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Self-interacting dark matter*

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Self-interacting dark matter (SIDM) is a hypothetical form of dark matter (DM), characterized by relatively strong (compared to the weak interaction strength) self-interactions (SIs), which has been proposed to resolve a number of issues concerning tensions between simulations and observations at the galactic or smaller scales. We review here some recent developments discussed at the 14th Marcel Grossmann Meeting (MG14), paying particular attention to restrictions on the SIDM (total) cross-section from using

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novel observables in merging galactic structures, as well as the rôle of SIDM on the Milky Way halo and its central region. We report on some interesting particle-physics inspired SIDM models that were discussed at MG14, namely the glueball DM, and a right-handed neutrino DM (with mass of a few tens of keV, that may exist in minimal extensions of the standard model (SM)), interacting among themselves via vector bosons mediators in the dark sector. A detailed phenomenology of the latter model on galactic scales, as well as the potential role of the right handed neutrinos in alleviating some of the small-scale cosmology problems, namely the discrepancies between observations and numerical simulations within standard ΛCDM and ΛWDM cosmologies are reported.

Keywords: Self-interacting dark matter; galactic astrophysics; cosmology; neutrinos.

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1. Introduction

The year 2015 celebrated a *century* since Einstein's proposal for *general relativity* (GR), which is an extremely successful classical theory of gravitation, providing an elegant explanation of a wide range of physical phenomena involving gravity. Its applicability ranges from local effects in the neighborhood of massive bodies, such as the gravitational redshift, to global ones, pertaining to cosmology and the evolution of the universe as a whole.

With respect to the latter, although the simple Friedmann–Lemaître–Robertson–Walker (FLRW) model for a homogeneous and isotropic universe works very well in describing several evolutionary aspects of it, nevertheless about $\sim 90\%$ of the total energy budget still remains a mystery. Only 5% of the energy budget of the cosmos, consisting of baryonic matter (that is, protons, neutrons and electrons), is well understood.[1] Unknown forms of matter (*dark matter* (DM) — 27%) and vacuum energy (*dark energy* (DE) — 71%) have to be included in order to explain the cosmological observations, in particular the ones based on cosmic light coming from a variety of sources, such as distant standard candles like 1A supernovae, cosmic microwave background (CMB) temperature fluctuations, baryon acoustic oscillations (BAOs) and weak and strong gravitational lensing data. All these models point towards the Λ (cosmological constant) cold-dark-matter (ΛCDM) model as a prototype which, in spite of its simplicity, can provide sufficient explanation for all the aforementioned data.

In this review of self-interacting dark matter (SIDM), we shall concentrate on the self-interaction (SI) aspects of DM.[2] From a particle physics point of view, DM has many candidates from physics beyond the Standard Model (SM), such as supersymmetric partners, axions, sterile neutrinos, etc. In fact the current cosmological data from Planck satellite,[3] and its predecessor WMAP,[4] on the amount of allowed DM abundance imply stringent constraints on collider searches for new particles, e.g. those implied by supersymmetry (SUSY). From an astrophysical point of view, DM is viewed simply as the "missing" amount of matter in the universe as compared with expectations from standard FLRW cosmology. It has been studied mostly in a model independent way, by performing N-body simulations of noninteracting Newtonian massive bodies. Such simulations aimed at reproducing the large scale structure of the (observable) universe, and are intimately linked with (though do not depend explicitly on) the particle mass range, nature, and physics of the particle decoupling from the primordial plasma. These properties lead to a characteristic free-streaming length of the dark candidate at the time of formation of the gravitationally bounded DM seeds. Typically, weakly interacting cold massive leptons, with masses of order $\sim 100\,\mathrm{GeV}$, decouple at nonrelativistic energy scales, implying a very small free-streaming length, far below the scale of dwarf galaxies, with the corresponding 'Jeans' mass of order $10^{-4}M_\odot$.[5] The lower bound on the mass of such DM is associated with a cutoff in the matter power spectrum at the lowest end of the free-streaming length (i.e. largest wavelength number k), which is supplemented by the effect of acoustic oscillations owing to the coupling of the radiation field with the CDM particles.[5] On the contrary, warmer (i.e. of keV mass) DM particles that decouple while still relativistic, can become nonrelativistic already at the radiation era, with free-streaming scales typically of a galaxy size, wiping out all possible structure below such a scale. Consequently, this will affect the building-up of DM halos (and the galaxies within), by lowering the inner halo mass concentration and by suppressing the number of small satellite halos.[6]

In this way, re-ionization studies based on N-body simulations of the universe at redshifts $z \sim 20$ (see Fig. 1),[7] when combined with the evidence from the WMAP observations,[4] exclude any WDM with mass less than $10\,\mathrm{keV}$ as being the dominant DM component in the universe (thereby excluding light gravitino models of DM with $m_X \simeq 5\,\mathrm{keV}$). This is so because the suppression of formation of such low mass objects in this early epoch makes the formation of primordial molecular hydrogen gas clouds very inefficient, and thus inconsistent with the large optical depth observed by the WMAP satellite.[7] Complementary constraints, confirming such exclusion regions, are those due to observations on the number of Milky Way satellites, which is about an order of magnitude greater than that predicted by WDM numerical simulations,[8] and the Ly-α forest constraints.[9]

It should be noted at this stage that such structure formation arguments can only place a lower bound on the mass of the WDM candidate. The reader should bear in mind that WDM with masses $m_X \gtrsim 100\,\mathrm{keV}$ becomes indistinguishable from CDM, as far as large-scale structure formation is concerned.[6] Hot DM (due to the three active left-handed neutrino species of the SM) is also excluded as the dominant source of DM in the universe by the upper limits for the corresponding contributions Ω_ν to the universe energy density, which according to the recent Planck 2015 data,[3] combined with lensing, BAO and Lyman α data, are bounded from above as follows:

$$\Omega_\nu h^2 = \frac{\sum_{i=1}^{3} m_i}{94.0\,\mathrm{eV}} \leq 0.0025 \tag{1}$$

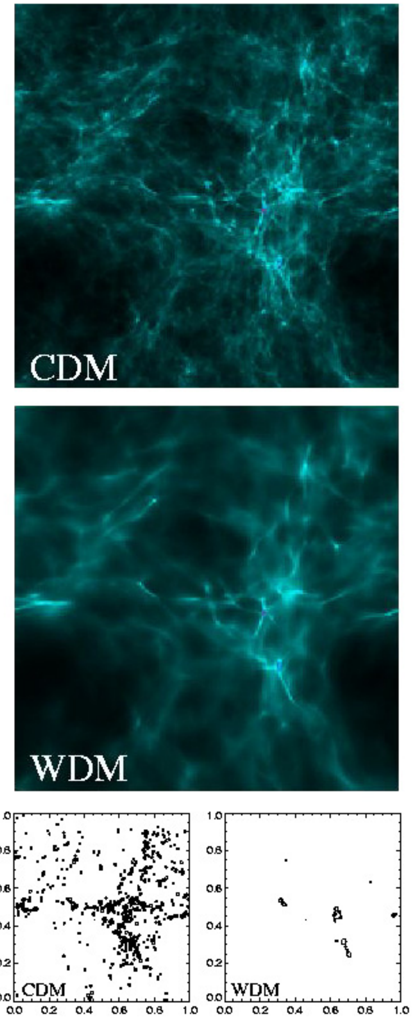

Fig. 1. Upper and Middle panels: Projected gas distributions (based on numerical simulations for structure formation at $z = 20$) in CDM (top) and warm dark matter (WDM) models with mass $m_{\rm WDM} = 10\,{\rm KeV}$ (middle). Lower bottom panels: the distribution of dark halos with mass greater than $10^5 M_\odot$ for the CDM (left) and for the WDM (right) model. There are serious discrepancies with observations therefore for the WDM model with DM mass $m_{\rm WDM} \le 10\,{\rm keV}$, in that it fails to account for the large structure of the universe at redshifts $z = 20$. Pictures taken from Ref. 7.

in a standard notation, where the energy densities are expressed in units of the critical density of the universe, and m_i denote the light neutrino masses. From (1) one also obtains a cosmological bound on the sum of the masses of the three light neutrino species $\sum_{i=1}^{3} m_i \le 0.23\,{\rm eV}$.

The exclusion of hot dark matter (HDM) and (few keV) WDM from being the dominant source of DM at the large scale universe has prompted a plethora of works, both in particle physics and astrophysics, on the CDM model, which depending on its thermal history may be constrained significantly not only by astrophysical observations but also at particle colliders, such as the large hadron collider (LHC) at CERN, assuming that DM is particle in origin.[10] One of the main reasons for the particle physicists taking a great interest in the DM problem was the so-called weakly interacting massive particle (WIMP) coincidence or 'miracle'.[11] This is associated with the fact that the observed relic abundance of DM in the universe,[3] $\Omega_\chi \sim 0.22$ for a neutral particle of mass m_χ, assumed to be thermal[a] having a freeze-out temperature[10] $k_B T \simeq m_\chi/20$ (in natural units, with k_B the Boltzmann constant), which is either stable or it has at least a proper life time longer than the age of the universe, can be achieved by particles with annihilation cross-sections to SM particles $\sigma(\chi\,\chi \to \mathrm{SM})$ of order of those of the weak interactions

$$\Omega_\chi \simeq \frac{0.1\,\mathrm{pb}\,\cdot\,\mathrm{c}}{\langle\sigma(\chi\,\chi \to \mathrm{SM}\,v\rangle} \simeq 0.22 \quad \text{for } \sigma(\chi\,\chi \to \mathrm{SM}\,v\rangle) \simeq 3\cdot 10^{-26}\,\mathrm{cm}^3\,\mathrm{s}^{-1}, \tag{2}$$

with v the relative velocity. A plethora of modern particle physics models, with DM masses ranging from a few hundreds of GeV to a few TeV, especially supersymmetric ones, have been tested against cosmological data at current colliders, such as the LHC, using (2), with no evidence for the existence of such particles. This may not come as a surprise. DM may not be thermal, e.g. it may be due to axions or other nonthermal particles, such as sterile neutrinos, and in general one may even face a situation where more than one dominant DM species exist in the universe. In the latter case, some of the stringent constraints obtained by comparing collider data to cosmology may be relaxed.[10]

In fact the ΛCDM paradigm for DM, on which the aforementioned WIMP miracle is based, in spite of offering a convincing explanation for the observed data,[3] namely CMB, BAOs, weak and strong lensing data, nevertheless it fails to account for observations at smaller scales. In particular, at galaxy scales, there are serious unresolved problems, related mainly to discrepancies in the distribution of matter in the inner halo regions of galaxies between numerical simulations based on ΛCDM and observations. These issues present serious challenges to the ΛCDM paradigm that call urgently for explanations.

Some of these problems have been identified at the 14th Marcel Grossmann Meeting and proposals for their resolutions have been presented. Some of these proposals go beyond the ΛCDM paradigm, including either SIs among the DM particles,[13–17] due to novel kind of forces exclusive to the dark sector, or studying viscous properties of DM,[18] which have important back reaction effects on the standard cosmological equations, or discussing novel DM candidates electrically charged with tiny fractions of the fundamental electric charge,[19] as well as presenting novel

[a] Although, it should be noted that nonthermal WIMP miracle models do exist in the literature.[12]

ideas on the DE front (the "running cosmic vacuum model"),[20] which challenges the ΛCDM cosmic-concordance model and can easily accommodate SIDM.

In this review, we shall concentrate on SIDM models and their galactic phenomenology. The structure of the *rapport* is the following: in the next Sec. 2, we outline the problems/challenges faced by the simulations based on the ΛCDM model as regards the observed distribution of matter at galactic scales and discuss potential resolutions, among which emphasis is given on SIs of DM. In Sec. 3, we present a particle physics model of self-interacting WDM provided by right-handed neutrinos, which addresses successfully these challenges in a way consistent with observations. The model is also consistent with the rest of the cosmological and particle physics data for right handed neutrinos. Finally, in the concluding Sec. 4, apart from recapitulating the main lessons to be learned from our discussion in the previous sections, we also report on other approaches to DM presented in this conference, associated with modifications of Friedman cosmological equations as a result of the viscous nature of DM, as well as with novel DM milicharged particles that may be searched in LHC or other experiments, and DE models based on the "running cosmic vacuum."

2. DM Challenges of the ΛCDM Model and Their Potential Resolutions via a Self-Interacting Dark Sector

There are three major problems/challenges to the ΛCDM model at galactic scales (which we can collectively call "*small-scale cosmology crisis* or *problems*"), we shall identify here, with resolution proposals discussed at MG14[13–16]:

(i) *The core-cusp problem* (or, as is also known, the cuspy-halo problem), refers to a discrepancy between the observed DM density profiles of low-mass galaxies and the density profiles predicted by cosmological N-body simulations. Characteristically, all the ΛCDM-based (DM only) simulations form DM halos which have "cuspy" DM distributions, with the density increasing steeply, i.e. as $\rho \propto r^{-1}$, at small radii. This is, e.g. evidenced in the standard Navarro–Frenk–White (NFW) DM profile.[21] On the contrary, the rotation curves of most of the observed dwarf galaxies indicate flat central density profiles ("cores").[22]

(ii) *The "missing satellite problem"* (or, as is also known, the dwarf galaxy problem), arises from a discrepancy between ΛCDM-based numerical cosmological simulations that predict the evolution of the distribution of matter in the universe — pointing towards a hierarchical clustering of DM (where smaller halos merge to form larger halos) — and observations. Although there seem to be enough observed normal-sized galaxies to account for such a numerical distribution, the number of dwarf galaxies is orders of magnitude lower than that expected from the simulations. As a concrete example, we mention that there were observed to be around 38 dwarf galaxies in the Local Group, and only

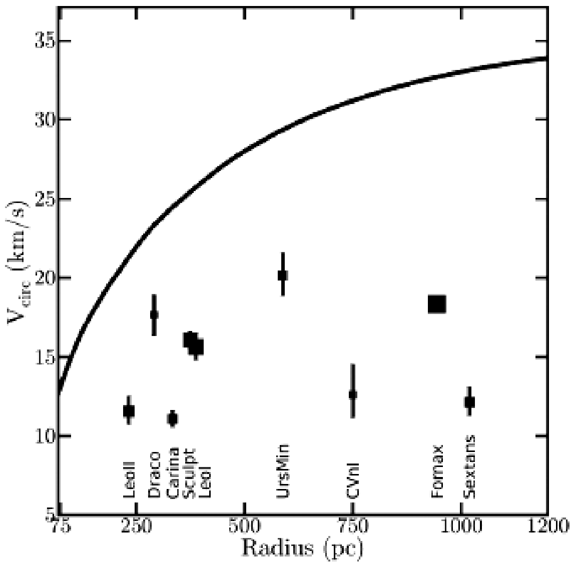

Fig. 2. The "too-big-to-fail problem": The continuous line denotes the rotation curve of typical largest subhalo of the Milky Way Galaxy, as simulated within the collisionless ΛCDM model. The data points pertain to observed circular velocities of the largest subhalos of the Milky Way at their half light radii. The discrepancy is obvious. Picture taken from Ref. 24.

around 11 orbiting the Milky Way, yet one DM simulation predicted around 500 Milky Way dwarf satellites.[23]

(iii) *The too-big-to-fail problem*, that is a discrepancy arising between the most massive subhaloes predicted in (dissipationless) ΛCDM simulations and the observed dynamics of the brightest dwarf spheroidal (dSph) galaxies in the Milky way (see Fig. 2). In other words, the ΛCDM simulations predict that the most massive subhaloes of the Milky way are too dense to host any of its bright satellites, with luminosity higher than 10^5 the luminosity of the Sun.[24]

All three problems have their root in the fact that the cold DM particles, which the ΛCDM simulations rely upon, have too short free streaming length during the epochs of galaxy formation, and therefore they form too clumped and too many structures compared to those observed.

As emphasized by Fairbairn in his talk at MG14,[13] understanding the shape and depth of the gravitational potential in dSph may have an important bearing in the understanding of these fundamental questions. In this respect, he started his talk by presenting a new method for estimating the dSph gravitational potential,[25] based on exploiting the higher order analogues of the projected Virial Theorem $2K_z + W_s = 0$, which provide global constraints on the moments of the velocity distribution ($\langle v_z^n \rangle$) by effectively integrating the spherical Jeans equations over all radii:

$$\begin{aligned} \text{second-order}: \int_0^\infty \Sigma(R)\langle v_z^2\rangle R dR &= \frac{2}{3}\int_0^\infty \nu \frac{d\Phi}{dr} r^3 dr \\ \text{fourth-order}: \int_0^\infty \Sigma(R)\langle v_z^4\rangle R dR &= \frac{2}{5}\int_0^\infty \nu(5-2\beta)\langle \mathrm{v}_r^2\rangle \frac{d\Phi}{dr} r^3 dr, \\ \int_0^\infty \Sigma(R)\langle v_z^2\rangle R^3 dR &= \frac{4}{35}\int_0^\infty \nu(7-6\beta)\langle \mathrm{v}_r^2\rangle \frac{d\Phi}{dr} r^5 dr, \end{aligned} \tag{3}$$

where r denotes the physical (three-dimensional (3D)) radius of a star from the center of its galaxy, $\nu(r)$ is the 3D number density of stars, R and $\Sigma(R) = \int_{-\infty}^{\infty} \nu(r)dz$ are projections of r and ν, respectively, onto the plane perpendicular to the line of sight (LOS); Φ is the gravitational potential and $\beta = \beta(r) = 1 - \frac{\sigma_t^2(r)}{\sigma_r^2(r)}$ is the stellar velocity anisotropy parameter, with σ_r^2 ($\sigma_t^2 = 2\sigma_\theta^2 = 2\sigma_\phi^2$) is the variance of the radial (tangential) velocity distributions. The quantities (3) carry more or less the same information as the corresponding Jeans equations.[25]

Based on such novel approaches, and defining appropriate Virial shape estimators from (3), the authors of Ref. 25 performed an illustrative analysis on the DM profile of the galaxy Sculptor, NGC 253, in the Sculptor group, using phenomenological expressions for $\nu(r)$ and the density profile $\rho(r)$, which is taken to be close to the NFW. This galaxy is a good study case, as argued by Fairbairn in his talk,[13] given that it is one of the brightest galaxies in the vicinity of the Milky Way, and is a starburst, that is, it undergoes intense star formation, which according to several astronomers is believed to have been caused by the collision with a dwarf galaxy a few hundred million years ago, disturbing its disc and started the currently observed starburst. There is also a belief that there is a supermassive black hole at its center which is slightly heavier that of the center of the Milky Way, Sagittarius A* (Sgr A*). The novel analysis of Ref. 25, where more general functions of $\beta(r)$ have been used, has demonstrated that one can fit the velocity dispersion of Sculptor using NFW profiles, leading to the conclusion that, contrary to the results of other analyses based on traditional Jeans equations, there is no extended core in the Sculptor in agreement with ΛCDM simulations. As Fairbairn emphasized in his talk, although from such a single study case one cannot make generic conclusions, nevertheless (s)he is tempted to conjecture that the dSphs do not have cores, in which case there should be no discrepancy with ΛCDM simulations. However, this point of view is not shared by the majority of the astrophysicists.

Adopting the standard point of view, several *possible solutions* to the core-cusp problem have been proposed. Many recent studies have shown that including baryonic feedback (particularly feedback from supernovae and active galactic nuclei) can "flatten out" the core of a galaxy's DM profile, since feedback-driven gas outflows produce a time-varying gravitational potential that transfers energy to the orbits of the collisionless DM particles.[26] Other works have shown that the core-cusp problem can be solved outside of the most widely accepted CDM paradigm: simulations with warm or SIDM also produce DM cores in low-mass galaxies.[27]

The missing satellite problem has two *potential solutions*.[28] One is that the smaller halos do exist but only a few of them end up becoming visible because they have not been able to attract enough baryonic matter to create a visible dwarf galaxy. In support of this, Keck observations in 2007 of eight newly discovered ultra-faint Milky Way dwarf satellites showed that six were almost exclusively composed of DM, around 99.9% (with a mass-to-light ratio of about 1000).[29] Such ultra-faint dwarfs substantially alleviate the discrepancy between the predicted and observed numbers of satellites around the Milky Way, but there are still discrepancies by a factor of about four too few dwarf galaxies over a significant range of masses. In Ref. 29, the authors argued that, if galaxy formation in low-mass DM halos is strongly suppressed after re-ionization, then the simulated circular velocity function of CDM subhalos can be brought into approximate agreement with the observed circular velocity function of Milky Way satellite galaxies. Other solutions may be that dwarf galaxies tend to be merged into or tidally stripped apart by larger galaxies due to complex interactions. This tidal stripping has been part of the problem in identifying dwarf galaxies in the first place, which is an extremely difficult task since these objects have low surface brightness and are highly diffused, so much that they are virtually unnoticeable.

Finally, the too-big-to-fail problem may also be tackled by the combined inclusion of SIs in DM, which tend to make the individual galaxies more cored and spherical, along with baryonic feedback.[14]

At this point, we stress that SIs have been argued to play an important rôle in galactic structure already in Ref. 30. The original idea of self-interacting DM (SIDM) was implemented for CDM particles with rest masses above $1\,\mathrm{MeV}/c^2$ (up to $10\,\mathrm{GeV}/c^2$), consistent with the nature of the effective interactions and the mean free paths considered in that work. This way of thinking regarding SIs was applied uniquely on DM halo scales with typical densities of $10^{-2}M_\odot/\mathrm{pc}^3$, suggesting that normalized total cross-sections of order $\sigma/m \sim 0.1$–$100\,\mathrm{cm}^2/\mathrm{g}$, would imply observational effects in the inner regions of the DM halos. It was also shown that a SIDM regime with these values of σ/m would generate shallower inner DM profiles, with a necessary reduction in the amount of sub-structures, thereby alleviating (or even solving) the core-cusp and the missing satellite problems of collisionless ΛCDM, as mentioned above. However, contemporaneously, some tension with upper limits in the DM cross-sections as obtained from lensing studies on galaxy cluster scales emerged. In a subsequent work,[31] motivated by updated analysis of the Bullet Cluster,[32] new cosmological simulations within CDM were performed, with the aim of further scrutinizing the effects of SIDM on inner halo cores of galaxies and galaxy clusters. The authors of Ref. 31 concluded that $\sigma/m \sim 0.2$ barn $\mathrm{GeV}^{-1} = 0.1\,\mathrm{cm}^2$ g^{-1} is consistent with all the observational constraints. In general, SIDM would make no difference from ΛCDM at large scales, but individual galaxies would appear more cored and spherical, with higher velocity dispersion (cf. Fig. 3). As emphasised

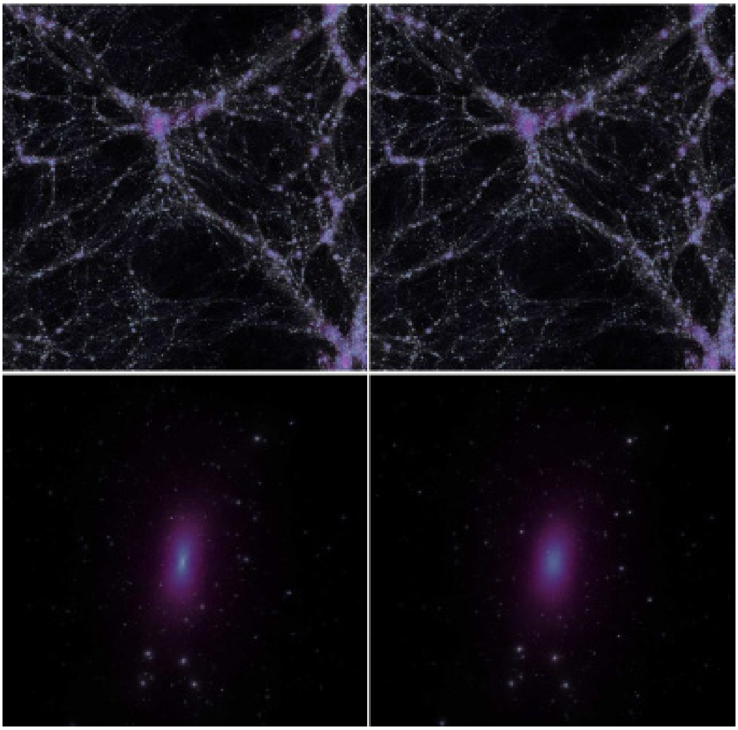

Fig. 3. Comparison of collisionless ΛCDM model simulations (left panels) with SIDM simulations with cross-section $\sigma = 1$ cm^2/g (right panels). The upper panels pertain to large scale structure, where the two models agree, while the lower panels refer to individual galaxies, where one observes that in SIDM models, galaxies appear more cored and spherical. Pictures taken from Ref. 31.

by Fairbairn (Ref. 13) and Harvey (Ref. 14) in their talks, observations from clusters, such as the above mentioned refined analyses in the bullet cluster,[32] constitute important tools to probe SIDM models.

Harvey concentrated in his talk on stringent constraints of the interaction cross-section of SIDM, by studying merging galaxies. He emphasized the need to include SIs to tackle the core-cusp and too-big-to-fail problems by reducing the central density and went on to discuss how one can probe the respective cross-sections in colliding clusters. In this latter respect, we mention the works of Ref. 33, according to which SIs can lead to both deceleration and evaporation of a DM halo when the latter moves through a background of DM particles. This results in a shift of the halo's centroid relative to the collisionless stars and galaxies. As Harvey emphasised in his talk, the substructure evaporation phenomenon is rare, and may be attributed to a short range dark force, e.g. due to a massive non-Abelian vector dark boson exchange graph in $\chi\chi \to \chi\chi$ DM scattering. This leads to isotropic scattering cross-sections σ. *Au contraire*, the exchange of long-range dark photons in the above SI, is frequent, leading to low-momentum transfer and directional scattering, in the sense that the cross-section is no longer isotropic but depends on the scattering angle θ,

$\sigma(\theta)$. This leads to substructure deceleration. By concentrating on such phenomena, harvey went on first to review (limited) constraints on SIDM cross-sections by the (conventional) study of several merging clusters of galaxies (see Fig. 4), without taking into account DM drag during the collision. Then he proceeded with defining new observables, by taking into account *the DM drag* (see Fig. 5), which yield more stringent constraints on the SIDM cross-section. In particular, the study of 72 mergers using this new technique imposed a more stringent constraint on the SIDM cross-section per unit DM mass $\sigma/m \leq 0.47\,\mathrm{cm}^2/\mathrm{g}$, which together with the lower bound $\sigma/m \geq 0.1\,\mathrm{cm}^2/\mathrm{g}$ imposed by the cosmology on galaxy scales, defines a new range for σ_{SIDM}/m

$$0.1 \leq \frac{\frac{\sigma_{\mathrm{SIDM}}}{m}}{\mathrm{cm}^2\ \mathrm{g}^{-1}} \leq 0.47 \tag{4}$$

to be considered in galactic studies. This leads to a possible resolution of the three "small scale cosmology problems" of DM.

A challenge, and some tension with the upper bound of (4) still remains,[34] as a result of observations in the Abell Cluster 3827. The observed separation between the DM halo and the stars of a galaxy moving through a region of large DM density (i.e. the core of Abell 3827 in this case), which is a characteristic feature of SIDM, suggests that this cluster may provide the first evidence of a SIDM.[14] However in Ref. 34, the authors estimated the DM SI cross-section needed to reproduce the observed effects, and argued that the sensitivity of Abell 3827 has been significantly overestimated in a previous study.[35] In fact, their basic point was that the model used in Ref. 35 to interpret the observations in terms of DM SIs failed to take into account the nonindependent development of the stars and the DM subhalo due to the initial gravitational bound of the former to the latter. Indeed, to achieve a star-subhalo separation, as observed, one would need external forces (such as SIs) comparable in strength to the gravitational attraction within the system. Another feature of the model used in Ref. 35 which the authors of Ref. 34 criticized, was the assumption that the effective DM drag force was constant throughout the evolution of the system. Such a feature might have been expected if the subhalo were on a circular orbit along the trajectory of the Abell 3827 cluster, which however is disfavored by observations. Moreover, the constant-drag-force assumption also contradicts the fact that a typical rate of DM SIs depends on the velocity of the subhalo relative to the cluster, as well as the DM density of the cluster. Both vary along the subhalo trajectory, and in fact the rate of DM SIs is negligibly small, as long as the subhalo is far away from the core of the cluster.

The corrected estimate for the SIDM cross-section per DM mass in the analysis of Ref. 34 is $\sigma_{\mathrm{SIDM}}/m \simeq 3\,\mathrm{cm}^2\ \mathrm{g}^{-1}$, when SIs result in a drag force, and $\sigma_{\mathrm{SIDM}}/m \simeq 1.5\,\mathrm{cm}^2\ \mathrm{g}^{-1}$ in the case of contact interactions, in tension with the upper bounds (4).

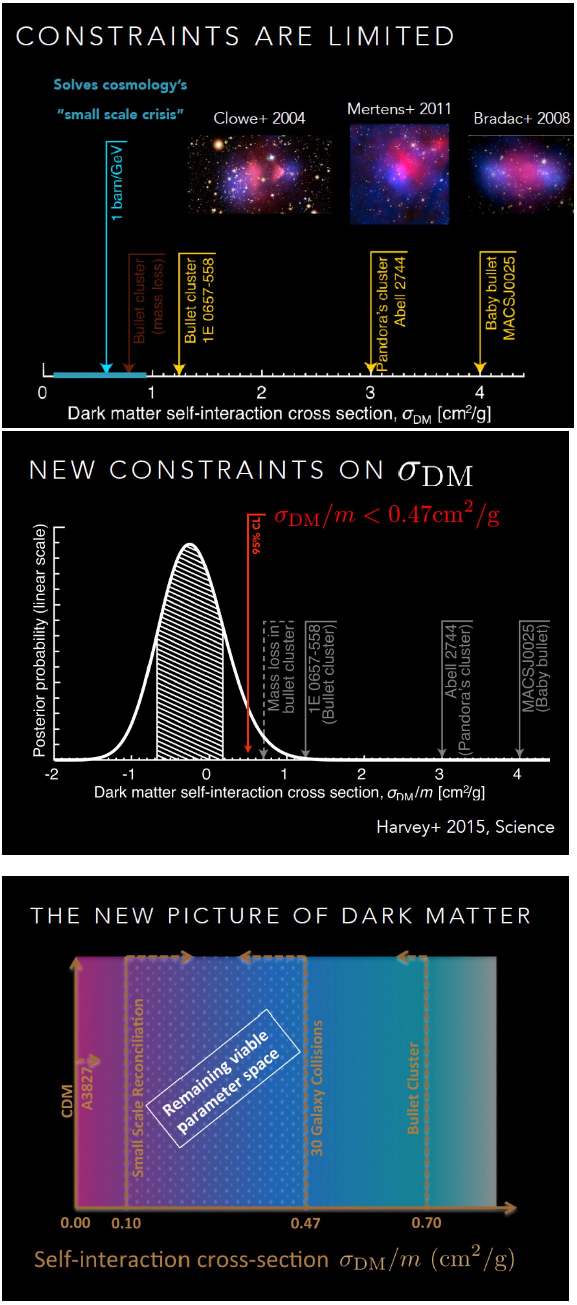

Fig. 4. Upper: Limited constraints on the SI total cross-section of DM from studies of merging clusters if drag of DM is ignored. We see that to explain the observed effects, one needs to invoke cross-sections $\sigma/m > 0.1\,{\rm cm}^2/{\rm g}$ (which is the order that **alleviates** the three "small-scale problems" mentioned previously). Middle and Lower: new constraints on SIDM after taking into account Drag of DM and the use of new relevant observables, from the study of 72 mergers. Pictures taken with permission from the talk at MG14 by Harvey, Ref. 14.

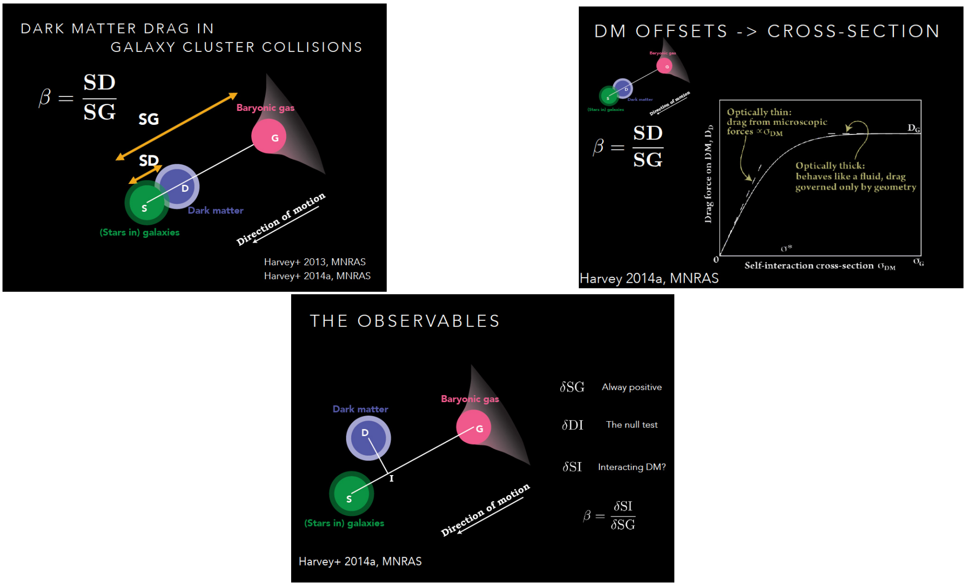

Fig. 5. Schematic view of the new observables using DM drag in colliding galaxies. Pictures taken with permission from the talk at MG14 by Harvey, Ref. 14.

The above example of Abell 3827 shows how active in research and yet inconclusive, due to both theoretical and observational challenges, is the field of SIs of DM from the generic astrophysical view point. Note however that no attempt is made in the above discussions to analyze microscopic interactions that may lead to phenomenologically acceptable SIDM cross-sections. A concrete example of SIDM, inspired from hadronic physics but applied to the dark sector, has been discussed in the talk by Fairbairn,[13] but with preliminary results so far. The example involved hidden sector glueballs in some extensions of the SM with non-Abelian gauge group in the hidden sector.[36] The model assumes no light quarks (by appropriately arranging the parameters), hence the lowest lying states are (stable) glueballs which can thus play the rôle of SIDM. The mass scale where the interaction coupling g_0 blows up is given by $\Lambda = \mu_0 e^{-8\pi^2/bg_0^2}$ and the theory is confining if $b > 0$. The mass scale μ_0 is a phenomenological parameter and may or may not be identified with the Planck mass scale M_{Pl}. The important point is that $\Lambda \ll M_{\mathrm{Pl}}$, so the effective cutoff scale is at low energies. The glueballs would have SIs as a result of the strong forces that bind them (glueball states could, e.g. be described by self-interacting scalar particles, although higher spin states are present). Dimensional analysis implies that the couplings of the various SI terms are proportional to Λ. If three to two glueball processes are ignored, the thermal relic DM density $\Omega_{\mathrm{DM}}\rho_{c0}$ (with ρ_{c0} the present era critical density of the universe) is related to Λ via:

$$\Lambda = \frac{\Omega_{\mathrm{DM}}\rho_{c0}}{Y_\infty s_{c0}}, \quad Y_\infty = \frac{3}{4}\frac{N_c^2 - 1}{g_S^\star(T = \Lambda)}, \quad \frac{\sigma}{m} \sim \Lambda^{-3}, \tag{5}$$

where N_c is the number of 'colors' in the hidden sector, s_0 the present era entropy density of the universe, and $g_S^\star(T)$ the number of degrees of freedom at temperature $T(= \Lambda)$. The simplest (vanilla) model fails to reproduce results on cosmological (early universe) DM compatible with SIDM phenomenology at galactic scales. However, $3 \to 2$ glueball processes are present, as emphasized by Fairbairn in his talk,[13] and they reduce the DM relic abundance to phenomenologically acceptable levels, as follows by solving (analytically) the corresponding Boltzmann equation

$$\frac{dn}{dt} + 3Hn = s\frac{dY}{dt} = -\langle \sigma v^2 \rangle_{3\to 2} n^3, \quad \langle \sigma v^2 \rangle_{3\to 2} \simeq \frac{16\pi}{3\Lambda^5},$$

where H is the Hubble parameter, s the entropy density and n the relic DM density. If such $3 \to 2$ processes were all there is, then one can estimate the relic DM density as

$$Y_\infty \sim 4.326 \times 10^{-10}\frac{\text{GeV}}{\Lambda}, \quad \Lambda = 0.132\frac{\frac{g_S^{\star\,2}}{3}}{g_\star^{1/6}}c^{1/3}\text{GeV},$$

and obtain phenomenologically viable scenarios for SIDM with $\sigma/m \sim 1\,\text{cm}^2/\text{g}$.

However, three to two processes lead unavoidable to $2 \to n$ glueball processes, since they are responsible for increasing the kinetic energy very rapidly, and thus it becomes energetically possible to produce multiple glueballs. Such multi-glueball processes affect severely the SIDM relic abundance in this model, complicating the problem of determining the correct Λ that could lead to the observed galaxy formation. Thus, such models of SIDM are still far from being completely studied, in the sense that a complete account of all possible effects that characterize them is still lacking. Nevertheless the involvement of non-Abelian gauge interactions in the dark sector, mimicking the successful use of SM gauge physics in the visible sector, in order to explain galactic distribution of DM is generically interesting and should be pursued further by searching for new models, or improving the existing ones, like the one described above.

Another concrete model of SIDM has been presented by C. R. Argüelles, based on work done in Refs. 16 and 17. It is based on a class of self-interacting models of massive right-handed neutrinos that exist in minimal extensions of the SM. It is important to stress that, in contrast to standard N-body simulations, the semi-analytical approach proposed in Refs. 16 and 17 includes in addition to gravity, other important physical ingredients, such as quantum (fermionic) physics and thermodynamics. Interestingly, such a scenario, in which the DM fermion (right-handed neutrino) has a mass of a order a few tens of keV, implies a universal and novel DM density profile (*compact core - dilute halo*), with important implications for the very central and halo regions of galaxies.[b] We shall discuss this model and its rôle in resolving the aforementioned three-problems of small scale cosmology in the next section.

[b]Similar core — halo DM density behavior have also been obtained within modern quantum-wave DM 3D N-body simulations.[37]

3. Self-Interacting Right-Handed Neutrino & DM Distribution in Galaxies

The right-handed neutrino model for SIDM (Ref. 16) is based on minimal (non-supersymmetric) extensions of the SM with sterile neutrinos. In such models, the DM may be provided by the lightest N_{R1} of three right-handed neutrino species that, e.g. appear in the νMSM model,[38] but this identification is not binding. However, unlike νMSM, we allow our right-handed neutrinos to be self-interacting. We introduce phenomenologically (Ref. 16) neutrino SIs through a massive-vector-meson V_μ mediator. For concreteness, we assume the fermions to be of Majorana type (nevertheless, the formalism is readily extendable to Dirac fermions). This is the common feature our model shares with the νMSM. As we shall argue in this work, there is an intriguing similarity in the allowed range (in the few tens of keV) of the sterile neutrino DM mass between the two models, despite the fact that these bounds have been obtained by quite different reasons.

The self-interacting model builds upon earlier work (Ref. 17) in which the authors argued that semi-degenerate self-gravitating fermion systems in thermodynamic equilibrium, with masses in the range of a few tens of keV, termed "*Inos*," provide good candidates for the DM in galaxies including the Milky Way. The numerical solutions for the DM density distributions arising from the original equilibrium equations in Ref. 17 showed a segregation of three physical regimes: (i) an inner core of almost constant density governed by degenerate quantum statistics; (ii) an intermediate region with a sharply decreasing density distribution followed by an extended plateau, implying the transition from quantum to classical dilute regime; (iii) an asymptotic, $\rho \propto r^{-2}$ classical Boltzmannian tail. Interestingly, while the Boltzmannian outer behavior (similar to core isothermal spheres) can account for halo observables from dwarf to big spiral galaxies, the dark quantum core may provide an alternative to the central massive black hole scenario. However, as demonstrated in Ref. 16 and argued by Argüelles in his talk,[15] the inclusion of SIs among the Inos opens up the possibility of achieving higher central degeneracy and larger compactness of the quantum core, thus explaining the dynamics of the very central star cluster around SgrA*. This has been taken as a case of detailed study in Ref. 16, where, in addition, the Inos are identified with the lightest of the right-handed neutrinos N_1 of νMSM, upon including sufficiently strong interactions in the dark sector. Other types of galaxies have also been considered in the study, and fitted successfully by the model, such as normal and big elliptical galaxies, harboring central dark compact objects of $\sim 10^9 M_\odot$. We next proceed to review briefly the features of this specific SIDM model.

The Lagrangian of the right-handed neutrino sector, including gravity, reads (in units $\hbar = c = 1$, which we use throughout here)[16]:

$$\mathcal{L} = \mathcal{L}_{\rm GR} + \mathcal{L}_{N_{R1}} + \mathcal{L}_V + \mathcal{L}_I, \tag{6}$$

where

$$\mathcal{L}_{\rm GR} = -\frac{R}{16\pi G}, \quad \mathcal{L}_{N_{R1}} = i\overline{N}_{R1}\gamma^\mu \nabla_\mu N_{R1} - \frac{1}{2} m \overline{N^c}_{R1} N_{R1},$$
$$\mathcal{L}_V = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m_V^2 V_\mu V^\mu, \quad \mathcal{L}_I = -g_V V_\mu J^\mu_V = -g_V V_\mu \overline{N}_{R1} \gamma^\mu N_{R1}, \tag{7}$$

with R the Ricci scalar for the static spherically symmetric metric background

$$g_{\mu\nu} = {\rm diag}(e^\nu, -e^\lambda, -r^2, -r^2 \sin^2\varphi), \tag{8}$$

where e^ν and e^λ depend only on the radial coordinate, r and φ denotes the polar angle. The quantity m is the mass of the sterile neutrino, $\nabla_\mu = \partial_\mu - \frac{i}{8}\omega_\mu^{ab}[\gamma_a, \gamma_b]$ is the gravitational covariant derivative acting on a Majorana spinor, with ω_μ^{ab} the spin connection. The right-handed sterile neutrinos N_{R1} satisfy the Majorana four-spinor condition, $\Psi^c = \Psi$, together with $\overline{\Psi} = \Psi^T C$, where the conjugate spinor field $\Psi^c = C\overline{\Psi}^T$ and C is the unitary ($C^\dagger = C^{-1}$) charge conjugation operator, flipping the fermion chirality, i.e. $(\Psi_L)^c = (\Psi^c)_R$ is right-handed (R), whilst $(\Psi_R)^c = (\Psi^c)_L$ is left-handed (L). The definition of chirality (handedness) is the standard one, $\Psi_{L(R)} = \frac{1}{2}\Big(1 \mp \gamma^5\Big)\Psi$, with the $+$ $(-)$ sign denoting right-(left)handed spinors, and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, with γ^μ the 4×4 Dirac matrices. The massive-vector-mesons V_μ should not be viewed as gauge bosons if the fermions are Majorana. As is well known, the Lorentz gauge condition $\partial^\mu V_\mu = 0$, which we assume here, emerges in that case as a consequence of their equations of motion. Latin indices denote pertain to flat tangent space and hence they are raised and lowered with the Minkowski η_{ab} metric. The microscopic origin of the vector meson mass m_V is not discussed here. It may well come from an appropriate Higgs mechanism in the dark sector.

For simplicity, we assume minimal-coupling of the vector field with the sterile neutrino current J^μ_V in the interaction term $\mathcal{L}_I$ (7). This current is conserved if decays of sterile neutrinos are ignored. Such a coupling may also arise from linearization of a Thirring-type four fermion vector current interaction $J^\mu_V J_{V\mu}$ by means of an auxiliary vector field A_μ (which acquires dynamics upon implementing quantum corrections). Such a four-fermion-contact-interaction model is also the effective low-energy approximation of the detailed model (7) for fermion energies much lower than the scale m_V.

In general one may add to (6) a Yukawa term, coupling the (three, in general) right-handed neutrinos to the active neutrino sector (see, e.g. the case of νMSM in Ref. 38)

$$\mathcal{L}_{\rm Yuk} = F_{\alpha I}\overline{\ell}_\alpha N_{RI}\phi^c + {\rm h.c.}, \quad I = 1, 2, 3 \tag{9}$$

where ℓ_α are the lepton doublets of the SM, $\alpha = e, \mu, \tau$, $F_{\alpha I}$ are appropriate Yukawa couplings, and ϕ^c is the SM conjugate Higgs field, i.e. $\phi^c = i\tau_2\phi^\star$, with τ_2 the 2×2 Pauli matrix. Upon considering such a coupling, one obtains the stringent X-ray and BBN constraints of the mixing angle and mass of N_{R1} depicted in Fig. 6, given that (9) implies decays of the heavy neutrinos $N_I \to \nu H$, where H denotes the

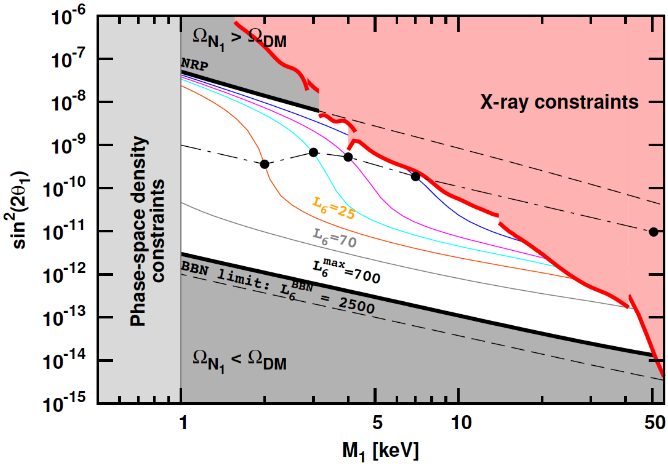

Fig. 6. Cosmological constraints on the mass (M_1) and mixing angle (θ_1) of the lightest right-handed neutrino state N_1 of the νMSM model, which plays the rôle of DM. Picture taken from Ref. 38.

Higgs excitation field, defined via: $\phi = \langle\phi\rangle + H$. In such a case J_V^μ is *not* conserved in time. However, in the context of μMSM, the lightest of the heavy neutrinos decay time is longer than the age of the universe, hence the latter can be considered as stable for all practical purposes, thus playing the rôle of DM.

For our purposes, as already mentioned, we concentrate here on this lightest neutrino and ignore such a mixing with the SM sector, setting $F_{\alpha 1} = 0$, in which case the lightest neutrino is absolutely stable. The important feature for us are the SIs of the right-handed neutrino, which will be used for ensuring phenomenologically correct values for the radius and mass of the galactic core. The inclusion of such interactions do not affect our conclusions, as justified in detail in Ref. 16. This is due to the very weak nature of the Yukawa coupling $F_{\alpha I}$, necessitated by the see saw mechanism. However, an interesting motivation to include coupling with the SM sector (active) neutrinos ν, is to obtain a possible indirect detection method for the 'Inos' through the decaying channel $N_{R1} \to \nu + \gamma$, with a potential enhancement due to their self-interacting nature. In this respect, we cannot resist of pointing out the recent observations by the Fermi satellite, providing evidence of a clear emission in the energy range 10–25 keV from the central region of the Galaxy.[39] The latter could find plausible explanation by means of a DM particle species with a mass of order $50\,\mathrm{keV/c^2}$, similar to the one obtained in Ref. 16 and discussed here.

The analysis in Ref. 16 has been performed within a relativistic mean field (RMF) approximation, according to which the system can be considered as corresponding to a static uniform matter distribution in its ground state. The vector meson and the temporal ("0") component of the corresponding current are replaced by their vacuum expectation valies (VEV), $J^0 \to \langle \overline{N_{R1}}\gamma^0 N_{R1}\rangle = \langle N_{R1}^\dagger N_{R1}\rangle$, $V_\mu \to \langle V_0\rangle = \frac{g_V}{m_V^2} J_0^V = n u_0$, with u_0 the temporal component of the (average) future-directed forward velocity ($u_\mu u^\mu = 1$) and $n = e^{-\nu/2}\langle \overline{N}_{R1}(k)\gamma^0 N_{R1}(k)\rangle = \frac{g}{(2\pi)^3}\int d^3k f(k)$, where T is the temperature of the heat bath, k_B the Boltzmann constant, $g(=1)$ a spin-degeneracy factor for the Majorana spinors, $f(k) = (\exp[(\epsilon(k)-\mu)/(k_BT)]+1)^{-1}$, with $\epsilon(k) = \sqrt{k^2+m^2} - m$ the particle kinetic energy, and μ is the chemical potential with the particle rest-energy subtracted off. The spatial components of the average current vanish, $\langle J_V^i\rangle = 0$, due to the mean field approximation, implying a nearly static and uniform matter distribution in its ground state and therefore invariant under spatial translations and rotations. In the RMF approximation, the kinetic terms of the vector field are thus not relevant, thereby allowing contact four-fermion interactions among the right-handed neutrinos of Nambu–Jona-Lasinio type to be studied in a similar way. Detailed thermodynamic equilibrium conditions (generalized with respect to the standard Tolman and Klein conditions by the effect of the meson mediator field, see Ref. 16) have been shown to be satisfied by the classical (i.e. obeying Lagrange equations of motion) interacting fields, entering the Lagrangian (6), (7).

Information about the strength of the SI coupling of the effective interactions of the fermions ('inos') and the mass of the vector-meson mediator is encoded in the quantity

$$C_V \equiv \frac{g_V^2}{m_V^2}, \tag{10}$$

which determines the order of magnitude of the corresponding cross-section, as we shall explain below. As we have discussed above, this is important for the small-scale DM distributions, an issue that we will come back to at the end of this section.

In Ref. 16, it was assumed that the SIs among the DM neutrinos occur only in the quantum regime and thus within the core, where the thermal de Broglie wavelength $\lambda_B = \frac{h}{\sqrt{2\pi m k_B T}}$ is larger than the inter-particle mean distance l at temperature T, $\lambda_B/l > 1$. That is we consider the ansatz:

$$C_V(r) = \begin{cases} C_0 & \text{at } r < r_m \text{ when } \dfrac{\lambda_B}{l > 1}, \\ 0 & \text{at } r \geq r_m \text{ when } \dfrac{\lambda_B}{l < 1}, \end{cases} \tag{11}$$

where C_0 is a positive constant and $r_m = r_c + \delta r$, with $\delta r \ll r_c$, is the core-halo matching point, with r_c the core radius and δr the thickness of the core-halo intermediate layer.[16] The region $r \geq r_m$, where the DM distribution is in a much more dilute state (i.e. $\lambda_B/l \ll 1$), marks the transition from the quantum degenerate state to the Boltzmann one.

There are three free parameters in the approach, evaluated at the galactic center: the DM temperature per unit mass ($\beta_0 = k_BT_0/m$), the degeneracy parameter at the center (depending on the chemical potential μ_0) $\theta_0 = \mu_0/(k_BT_0)$, and the SI constant C_0 (coupling). All these are embedded in a dimensionless set of combined Einstein equations of motion and thermodynamic conditions, supplemented with

Table 1. Set of right-handed SIDM model parameters for three different galaxy types analyzed in Ref. 16 that satisfy all the appropriate core and halo conditions.

m (keV)	$\overline{C}_0$	θ_0	β_0	r_c (pc)	δr (pc)	$\theta(r_m)$
Milky Way ($M_c = 4.4 \times 10^6 M_\odot$)						
47	2	3.70×10^3	1.065×10^{-7}	6.2×10^{-4}	2.1×10^{-4}	-29.3
	10^{14}	3.63×10^3	1.065×10^{-7}	6.2×10^{-4}	2.2×10^{-4}	-29.3
	10^{16}	2.8×10^3	1.065×10^{-7}	6.3×10^{-4}	2.4×10^{-4}	-29.3
350	1	$2.40 \times 10^{6(\dagger)}$	1.431×10^{-7}	1.3×10^{-6}	6.7×10^{-7}	-37.3
	10^{14}	1.27×10^5	1.104×10^{-7}	5.9×10^{-6}	9.4×10^{-7}	-37.3
	4.5×10^{18}	1.7×10^1	1.065×10^{-7}	5.9×10^{-4}	2.0×10^{-4}	-37.3
Elliptical ($M_c^{\rm cr} = 2.3 \times 10^8 M_\odot$)						
47	2	$1.76 \times 10^{5(\dagger)}$	1.7×10^{-6}	7.9×10^{-5}	3.9×10^{-5}	-31.8
	10^{14}	5.8×10^4	1.4×10^{-6}	1.4×10^{-4}	4.8×10^{-5}	-31.8
	10^{16}	1.5×10^4	1.3×10^{-6}	3.0×10^{-4}	7.0×10^{-5}	-31.8
Large Elliptical ($M_c = 1.8 \times 10^9 M_\odot$)						
47	10^{16}	1.02×10^4	3.0×10^{-6}	3.8×10^{-4}	1.8×10^{-5}	-32.8

the appropriate boundary conditions, which are solved semianalytically as detailed in Ref. 16. Following the above procedure, one can then constrain the degeneracy parameters $\beta \equiv k_B T/m = \beta_0 e^{(\nu_0 - \nu(r))/2}$ and $\theta \equiv \mu/(k_B T)$ at the core (β_0, θ_0), together with the sterile neutrino mass m and coupling C_V. The novel DM constraints, as derived from the detailed numerical analysis in Ref. 16 and outlined in Argüelles talk are summarized in Table 1 for SgrA* and some large elliptical galaxies.[15] The corresponding density profiles and degeneracy parameters are also depicted in Fig. 7, where for comparison (in the bottom panel) we also plot the corresponding results for the noninteracting case ($C_V = 0$) discussed in Ref. 17. The calculations were done for the maximum allowed range of the interaction constant $\overline{C}_0$, and the corresponding central degeneracy θ_0, temperature β_0 and ino mass m. Even if the upper limit in the sterile neutrino mass ($m \lesssim 50\,\mathrm{keV}/c^2$) is imposed by cosmological and astrophysical constraints under the assumption of mixing with the SM sector (9) within the context of the νMSM (cf. Fig. 6), the authors of Ref. 16 explored larger values of the ino mass, which is possible for sterile neutrinos that do not interact or have negligible interactions through a Higgs portal (9) with the *active* sector.

From the results presented in Table 1 one can see that for DM mass $m < 47\,\mathrm{keV}/c^2$ or $m > 350\,\mathrm{keV}/c^2$ there is no pair of parameters $(\overline{C}_0, \theta_0)$ that can fit the Milky Way observables. Whilst $m = 47\,\mathrm{keV}/c^2$ is the lower bound for the particle mass that satisfies the observed core constraints (within the observational errors), $m = 350\,\mathrm{keV}/c^2$ is the uppermost bound set by reaching the critical core mass for gravitational collapse, $M_c^{\rm cr} \propto M_{\rm Pl}^3/m^2 \approx 4.4 \times 10^6 M_\odot$ (see Ref. 40).

From Fig. 7, one can also see that the inclusion of sufficiently strong interactions in the dark sector among the sterile neutrinos can lead to significantly more compact cores and higher central degeneracies than the free case. Within this theoretical approach, one can establish a direct link between the total cross-section σ and

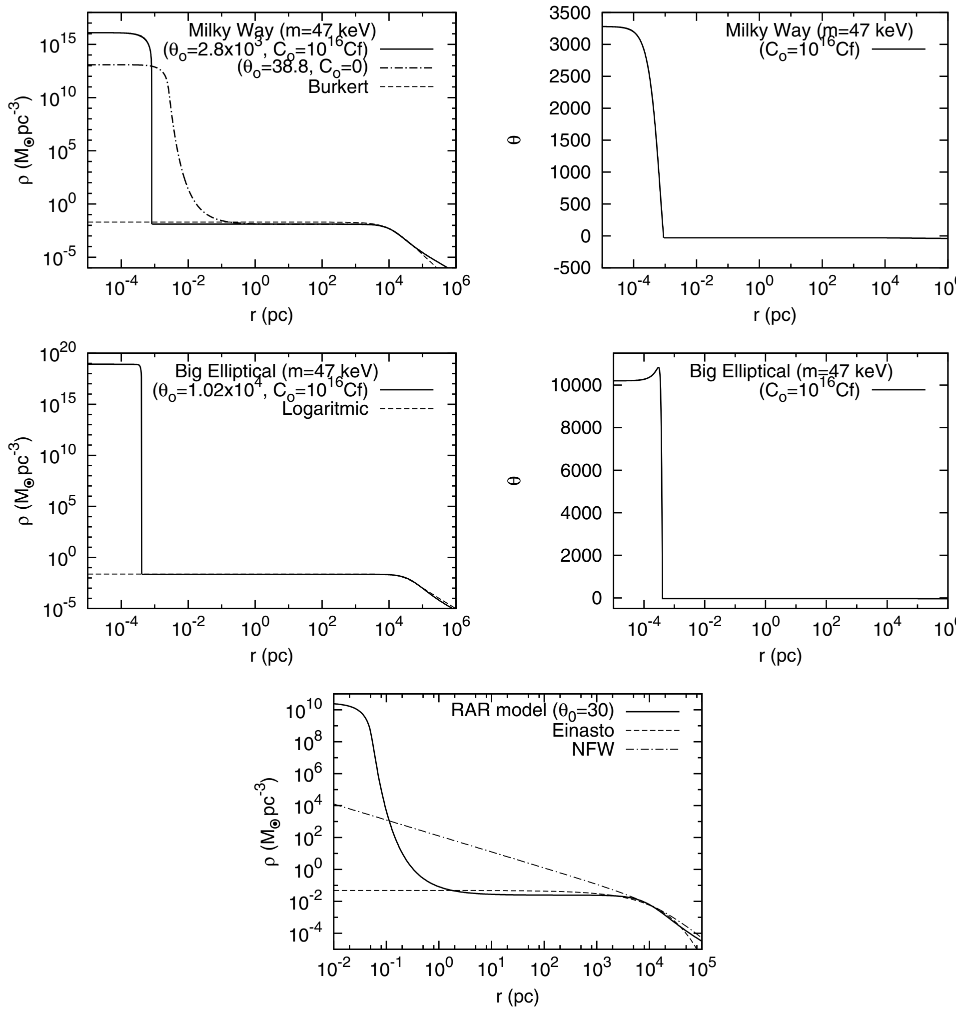

Fig. 7. First two upper panels: Mass density ρ and degeneracy θ profiles (versus distance r from the center of the galaxy) for $m = 47\,\mathrm{keV}/c^2$ in the interaction regime $\overline{C}_0 = 10^{16}$, where core and halo Milky Way observational constraints are fulfilled, compared with the noninteracting case ($\overline{C}_0 = 0$) for the same ino mass. Both the interacting and noninteracting cases use the Ruffini–Argüelles–Rueda (RAR) DM density profile,[17] which is in good agreement with Burkert profile. Middle two panels: The same as in previous case but for the case of a large elliptical galaxy. Lower panel: The RAR profile (in the noninteracting case $C_V = 0$) and its comparison with the NFW and a cored Einasto profile for a typical spiral galaxy. The picture at the bottom panel was taken from Ref. 17, while the rest from Ref. 16.

the interaction strength $C_0 = (g_V/m_V)^2$. This allows for a comparison of these results with the generic ones existing in the literature and mentioned in the previous section, on the required range of the total cross-section per unit DM mass, σ/m, in order to resolve the small-scale Cosmology "crisis" (see Fig. 4 and Eq. (4)). The total N_{R1}-N_{R1} scattering cross-section in the quantum core of the Galaxy has been

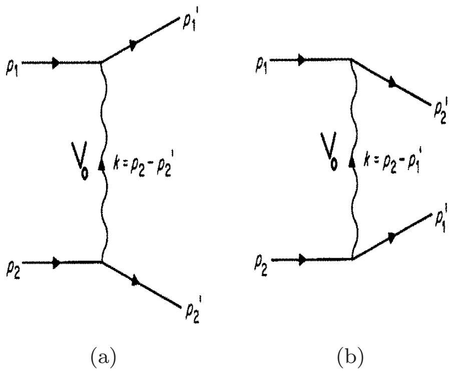

Fig. 8. Typical tree-level Feynman diagrams for calculating the cross-section for the right-handed neutrino-neutrino (straight lines) scattering via a massive vector field (wavy line) exchange. From Ref. 16.

calculated in Ref. 16 in a perturbative regime $g < 1$ for the dimensionless interaction coupling, using the diagrams of Fig. 8. In this region, the typical momentum of the DM particles is much smaller than the vector meson mass, $p^2 \ll m_V^2$, in fact they are even much smaller than the DM particle mass, due to the very low temperatures, hence effectively the SIs are replaced by four-fermion contact interactions:

$$\sigma_{\rm core}^{\rm tot} \approx \frac{\left(\frac{g_V}{m_V}\right)^4}{4^3\pi} 29m^2 \quad \left(\frac{p^2}{m^2 \ll 1}\right). \tag{12}$$

To put things in perspective, one can normalize the interaction field strength in terms of the visible sector (SM) weak interaction dimensionful coupling, the Fermi "constant" G_F, by defining and estimating the quantity $\overline{C}_V = (\frac{g_V}{m_V})^2 G_F^{-1}$, as done in Ref. 16. Thus, if, e.g. one constrain the total cross-section to the N-body simulation value $\sigma^{\rm tot}/m = 0.1\,{\rm cm}^2/{\rm g}$ (or in general to lie in the region (4) discussed above, the coupling constant $\overline{C}_V$ would be constrained to the value

$$\overline{C}_V \in (2.6 \times 10^8, 7 \times 10^8), \tag{13}$$

for ino masses in the range $m \in (47, 350)\,{\rm keV}$. It worths noticing that for $C_V \sim 10^8 G_F$, the mass of the massive-vector meson would be constrained to values $m_V \lesssim 3 \times 10^4\,{\rm keV}$, in order to satisfy $g_V \lesssim 1$ as requested by the self-consistency of the perturbation scheme we have applied to compute the cross-section.

A conservative lower bound of C_V has been obtained in Ref. 16 by requiring that the cross-section σ be sufficiently large so that a scattering probability among the inos occur at least once during the age of the galaxy ($t_{\rm age}$), that is, the product of the scattering-rate per particle (Υ) times $t_{\rm age}$ be larger than unity: $\Upsilon t_{\rm age} \gtrsim 1$. By assuming a typical $t_{\rm age} \sim 10^{16}$ s (i.e. redshift $z \sim 10$ at galaxy formation epochs), one obtains, for $m = 47\,{\rm keV}$ and $\rho_0 \sim 10^{16} M_\odot/{\rm pc}^3$ (for the Milky Way case):

$$\frac{\sigma}{m} \gtrsim 10^{-18}\,{\rm cm}^2/{\rm g}, \tag{14}$$

directly implying from the cross-section formula within the low energy approximation (12), that $C_V \gtrsim 2G_F$, that is the interaction strength cannot be smaller than the weak interaction (Fermi coupling) in the visible sector.

The value $\overline{C}_0 = 10^{16}$ constitutes a very interesting case, allowing for a successful *universal* application of the model all the way from spiral to large elliptical galaxies. However, if the interaction constant is forced to agree with the N-body simulation results for the total DM cross-section (i.e. $\overline{C}_0 = 7 \times 10^8$ for $m = 47\,\text{keV}$), then the applicability of the model is reduced up to elliptical galaxies with dark compact cores of $\sim 2 \times 10^8 M_\odot$. It was further remarked in Ref. 16 that, as the value of the coupling constant $\overline{C}_0$ increases from unity, the contributions to the total energy and pressure due to the meson-vector field ($\sim C_0 n^2$) becomes more and more relevant. For example, as can be seen from Table 1, in the Milky Way case, for $\overline{C}_0 \sim 10^{14}$ and $m = 47$ keV, a somewhat lower value for the central degeneracy is needed to acquire the same core mass as in the $\overline{C}_0 \sim 1$ regime. Or equivalently, if the same central degeneracy as in $\overline{C}_0 \sim 1$ case is applied, an increase of $\sim$ few % in the mass of the core M_c would appear.

As stressed in Refs. 15 and 16, the inclusion of baryonic matter is not expected to change the basic conclusions that the introduction of WDM fermion SIs affects the core/halo structure and in particular induces higher central degeneracies and higher compactness of the inner quantum core of galaxies. Moreover, the model provides a natural resolution to the core-cusp problem of DM at small scales, because the density profiles based on fermionic phase-space distributions develop always an extended plateau on halo scales, in a way that resemble Burkert or cored Einasto profiles (cf. Fig. 7). Another important feature is the fact that the right-handed neutrino DM mass is 'colder' by a few keV as compared to most of the WDM models available in the literature, which implies that the model does not suffer from such standard WDM problems (see for instance discussions on exclusion of WDM mentioned in the introduction of the review). It goes without saying, though, that the presence of self-interacting right-handed neutrinos with a mass range of a few tens of keV may co-exist harmonically with other types of DM, given that the latter may consist (like ordinary matter) of more than one dominant species.

A final observation regarding the range of the self-interacting ino masses, $m \geq 47\,\text{keV}/c^2$ was stressed in Ref. 16. This is, by identifying the inos with the (lightest) right-handed neutrino of the νMSM model,[38] the latter should have a very weak mixing angle with the SM lepton sector, and its mass be less than $50\,\text{keV}/c^2$, otherwise the model would not be consistent with current phenomenology, as can be seen from Fig. 6. The above considerations, then, leave a narrow range of the self-interacting 'ino' mass $47 \leq m \leq 50\,\text{keV}/c^2$, for the right-handed neutrino to play both a rôle as a WDM candidate and a provider of a core-halo galactic structure in better accordance with observations. It was finally pointed out in Ref. 16, that such constraints will be alleviated if any mixing of the ino with the SM sector is avoided, making this issue therefore interesting to look for independent tests of this model, for instance, in neutrino oscillation or other relevant particle physics experiments.

4. Conclusions and Outlook

There have been very interesting results lately, some of which have been reviewed here based on excellent (invited) presentations in the interacting DM session of MG14, concerning the rôle of self-interactions in the dark matter sector of the universe in bridging the gap between observations and the numerical simulations based on the ΛCDM paradigm.

We have heard talks on how interacting DM models may tackle the three basic challenges for N-body simulations based on ΛCDM cosmology, associated with the DM distribution in galaxies ('small-scale Cosmology crisis). We have also seen how the situation at present is far from being conclusive, given that there are always observed cases, like the aforementioned Abell 3827 cluster, which challenge even the most successful of models.

DM may, like matter, consist of more than one dominant species, which co-exist harmonically at various scales, in such a way that, for instance CDM may provide a good explanations for large scale structure, but other species, such as right-handed neutrinos or dark glueballs (to name two concrete examples that have been discussed here) play also an important rôle for explaining the observed details in core-halo structures of a galaxy. In this respect, one should also mention another class of interesting DM models, that of *inelastic dark matter* (IDM),[41] according to which there are DM candidates with electroweak interactions that scatter inelastically off matter nuclei in such a way so as to be able to provide explanations for the DAMA signal,[42] while not contradicting the (negative) results of other direct detection searches. One should also bear in mind that more mundane astrophysical explanations may be in operation together with SIs, in order to account for a complete set of observations.

Before closing we should also mention three invited talks on the dark sector of the universe, that suggest serious modifications to the ΛCDM model, which though appear to be in agreement with standard phenomenology, make interesting predictions (as argued by the presenters of the respective talks) and can accommodate SIDM models in their structure. As we have heard from Tetradis,[18] DM may constitute a viscous fluid as a consequence of back-reaction effects of matter perturbations (of sufficiently long wavelengths) on the cosmological (Friedman) equations. Dissipative damping of velocity perturbations by bulk and shear viscosity in the dark sector can affect the history of the universe as well as the large scale structure. Up to scales $k \sim 0.2\,\mathrm{h/Mpc}$ (that is distance scales of $10\,\mathrm{Mpc}$ in order of magnitude) there is very good agreement of such ideas with the matter power spectrum from large N-body simulations, as discussed in the talk. However, as we discussed here, the latter fail to account for the observed DM distributions in galaxies, and hence in our opinion the 'small-scale-cosmology crisis issues' were not addressed. It would be very interesting to see how this approach can tackle them, for instance how it can accommodate SIDM models. In principle such SIDM models can be included in the approach, given that the latter is pretty generic for long wavelength matter perturbations, and the whole analysis is based on fluid dynamics and effective field theories, which are both well tested theoretical frameworks. The challenge of a concrete microscopic models to be matched with the viscous DM approach is therefore a very interesting one.

In addition to generic cosmological DM models, a particularly interesting candidate from particle physics of DM (which characterizes certain particle physics models some of them within the framework of higher dimensional space times, and which was discussed in the session from Pinfold[19] from the point of view of its experimental detection at LHC[43]), is that of millicharge DM, that is, electrically charged particles with a tiny fraction of the electrons charge, $q < 10^{-3}e$, which may be stable enough, and characterized by weak (due to the smallness of their charge) electromagnetic interactions, so that they may constitute reasonable candidates for DM. Pinfold, who is the spokesperson of the MoEDAL experiment at the LHC, that searches for highly ionizing particles,[44] presented plans for the installation of new devices in the general area of the experiment, or in other 'cosmic' versions of it, which would be capable of detecting such millicharge particles. Millicharge DM alone is unlikely to be able to resolve the large and small scale structure of the cosmos, nevertheless may coexist with other DM particles, as mentioned earlier, and hence it worths searching for it, especially because it is also predicted by theoretical models.

Finally, one should accommodate in all the above models the DE sector, which after all dominates the energy budget of the Cosmos in the current era.[3] Leaving aside interesting suggestions in the literature that DM and DE may be entangled, the complete lack at present of a microscopic understanding of the origin and nature of DE makes this topic quite challenging. One particularly interesting, and unconventional, model for DE was presented in our session by Sola,[20] who was one of the pioneers of the so-called 'running cosmic vacuum' model. The latter model involves a running (with the cosmic time) vacuum energy, in a way reminiscent of a 'renormalization group (RG) scale' running of couplings in a field theory model. In such a model, the temporal evolution of the effective vacuum energy density $\rho_\Lambda(t) = \rho_\Lambda(\mu_c(t))$ is assumed inherited from its dependence on a characteristic cosmic scale variable $\mu_c = \mu_c(t)$, which is thus analogous to a running (mass) scale in the RG approach. A natural candidate for such scale in FLRW cosmology is the Hubble parameter $H(t)$ and the proposed RG equation is[20]:

$$\frac{d\rho_\Lambda(t)}{d\ln H^2} = \frac{1}{(4\pi)^2}\sum_i \left[a_i M_i^2 H^2 + b_i H^4 + c_i \frac{H^6}{M_i^2} + \cdots\right]. \tag{15}$$

In general μ_c^2 can be associated to a linear combination of H^2 and $\dot{H}$, in which there is a richer structure on the right-hand-side of (15). The solutions of such RG equations for the vacuum energy density read:

$$\rho_\Lambda(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha \frac{H^4}{H_I^2}\right), \tag{16}$$

where c_0 is an integration constant which can be fixed from the low energy data of the current universe,[20] and the other two dimensionless coefficients are given as follows:

$$\nu = \frac{1}{48\pi^2}\sum_{i=F,B} a_i \frac{M_i^2}{M_{\rm pl}^2}, \quad \alpha = \frac{1}{96\pi^2}\frac{H_I^2}{M_{\rm Pl}^2}\sum_{i=F,B} b_i. \tag{17}$$

where $i = F, B$ runs over bosonic and fermionic species of the matter quantum-field theory. Despite the time dependence of the vacuum, its equation-of-state remains that of a cosmological constant, $w_\Lambda = -1$, while the matter excitations over the running cosmic vacuum have an equation-of- state $w_m \neq 0$, as a consequence of an exchange between energy and matter in the model. The flow equation for matter excitations reads (the overdot denotes derivative with respect to the cosmic time):

$$\dot{\rho}_m + 3(1 + \omega_m) H \rho_m = -\dot{\rho_\Lambda}, \tag{18}$$

from which one may derive interesting scaling laws for the Hubble parameter at various eras. Such models constitute challenges for the prediction of the cosmic concordance ΛCDM model, as emphasized in the talk, and discussed in more detail recently in Ref. 20. Given that the matter content of the running vacuum model was not specified, it can naturally accommodate SIDM, thus offering interesting new perspectives in the small-scale cosmology. This remains an interesting avenue of research to be pursued.

In closing, we would like to re-iterate our point of view that the DM and DE aspects of the universe present serious challenges to current wisdom. Despite the popularity of the ΛCDM model, there are many unresolved problems associated not only with the galactic structure and the DM distribution in galaxies, but also with the very nature and microscopic origin of the dark sector, which the generic and simple ΛCDM paradigm does not 'even attempt' to answer. One should definitely go beyond ΛCDM for a variety of reasons, both observational and foundational. Using (self) interacting dark matter models, which constituted the topic of our discussion here, is one way forward in this respect, and appears not only natural from the point of view of microscopic particle physics models which may contain complicated hidden sectors, but also desirable, as it seems that such models can resolve several of the small-scale cosmology problems. Of course, they themselves have their own open issues, but it is our strong belief that simple and natural particle physics inspired candidates of DM, such as self-interacting right handed neutrinos or glueball DM, discussed here, may play an important role not only in the microcosmos (particle physics scales) but also in the macrosmos (galactic and large scale structure of the universe).

The spectacular advances in instrumentation and the construction of high precision experimental apparatuses in both particle physics and astrophysics, which we have witnessed during the past decade and which seem to continue at a greater pace for the years to come, make the fields of particle physics and astrophysics walk hand in hand in our quest for a better understanding of the universe we live in. Soon we may have some answers concerning the nature of its dark sector. Having mentioned keV-mass right-handed neutrinos as DM candidates, that play a crucial rôle in the galactic structure, motivates the search for such particles in next generation proposed experiments, such as SHIP,[45] provided of course that there is appreciable mixing between the hidden and dark sectors, as happens in the νMSM model.[38] *To be continued....*

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