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# Defining the effective temperature of a quantum driven system from current-current correlation functions

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**Abstract.** We calculate current-current correlation functions and find an expression for the zero-frequency noise of multiterminal systems driven by harmonically time-dependent voltages within the Keldysh non-equilibrium Green's functions formalism. We also propose a fluctuation-dissipation relation for current-current correlation functions to define an effective temperature. We discuss the behavior of this temperature and compare it with the local temperature determined by a thermometer and with the effective temperature defined from a single-particle fluctuation-dissipation relation. We show that for low frequencies all the definitions of the temperature coincide.

## 1 Introduction

Over the last years we have witnessed a technological trend towards miniaturization of electronic circuits. This tendency has been accompanied by a growing research activity focused on achieving a better understanding of the mechanisms for heat dissipation and energy flow in mesoscopic systems. However, the motivation for the research in this area is not only technological, because the very fundamental concepts of standard statistical mechanics and thermodynamics are put into test when studying these systems, even more when the process under consideration corresponds to an out-of-equilibrium situation.

Several efforts have been made towards the extension of standard thermodynamical concepts to the out-of-equilibrium evolution of different systems. Some well-known examples are the aging regime of glassy systems, sheared glasses, granular materials and colloids [1–11]. A very successful achievement in the characterization of such nonequilibrium states has been the identification of an *effective temperature*, i.e., a parameter with the same properties of the temperature of a system at equilibrium that is useful to describe the evolution of nonequilibrium systems. For instance, for glassy systems the definition of effective temperature was introduced [1,2] by means of a generalization of the equilibrium fluctuation-dissipation relations (FDR) and the physical meaning of this concept was supported by showing that such a temperature would coincide with the one measured by a thermometer [3,4]. The definition of an effective temperature from a FDR was introduced for quantum glassy systems in references [12,13] and later analyzed in electronic systems [14]. More recently, these temperatures were studied in an Ising chain after a sudden quench [15].

The physics of the mesoscopic scale is ruled by the quantum coherence of the particle propagation. This originates non-trivial interference mechanisms and surprising effects. Well known examples are the violation of the Fourier's Law in low-dimensional phononic systems [16,17] as well as the $2k_F$ oscillations of the local voltage and the negative electrical resistance [18–20].

In the past few years there have been many experimental attempts to locally characterize the heat flow in non-equilibrium systems. For example, Pothier et al. [21] measured the local energy distribution function in metallic diffusive wires in a stationary out-of-equilibrium situation. More recently, Altimiras et al. [22,23] measured the electron energy distribution in an integer quantum Hall regime with one of the edge channels driven out-of-equilibrium. Chiral heat transport has been investigated in the quantum Hall regime using micron-scale thermometers [24] and later explained with the introduction of a local temperature along the edge [25]. The idea of defining a non-equilibrium local temperature has been useful to study out-of-equilibrium transport in other mesoscopic systems. For example, thermoelectric transport has been studied with the aid of the local temperature determined by an ideal thermometer [26,27]. Also the concept of effective temperature has been useful to study heat exchange between a nanojunction and its environment, which can act as a freezing agent [28], and to study mesoscopic superconductors [29,30]. Another example is the prediction that a superconducting wire can remain in superconducting state even in contact with a bath that greatly exceeds the critical temperature if the effective local temperature is maintained below the critical value [31]. A local temperature has also been defined to characterize the heat

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transport in molecular devices [32]. Additional studies have been reviewed in reference [33].

In a previous work [34] we defined *local* and *effective* temperatures in electronic quantum systems driven out of equilibrium by external ac potentials. Examples of such systems are quantum dots with ac voltages acting at their walls (quantum pumps) [35–38] and quantum capacitors [39,40]. In that work we presented two concepts, which are the *local* and the *effective* temperatures. The local temperature was introduced following a procedure inspired in a work by Engquist and Anderson [41]. The idea is to include a thermometer in the microscopic description of the system. On the other hand the effective temperature is defined from a local FDR involving single-particle Green's functions. We showed that for low driving frequencies both ways of defining the temperature coincide. In a more recent work [42] we slightly generalized the definition of the thermometer to consider the possibility of simultaneously sensing the local temperature and the local chemical potential of the sample. We showed that the new local temperature determined by this new definition coincides with the previous one. Even more, we showed that such a parameter verified the thermodynamical properties of a temperature, meaning that its gradient signals the direction for heat flow at the contacts.

The aim of this work is to analyze the role of effective temperatures within the context of a FDR for current-current correlation functions. The motivation is twofold. On one hand we are interested in testing the robustness of the definition of an effective temperature from a FDR, at the level of a correlation function different from the one we have considered in our previous work. On the other hand, current-current correlation functions are particularly appealing quantities since they are related to noise, which can be experimentally measured and contain valuable information on the nature of the elementary particles that take part in the transport process. The zero-frequency noise is usually used to characterize the correlations between particles in mesoscopic systems [43]. Additionally in quantum pumps, noise is related to the possibility of having quantized pumping [44] and it contains information that cannot be extracted from the time-averaged current [45]. Current correlations in mesoscopic coherent conductors were first discussed by Büttiker in reference [46] and since then an extensive theoretical literature on noise in mesoscopic systems analyzed within the scattering matrix formalism has been developed [45,47–52]. We use here another approach, which is based Keldysh formalism. For non-interacting systems both treatments were proved to coincide at the level of the description of the current for dc [53] and ac-driven systems [54–56]. In the present work we show that this is also the case for the current fluctuations correlations. The main goal of this work is to show that the effective temperature obtained from a fluctuation-dissipation relation for current-current correlation functions coincides with the local temperature defined using a thermometer and thus verifies the same thermodynamical properties of the latter.

This paper is organized as follows. In Section 2, we present the model and summarize the theoretical treatment. In Section 3 we review three definitions of temperature addressed in recent works [34,42]. In Section 4 we derive general expressions for current-current correlation functions and an explicit expression for the zero-frequency noise within the Keldysh Green's functions formalism. In Section 5 we present numerical results for a particular system. Section 6 is devoted to discussion and conclusions. We give some details of the calculation in the Appendix.

## 2 Model and theoretical treatment

In Figure 1 we display the same setup as in references [34,42] representing a quantum driven system, with the Hamiltonian $H_{sys}(t)$, connected to a probe characterized by $H_P$. The total system is then described by

$$H(t) = H_{sys}(t) + H_{cP} + H_P, \tag{1}$$

with $H_{cP}$ implementing the local coupling between the system and the probe. The Hamiltonian corresponding to the driven system can in turn be written as

$$H_{sys}(t) = H_L + H_{cL} + H_C(t) + H_{cR} + H_R, \tag{2}$$

where $H_C(t)$, $H_L$ and $H_R$ stand for the Hamiltonians of the central part and left and right reservoirs, coupled among themselves via the Hamiltonians $H_{cL}$ and $H_{cR}$.

The Hamiltonian describing the central system ($C$) contains the ac fields and can be written as $H_C(t) = H_0 + H_V(t)$. We assume that $H_0$ is a Hamiltonian for non-interacting electrons while $H_V(t)$ is harmonically time dependent with a fundamental driving frequency $\Omega_0$. We leave further details of the model undetermined as much of the coming discussion is model independent.

All three reservoirs (left, right and the probe) are modeled by systems of non-interacting electrons with many degrees of freedom, i.e., $H_\alpha = \sum_{k\alpha} \varepsilon_{k\alpha} c^\dagger_{k\alpha} c_{k\alpha}$, where $\alpha = L, R, P$. The corresponding contacts are described by $H_{c\alpha} = w_{c\alpha} \sum_{k\alpha} (c^\dagger_{k\alpha} c_{l\alpha} + c^\dagger_{l\alpha} c_{k\alpha})$, where $l\alpha$ denotes the coordinate of $C$ where the reservoir $\alpha$ is connected. As in previous works [18–20,34,42,57–59], we consider non-invasive probe and we treat $w_{cP}$ at the lowest order of perturbation theory when necessary.

We will analyze the out-of-equilibrium dynamics of this system within the Schwinger-Keldysh Green's functions formalism. Within this formalism, instead of the usual time-ordering operator used in equilibrium theory a contour-ordering operator which orders time-labels according to their order on the Keldysh contour is introduced. The single particle propagator reads

$$iG_{j,j'}(t,t') = \langle T_{\mathcal{C}}[c_j(t) c^\dagger_{j'}(t')] \rangle. \tag{3}$$

The contour-ordered Green's function contains four different functions depending on where the times $t$ and $t'$ are over the Keldysh contour [60]. It is easy to see that

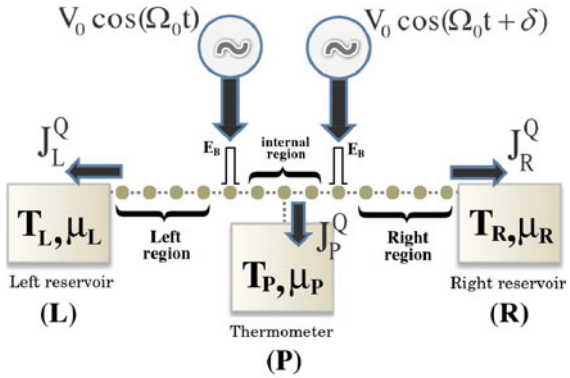


**Fig. 1.** (Color online) Scheme of the setup. The central device is a wire with two barriers of height $E_B$ connected by its ends to two reservoirs ($L$ and $R$). The third reservoir ($P$) represents the probe, which consists of a macroscopic system weakly coupled to a given point of the central device. In this setup, transport is induced by two oscillating ac fields (both with the same amplitude and frequency but with a phase lag) applied at the points where the barriers are located. The left and right regions depicted in this scheme are related to the heat current that flows into the respective reservoirs [42,57].

they are not all independent. We then consider the *lesser*, *greater* and *retarded* Green's functions,

$$\begin{aligned} iG^{<}_{j,j'}(t,t') &= -\langle c^{\dagger}_{j'}(t')c_j(t)\rangle, \\ iG^{>}_{j,j'}(t,t') &= \langle c_j(t)c^{\dagger}_{j'}(t')\rangle, \\ iG^{R}_{j,j'}(t,t') &= \Theta(t-t')\langle\left[c_j(t), c^{\dagger}_{j'}(t')\right]_{+}\rangle, \end{aligned} \tag{4}$$

where $[,]_+$ denote the anticommutator of the fermionic operators, $\langle ...\rangle$ is the quantum statistical average and the indexes $j, j'$ denote spatial coordinates of the system. These Green's functions can be evaluated after solving the Dyson equations.

In this work we will focus on current-current correlation functions. The current in reservoir $\alpha$ at time $t$ is defined by the operator [54–56]

$$\hat{J}_\alpha(t) = iw_{c\alpha}\sum_{k\alpha}\left(\hat{c}^{\dagger}_{k\alpha}(t)\hat{c}_{l\alpha}(t) - \hat{c}^{\dagger}_{l\alpha}(t)\hat{c}_{k\alpha}(t)\right), \tag{5}$$

which obeys bosonic commutation rules. The ensuing connected contour-ordered propagator reads in this case

$$iC_{\alpha\beta}(t,t') = \langle T_{\mathcal{C}}[\hat{J}_\alpha(t)\hat{J}_\beta(t')]\rangle - \langle\hat{J}_\alpha(t)\rangle\langle\hat{J}_\beta(t')\rangle, \tag{6}$$

while the *lesser*, *greater* and *retarded* Green's functions are

$$\begin{aligned} iC^{<}_{\alpha\beta}(t,t') &= \langle\hat{J}_\beta(t')\hat{J}_\alpha(t)\rangle - \langle\hat{J}_\alpha(t)\rangle\langle\hat{J}_\beta(t')\rangle, \\ iC^{>}_{\alpha\beta}(t,t') &= \langle\hat{J}_\alpha(t)\hat{J}_\beta(t')\rangle - \langle\hat{J}_\alpha(t)\rangle\langle\hat{J}_\beta(t')\rangle, \\ iC^{R}_{\alpha\beta}(t,t') &= \Theta(t-t')\langle\left[\hat{J}_\alpha(t), \hat{J}_\beta(t')\right]_{-}\rangle. \end{aligned} \tag{7}$$

where $[,]_-$ denote the commutator of the currents.

For the case of harmonic driving it is convenient to use the Floquet-Fourier representation of the Green's functions [54–56]:

$$A_{j,j'}(t,t-\tau) = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i(k\Omega_0 t+\omega\tau)}A_{j,j'}(k,\omega). \tag{8}$$

where $A$ stands for single-particle (3) or current-current (6) propagators.

In general the *Keldysh* and *retarded* Green's functions, can be expressed in terms of the *lesser* and *greater* Green's functions via

$$\begin{aligned} A^{K}_{j,j'}(t,t') &= A^{>}_{j,j'}(t,t') + A^{<}_{j,j'}(t,t'), \\ A^{R}_{j,j'}(t,t') &= \Theta(t-t')\left[A^{>}_{j,j'}(t,t') - A^{<}_{j,j'}(t,t')\right]. \end{aligned} \tag{9}$$

From the definition given in equation (8) it is straightforward to see that the Floquet-Fourier components of these functions can be written as

$$\begin{aligned} A^{K}_{j,j'}(k,\omega) &= A^{>}_{j,j'}(k,\omega) + A^{<}_{j,j'}(k,\omega), \\ A^{R}_{j,j'}(k,\omega) &= i\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{A^{>}_{j,j'}(k,\omega') - A^{<}_{j,j'}(k,\omega')}{\omega-\omega'+i0^{+}}. \end{aligned} \tag{10}$$

## 3 Defining the temperature

### 3.1 Local temperature determined by a probe

In reference [34] we defined the local temperature ($T_{lP}$) of the site $lP$ of the system as the value of the temperature of the probe such that the time-averaged heat exchange between the central system and the probe vanishes.

It can be shown [61] that, given $H_C(t)$ without many-body interactions, the dc component of the heat current flowing from the central system to the thermometer can be expressed as ($\hbar = k_B = e = 1$)

$$\begin{aligned} J^{Q}_{P} = \sum_{\alpha=L,R,P}\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\Big\{&[f_\alpha(\omega) - f_P(\omega_k)] \\ &\times(\omega_k-\mu)\Gamma_P(\omega_k)\Gamma_\alpha(\omega)\left|G^{R}_{lP,l\alpha}(k,\omega)\right|^2\Big\}, \end{aligned} \tag{11}$$

where $\omega_k = \omega + k\Omega_0$, while $\Gamma_\alpha(\omega) = -2\pi|w_\alpha|^2\sum_{k\alpha}\delta(\omega - \varepsilon_{k\alpha})$ are the spectral functions characterizing the reservoirs ($\alpha = L, R, P$), and $f_\alpha(\omega) = 1/[e^{\beta_\alpha(\omega-\mu_\alpha)}+1]$ is the Fermi function, which depends on $T_\alpha = 1/\beta_\alpha$ and $\mu_\alpha$ respectively the temperature and the chemical potential of the reservoir $\alpha$. Thus, the local temperature $T_{lP}$ corresponds to the solution of the equation

$$J^{Q}_{P}(T_{lP}) = 0. \tag{12}$$

In general, equation (12) must be solved numerically, but under certain conditions, an analytical expression can be found. In particular, for the low temperature weak-driving adiabatic regime, which corresponds to small amplitudes and frequencies of the driving potential, and for $\Omega_0 \ll T$ [34],

$$T_{lP} = T\left[1 + \lambda^{(1)}_{lP}(\mu)\Omega_0\right], \tag{13}$$

where

$$\lambda_l^{(n)}(\omega) = \frac{1}{\sum_{k=-1}^{1}\varphi_l(k,\omega)} \sum_{k=-1}^{1} (k)^{n+2} \frac{d^n[\varphi_l(k,\omega)]}{d\omega^n}, \quad (14)$$

$$\varphi_l(k,\omega) = \sum_{\alpha=L,R} \left|G^R_{l,l\alpha}(k,\omega)\right|^2 \Gamma_\alpha(\omega). \quad (15)$$

An alternative definition of local temperature was discussed in reference [42], where the fact that the heat current is related to the charge current was taken into account. Then, the local temperature ($T^*_{lP}$) and the local chemical potential ($\mu^*_{lP}$) were defined from the condition of simultaneously vanishing of the time-averaged charge and heat currents between the probe and the system. That is

$$\begin{cases} J^Q_P(T^*_{lP}, \mu^*_{lP}) = 0, \\ J^e_P(T^*_{lP}, \mu^*_{lP}) = 0, \end{cases} \quad (16)$$

where (see Refs. [54–56,58,59])

$$J^e_P = \sum_{\alpha=L,R,P} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \Big\{ [f_\alpha(\omega) - f_P(\omega_k)] \times \Gamma_P(\omega_k)\Gamma_\alpha(\omega) \left|G^R_{lP,l\alpha}(k,\omega)\right|^2 \Big\}, \quad (17)$$

is the dc component of the charge current flowing through the contact between the system and the probe.

The simultaneous equations given in equation (16) can be solved numerically for any situation, but an analytical expression can be found within the low temperature weak-driving adiabatic regime, when $\Omega_0 \ll T$, and leads to $T^*_{lP} = T_{lP}$, given in equation (13).

### 3.2 Effective temperature from a single-particle fluctuation-dissipation relation (FDR)

For systems in equilibrium, the fluctuation-dissipation theorem establishes a relation between the Keldysh (correlation) and the retarded Green's functions. In reference [34], we defined a local FDR involving single-particle Green's functions from which an effective temperature for the site $l$ ($T^{eff}_l = 1/\beta^{eff}_l$) can be extracted,

$$iG^K_{l,l}(0,\omega) - iG^K_{l,l}(0,\mu) = \tanh\left[\frac{\beta^{eff}_l(\omega-\mu)}{2}\right] \overline{\varphi}_l(\omega), \quad (18)$$

with $\overline{\varphi}_l(\omega) = -2\,\mathrm{Im}[G^R_{l,l}(0,\omega)] = \sum_k \varphi_l(k,\omega_{-k})$. In general, equation (18) defines an effective temperature that might depend on $\omega$, so the limit $\omega \to \mu$ is taken. An extra term is added to the lhs of equation (18) because the rhs is always zero at $\omega = \mu$ but $G^K_{l,l}(0,\mu)$ is not necessarily zero in an arbitrary out-of-equilibrium situation.

Within the low temperature weak-driving adiabatic regime, when $\Omega_0 \ll T$, we showed [34] that $T^{eff}_{lP} = T_{lP}$. Then, the conclusion of our previous investigations is that for the weak driving adiabatic regime the effective temperature defined from a single-particle FDR coincides with that determined by a thermometer.

## 4 Current-current correlation functions and effective temperature

### 4.1 A non-equilibrium fluctuation-dissipation relation

We analyze the role of effective temperatures ($T^{eff*}$) from a FDR in the framework of two-particle correlation functions. As a priori they are not necessarily the same as the effective temperatures defined above we use an asterisk to refer to them. We will focus on current-current correlation functions since they are more easily accessible from an experimental point of view. We are particularly interested in a local relation, that is both currents evaluated at the same point.

As in the case of the single-particle FDR we focus on the dc components of the correlation functions to define the effective temperature. This corresponds to assuming that an equilibrium-like FDR holds for the $k = 0$ Floquet component with $\beta^{eff*}$ playing the role of the inverse of temperature,

$$C^R_{\alpha\alpha}(0,\omega) = i\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{C^K_{\alpha\alpha}(0,\omega')}{\omega-\omega'+i0^+} \tanh\left[\frac{\beta^{eff*}_{l\alpha}\omega'}{2}\right]. \quad (19)$$

An equivalent expression for the FDR given in equation (19) is obtained by considering the imaginary part, which leads to

$$iC^K_{\alpha\alpha}(0,\omega) = \coth\left[\frac{\beta^{eff*}_{l\alpha}\omega}{2}\right] \overline{\varphi}^*_\alpha(\omega), \quad (20)$$

where $\overline{\varphi}^*_\alpha(\omega) = -2\,\mathrm{Im}\left[C^R_{\alpha\alpha}(0,\omega)\right]$. (Notice that the real part is simply derived by means of Kramers-Kronig relations.) As in the case of the single-particle FDR in equation (18), equation (20) defines an effective temperature that might depend on $\omega$, so the limit $\omega \to 0$ is taken.

It is important to notice the similarity of this expression with the one shown in equation (18) for single-particle Green's functions (fermionic operators). In this case the hyperbolic tangent is replaced by an hyperbolic cotangent due to the bosonic statistic of current operators.

### 4.2 Current-current correlation and noise

Although we are more interested in the case of local current correlations, let us start by considering the more general case of correlation at different points. If we consider two reservoirs ($\alpha$ and $\beta$) and two times (an absolute time $t$ and a relative time $\tau$) we can define the correlation function of currents as

$$P_{\alpha\beta}(t,t-\tau) = \frac{1}{2}\langle \Delta\hat{J}_\alpha(t)\Delta\hat{J}_\beta(t-\tau) + \Delta\hat{J}_\beta(t-\tau)\Delta\hat{J}_\alpha(t)\rangle, \quad (21)$$

where $\Delta\hat{J}_\alpha(t) = \hat{J}_\alpha(t) - \langle\hat{J}_\alpha(t)\rangle$.

With the definition of the contour-ordered current-current correlation function given in equation (6), the correlation function of currents given in equation (21) can be

expressed as

$$P_{\alpha\beta}(t,t-\tau) = \frac{i}{2}\left(C^{>}_{\alpha\beta}(t,t-\tau) + C^{<}_{\alpha\beta}(t,t-\tau)\right) = \frac{i}{2}C^{K}_{\alpha\beta}(t,t-\tau). \quad (22)$$

If instead of a symmetrized current-current correlation we are interested in a non-symmetrized one,

$$P^{ns}_{\alpha\beta}(t,t-\tau) = \langle \Delta\hat{J}_{\alpha}(t)\Delta\hat{J}_{\beta}(t-\tau)\rangle, \quad (23)$$

the correlation becomes

$$P^{ns}_{\alpha\beta}(t,t-\tau) = iC^{<}_{\beta\alpha}(t-\tau,t). \quad (24)$$

In this work we will give results for the symmetrized correlation only but it is straightforward to obtain the results for the non-symmetrized one.

Since experimentally the noise spectrum is averaged over the absolute time $t$, the relevant quantity here is

$$\mathcal{P}_{\alpha\beta}(\omega) = 2\int d\tau \langle P_{\alpha\beta}(t,t-\tau)\rangle_t e^{i\omega\tau} \quad (25)$$

where $\langle\ldots\rangle_t$ denotes the time average. From the definition of the Floquet-Fourier components given in equation (8) it is easy to see that

$$\mathcal{P}_{\alpha\beta}(\omega) = iC^{K}_{\alpha\beta}(0,\omega). \quad (26)$$

Hence, the only relevant Floquet-Fourier component is the one with $k = 0$.

As we are considering non-interacting electrons the contour-ordered propagator given in equation (6) can be exactly evaluated in terms of single-particle propagators (3). Using Wick's theorem (see the Appendix for the details), this contour-ordered function can be written as

$$\begin{aligned} iC_{\alpha\beta}(t,t') = -w_{c\alpha}w_{c\beta}\sum_{k\alpha,k\beta}\{&G_{l\beta,k\alpha}(t',t)G_{l\alpha,k\beta}(t,t') \\ &-G_{k\beta,k\alpha}(t',t)G_{l\alpha,l\beta}(t,t') \\ &-G_{l\beta,l\alpha}(t',t)G_{k\alpha,k\beta}(t,t') \\ &+G_{k\beta,l\alpha}(t',t)G_{k\alpha,l\beta}(t,t')\}. \end{aligned} \quad (27)$$

In the Appendix we show the detailed calculation leading from this expression to the Floquet-Fourier components $C^{\gtrless}_{\alpha\beta}(0,\omega)$. Here we only reproduce the results for two cases of particular interest.

The first case is the zero-frequency limit of $C^{K}_{\alpha\beta}(0,\omega)$, which reads

$$P_{\alpha\beta} \equiv \frac{i}{2}C^{K}_{\alpha\beta}(k=0,\omega=0) = \delta_{\alpha\beta}P_{\alpha} + P_{\alpha\neq\beta}, \quad (28)$$

where

$$\begin{aligned} P_{\alpha} = &\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\Gamma_{\alpha}(\omega')\sum_{k=-\infty}^{\infty}\sum_{\gamma}\Gamma_{\gamma}(\omega'_k)f_{\alpha\gamma}(\omega',\omega'_k) \\ &\times|G^{R}_{l\alpha,l\gamma}(-k,\omega'_k)|^2, \\ P_{\alpha\neq\beta} = &-\frac{1}{2}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\Gamma_{\alpha}(\omega')\sum_{k=-\infty}^{\infty}\Gamma_{\beta}(\omega'_k)\Big\{f_{\alpha\beta}(\omega',\omega'_k) \\ &\times\mathrm{Re}\Big[G^{R}_{l\beta,l\alpha}(k,\omega')G^{R}_{l\alpha,l\beta}(-k,\omega'_k)\Big] \\ &-2\sum_{k=-\infty}^{\infty}\sum_{\gamma}\Gamma_{\gamma}(\omega'_{k'})f_{\alpha\gamma}(\omega',\omega'_{k'})\mathrm{Im}\Big[G^{R}_{l\beta,l\alpha}(k,\omega') \\ &\times G^{R}_{l\alpha,l\gamma}(-k',\omega'_{k'})G^{R}_{l\beta,l\gamma}(k-k',\omega'_{k'})^*\Big] \\ &+G^{>}_{l\beta,l\alpha}(k,\omega')G^{<}_{l\beta,l\alpha}(k,\omega')^*\Big\} \\ &+\Big\{\text{same with } \alpha\leftrightarrow\beta\Big\}, \end{aligned} \quad (29)$$

being

$$f_{\alpha\beta}(\omega,\omega') = f_{\alpha}(\omega)(1-f_{\beta}(\omega')) + f_{\beta}(\omega')(1-f_{\alpha}(\omega)). \quad (30)$$

It is important to notice that this is a general result for multiterminal quantum driven systems and the sum over $\gamma$ extends over all reservoirs connected to the central system. For this work we chose a two terminal system, but this result is completely general as no assumption concerning the reservoirs was made in the calculation.

At this point it is interesting to compare with previous results obtained within the scattering matrix formalism (see reference [50]). In order to do so, we need to assume that all reservoirs are at equal temperature and chemical potential (unbiased pump). We split the zero frequency noise into two contributions

$$P_{\alpha\beta} \equiv P^{(th)}_{\alpha\beta} + P^{(sh)}_{\alpha\beta}, \quad (31)$$

where

$$\begin{aligned} P^{(th)}_{\alpha\beta} = &\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}f(\omega')(1-f(\omega'))\Gamma(\omega')\sum_{k=-\infty}^{\infty}\Gamma(\omega'_k) \\ &\times\Big\{\delta_{\alpha\beta}\sum_{\gamma}\Big(|G^{R}_{l\alpha,l\gamma}(k,\omega')|^2 + |G^{R}_{l\alpha,l\gamma}(-k,\omega'_k)|^2\Big) \\ &-|G^{R}_{l\alpha,l\beta}(k,\omega')|^2 - |G^{R}_{l\beta,l\alpha}(k,\omega')|^2\Big\}, \\ P^{(sh)}_{\alpha\beta} = &\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\Gamma(\omega')\sum_{k=-\infty}^{\infty}\Gamma(\omega'_k)\Big\{\delta_{\alpha\beta}\left(f(\omega')-f(\omega'_k)\right)^2 \\ &\times|G^{R}_{l\alpha,l\gamma}(k,\omega')|^2 - f(\omega')^2\Big(|G^{R}_{l\alpha,l\beta}(k,\omega')|^2 \\ &+|G^{R}_{l\beta,l\alpha}(k,\omega')|^2\Big) + 2f(\omega')f(\omega'_k)\mathrm{Re}\Big[G^{R}_{l\beta,l\alpha}(k,\omega') \\ &\times G^{R}_{l\alpha,l\beta}(-k,\omega'_k)\Big] - 2\mathrm{Re}\Big[\Big(f(\omega')G^{R}_{l\beta,l\alpha}(k,\omega') \\ &-f(\omega'_k)G^{R}_{l\alpha,l\beta}(-k,\omega'_k)^*\Big)G^{<}_{l\beta,l\alpha}(k,\omega')^*\Big] \\ &-|G^{<}_{l\beta,l\alpha}(k,\omega')|^2\Big\}. \end{aligned} \quad (32)$$

The first term $P_{\alpha\beta}^{(th)}$ is the Nyquist-Johnson noise while $P_{\alpha\beta}^{(sh)}$ is the shot noise. Using the relation between the Floquet $S$-matrix and Green's functions [54–56]

$$S_{F,\alpha\beta}(\omega_m,\omega_n) = \delta_{\alpha\beta}\delta_{n,m} - i\sqrt{\Gamma(\omega_m)\Gamma(\omega_n)} \times G^R_{l\alpha,l\beta}(m-n,\omega_n), \quad (33)$$

it is easy to show that the result given in equation (32) coincides with the one obtained using the Floquet $S$-matrix formalism in reference [50].

The other case of interest is the one in which $\alpha = \beta = P$, i.e. we concentrate in current fluctuations of the probe. Using the fact that the probe is noninvasive, we only keep terms to the lowest order in the coupling $w_{cP}$ between the system and the thermometer,

$$iC^K_{PP}(0,\omega) = \Gamma_P \int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\sum_{k=-\infty}^{\infty}\sum_{\gamma=L,R}\Gamma_\gamma(\omega')\Big\{f_\gamma(\omega') \times\big[2 - f_P(\omega'_k+\omega) - f_P(\omega'_k-\omega)\big] + \big[f_P(\omega'_k+\omega) + f_P(\omega'_k-\omega)\big] \times(1-f_\gamma(\omega'))\Big\}|G^R_{lP,l\gamma}(k,\omega')|^2. \quad (34)$$

On the other hand we need $\overline{\varphi}^*_P(\omega)$, which is

$$\overline{\varphi}^*_P(\omega) = \Gamma_P \int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\sum_{k=-\infty}^{\infty}\sum_{\gamma=L,R}\Gamma_\gamma(\omega') \times\big[f_P(\omega'_k-\omega) - f_P(\omega'_k+\omega)\big] \times|G^R_{lP,l\gamma}(k,\omega')|^2. \quad (35)$$

The functions entering equations (34) and (35) are the ones involved in the definition of effective temperature given in equation (20).

## 5 Results

In this section we present results for a central device consisting of non-interacting electrons in a one-dimensional lattice:

$$H_0 = -w\sum_{l,l'}(c_l^\dagger c_{l'} + H.c.), \quad (36)$$

where $w$ denotes a hopping matrix element between neighboring positions $l, l'$ on the lattice. The driving term is chosen as

$$H_V(t) = \sum_{j=1}^{2} eV_j(t)c_{lj}^\dagger c_{lj}, \quad (37)$$

with $V_j(t) = E_B + V_0\cos(\Omega_0 t + \delta_j)$, $lj$ being the positions where two oscillating fields with frequency $\Omega_0$ and phase-lag $\delta$ are applied. This defines a simple model for a quantum pump where two ac gate voltages are applied at the walls of a quantum dot [35–38,54–56,62,63].

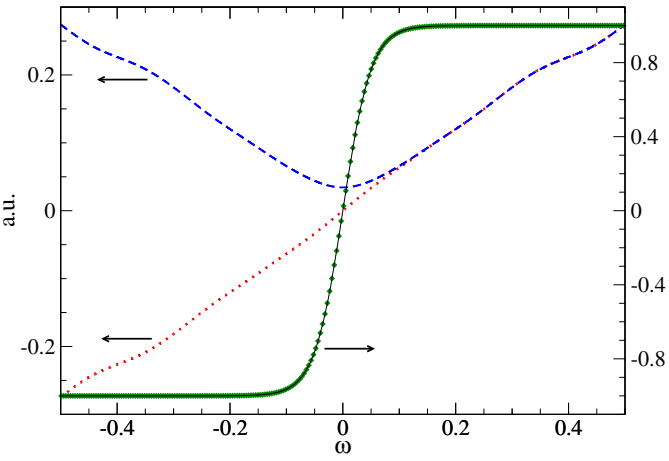


**Fig. 2.** (Color online) Current-current correlation functions $\overline{\varphi}^*_P(\omega)$ (dotted red), $iC^K_{PP}(0,\omega)$ (dashed blue), their quotient (green diamonds), and $\tanh[\beta^{eff*}\omega/2]$ (solid black) as a function of $\omega$. The reservoirs have chemical potential $\mu = 0.2$ and temperature $T = 0.025$. The driving frequency is $\Omega_0 = 0.01$, the amplitude is $V_0 = 0.05$ and $E_B = 0.2$.

### 5.1 Equivalence between effective and local temperature at weak driving

As in references [34,42] we are interested in the weak driving regime, which corresponds to a situation where the ac voltage amplitudes are lower than the kinetic energy of the electrons in the structure and the driving frequency is much smaller than the inverse of the dwell time of these electrons. We have shown that in this regime the local temperature defined from equation (12), with the chemical potential of the probe fixed, is identical to the local temperature defined from equation (16), where the chemical potential of the probe has to be determined in order to satisfy both equations, and it is also identical to an effective temperature defined from a local fluctuation-dissipation relation of single-particle Green's functions (see Eq. (18)).

We now turn our attention to the effective temperature $T^{eff*}$ defined in equation (20), involving current-current correlation functions. The correlation functions given in equations (34) and (35) depend on the temperature $T_P$ and the chemical potential $\mu_P$ of the probe via the Fermi function $f_P$. Thus, the effective temperature $T^{eff*}$, so calculated, also depends on $T_P$ and $\mu_P$. There are many possible reasonable choices for the latter quantities. In this subsection we will concentrate in only one choice and leave for the next subsection the analysis of other possibilities. We choose $\mu_P$ equal to the chemical potential $\mu$ of the reservoirs and $T_P$ equal to the local temperature $T_{lP}$, i.e. the one for which the heat flow between the system and the probe vanishes.

In Figure 2 we show a typical plot for $iC^K_{PP}(0,\omega)$, $\overline{\varphi}^*_P(\omega)$ and their ratio as a function of $\omega$. According to the definition of effective temperature given in equation (20), the derivative of this ratio at $\omega = 0$ corresponds to $\beta^{eff*}/2$. This derivative is calculated numerically. In the same figure we plot $\tanh\left[\beta^{eff*}\omega/2\right]$ and we see that the quotient

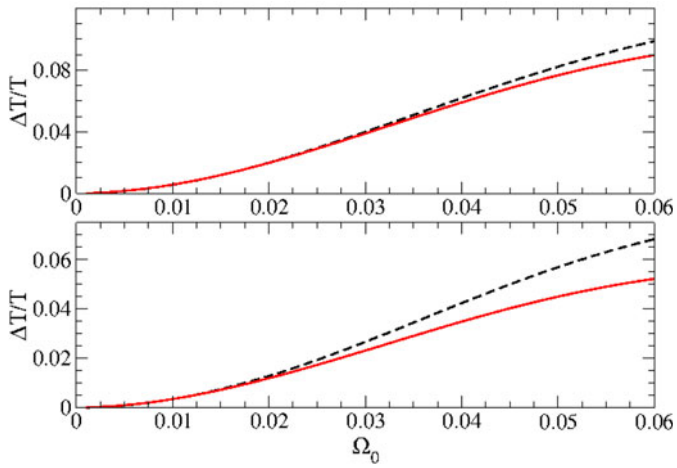


**Fig. 3.** (Color online) Local temperature difference $\Delta T_{lP}$ (dashed black) and effective temperature difference $\Delta T_{lP}^{eff*}$ (solid red), relative to the temperature $T$ of the reservoirs ($\Delta T/T$), for the site $lP = lL$ (i.e. the site connected to the left reservoir) as a function of driving frequency $\Omega_0$. The reservoirs have chemical potential $\mu = 0.2$, the driving amplitude is $V_0 = 0.05$ and $E_B = 0.2$. The upper panel corresponds to $T = 0.016$, while the lower panel corresponds to $T = 0.005$.

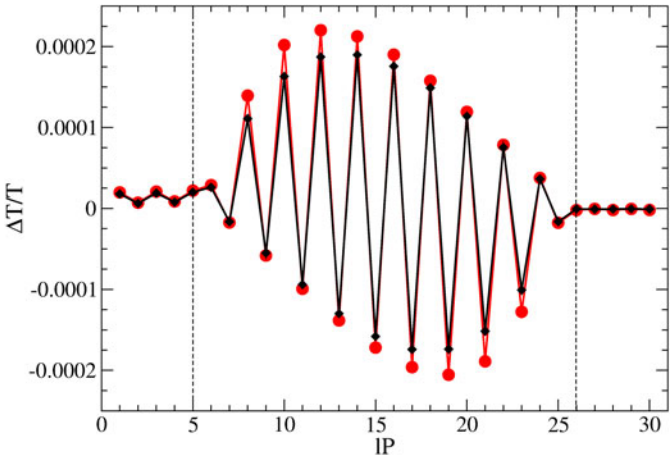


**Fig. 4.** (Color online) Local temperature difference $\Delta T_{lP}$ (black diamonds) and effective temperature difference $\Delta T_{lP}^{eff*}$ (red circles) along a one-dimensional model of $N = 30$ sites with two ac fields operating with a phase lag of $\delta = \pi/2$ at the positions indicated by dotted lines. The system is in contact with reservoirs with chemical potentials $\mu = 0.2$ and temperature $T = 0.02$. The driving frequency is $\Omega_0 = 0.001$, the amplitude is $V_0 = 0.05$ and $E_B = 0.2$.

$\overline{\varphi}_P^*(\omega)/iC_{PP}^K(0,\omega)$ is well fitted by a FDR-type relation for a reasonably large frequency interval.

At this point we find that it is convenient to define local and effective temperature differences with respect to the temperature $T$ of the reservoirs respectively as $\Delta T_{lP} = T_{lP} - T$ and $\Delta T_{lP}^{eff*} = T_{lP}^{eff*} - T$. In Figure 3 we show the behavior of $\Delta T^{eff*}$ and $\Delta T_{lP}$, relative to the temperature $T$ of the reservoirs, calculated for the site connected to the left reservoir, as a function of the driving frequency $\Omega_0$ for two different values of $T$. This analysis can be done for any site of the central system but we chose this particular site because its local temperature determine the heat current that flows into the left reservoir [42]. Results for any other site of the central system are similar. In Figure 3 the upper panel corresponds to $T = 0.016$, while the lower corresponds to $T = 0.005$. We see that both ways of defining the temperature coincide at low frequencies. This supports the idea that, for a given temperature $T$ of the reservoirs, $T^{eff*}$ is a *bona fide* temperature within the low driving regime. As we can see from Figure 3, the higher the temperature $T$ of the reservoirs, the broader the region of low driving frequency $\Omega_0$ in which the two definitions of the temperature agree.

As we mentioned earlier, the definition given in equation (20) can be used to calculate the effective temperature in any site of the central system. In Figure 4 we show the comparison between the local temperature difference $\Delta T_{lP}$ and the effective temperature difference $\Delta T^{eff*}$ all along the sample. The values of $\Delta T_{lP}$ and $\Delta T^{eff*}$ are plotted for each point of a linear chain of $N = 30$ sites, relative to the temperature $T$ of the reservoirs, for $T_L = T_R = T = 0.02$, $\mu_L = \mu_R = \mu = 0.2$ and a particular low value of the driving frequency $\Omega_0 = 0.001$. We can see that there is a good agreement between the two temperatures along the whole structure and an almost perfect agreement within the "Left" and "Right" regions (defined in Fig. 1), which are the ones from where we can determine the heat flow between the system and each one of the reservoirs (see Refs. [42,57]). Due to the location of the barriers and the absence of temperature and chemical potential bias between the reservoirs, the system may seem L-R symmetric. However the existence of a phase lag $\delta \neq 0$ breaks this symmetry and thus the temperature profile in Figure 1 is L-R asymmetric. It is also important to notice the existence of $2k_F$ Friedel-like oscillations, $k_F$ being the Fermi vector of the electrons leaving the reservoirs. These oscillations are an indication of quantum interference. They were previously reported for exactly the same setup we study in this work [34,42] and also predicted in other mesoscopic systems under a stationary driving [64].

### 5.2 Different choices of $T_P$ and $\mu_P$

The effective temperature $T^{eff*}$ depends on the values of $T_P$ and $\mu_P$ (respectively the temperature and chemical potential of the probe). The choice analyzed in the previous section was $\mu_P$ equal to the chemical potential $\mu$ of the reservoirs and $T_P$ equal to the local temperature $T_{lP}$. We will call this choice Case I. Another suitable choice (Case II) could be to choose $\mu_P = \mu$, as in the previous case but $T_P$ such that $T^{eff*} = T_P$. A third choice (Case III) could be to choose $T_P$ such that $T^{eff*} = T_P$ but at the same time $\mu_P = \mu_{lP}$ (the local voltage) in order to have a vanishing charge current between the system and the probe at that temperature. In this work we will only deal with these three possibilities.

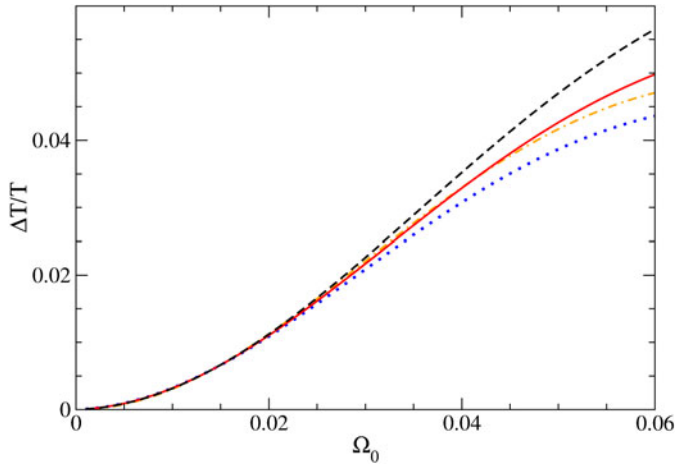


**Fig. 5.** (Color online) Local temperature difference $\Delta T_{lP}$ (dashed black) and effective temperature difference $\Delta T_{lP}^{eff*}$ for Case I (solid red), Case II (dotted blue) and Case III (dashed and dotted orange), relative to the temperature $T$ of the reservoirs, for the site $lP = lL$ as a function of driving frequency $\Omega_0$. The reservoirs have chemical potential $\mu = 0.2$ and temperature $T = 0.01$.

If Figure 5 we show the three different effective temperatures differences $\Delta T_{lP}^{eff*}$ corresponding to the above mentioned cases together with the local temperature difference $\Delta T_{lP}$, relative to the temperature $T$ of the reservoirs, as functions of the driving frequency $\Omega_0$ for a given temperature $T$ of the reservoirs. As we can see, all three cases give a good estimate of the local temperature in the regime of interest (i.e. low driving frequencies). This behavior supports the robustness of the definition of the local temperature from a FDR.

## 6 Summary and conclusions

In this work we have calculated the current-current correlation functions for quantum driven systems and found an explicit expression for the zero-frequency noise within the Schwinger-Keldysh Green's functions formalism. In the particular case of multiterminal unbiased quantum driven systems our result is in agreement with previous results obtained within the scattering matrix approach [50]. For non-interacting systems both descriptions agree, while the Green's functions has the advantage of providing a systematic framework for the study of interacting systems.

We have also defined an effective temperature from a local fluctuation-dissipation relation for current-current correlation functions and showed that for low frequencies it coincides with the local temperature defined with a thermometer and from a FDR at the level of single-particle propagators. In this work we studied the case of unbiased reservoirs but our treatement can be easily extended to deal with finite chemical potencial and/or finite temperature bias and we expect that the general conclusion will also hold for those cases. This result opens the possibility of using current-current correlation in real experiments, in order to define the local temperature of a driven sample.

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## Appendix: Analytical expressions for current-current correlation functions

Using equation (5), the connected contour-ordered current-current correlation (see Eq. (6)) can be written in terms of electron operators as

$$\begin{aligned} iC_{\alpha\beta}(t,t') = -w_{c\alpha}w_{c\beta}\sum_{k\alpha,k\beta}\Big(&D_{k\alpha,l\alpha,k\beta,l\beta}(t,t') \\ &- D_{k\alpha,l\alpha,l\beta,k\beta}(t,t') \\ &- D_{l\alpha,k\alpha,k\beta,l\beta}(t,t') + D_{l\alpha,k\alpha,l\beta,k\beta}(t,t')\Big), \end{aligned} \tag{A.1}$$

where

$$\begin{aligned} D_{i,j,k,l}(t,t') = &\langle T_{\mathcal{C}}[\hat{c}_i^\dagger(t)\hat{c}_j(t)\hat{c}_k^\dagger(t')\hat{c}_l(t')]\rangle \\ &-\langle\hat{c}_i^\dagger(t)\hat{c}_j(t)\rangle\langle\hat{c}_k^\dagger(t')\hat{c}_l(t')\rangle. \end{aligned} \tag{A.2}$$

Using Wick's theorem and the definition of the contour-ordered Green's function (Eq. (3)) we can rewrite equation (A.2) as

$$D_{i,j,k,l}(t,t') = G_{l,i}(t',t)G_{j,k}(t,t'). \tag{A.3}$$

The substitution of equation (A.3) into equation (A.1) gives the result shown in equation (27).

The next step is to calculate the lesser and greater Green's functions. In order to have the lesser (greater) Green's function we need $t \in C_1$ and $t' \in C_2$ ($t \in C_2$ and $t' \in C_1$). If $t \in C_1$ and $t' \in C_2$, then

$$D^{<}_{i,j,k,l}(t,t') = G^{>}_{l,i}(t',t)G^{<}_{j,k}(t,t'). \tag{A.4}$$

Using equation (A.4) we can write the lesser Green's function as

$$\begin{aligned} iC^{<}_{\alpha\beta}(t,t') = w_{c\alpha}w_{c\beta}\sum_{k\alpha,k\beta}\Big\{&\Big(G^{>}_{k\alpha,l\beta}(t,t')^*G^{<}_{l\alpha,k\beta}(t,t')\Big) \\ &-\Big(G^{>}_{k\alpha,k\beta}(t,t')^*G^{<}_{l\alpha,l\beta}(t,t')\Big) \\ &-\Big(G^{>}_{l\alpha,l\beta}(t,t')^*G^{<}_{k\alpha,k\beta}(t,t')\Big) \\ &+\Big(G^{>}_{l\alpha,k\beta}(t,t')^*G^{<}_{k\alpha,l\beta}(t,t')\Big)\Big\}. \end{aligned} \tag{A.5}$$

where the property $G^{<,>}_{j,j'}(t,t') = -G^{<,>}_{j',j}(t',t)^*$ was used. Notice that the last term is equal to the first interchanging $<$ with $>$ and conjugating. The same situation arises with

the second and third terms. The result for the greater Green's function is obtained by switching $<$ with $>$.

We are interested in the Floquet-Fourier components of this Green's functions. So, according to equation (8) we need to calculate

$$C^{<}_{\alpha\beta}(t,t') = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i[k\Omega_0 t+\omega(t-t')]}C^{<}_{\alpha\beta}(k,\omega). \tag{A.6}$$

Substituting with the Floquet-Fourier expansions of the Green's functions (see Eq. (8)) into equation (A.5) and rewriting into the form of equation (A.6) we can obtain the $k=0$ Floquet-Fourier component, which is

$$C^{<}_{\alpha\beta}(0,\omega)=i\left[\mathcal{A}^{<}_{\alpha\beta}(\omega)+\mathcal{A}^{>}_{\alpha\beta}(-\omega)^{*}+\mathcal{B}^{<}_{\alpha\beta}(\omega)+\mathcal{B}^{>}_{\alpha\beta}(-\omega)^{*}\right], \tag{A.7}$$

where

$$\begin{aligned}\mathcal{A}^{<}_{\alpha\beta}(\omega) &= \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}w_{c\alpha}\sum_{k\alpha}G^{>}_{l\beta,k\alpha}(-k,\omega'_k)\\ &\quad\times w_{c\beta}\sum_{k\beta}G^{<}_{l\alpha,k\beta}(k,\omega'+\omega),\\ \mathcal{B}^{<}_{\alpha\beta}(\omega) &= \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}G^{>}_{l\alpha,l\beta}(k,\omega')^{*}\\ &\quad\times w_{c\alpha}w_{c\beta}\sum_{k\alpha,k\beta}G^{<}_{k\alpha,k\beta}(k,\omega'+\omega),\end{aligned}$$

and $\mathcal{X}^{>}$ is obtained from $\mathcal{X}^{<}$ by switching $<$ with $>$ ($\mathcal{X}=\mathcal{A},\mathcal{B}$). Since the greater Floquet-Fourier component is obtained from equation (A.5) by switching $<$ with $>$, it can be written as:

$$C^{>}_{\alpha\beta}(0,\omega) = i\left[\mathcal{A}^{>}_{\alpha\beta}(\omega)+\mathcal{A}^{<}_{\alpha\beta}(-\omega)^{*}+\mathcal{B}^{>}_{\alpha\beta}(\omega)+\mathcal{B}^{<}_{\alpha\beta}(-\omega)^{*}\right]. \tag{A.8}$$

We want to obtain an expression of the previous quantities in terms of $G^{R}_{l,l'}(k,\omega)$ with $l,l'$ in the central system. In order to do so we need to calculate the Floquet-Fourier components of

$$\sum_{k\alpha}G^{<}_{l\beta,k\alpha}(t,t') = \frac{1}{w_{c\alpha}}\int dt_1\left[G^{R}_{l\beta,l\alpha}(t,t_1)\Sigma^{<}_{\alpha}(t_1,t')+G^{<}_{l\beta,l\alpha}(t,t_1)\Sigma^{A}_{\alpha}(t_1,t')\right], \tag{A.9}$$

and

$$\begin{aligned}\sum_{k\alpha,k\beta}G^{<}_{k\alpha,k\beta}(t,t') = \frac{1}{w_{c\alpha}w_{c\beta}}\Big\{&\delta_{\alpha\beta}\,\Sigma^{<}_{\alpha}(t,t')+\int dt_1dt_2\\ &\times\Big[\Sigma^{R}_{\alpha}(t,t_1)G^{R}_{l\alpha,l\beta}(t_1,t_2)\Sigma^{<}_{\beta}(t_2,t')\\ &+\Sigma^{R}_{\alpha}(t,t_1)G^{<}_{l\alpha,l\beta}(t_1,t_2)\Sigma^{A}_{\beta}(t_2,t')\\ &+\Sigma^{<}_{\alpha}(t,t_1)G^{A}_{l\alpha,l\beta}(t_1,t_2)\Sigma^{A}_{\beta}(t_2,t')\Big]\Big\},\end{aligned} \tag{A.10}$$

where

$$\begin{aligned}\Sigma^{<}_{\alpha}(t,t') &= |w_{c\alpha}|^2\sum_{k\alpha}g^{0,<}_{k\alpha,k\alpha}(t,t'),\\ \Sigma^{R}_{\alpha}(t,t') &= |w_{c\alpha}|^2\sum_{k\alpha}g^{0,R}_{k\alpha,k\alpha}(t,t'),\\ \Sigma^{A}_{\alpha}(t,t') &= \Sigma^{R}_{\alpha}(t',t)^{*},\end{aligned} \tag{A.11}$$

with their respective Fourier transforms

$$\begin{aligned}\Sigma^{<}_{\alpha}(\omega) &= if_{\alpha}(\omega)\Gamma_{\alpha}(\omega),\\ \Sigma^{R}_{\alpha}(\omega) &= \int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Gamma_{\alpha}(\omega')}{\omega-\omega'+i0^{+}},\\ \Sigma^{A}_{\alpha}(\omega) &= \Sigma^{R}_{\alpha}(\omega)^{*}.\end{aligned} \tag{A.12}$$

Using the Fourier transforms of $\Sigma^{<}$ and $\Sigma^{R}$ together with the Floquet-Fourier expansion for the Green's functions (see Eq. (8)) we obtain

$$\sum_{k\alpha}G^{<}_{l\beta,k\alpha}(k,\omega) = \frac{1}{w_{c\alpha}}\Big\{G^{R}_{l\beta,l\alpha}(k,\omega)\Sigma^{<}_{\alpha}(\omega)+G^{<}_{l\beta,l\alpha}(k,\omega)\Sigma^{R}_{\alpha}(\omega)^{*}\Big\}, \tag{A.13}$$

where

$$G^{\lessgtr}_{l\beta,l\alpha}(k,\omega') = \sum_{m=-\infty}^{\infty}\sum_{\gamma}\Sigma^{\lessgtr}_{\gamma}(\omega'_m)G^{R}_{l\alpha,l\gamma}(-m,\omega'_m)^{*}\times G^{R}_{l\beta,l\gamma}(k-m,\omega'_m), \tag{A.14}$$

with $\Sigma^{\lessgtr}_{\gamma}(\omega)=\Gamma_{\gamma}(\omega)\lambda^{\lessgtr}_{\gamma}(\omega)$, and

$$\begin{aligned}\lambda^{<}_{\gamma}(\omega) &= if_{\gamma}(\omega),\\ \lambda^{>}_{\gamma}(\omega) &= -i(1-f_{\gamma}(\omega)).\end{aligned} \tag{A.15}$$

On the other hand,

$$\begin{aligned}\sum_{k\alpha,k\beta}G^{<}_{k\alpha,k\beta}(k,\omega) = \frac{1}{w_{c\alpha}w_{c\beta}}\Big\{&\delta_{\alpha\beta}\delta_{k0}\Sigma^{<}_{\alpha}(\omega)\\ &+G^{R}_{l\alpha,l\beta}(k,\omega)\Sigma^{<}_{\beta}(\omega)\Sigma^{R}_{\alpha}(\omega_k)\\ &+\Big[G^{R}_{l\beta,l\alpha}(-k,\omega_k)^{*}\Sigma^{<}_{\alpha}(\omega_k)\\ &+G^{<}_{l\alpha,l\beta}(k,\omega)\Sigma^{R}_{\alpha}(\omega_k)\Big]\Sigma^{R}_{\beta}(\omega)^{*}\Big\}.\end{aligned} \tag{A.16}$$

By substituting equations (A.13) and (A.16) into equation (A.8) and then into equation (A.7) we obtain an expression for $C^{<}_{\alpha\beta}(0,\omega)$. The expression for the greater Green's function is obtained by switching $<$ with $>$.

We are interested in the case where $\alpha=\beta=P$, i.e. we are interested in fluctuations in the current flowing to the probe. As we are dealing with a non-invasive probe, we only keep terms up to the lowest order in the coupling between the system and the probe. Since $\Gamma_{\alpha},H_{\alpha}\propto|w_{c\alpha}|^2$

it is easy to see that

$$\mathcal{A}^{<}_{PP}(\omega) = O\left(|w_{cP}|^4\right),$$
$$\mathcal{B}^{<}_{PP}(\omega) = \int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}G^{>}_{lP,lP}(0,\omega')^{*}\lambda^{<}_{P}(\omega'+\omega)\Gamma_P(\omega'+\omega) + O\left(|w_{cP}|^4\right). \quad \text{(A.17)}$$

Thus, $\mathcal{B}^{<}_{PP}(\omega)$ and $\mathcal{B}^{>}_{PP}(\omega)$ are the only terms with contributions of order $|w_{cP}|^2$. Using equations (A.14) and (A.15) we can write, up to this order,

$$C^{K}_{PP}(0,\omega) = -i\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\sum_{k=-\infty}^{\infty}\sum_{\gamma=L,R}|G^{R}_{lP,l\gamma}(-n,\omega'_n)|^2 \times\Big\{f_{\gamma P}(\omega'_n,\omega'+\omega)\Gamma_P(\omega'+\omega) + f_{\gamma P}(\omega'_n,\omega'-\omega)\Gamma_P(\omega'-\omega)\Big\}\Gamma_\gamma(\omega'_n). \quad \text{(A.18)}$$

To obtain an expression for $\overline{\varphi}^{*}_{P}(\omega)$ we can use the following identity, easily deduced from equation (10),

$$\overline{\varphi}^{*}_{P}(\omega) \equiv -2\mathrm{Im}[C^{R}_{PP}(0,\omega)] = i\left(C^{>}_{PP}(0,\omega) - C^{<}_{PP}(0,\omega)\right), \quad \text{(A.19)}$$

which leads to

$$\overline{\varphi}^{*}_{P}(\omega) = \int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\sum_{k=-\infty}^{\infty}\sum_{\gamma=L,R}\Gamma_\gamma(\omega'_n)|G^{R}_{lP,l\gamma}(-n,\omega'_n)|^2 \times\Big\{\Big[f_\gamma(\omega'_n) - f_P(\omega'+\omega)\Big]\Gamma_P(\omega'+\omega) - \Big[f_\gamma(\omega'_n) - f_P(\omega'-\omega)\Big]\Gamma_P(\omega'-\omega)\Big\}. \quad \text{(A.20)}$$

If we additionally consider a constant density of states in the probe to obtain a result independent of any particular probe, we arrive to the expressions given in equations (34) and (35).

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