effective Hamiltonians to describe the low-energy physics of a system is very common in condensed matter physics. One example is the canonical transformation that eliminates the electron–phonon interaction, giving rise to an attractive interaction between electrons, which in turn leads to superconductivity [15]. Another example is the canonical transformation that eliminates the hopping in the half-filled Hubbard model (or the ionic Hubbard model), giving rise to a Heisenberg interaction [16, 17]. In several transition-metal oxides, including the superconducting cuprates, a low-energy Hamiltonian has been obtained using perturbation

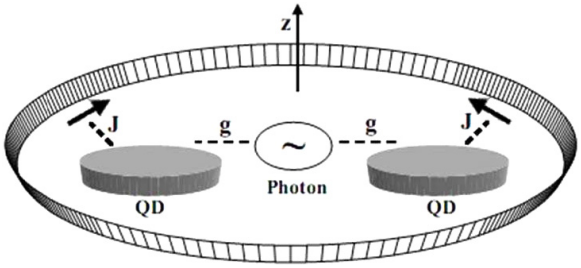

Figure 1. Scheme of the system: a 0D cavity encloses two QDs, each coupled to a single impurity (spin) of arbitrary magnitude.

methods [18–21, 23–25]. It has been found that more accurate effective parameters are obtained if the perturbative expressions are replaced by a fit of the low-energy levels in a simple system [21].

It is interesting to note that a spin-1/2 Heisenberg model with anisotropic exchange interactions has been derived as an effective model for atoms trapped in microcavities [22]. In this case, the spin degree of freedom represents two long-lived atomic levels instead of actual spin states.

In this work we extend the results derived previously by one of us for two spin 1/2 impurities [13] and study the effective Hamiltonian $H_{\rm eff}$ for the interaction of two arbitrary spins in a 0D cavity, each of which has an exchange interaction with electrons and holes of a QD (see figure 1). The generalization to arbitrary spin is important to apply the model to transition-metal impurities, as discussed in section 4.

In section 2 we present the model and derive the effective interaction. Section 3 discusses the results, together with the limitations and validity of the model. We give an account of the scope of the present research to real systems in section 4, where we address the technologically relevant case of Mn impurities embedded in CdTe/ZnTe QDs inside a micropillar. Section 5 contains a summary and a discussion.

2. Model

The core of the model contains the cavity-photon mode, and the excitonic degrees of freedom of both QDs represented by spin 1/2 and the coupling between them. This is a simple generalization of the Jaynes–Cummings model [26] to two excitonic degrees of freedom. In addition, the electron and hole of each exciton have a spin exchange interaction with a localized spin. A scheme of the system is shown in figure 1. We remind the reader that the symmetry of the cavity splits heavy holes (HHs) with angular momentum projection $j_z = \pm 3/2$ from the light holes (LHs) with $j_z = \pm 1/2$, which lie at higher energy. We assume that the light is circularly polarized with spin projection $j_z = 1$. Thus, there is only one possible low-lying bright exciton which can be excited by this light, and corresponds to $j_z = -1/2$ (3/2) for the electron (HH). Therefore, the polarization subscripts can be dropped. The dark excitons with total angular momentum projection $j_z = \pm 2$ do not mix with the light and can be disregarded at low enough temperature. Furthermore, they cannot lead to an effective interaction between localized spins and we neglect them. The spin–spin interaction between excitons and localized spins includes two types of exchange interactions, the anisotropic one between the heavy hole and the localized spin, and the isotropic (Heisenberg) one between the electron and the localized spin [27]. The spin-flip terms lead to a mixing of the bright and dark exciton states, or even with excitons containing LHs. The latter are irrelevant at low temperatures[3], while the former can be neglected under certain conditions explained in sections 3 and 4. Under these conditions, assumed in the rest of this work, only an Ising type interaction between the exciton and the localized spin needs to be considered.

From the above considerations, the Hamiltonian takes the form

$$H = E_{\rm c} a^\dagger a + \sum_{i=1}^{2} [E_{\rm x}\sigma_i^z + g(\sigma_i^- a^\dagger + {\rm Hc}) + J(\sigma_i^z + 1/2)S_i^z], \quad (1)$$

where $a^\dagger$ is the creation operator of the cavity mode, σ_i^z, σ_i^+, and σ_i^- are spin operators for the two level system of the QD i, with ground ($|i\downarrow\rangle$) and excited ($|i\uparrow\rangle$) states which represent zero and one exciton respectively, and S_i^z is the spin projection of the localized spin i. For simplicity we assume a symmetric device. The extension of the results when the excitonic energy $E_{\rm x}$ and the couplings g and J depend on i is rather simple (see section 3.5). A particular case is discussed in section 3.4. Without loss of generality we assume $g > 0$ (the phase of $a^\dagger$ can be changed).

We define the detuning as the difference between cavity and exciton energies $\delta = E_{\rm c} - E_{\rm x}$.

3. Results

The Hamiltonian equation (1) conserves the total number of light plus excitonic excitations

$$N_{\rm e} = \sum_{i=1} (\sigma_i^z + 1/2) + a^\dagger a. \quad (2)$$

Following [13], we assume that the intensity of radiation exciting the system from outside is such that we may restrict ourselves to the subspace of $N_{\rm e} = 1$. This subspace contains $3(2S+1)^2$ states (the factor three comes from either one of the two excitons or the photon, and $2S+1$ is the degeneracy of each spin). For small enough J, an effective Hamiltonian for the lowest $(2S+1)^2$ states can be derived from perturbation theory in J up to second order.

3.1. Perturbations in J

Here we assume small J and small temperature T. More specifically, the conditions of validity of perturbation theory are the following: (i) for negative detuning δ, and $-\delta > g$, one must have $J, T \ll -\delta$, (ii) for $|\delta| < g$ one must have $J, T \ll g$, (iii) for large positive δ, $J, T \ll 2g^2/\delta$, which is harder to satisfy. Studies of situations when this condition is not met are discussed in the next subsections. The perturbation theory

[3] The effect of these spin-flip terms can be taken into account in second-order perturbation theory in the exchange interaction. They renormalize the exciton energy, but do not lead to effective interactions between the localized spins.

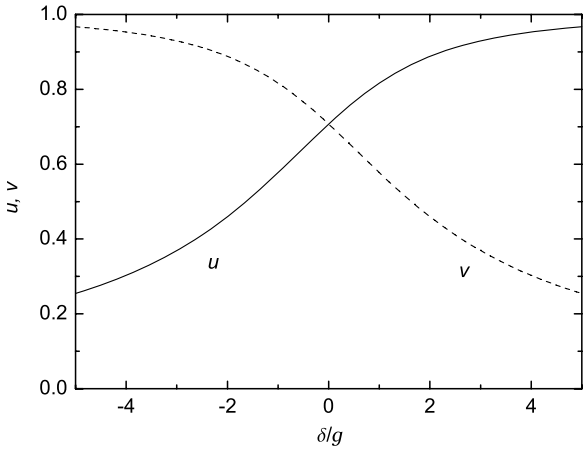

Figure 2. Coefficients of the eigenstates $|1\rangle$ and $|3\rangle$ given by equations (3).

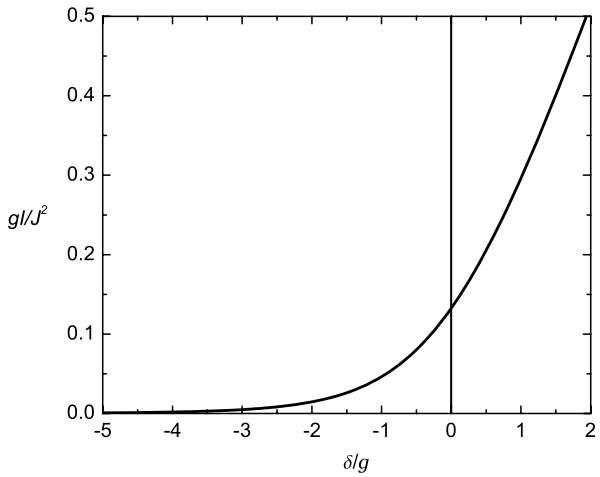

Figure 3. Effective interaction as a function of detuning for small J and T.

approach up to order J^2 allows us to ignore the spin-flip term in the Mn–electron interaction, since this process would convert a bright exciton into a dark exciton, which cannot lead to an effective spin–spin interaction. Only through a second electron spin-flip would the bright exciton be recovered. However, this process of order J^2, involves only one spin and does not affect the other. Processes involving dark excitons, which lead to effective spin–spin interactions, are of order higher than J^2 and we do not consider them in this section.

For $J = 0$, the Hamiltonian can be diagonalized and is resolved into three subspaces, each one $(2S+1)^2$ times degenerate, since the energy of the states are independent of the spin projections. In increasing order of energies, the light and excitonic part of the eigenstates and their energies are given by

$$\begin{aligned}
|1\rangle &= \frac{u}{\sqrt{2}}(|0\uparrow\downarrow\rangle + |0\downarrow\uparrow\rangle) - v|1\downarrow\downarrow\rangle,\\
|2\rangle &= \frac{1}{\sqrt{2}}(|0\uparrow\downarrow\rangle - |0\downarrow\uparrow\rangle),\\
|3\rangle &= \frac{v}{\sqrt{2}}(|0\uparrow\downarrow\rangle + |0\downarrow\uparrow\rangle) + u|1\downarrow\downarrow\rangle,\\
E_{1(3)} &= \frac{\delta}{2} \mp r, \qquad r = \sqrt{\delta^2/4 + 2g^2},\\
E_2 &= 0,\\
u^2 &= \frac{1}{2} + \frac{\delta}{4r}, \qquad v^2 = 1 - u^2, \qquad u, v > 0,
\end{aligned} \tag{3}$$

where the ket $|CX_1X_2\rangle$ represents the state of the whole system with C cavity photons ($C = \{0, 1\}$), and each exciton in the state X_i ($X_i = \{\downarrow, \uparrow\}$). The state $|2\rangle$ is completely of excitonic nature, while the states $\{|1\rangle, |3\rangle\}$ are an admixture of excitons and light. Under strong coupling conditions, they form a polariton. The values of the parameters $\{g, \delta\}$ determine how strong the matter-light admixture in states $\{|1\rangle, |3\rangle\}$ is, see figure 2. For example, when the detuning δ is large, the state $|1\rangle$ ($|3\rangle$) becomes excitonic (photonic)-like.

Including the degrees of freedom which describe the localized spin, the low-energy manifold is constituted by the states $|1\rangle|S_1^z\rangle|S_2^z\rangle$, $-S \leqslant S_i^z \leqslant S$. The term in J of equation (1) introduces diagonal corrections and mixes these states with $|2\rangle|S_1^z\rangle|S_2^z\rangle$ and $|3\rangle|S_1^z\rangle|S_2^z\rangle$, conserving S_1^z and S_2^z. This fact simplifies the use of degenerate perturbation theory up to second order, or a canonical transformation [15] that leads to the effective Hamiltonian of the low-energy subspace H_{eff}. Except for an irrelevant additive constant (E_1), H_{eff} takes the form

$$H_{\text{eff}} = B(S_1^z + S_2^z) - D[(S_1^z)^2 + (S_2^z)^2] + I S_1^z S_2^z. \tag{4}$$

This is a usual form of an interaction Hamiltonian in quantum magnetism, where B, D and I play the role of an effective magnetic field, axial anisotropy and Ising interaction respectively. The presence of an effective magnetic field is expected due to the symmetry breaking caused by the circularly polarized light, and has been found in previous studies of spin–spin indirect interactions in doped semiconductors [28].

The effective parameters are given by

$$\begin{aligned}
B &= \frac{u^2 J}{2}, \qquad D = \frac{J^2}{2}\left(\frac{u^2}{2r-\delta} + \frac{g^2}{8r^3}\right),\\
I &= J^2\left(\frac{u^2}{2r-\delta} - \frac{g^2}{8r^3}\right).
\end{aligned} \tag{5}$$

It can be easily checked that H_{eff} reproduces the energies of all the $(2S+1)^2$ lowest-lying eigenstates of H up to second order in J. Thus H_{eff} is a good representation of H as long as J and the temperature are smaller than the energy separation between the levels given by equations (3).

The evolution of I as a function of detuning is shown in figure 3. It is always antiferromagnetic and increases with detuning δ. The fact that for $\delta > 0$, perturbation theory in J yields a large interaction I for $g \to 0$ might seem surprising, since for $g = 0$ (no coupling of excitons with the light) there is no interaction between spins ($I = 0$). Indeed, the perturbative expression is not valid for $g \to 0$, since the states $|2\rangle$ and $|3\rangle$ become degenerate (their separation is $\sim 2g^2/\delta$) and perturbation theory requires that J is smaller than this separation to be valid, as stated at the beginning of this section. The case of large J is discussed in section 3.3.

3.2. Small J and higher T

For large detuning δ, the separation between the two lowest-energy manifolds, which is of the order of $2g^2/\delta$, becomes small. When the temperature T is larger than this separation, the effective Hamiltonian equation (4) is not enough to describe the low-energy physics because the size of the Hilbert space is too small. Nevertheless one might ask if expectation values of the spin operators can be calculated using an effective density matrix $\rho_{\rm eff}$ derived from some $H_{\rm eff}$

$$\rho_{\rm eff} = \frac{\exp(-\beta H_{\rm eff})}{{\rm Tr}_s\{\exp(-\beta H_{\rm eff})\}}, \tag{6}$$

where $\beta = 1/T$ and ${\rm Tr}_s$ is the trace over spin variables (S_1^z, S_2^z). The effective spin density matrix is obtained from the partial trace ${\rm Tr}_{\rm x}$ over exciton and photon variables

$$\rho_{\rm eff} = \frac{{\rm Tr}_{\rm x}\{\exp(-\beta H)\}}{{\rm Tr}_s{\rm Tr}_{\rm x}\{\exp(-\beta H)\}}. \tag{7}$$

Equations (6) and (7) are an indirect definition of $H_{\rm eff}$.

Assuming $g \ll \delta$ and also $J \ll 2g^2/\delta$, so that perturbation theory in J is valid, and correcting up to second order in J, the energies of the states $|1\rangle|S_1^z\rangle|S_2^z\rangle$ and $|2\rangle|S_1^z\rangle|S_2^z\rangle$ (see equations (3)) we obtain

$$\rho_{\rm eff} = \frac{\sum_{S_1^z,S_2^z} f(S_1^z,S_2^z)|S_1^z\rangle|S_2^z\rangle\langle S_1^z|\langle S_2^z|}{\sum_{S_1^z,S_2^z} f(S_1^z,S_2^z)}, \tag{8}$$

with

$$f(S_1^z,S_2^z) = 2\exp[-\beta J(S_1^z+S_2^z)/2] \times \cosh\left[\beta\frac{g^2}{\delta} + \frac{\beta J^2\delta(S_1^z-S_2^z)^2}{8g^2}\right]. \tag{9}$$

Except for an irrelevant constant, the effective Hamiltonian $H_{\rm eff}$ can be defined taking the logarithm of the effective density matrix $\rho_{\rm eff}$ given by equation (8), so that inserting $H_{\rm eff}$ in equation (6) gives the correct $\rho_{\rm eff}$. The resulting effective Hamiltonian reads

$$H_{\rm eff} = -T\sum_{S_1^z,S_2^z} \ln[f(S_1^z,S_2^z)]|S_1^z\rangle|S_2^z\rangle\langle S_1^z|\langle S_2^z|. \tag{10}$$

As a difference to other cases in which the density matrix may be approximated by a product of density matrices for two subsystems (e.g. [29]), we note that the eigenstates of this $H_{\rm eff}$ are not eigenstates of the isolated spin subsystem. Instead, each eigenvalue gives the temperature dependent expectation value of the whole Hamiltonian H for the spin configuration of the corresponding eigenstate, except for a temperature dependent constant. This constant is irrelevant since one is interested in differences of the total energy for different spin configurations.

For $T \ll 2g^2/\delta$, the expression equation (10) for $H_{\rm eff}$ coincides with equation (4) in the limit $g \ll \delta$. Instead, for $T \gg 2g^2/\delta$, $H_{\rm eff}$ retains the same form except for a temperature dependent constant, but the effective parameters become $B = J/2$, $I = 2D = J^2/(4T)$. While B coincides with the low-temperature result equations (5) in the limit $g \ll \delta$, the anisotropy and Ising interaction decrease linearly with temperature. At intermediate temperatures and for general spin, it is not possible to describe the system by a simple effective Hamiltonian-like equation (4). An exception is the case of spin 1/2, for which, except for a constant, $H_{\rm eff}$ is given by equation (4) with $B = J/2$, $D = 0$ and $I = 2T\{\ln[\cosh(\beta g^2/\delta + \beta J^2\delta/8g^2)] - \ln[\cosh(\beta g^2/\delta)]\}$.

3.3. Case $J \gg 2g^2/\delta$

In this case, the separation between the energy of the lowest-lying manifolds for $J = 0$ is smaller than J and perturbation theory in J is no longer valid. Here we assume that g is small enough so that we can treat this term perturbatively. Standard quasi-degenerate perturbation theory up to second order leads to the diagonalization of the following matrix, which mixes the states $|1\rangle|S_1^z\rangle|S_2^z\rangle$ and $|2\rangle|S_1^z\rangle|S_2^z\rangle$

$$\begin{pmatrix} F_1 & -V/2 \\ -V/2 & F_2 \end{pmatrix}, \qquad F_i = E_{\rm x} + JS_i^z - \frac{g^2}{\delta - JS_i^z}, \qquad V = \sum_i \frac{g^2}{\delta - JS_i^z}. \tag{11}$$

Proceeding as above, neglecting terms of order Jg^2/δ^2 or greater in J or g, we obtain $H_{\rm eff}$ of the form of equation (10) but with

$$f(S_1^z,S_2^z) = 2\exp[-\beta J(S_1^z+S_2^z)/2] \times \cosh\left[\frac{\beta J(S_1^z-S_2^z)}{2}\right]. \tag{12}$$

Interestingly, for $T \gg J$, $H_{\rm eff}$ takes the same form as in section 3.2 for $T \gg 2g^2/\delta$ (equation (4) with $B = J/2$, $I = 2D = J^2/(4T)$). However for $T = 0$, we find that up to terms of order g^2/δ and any order in J, the effective interaction I vanishes.

3.4. Extension to asymmetric systems

If both spins, their corresponding exchange constants J_i and both excitonic energies $E_{{\rm x}i}$ are different, in general, one has to solve a 3×3 matrix for each S_1^z and S_2^z. For the case in which perturbation theory in J_i is valid (strong coupling or negative detuning, small temperatures), if the separation Δ between excitonic energies is small ($E_{\rm x1} = E_{\rm x} + \Delta/2$, $E_{\rm x2} = E_{\rm x} - \Delta/2$), it can also be treated perturbatively and the lowest energy for given S_1^z and S_2^z becomes

$$E = E_1 + \frac{u^2}{2}\sum_i J_iS_i^z - \frac{g^2}{16r^3}\left(\sum_i J_iS_i^z\right)^2 - \frac{u^2(\Delta + J_1S_1^z - J_2S_2^z)^2}{4r - 2\delta}, \tag{13}$$

with E_1, r and u given by equation (3). The effective Hamiltonian becomes

$$H_{\rm eff} = \sum_i [B_i S_i^z - D_i (S_i^z)^2] + I S_1^z S_2^z,$$
$$B_1 = u^2 J_1 \left(\frac{1}{2} + \frac{\Delta}{2r - \delta} \right),$$
$$B_2 = u^2 J_2 \left(\frac{1}{2} - \frac{\Delta}{2r - \delta} \right) \qquad (14)$$
$$D_i = \frac{J_i^2}{2} \left(\frac{u^2}{2r - \delta} + \frac{g^2}{8r^3} \right),$$
$$I = J_1 J_2 \left(\frac{u^2}{2r - \delta} - \frac{g^2}{8r^3} \right).$$

3.5. Exact solution in the presence of a external magnetic field

Due to the Zeeman effect, a static magnetic field causes the splitting of the energy of both impurity spins and exciton states according to their spin angular momentum projection. In the case of a pair of bright and dark excitons differing in the projection of the electronic spin, the external magnetic field pointing in the z direction, $B_{\rm ext}$, causes a splitting by an energy of $\Delta E_Z = g_e \, \mu_{\rm B} B_{\rm ext}$. If the strength of $B_{\rm ext}$ is such that $\Delta E_Z \gg J$, the impurity–electron interaction cannot efficiently produce spin-flips. Thus, under such a strong magnetic field, we can assume that the effective interaction between the localized spin and the electron is of the Ising form, as given in equation (1). Besides, the argument leading to the neglect of the spin-flip term in the interaction between the localized spin and the hole remains valid, since the splitting between HH and LH [30] is much larger than the Zeeman splitting. This analysis indicates that the Hamiltonian equation (1) can be diagonalized exactly, all orders in the coupling constant J, in the subspace of one excitation: photon, exciton 1 or exciton 2, and localized spin states S_{1z} and S_{2z}. In matrix form, the reduced Hamiltonian reads

$$H = \begin{pmatrix} \frac{E_1' - E_2'}{2} + F & 0 & g_1 \\ 0 & -\frac{E_1' - E_2'}{2} + F & g_2 \\ g_1 & g_2 & E_{\rm c} + F \end{pmatrix} \qquad (15)$$

where $E_i' = E_{{\rm x}i} + J_i S_i^z + \mu_{\rm B}(3g_h/2 - g_e/2) B_{\rm ext}$, and $F = 2\,\mu_{\rm B} B_{\rm ext}(S_1^z + S_2^z)$ account for the Zeeman splitting of localized spins. If only the states with maximum absolute value of the localized spin projection $\pm S$ are considered (see section 4), from a numerical diagonalization, an effective magnetic field $B_{\rm eff}$ and spin–spin coupling $I_{\rm eff}$ can be deduced. $I_{\rm eff}$ coincides of course with the perturbative result I for small enough J_i, but it is smaller in the general case. In section 4 we will consider a particular problem and provide a numerical value for the effective coupling constant $I_{\rm eff}$.

4. Applications

In this section we show how our model can be used to study the spin–spin indirect interaction in a real system of possible technological relevance. We are motivated, on the one hand, by the experimental progress in fabrication, state manipulation and state detection of QDs containing single Mn impurities [31, 32], and, on the other hand, by the fabrication of high Q-factor micropillar cavities [33]. Put together, they form a highly promising system for spintronic applications, and in particular for quantum information technology.

More specifically, we have in mind CdTe/ZnTe QDs containing (or having in its vicinity) a single Mn ($S = 5/2$) magnetic impurity, all embedded in a II–VI semiconductor-based micropillar. These micropillar structures, of a diameter of a few micrometers, can be made to embed CdTe/ZnTe QDs and exhibit large Q-factors [34]. Due to their size, the separation between different photon modes is large enough to consider them as single-mode systems, for example, a micropillar of height $h = 0.1$ μm and diameter $\phi = 2$ μm has energy levels separated by about $\Delta E = 10$ meV, larger than any energy of the system.

Mn interacts with the exciton through exchange with the electron and hole; in principle, this interaction contains both transverse (spin-flip) and Ising terms. The spin-flip term in the Mn–hole interaction becomes important when there is mixing between LH and HH bands, a fact that is related to factors such as the geometry of the QD and the splitting between LH and HH bands [35, 36]. As reported by Leger *et al* [37], the mixing of LH–HH bands is small enough to be disregarded as a first approximation to the problem. In addition, the spin-flip term of the Mn–exciton interaction leads to the conversion of bright to dark excitons, which cannot radiate into cavity photons. Thus, a single spin-flip does not contribute to the effective interaction between different Mn. By subsequent flips in the same QD, a bright exciton can be recovered, leading to a contribution to the effective Mn–Mn coupling of order larger than J^2 (see equation (1)). Therefore, in the perturbation theory approach to order J^2, one can retain the Ising Mn–exciton interaction alone.

If an external magnetic field splits bright and dark excitons, such that the difference in energy is much larger that J, only the Ising term in Mn–electron interaction will be relevant, and the Hamiltonian can be diagonalized exactly in the subspace of one excitation. The Zeeman splitting of LHs cannot cause these states to significantly approach the HHs, since the separation between HH and LH levels is much larger than the Zeeman splitting ([30] reports HH–LH splitting of 30 meV). An additional advantage offered by the application of a static magnetic field is that decoherence times for exciton and Mn spins become larger [31, 39]. Finally, the strength of the Mn–hole interaction is about a factor five greater than that of the Mn–electron [27, 38]. For the arguments given above, we can describe the Mn–exciton interaction as an Ising one, having a strength given by the sum of the Mn–hole and Mn–electron exchange interactions.

We now estimate the feasibility of a two-qubit operation. Quantum computing with qubits larger than 1/2 has been analyzed by Bertaina *et al* [40], who demonstrated the possibility of controlling specific transitions between Mn spin states, for example the $S_z = -5/2 \rightleftarrows 5/2$. We thus envisage that our system can be controlled in a similar way, and quantum computing can be done in a pair of transitions, say the $S_z = \pm 5/2$. Reasonable values for the coupling constants are $J = 0.1$–1 meV, $g < 0.5$ meV [27, 36–38].

While finding a QD with the sought detuning—or tuning it by changing the temperature of the system [2]—is possible, we acknowledge here that with current technology the likelihood of obtaining two QDs with the same detuning is very low; nevertheless, and in order to proceed, based on equations (14) we assume that our results are not drastically changed by taking the symmetric system with a single detuning of $\delta = 1$ meV. We use the temperature $T = 1$–60 K of typical experiments [41, 34], which is also compatible with the temperature range for tuning QDs to cavity resonances [2]. This temperature is low enough such that the results of equations (14) can be used. The use of our perturbation theory result from section 3.1, and our exact numerical result in the presence of a static magnetic field from section 3.5, yield a coupling constant I and an estimate for the time $\tau_I = \pi\hbar/(2IS_1^zS_2^z)$ required for the two-qubit operation.

The time τ_I ought to be compared with decoherence times. In our model different decay channels exist: exciton recombination, excitonic spin dephasing, photon leakage and Mn decoherence. In the case of excitons, the radiative lifetime is reported to be 290 ps [41], while the dephasing of the excitonic spin may be taken as 30 ps [42]. These should be compared not with the total time for an operation, but rather with the time associated with the interaction between the exciton in a single QD and the Mn impurity: $\tau_J \simeq 2\pi\hbar/J = 5$–$25$ ps. For low concentrations, the decoherence time of Mn spins can be larger than 10 μs [32]. Photon leakage can be estimated from Q-factors; assuming a conservative value of $Q \simeq 5000$ yields a $\tau_{\rm ph} = 2Q/\omega \simeq 5$ ps. Thus, we see that the limitation is given by the leakage of photons out of the cavity. However, we have used a conservative value for the Q-factor, and, as it is reported for other materials, it could be as large as $Q = 160\,000$ [1, 43].

We calculate the spin–spin coupling constant I and its associated time by two methods. From perturbation theory, we choose the values $J = 0.1$ meV and $g = 0.5$ meV, compatible with the experimental data and the validity of perturbation theory. The resulting effective interaction is $I = 2.7 \times 10^{-3}$ meV, with a corresponding time $\tau_I = 46$ ps for a two-qubit operation. Under these conditions, we see that the time required to perform the operation exceeds the decay time for photons inside the cavity, and micropillars with larger Q-factor will be needed. From the exact diagonalization, the application of a magnetic field of $B = 5$ T would produce a splitting of dark and bright excitons of $\Delta E \simeq 0.6$ meV; we then choose the values $J = 0.5$ meV and $g = 0.5$ meV. This yields a Mn–Mn coupling of $I_{\rm eff} = 6.7 \times 10^{-2}$ meV with a corresponding time $\tau_I \simeq 1.8$ ps. Then, the use of the magnetic field allows us to consider a case where the two-qubit operation, based on Ising Mn–exciton interaction, becomes feasible.

5. Summary and discussion

We have derived an effective Hamiltonian for the effective interaction between two arbitrary spins, each one interacting with the exciton of a QD, and both dots located inside a zero-dimensional cavity. Both excitons are coupled strongly with the cavity mode. Our results are more robust in the strong coupling limit (large g) and for small or negative detuning $\delta = E_{\rm c} - E_{\rm x}$. In this case, when the exciton–spin exchange energies J_i, temperature T and the separation between excitonic energies Δ is small enough, the effective Hamiltonian is given by equations (14) and (3). It has a rather simple form, with an effective magnetic field and two different gyromagnetic factors (B_i), different anisotropies D_i and an Ising interaction I between the spins. For positive large δ and moderate T, it is not possible in general to describe the spin dynamics with a simple effective Hamiltonian, except at large temperatures or for two spins 1/2. In these cases, one has to pay the price of the temperature dependence of the anisotropy and interaction terms. As found earlier for two 1/2 spins [13], the interaction I decreases for higher δ and T.

If only states with maximum absolute value of the spin projection $S_i^z = \pm S$ are considered, a realistic effective interaction $H_{\rm eff}S_1^zS_2^z$ between these states can be obtained for general parameters and not too high δ, provided a magnetic field is applied to increase the energy of the dark excitons, and the excitation energy is low enough, so that no more than one exciton or photon is excited.

For large $|\delta|$ terms that do not conserve the number of excitations, equation (2) might be important [9, 26]. Also, for large positive δ and not too small temperature T, the dark excitons, not included in the present treatment, may approach the energy of the bright excitons. Fortunately, this does not affect our results in the strong coupling limit or under a strong enough applied magnetic field.

In the present treatment, we have not included decoherence terms due to finite lifetime of the excitons or the cavity mode. We expect these effects to be minor as long as their energy scales are smaller than the gap $E_2 - E_1$ (see equations (3)), as discussed in section 4.

Last, we have applied our model to a system consisting of Mn impurities in CdTe/ZnTe QDs, all embedded in micropillars. Very similar systems are currently investigated for their promising applications to spintronics, and a possible application of the system we studied here is to quantum computing. Regarding the Mn impurities as qubits, we calculated the time required for a two-qubit operation to be performed between them, and compared it to experimental values of decay and dephasing of the various degrees of freedom in the system. We conclude that this indirect interaction, based on Ising Mn–exciton coupling, can be used as a two-qubit operation, either when the Q-factor of the cavity exceeds 50 000, or when a static magnetic field is present.

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