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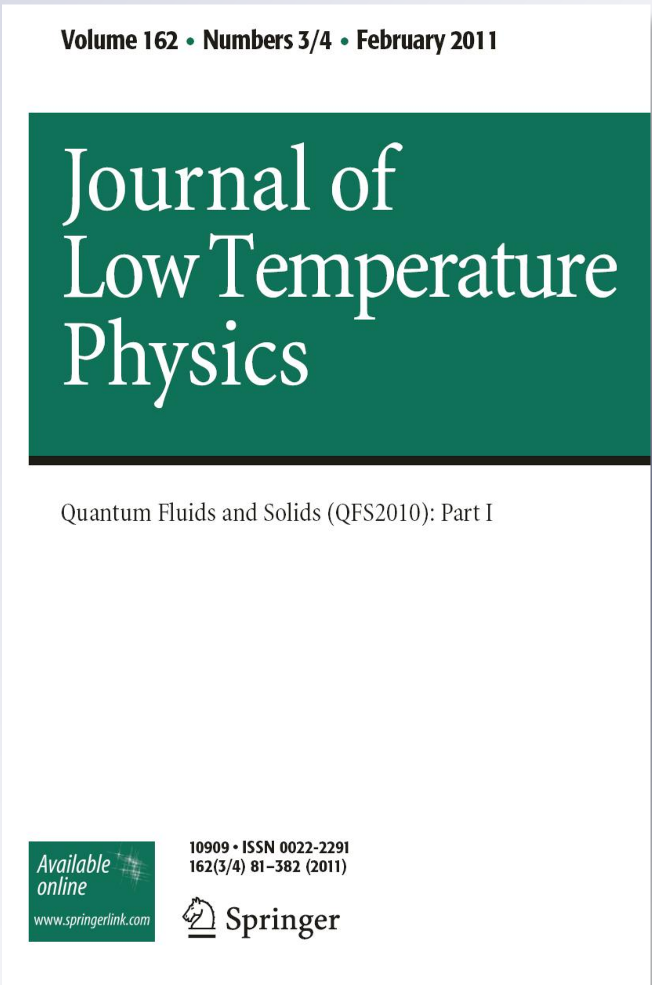

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Extended Fluid–Dynamics and Collective Motion of Two Trapped Fermion Species with Pairing Interactions

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Abstract We extend our earlier fluid–dynamical description of fermion superfluids incorporating the particle energy flow together with the equation of motion for the internal kinetic energy of the pairs. The formal scheme combines a set of equations similar to those of classical hydrodynamics with the equations of motion for the anomalous density and for its related momentum density and kinetic energy density. This dynamical frame represents a second order truncation of an infinite hierarchy of equations of motion isomorphic to the full time dependent Hartree–Fock–Bogoliubov equations in coordinate representation. We analyze the equilibrium solutions and fluctuations for a homogeneous, unpolarized fermion system of two species, and show that the collective spectrum presents the well-known Anderson–Bogoliubov low energy mode of homogeneous superfluids and a pairing vibration near the gap energy.

Keywords Fermion superfluids · Collective spectrum · Gapped mode

1 Introduction

One of the most peculiar aspects of fermion superfluids is their excitation spectrum for small amplitude perturbations, early investigated by Anderson [1] and Bogoliubov [2]. In the frame of the Random–Phase–Approximation (RPA) for homogeneous Fermi systems in the high density and/or weak coupling regime, it has been shown that these superfluids exhibit a first sound-like mode, a low energy density

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fluctuation, namely the Anderson–Bogoliubov (AB) mode, where the magnitude of the order parameter remains constant, and a massive excitation mode close to the superfluid gap energy that reflects the internal motion of the pairs, thus compromising the structure of the order parameter. Accordingly, any description of the collective spectrum of a trapped fermion superfluid should contain the above modes, when carried towards the thermodynamic limit.

This necessity calls for some attention when dealing with hydrodynamical models, since a description in terms of the dynamics of conserved quantities plus, eventually, quantities associated to the new degrees of freedom in the presence of broken symmetries, can only account for gapless spectra in the large wavelength regime [3]. When the excitation energy approaches the energy required to break a pair, the fluid of thermal excitations and the pair condensate are no longer clearly separated, thus invalidating hydrodynamics. Alternatively, a dynamical frame based on the Hartree–Fock–Bogoliubov (HFB) or Bogoliubov–de Gennes (BdG) approach should keep track of microscopic aspects of the system to a larger extent, as does the RPA or linearized HFB approach.

Recently, we proposed a fluid–dynamical (FD) description [4, 5] where both the particle and the pair density are viewed as fields that evolve with the restriction of the HFB dynamical matrices to their diagonal terms. This approach corresponds to the simplest truncation of an infinite hierarchy for the successive gradients and Laplacians of the densities, computed with respect to the relative coordinate of the pair, that in order to be cast as a closed system of equations of motion (EOM's) demands a criterion to express the Laplacian of the pair density in terms of the mass density and pair density itself, both in local equilibrium. This constitutes a local density approximation (LDA) that allowed us to investigate the particle and gap profiles for two trapped fermion species with an attractive contact interaction, in the weak coupling regime. The new aspect in this calculation is the presence of the gap pressure or gap kinetic energy, usually disregarded in the standard LDA (*i.e.*, Thomas–Fermi + BCS) and shown to be responsible of oscillations of the gap density near the center of the trap. However, this truncation criterion does not lead to the correct excitation spectrum of a homogeneous superfluid. In this paper we extend the FD scheme to include the particle and pair kinetic energies, and show that the truncation of the hierarchy has to be carried to higher momenta in order to obtain a gapped mode that resembles the pairing vibration of a fermion superfluid.

2 Fermion Fluid–Dynamics

The formalism developed in [4] follows the FD scheme of nuclear physics [6] in spatial representation $\mathbf{r}, \mathbf{r}'$ with $\mathbf{s} = \mathbf{r} - \mathbf{r}'$ and $\mathbf{R} = (\mathbf{r} + \mathbf{r}')/2$ the relative and the center-of-mass coordinate, respectively. In terms of the field operators $\Psi_\sigma^\dagger(\mathbf{r})$, $\Psi_\sigma(\mathbf{r})$, the particle and pair densities of two species denoted by a spin label $\sigma = \pm$ are

$$\rho_\sigma(\mathbf{r}, \mathbf{r}') = \langle \Psi_\sigma^\dagger(\mathbf{r}') \Psi_\sigma(\mathbf{r}) \rangle, \tag{1}$$

$$\kappa(\mathbf{r}, \mathbf{r}') = \langle \Psi_+(\mathbf{r}) \Psi_-(\mathbf{r}') \rangle \tag{2}$$

and employing the EOM's for the field operators [4, 5] one obtains the HFB EOM's for the above densities, which after regularization of the otherwise diverging pair density matrix, read

$$i\hbar\frac{\partial\rho(\mathbf{r},\mathbf{r}')}{\partial t}=\left[H_\sigma(\mathbf{r})-H_\sigma(\mathbf{r}')\right]\rho_\sigma(\mathbf{r},\mathbf{r}')+g\left[\kappa(\mathbf{r})\kappa^*(\mathbf{r},\mathbf{r}')-\kappa^*(\mathbf{r}')\kappa(\mathbf{r},\mathbf{r}')\right], \tag{3}$$

$$i\hbar\frac{\partial\kappa(\mathbf{r},\mathbf{r}')}{\partial t}=\left[H_\sigma(\mathbf{r})+H_\sigma(\mathbf{r}')\right]\kappa(\mathbf{r},\mathbf{r}')-g\left[\kappa(\mathbf{r})\rho_-(\mathbf{r}',\mathbf{r})+\kappa(\mathbf{r}')\rho_+(\mathbf{r},\mathbf{r}')\right] \tag{4}$$

We first check this dynamics for an homogeneous system in the presence of fluctuations $\delta\rho_\sigma(\mathbf{s}),\delta\kappa(\mathbf{s})$. The linearized EOM's possess the same term-by-term structure as the RPA equations derived by Anderson in the momentum representation, as expected for a unitary change of basis; furthermore, assuming a long-wavelength behavior $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ of all fluctuations, one can work out the algebraic system and verify that the roots are $\omega=0$, corresponding to the AB mode, and $\hbar\omega=2E_k$ with the quasiparticle energy $E_k=\sqrt{\hbar^2k^2/2m+\Delta^2}$, being $\Delta=-g\kappa$ the superfluid gap.

However, when one restricts the dynamical HFB–BdG EOM's (3, 4) to the diagonal of the matrices, one is forced to employ the relations $-(\hbar^2/2m)(\nabla^2_{\mathbf{r}}-\nabla^2_{\mathbf{r}'})=-(\hbar^2/m)\nabla_{\mathbf{R}}\cdot\nabla_{\mathbf{s}}$ and $-(\hbar^2/2m)(\nabla^2_{\mathbf{r}}+\nabla^2_{\mathbf{r}'})=-(\hbar^2/4m)\nabla^2_{\mathbf{R}}-(\hbar^2/m)\nabla^2_{\mathbf{s}}$, along with limiting processes $\lim_{s\longrightarrow 0}$ that incorporate a contribution from the internal kinetic energy of the pairs, defined in the next section, into the dynamics of the pair density. If this contribution is approximated by an LDA [4], the linearization scheme leads to gapless modes, as in a hydrodynamical description [3]. One can elaborate further into the structure of the fluctuations and recognize that since local fluctuations are, in fact, double Fourier transforms of fluctuations in momentum space, the only possible outcome of a linearized FD is a combination of gapless modes. We now examine higher order contributions to the hierarchy of EOM's, for the purpose of detecting the physical agent for the onset of the gapped pairing vibration.

3 Conservation Laws and Moments of the Order Parameter

We now introduce the particle current and particle kinetic energy densities of the σ-species

$$\mathbf{j}_\sigma(\mathbf{r},\mathbf{r}')=\frac{\hbar}{im}\nabla_{\mathbf{s}}\rho_\sigma(\mathbf{r},\mathbf{r}'), \tag{5}$$

$$\tau_\sigma(\mathbf{r},\mathbf{r}')=-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{s}}\rho_\sigma(\mathbf{r},\mathbf{r}')\equiv -i\hbar\nabla_{\mathbf{s}}\cdot\mathbf{j}_\sigma(\mathbf{r},\mathbf{r}') \tag{6}$$

and similarly for the pair densities

$$\mathbf{j}_\kappa(\mathbf{r},\mathbf{r}')=\frac{\hbar}{im}\nabla_{\mathbf{s}}\kappa(\mathbf{r},\mathbf{r}'), \tag{7}$$

$$\tau_\kappa(\mathbf{r},\mathbf{r}') = -\frac{\hbar^2}{2m}\nabla_{\mathbf{s}}^2\kappa(\mathbf{r},\mathbf{r}') \equiv -i\hbar\nabla_{\mathbf{s}}\cdot\mathbf{j}_\kappa(\mathbf{r},\mathbf{r}') \tag{8}$$

Once again, from the EOM's for the field operators one can derive

$$\begin{aligned}
i\hbar\frac{\partial \mathbf{j}_\sigma(\mathbf{r},\mathbf{r}')}{\partial t} &= \left[H_\sigma(\mathbf{r}) - H_\sigma(\mathbf{r}')\right]\mathbf{j}_\sigma(\mathbf{r},\mathbf{r}') + g\left[\kappa(\mathbf{r})\mathbf{j}_\kappa^*(\mathbf{r}',\mathbf{r}) - \kappa^*(\mathbf{r}')\mathbf{j}_\kappa(\mathbf{r},\mathbf{r}')\right]\\
&\quad + \frac{\hbar}{2mi}\left\{\nabla_{\mathbf{r}}\left[V_\sigma(\mathbf{r}) + g\rho_{-\sigma}(\mathbf{r})\right] + \nabla_{\mathbf{r}'}\left[V_{-\sigma}(\mathbf{r}') + g\rho_\sigma(\mathbf{r}')\right]\right\}\rho_\sigma(\mathbf{r},\mathbf{r}')\\
&\quad + g\frac{\hbar}{2mi}\left[\kappa^*(\mathbf{r}',\mathbf{r})\nabla_{\mathbf{r}}\kappa(\mathbf{r}) + \kappa(\mathbf{r},\mathbf{r}')\nabla_{\mathbf{r}'}\kappa^*(\mathbf{r}')\right],
\end{aligned} \tag{9}$$

$$\begin{aligned}
i\hbar\frac{\partial \tau_\sigma(\mathbf{r},\mathbf{r}')}{\partial t} &= \left[H_\sigma(\mathbf{r}) - H_\sigma(\mathbf{r}')\right]\tau_\sigma(\mathbf{r},\mathbf{r}') + g\left[\kappa(\mathbf{r})\tau_\kappa^*(\mathbf{r}',\mathbf{r}) - \kappa^*(\mathbf{r}')\tau_\kappa(\mathbf{r},\mathbf{r}')\right]\\
&\quad - \frac{\hbar^2}{4m}\left\{\nabla_{\mathbf{r}}^2\left[V_\sigma(\mathbf{r}) + g\rho_{-\sigma}(\mathbf{r})\right] - \nabla_{\mathbf{r}'}^2\left[V_{-\sigma}(\mathbf{r}') + g\rho_\sigma(\mathbf{r}')\right]\right\}\\
&\quad - g\frac{\hbar^2}{4m}\left[\kappa^*(\mathbf{r}',\mathbf{r})\nabla_{\mathbf{r}}^2\kappa(\mathbf{r}) - \kappa(\mathbf{r},\mathbf{r}')\nabla_{\mathbf{r}'}^2\kappa^*(\mathbf{r}')\right]\\
&\quad - i\hbar\left\{\nabla_{\mathbf{r}}\left[V_\sigma(\mathbf{r}) + g\rho_{-\sigma}(\mathbf{r})\right] + \nabla_{\mathbf{r}'}\left[V_{-\sigma}(\mathbf{r}') + g\rho_\sigma(\mathbf{r}')\right]\right\}\mathbf{j}_\sigma(\mathbf{r},\mathbf{r}')\\
&\quad - i\hbar g\left[\mathbf{j}_\kappa^*(\mathbf{r}',\mathbf{r})\nabla_{\mathbf{r}}\kappa(\mathbf{r}) + \mathbf{j}_\kappa(\mathbf{r},\mathbf{r}')\nabla_{\mathbf{r}'}\kappa^*(\mathbf{r}')\right],
\end{aligned} \tag{10}$$

$$\begin{aligned}
i\hbar\frac{\partial \mathbf{j}_\kappa(\mathbf{r},\mathbf{r}')}{\partial t} &= \left[H_+(\mathbf{r}) + H_-(\mathbf{r}')\right]\mathbf{j}_\kappa(\mathbf{r},\mathbf{r}') - g\left[\kappa(\mathbf{r})\mathbf{j}_-(\mathbf{r}',\mathbf{r}) + \kappa(\mathbf{r}')\mathbf{j}_+(\mathbf{r},\mathbf{r}')\right]\\
&\quad + \frac{\hbar}{2mi}\left\{\nabla_{\mathbf{r}}\left[V_\sigma(\mathbf{r}) + g\rho_{-\sigma}(\mathbf{r})\right] - \nabla_{\mathbf{r}'}\left[V_{-\sigma}(\mathbf{r}') + g\rho_\sigma(\mathbf{r}')\right]\right\}\kappa(\mathbf{r},\mathbf{r}')\\
&\quad - g\frac{\hbar}{2mi}\left[\rho_-(\mathbf{r}',\mathbf{r})\nabla_{\mathbf{r}}\kappa(\mathbf{r}) - \rho_+(\mathbf{r},\mathbf{r}')\nabla_{\mathbf{r}'}\kappa(\mathbf{r}')\right],
\end{aligned} \tag{11}$$

$$\begin{aligned}
i\hbar\frac{\partial \tau_\kappa(\mathbf{r},\mathbf{r}')}{\partial t} &= \left[H_+(\mathbf{r}) + H_-(\mathbf{r}')\right]\tau_\kappa(\mathbf{r},\mathbf{r}') - g\left[\kappa(\mathbf{r})\tau_-(\mathbf{r}',\mathbf{r}) + \kappa(\mathbf{r}')\tau_+(\mathbf{r},\mathbf{r}')\right]\\
&\quad - \frac{\hbar^2}{4m}\left\{\nabla_{\mathbf{r}}^2\left[V_+(\mathbf{r}) + g\rho_-(\mathbf{r})\right] + \nabla_{\mathbf{r}'}^2\left[V_-(\mathbf{r}') + g\rho_+(\mathbf{r}')\right]\right\}\kappa(\mathbf{r},\mathbf{r}')\\
&\quad + g\frac{\hbar^2}{4m}\left[\rho_-(\mathbf{r}',\mathbf{r})\nabla_{\mathbf{r}}^2\kappa(\mathbf{r}) + \rho_+(\mathbf{r},\mathbf{r}')\nabla_{\mathbf{r}'}^2\kappa(\mathbf{r}')\right]
\end{aligned} \tag{12}$$

$$\begin{aligned}
&\quad - i\hbar\left\{\nabla_{\mathbf{r}}\left[V_+(\mathbf{r}) + g\rho_-(\mathbf{r})\right] - \nabla_{\mathbf{r}'}\left[V_-(\mathbf{r}') + g\rho_+(\mathbf{r}')\right]\right\}\cdot\mathbf{j}_\kappa(\mathbf{r},\mathbf{r}')\\
&\quad + i\hbar g\left[\mathbf{j}_-(\mathbf{r}',\mathbf{r})\nabla_{\mathbf{r}}\kappa(\mathbf{r}) - \mathbf{j}_+(\mathbf{r},\mathbf{r}')\nabla_{\mathbf{r}'}\kappa(\mathbf{r}')\right]
\end{aligned} \tag{13}$$

The FD scheme is now complete in terms of the conserved quantities of the fermion species, the order parameter and its first two moments, as

$$\frac{\partial\rho_\sigma}{\partial t} = -\nabla\cdot\mathbf{j}_\sigma, \tag{14}$$

$$\frac{\partial\mathbf{j}_\sigma}{\partial t} = -\nabla\mu_\sigma + g\frac{\kappa\mathbf{j}_\kappa^* - \kappa^*\mathbf{j}_\kappa}{i\hbar}, \tag{15}$$

$$\frac{\partial \tau_\sigma}{\partial t} = -\nabla \cdot \mathbf{j}_\tau - \nabla \left(V_T + g\rho_T\right) \cdot \mathbf{j}_\sigma + g\frac{\kappa \tau_\kappa^* - \kappa^* \tau_\kappa}{i\hbar} - g\left(\mathbf{j}_\kappa^* \cdot \nabla\kappa + \mathbf{j}_\kappa \cdot \nabla\kappa^*\right)$$
$$- \frac{\hbar^2}{4m}\nabla^2\left(V_\sigma + g\rho_{-\sigma} - V_{-\sigma} - g\rho_\sigma\right) - g\frac{\hbar^2}{4m}\frac{\kappa^*\nabla^2\kappa - \kappa\nabla^2\kappa^*}{i\hbar}, \tag{16}$$

$$i\hbar\frac{\partial \kappa}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_T - \mu_T\right)\kappa + \tau_\kappa, \tag{17}$$

$$i\hbar\frac{\partial \mathbf{j}_\kappa}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_T + g\rho_T - \mu_T\right)\mathbf{j}_\kappa - \lim_{s\to 0}\left(-\frac{\hbar^2}{m}\nabla_{\mathbf{s}}^2\mathbf{j}_\kappa\right)$$
$$+ \frac{\hbar}{2mi}\nabla\left(V_+ + g\rho_- - V_- - g\rho_+\right) - g\mathbf{j}_T\kappa - g\frac{\hbar}{2mi}\left(\rho_- - \rho_+\right)\nabla\kappa, \tag{18}$$

$$i\hbar\frac{\partial \tau_\kappa}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_T + g\rho_T - \mu_T\right)\tau_\kappa - \lim_{s\to 0}\left(-\frac{\hbar^2}{m}\nabla_{\mathbf{s}}^2\tau_\kappa\right)$$
$$- i\hbar\nabla\left(V_+ + g\rho_- - V_- - g\rho_+\right)\cdot\mathbf{j}_\kappa + i\hbar g\nabla\cdot\left(\mathbf{j}_- - \mathbf{j}_+\right)$$
$$- g\tau_T\kappa - \frac{\hbar^2}{4m}\left[\nabla^2\left(V_T + g\rho_T\right)\kappa - g\rho_T\nabla^2\kappa\right] \tag{19}$$

In these equations the label T indicates the total quantity corresponding to the summation for $\sigma = \pm$, and in (16), we have introduced the energy flow vector $\mathbf{j}_\tau = (\hbar/mi)\lim_{s\longrightarrow 0}\nabla\tau$. We realize that an infinite hierarchy of higher gradients of the pair density is created by the limiting process $\lim_{s\longrightarrow 0}$, starting with (4). This hierarchy is equivalent to an expansion of the EOM (4) around $\mathbf{r} = \mathbf{r}'$; the set (14) to (19) can be closed only by a truncation that replaces the given limits by some local functions in terms of a selected equation of state. In this form one obtains an approximation to the RPA, which corresponds to the infinite hierarchy. In what follows we show that for a symmetric, homogeneous fermion system, a convenient choice of these terms yields eigenfrequencies for small amplitude oscillations compatible with the appearance of a gapped pairing vibration mode.

4 Symmetric and Homogeneous System of Paired Fermions

We now consider a homogeneous system of paired fermions *i.e.*, $f_+ = f_- \equiv f$ for any of the quantities in the FD EOM's, and linearize the latter for small amplitude fluctuations $\delta\rho_\sigma, \delta\mathbf{j}_\sigma, \delta\tau_\sigma, \delta\kappa, \delta\mathbf{j}_\kappa, \delta\tau_\kappa$ with a common phase factor $e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$. In the spirit of Ref. [4], we adopt for $\lim_{s\to 0}[(-\hbar^2/m)\nabla_{\mathbf{s}}^2\mathbf{j}_\kappa]$ and for $\lim_{s\to 0}[(-\hbar^2/m)\nabla_{\mathbf{s}}^2\tau_\kappa]$ the rigorous results that can be computed from the EOS derived by Papenbrock and Bertsch [7], considering in addition that all currents vanish in equilibrium. Equations (14) to (19) then give rise to the following algebraic system for the fluctuations

$$\omega\delta\rho = \mathbf{k}\cdot\delta\mathbf{j}, \tag{20}$$

$$\omega\delta\mathbf{j} = \mathbf{k}\frac{\partial\mu}{\partial\rho}\delta\rho + g\frac{\kappa\delta\mathbf{j}_\kappa^* - \kappa^*\delta\mathbf{j}_\kappa}{i\hbar}, \tag{21}$$

$$\omega\delta\tau = -\mathbf{k}\cdot\frac{\partial \mathbf{j}_\tau}{\partial\rho}\delta\rho - g\mathbf{k}\cdot\mathbf{j}\delta\rho_T - g\frac{\hbar k^2}{4m}\left(\kappa^*\delta\kappa - \kappa\delta\kappa^*\right) + g\frac{\delta\kappa\tau_\kappa^* + \kappa\delta\tau_\kappa^* - \delta\kappa^*\tau_\kappa - \kappa^*\delta\tau_\kappa}{\hbar}, \tag{22}$$

$$\hbar\omega\delta\kappa = \left(\frac{\hbar^2k^2}{4m} - \mu_T\right)\delta\kappa + \delta\tau_\kappa, \tag{23}$$

$$\hbar\omega\delta\mathbf{j}_\kappa = \left(\frac{\hbar^2k^2}{4m} + g\rho_T - \mu_T\right)\delta\mathbf{j}_\kappa + \frac{\partial}{\partial\rho}\lim_{s\to 0}\left(-\frac{\hbar^2}{m}\nabla_{\mathbf{s}}^2\mathbf{j}_\kappa\right)\delta\rho + g\frac{\hbar}{2m}\mathbf{k}\left[\delta\left(\rho_- - \rho_+\right)\kappa - \left(\rho_- - \rho_+\right)\delta\kappa\right] - g\kappa\delta\mathbf{j}_T, \tag{24}$$

$$\hbar\omega\delta\tau_\kappa = \left(\frac{\hbar^2k^2}{4m} + g\rho_T - \mu_T\right)\delta\tau_\kappa + \frac{\partial}{\partial\rho}\lim_{s\to 0}\left(-\frac{\hbar^2}{m}\nabla_{\mathbf{s}}^2\tau_\kappa\right)\delta\rho + g\frac{\hbar^2k^2}{4m}\left(\kappa\delta\rho - \rho_T\delta\kappa\right) - g\left[\kappa\delta\tau_T + \tau_T\delta\kappa + \hbar\mathbf{k}\cdot(\mathbf{j}_+ - \mathbf{j}_-)\delta\kappa\right] \tag{25}$$

In these expressions, all partial derivatives and nonfluctuating quantities are those in equilibrium. By examining (20) and (21), one can see that a first sound-like mode for the particle density, corresponding to vanishing pair currents, can be always encountered.

We are thus interested in the characteristics of pairing vibrations with $\delta\rho = 0$. Since our goal is to detect the appearance of a gap in the excitation spectrum, we concentrate on $k = 0$. It is then easy to see that the fluctuating particle current $\delta\mathbf{j}$ and pair current $\delta\mathbf{j}_\kappa$ vanish and one is left with a closed algebraic system for the anomalous perturbation and their complex conjugates. The eigenvalues $\hbar\omega$ then correspond to the roots of the determinant

$$D = \begin{pmatrix} \mu_T + \hbar\omega & 0 & -1 & 0 & 0 & 0 \\ 0 & \mu_T + \hbar\omega & 0 & -1 & 0 & 0 \\ g\tau_T & 0 & \mu_T - g\rho_T + \hbar\omega & 0 & g\kappa & 0 \\ 0 & g\tau_T & 0 & \mu_T - g\rho_T + \hbar\omega & 0 & g\kappa \\ -2g\tau_\kappa^* & 2g\tau_\kappa & 2g\kappa^* & -2g\kappa & \hbar\omega & 0 \\ 2g\tau_\kappa^* & -2g\tau_\kappa & -2g\kappa^* & 2g\kappa & 0 & \hbar\omega \end{pmatrix} \tag{26}$$

which can be computed analytically, yielding one vanishing eigenvalue, two complex conjugate solutions and three nonvanishing real ones, two of them being

$$\hbar\omega = \frac{1}{2}\left[-2\mu + g\rho - \pm\sqrt{g^2\rho^2 - 4\tau}\right] = \frac{1}{2}\left[-2\mu + g\rho - \pm\sqrt{\frac{2}{5}g^2\rho^2 - \frac{12}{5}g\rho\mu + \frac{24}{5}\Delta^2}\right] \tag{27}$$

The latter line employs the expression for the particle kinetic energy that can be derived with the procedure indicated in Ref. [7], namely $\tau = (3/5)(\rho\mu - 2\Delta^2/g)$. Note that the gap energy Δ is introduced into these eigenvalues by the kinetic energy; being the current scheme an approximation based on a truncation of an infinite hierarchy, we are now confident that already at this level the dynamics indicates the presence of a massive mode at around the correct energy.

5 Summary

We have extended the FD description of fermion superfluids by incorporating, in addition to EOM's for densities and currents [4, 5], the dynamics of the particle kinetic energy and the internal kinetic energy of the paired fermions. The latter in turn contain higher gradients and Laplacians taken with respect to the relative coordinate, that keep track of the microscopic relative dynamics of the pairs. We have shown that this infinite hierarchy, if truncated by introducing an LDA for the higher order terms, in the case of an homogeneous system gives rise to a gapped excitation with energy comparable to the pair breaking energy.

We have shown that the exact RPA spectrum appears as the eigenspectrum of the infinite hierarchy, equivalent to the full time dependent HFB-BdG matrix equations in coordinate representation. The approximation scheme here proposed presents the advantage of relying on macroscopic fields, in the spirit of hydrodynamics, and calls attention on the fact that superfluid dynamics in the weak coupling regime is fully described by both particle and pair densities. These are related in equilibrium, but can fluctuate independently giving rise to the AB and the pairing vibration mode. We have shown that the particle kinetic energy is the conserved quantity responsible for the appearance of the latter.

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