LINGUISTIC MODEL OF AFFINITY GROUPING TO THE STUDY OF POVERTY

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ABSTRACT

The problem of getting an homogeneous group of elements linked to various kinds of fuzzy relations is presented often in several decision areas. It is possible to solve it by applying the affinity concept that allows grouping items that meet certain characteristics in a degree.

Poverty is a key issue in the analysis of the countries' social situation. This has encouraged several researches in order to quantify this phenomenon, to characterize it and to compare it intertemporally into the country or with other nations.

To target households considered poor due to unsatisfied basic needs it is useful to find an optimal segment, resulting from the crossing of several criteria that identify better the shortcomings of these households to group them by affinity.

In this paper we introduce a linguistic model that uses affinity theory that permits grouping poor households depending on the degree of dissatisfaction of certain basic needs and the importance of the application of public policies to diminish them.

Keywords: *linguistic model, fuzzy linguistic relations, affinity grouping, Galois lattice, poverty measures, basic needs*

1. INTRODUCTION

The fuzzy relations between two sets of elements are part of the social and economic status of individuals. Studying them makes it possible to look for new ways to solve matters of decision in these areas. It allows raising and solving decision problems, such as allocation, grouping and sorting individuals, resources, investment and financing funds.

In particular, grouping in an homogeneous way elements of a set that meet certain characteristics in different degrees is a frequent problem in many situations for those who must take decisions (Gil Aluja, 1999).

When we want to allocate resources in an efficient way or to manage adequately certain public policies or marketing strategies, it is essential to identify those individuals with similar characteristics. In these cases before adopting a decision it is very useful to form affine subgroups.

Grouping must be such as to allow, not only to assemble agents or objects as homogeneous as possible, but also to establish a structure consistent with the objectives pursued, trying to collect at each group as many of them, and to make them compatible with certain requirements previously established. The affinity theory enables this kind of grouping using flexible and adaptive algorithms to solve these problems (Gil Aluja, 1999).

If we consider an environment of uncertainty it is appropriate to represent each item using a fuzzy subset whose referential is made of the characteristics or qualities that configure the aspects that defines the homogeneity looked for.

Poverty is a key issue in the analysis of the countries´ social situation. This has encouraged several researches to quantify this phenomenon, to characterize it and to compare it intertemporally into the country or with other nations (INDEC, 2003).

To implement efficiently policies that will help to improve the situation of poverty it is convenient that the households considered poor could be grouped according to some criterion. The problem consist in finding an optimal segment, which results from intersecting several criteria that identify better the households´ needs to group them by affinity (Lazzari & Fernandez, 2008).

In this paper we present a linguistic model that uses affinity theory that makes possible to group households depending on the degree of dissatisfaction of certain basic needs and the relevance of the application of public policies to diminish them.

This segmentation will permit characterizing better poor households and will help to achieve a more efficient allocation of available resources to diminish these population's needs.

The paper is organized as follows. In Section 2 we present a review of the affinity concept and an algorithm to get the affinity subrelations from a fuzzy binary relation. In Section 3 we introduce the concept of fuzzy linguistic binary relations.

In Section 4 we apply the affinity theory to group poor households from the level of dissatisfaction of basic needs (BN) considered at this research and we introduce a model to link households considered poor with the need to implement public policies to help to mitigate the BN dissatisfaction and based on that criterion to group them by affinity.

Finally, in Section 5 some concluding remarks are pointed out.

2. AFFINITY CONCEPT

The problem of obtaining an homogeneous group of elements connected to various kinds of fuzzy relations is presented frequently in many decision-making areas, where many times it is necessary to put together elements in blocks with different appearance. This problem can be solved by the achievement of the affinity classes and the corresponding Galois lattice.

Gil Aluja (1999) defines the affinities as "... those homogeneous groupings at certain levels, structured in a certain order, which link elements of two sets of different nature, related by the very nature of the phenomena that they represent."

The affinity notion (Kaufmann & Gil Aluja, 1991) arises from the need to be able to approach the study of relationships represented through rectangular matrixes that allow linking the elements of a set with those of another. Mathematically it consists in obtaining maximum subrelations, but it is not similarity because they are relations established between elements of two different sets, or to find a coverage of a relationship $R \subseteq H \times T$. It can be shown that the subrelations of this coverage lead to a Galois lattice (Gil Aluja, 1999; Kaufmann & Gil Aluja, 1993).

There are three aspects that configure the affinity concept. The first one is that the homogeneity of each group is linked to the chosen level, in other words, depending on the characteristics; a standard that defines a threshold from which the searched affinity will be assigned. This is reflected on the possibility of obtaining from α-cuts of a fuzzy relationship, a range of Boolean matrixes capable of allowing the adaptability needed to form clusters with the desired levels of homogeneity.

The second one expresses the need that the elements of each set shall be linked to each other by certain rules of nature or by human will.

The third one requires the construction of a constitutive structure with a certain order that could allow the subsequent decision. The purpose of the grouping and the type and strength of the relationship between the elements of these sets will determine all the possible groupings uniquely (Gil Aluja, 1999).

2.1. Algorithm For Obtaining Affinities

To obtain the affinity subrelations we begin with the formation of fuzzy sets that define an object through certain characteristics or features. These fuzzy sets get together to form a fuzzy relation that describes the agents, elements or products to be grouped by affinity, where the different valuations about the characteristics considered are included.

With the objective of operationalizing the affinity concept a procedure for obtaining from a fuzzy relation $R : H \times T \rightarrow [0,1]$ will be established, the maximum subrelation or those affinity looked for (Gil Lafuente, 2001; Lazzari, 1999).

i) It is established a minimum level at which we consider the existence of affinity for each characteristic, that sets a limit or threshold $\alpha \in [0,1]$.

ii) It is obtained the α-cut of the relation R: $R_\alpha = \{(x, y) \in H \times T / \mu_R(x, y) \geq \alpha\}$. Notice that R_α is a crisp relation.

iii) The power set of H is calculated. $\#(\wp(H)) = 2^n$, where n is the cardinal, in other words, the number of elements of H .

iv) Each element of $\wp(H)$ is correlated with the elements of the set T, which are related with a level greater than or equal to the one selected in i)[1].

v) The empty subsets and those included into another are discarded, and we obtain complete and maximum matrixes.

The subsets obtained are the so-called affinity subrelations, that form a Galois lattice, which apart from showing a range of homogeneous groupings, it allows structuring them in a perfectly way (Gil Aluja, 1999; Gil Lafuente, 2001).

Thus the agents or elements of set H are grouped by common characteristics at a level greater than or equal to the one chosen. The level to be considered is arbitrary and depends on the case that is being analyzed. They can also be studied the affinities at different levels.

3. FUZZY LINGUISTIC APPROACH

The fuzzy linguistic approach is a technique appropriate to deal with qualitative aspects of problems (Zadeh, 1975).

We consider a finite and totally ordered label set $L=\{l_0, l_1, .., l_t\}$, in the usual sense: $l_i \geq l_j$ if $i \geq j$. Each term l_i represents a possible value for a linguistic variable.

There is an intermediate label which represents "approximately 0.5", and the rest of labels are defined around it in a symmetric way; so, the number of labels, t+1, will be odd and not more than 11 or 13 (Xu, 2008; Delgado et al., 1999).

In addition, the term set satisfies the following properties (Martínez et al., 2008; Herrera-Viedma et al., 2006; Delgado et al., 1999; among others):

- There is the negation operator: NEG $(l_i) = l_j$ such that $j = t - i$.
- There is the maximization operator: MAX $(l_i, l_j) = l_i$ si $l_i \geq l_j$.
- There is the minimization operator: MIN $(l_i, l_j) = l_i$ si $l_i \leq l_j$.

The semantic of labels is given by fuzzy numbers on the $[0,1]$ interval, which are described by membership functions. According to Zadeh (1975), fuzzy sets are the most appropriate tool for this purpose. In this paper we use trapecial fuzzy numbers with a representation based on parameters of its membership function (Bonissone & Decker, 1986). This representation is achieved by the 4-tuple (a, b, c, d), where a and b indicate the interval in which the membership function values is 1, c and d indicate the left and right widths of the distribution.

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[1] It is also called "connection to the right" by Gil Aluja (1999)

The terms of the set L will be used to express the level of dissatisfaction of de BN of the households and to show the requirement of apply public policies to diminish this lacks.

Now, we will extend the concept of fuzzy binary relations to fuzzy linguistic binary relations.

Assuming this linguistic framework and two finite sets H and T, the fuzzy linguistic binary relations are defined.

Definition 1. We say that $R \subseteq H \times T$ is a *fuzzy linguistic binary relations* based on L if and only if R is a fuzzy binary relation with membership function $\mu_R : H \times T \to L$.

Any fuzzy linguistic binary relation R based on L can be associated with a matrix $n\,x\,m$: $M_R = (r_{ij}),\ i = 1,..., n;\ j = 1,..., m$ where $r_{ij} = \mu_R(x_i, x_j) \in L$.

4. MODEL FOR THE STUDY OF POVERTY

4.1. BASIC NEEDS

In Latin America, the direct method more widely used for measuring poverty is known as the "Unsatisfied Basic Needs" or UBN. This method consists in checking whether the households have met a number of requirements previously defined and considers that a household is poor if they have not succeeded in accomplishing them. This method uses only *ex post* information, since it does not consider the ability of the household to meet these needs in the future (Feres & Mancero, 2001).

It refers to those symptoms which point out the lack of access to certain types of goods or services such as housing, drinking water, electricity, education and health among others. This method requires the definition of minimum levels that indicates a subjective valuation of the different degrees of satisfaction of the needs considered as basic at some stage of development of a society. This method will consider "poor" those households that fail to meet some of the needs defined as basic (Minujin, 1995).

That is how poor households are classified if they fail to meet some of their needs and welfare is directly related to the satisfaction of the BN. The indicators provide detailed information about the kind of shortages that they have and they are useful when identifying target groups for policies that could diminish these needs (Feres & Mancero, 2001).

One of the disadvantages of this method is that synthesizing into a single indicator the diverse needs and the degree of how they are met, requires a lot of subjectivity when setting the "cut" level of each variable. Another aspect to consider is that in practice it manages to cover partially the different dimensions surrounding the poverty phenomenon (Feres & Mancero, 2001).

The available information about the level of dissatisfaction of the BN will allow us to group households. To identify properly population's needs and to target their necessities will help to achieve a more efficient allocation of public resources available for social assistance.

4.2. HOUSEHOLDS – BASIC NEEDS MATRIX

We will show the implementation of the proposed model in Argentina. With the information collected by the Argentineans National Census and the Household Permanent Survey a fuzzy linguistic binary relation R based on L is built, whose rows are the households surveyed and the columns are the basic needs considered (Lazzari & Fernandez, 2008).

Each element of the relation R will indicate the linguistic valuation of the level of dissatisfaction of each BN considered for the household in question, expressed through a linguistic term of L.

In this paper the linguistic term set used to value the degree of dissatisfaction of each BN is: $L = \{l_0, l_1, l_2, l_3, l_4, l_5, l_6\}$ (Table 1) and the Basic Needs considered for each household will be those that are shown in table 2 (used by de INDEC in Argentina since de 80´s).

Table 1. Linguistic Labels

Label	Meaning	Semantic
l_0	null	(0.00, 0.00, 0.00, 0.00)
l_1	very low	(0.05, 0.10, 0.05, 0.10)
l_2	low	(0.20, 0.30, 0.10, 0.15)
l_3	medium	(0.45, 0.55, 0.15, 0.15)
l_4	high	(0.70, 0.80, 0.15, 0.10)
l_5	very high	(0.90, 0.95, 0.10, 0.05)
l_6	absolute	(1.00, 1.00, 0.00, 0.00)

Table 2. Basic Needs

	Components	Indicators
1	Overcrowding	Households with three or more persons per room
2	Housing	Type of housing. Material of the floors. Material of the outer walls. Outside cover of the roof . Ceiling. Place for cooking with water installation. Cooking fuel. Home and / or ground ownership.

3	Sanitary conditions	Accessibility to safe drinking water. Bathroom or latrine and its exclusivity. WC with water discharge.
4	School attendance	School attendance of children between 6 and 12 years.
5	Ability of subsistence	More than four people per occupied member. Head of household without third year of primary school complete.

To make clear the methodology to be applied a case of a little number of households is considered as well as the five BN of Table 2.

Let $H = \{ h_1, h_2, h_3, h_4, h_5 \}$ be the set of households and $T = \{ t_1, t_2, t_3, t_4, t_5 \}$ be the set of BN.

Where t_1: Overcrowding; t_2: Housing; t_3: Sanitary conditions; t_4: School attendance and t_5: Ability of subsistence.

With the available information we construct a matrix associated with the relation $R : HxT \to L$ (Figure 1).

$$\begin{array}{c c} R & \begin{matrix} t_1 & t_2 & t_3 & t_4 & t_5 \end{matrix} \\ \begin{matrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{matrix} & \begin{pmatrix} l_3 & l_6 & l_5 & l_2 & l_5 \\ l_2 & l_4 & l_2 & l_0 & l_5 \\ l_0 & l_2 & l_3 & l_0 & l_4 \\ l_1 & l_3 & l_4 & l_1 & l_3 \\ l_0 & l_1 & l_1 & l_0 & l_1 \end{pmatrix} \end{array}$$

Figure 1. UBN Matrix

To understand the significance of R we can give an example: the label l_2 on the NB t_2 ("housing") of the household h_3 shows that this household has "low dissatisfaction regarding housing conditions."

4.3. GROUPING POOR HOUSEHOLDS ACCORDING TO UBN

Once the matrix R is raised, the affinities can be studied to group households according to common characteristics. This information can be used to segment the population or to understand better the surveyed households` status for the purpose of implementing differentiated social policies.

Having into account the characteristics of the linguistic framework adopted in this paper to define the fuzzy linguistic binary relations, the procedure to find the affinities is the one shown at 2.1.

i) The consulted experts consider convenient to study the affinities at a level greater than or equal to "high" (l_4).

ii) A crisp relation of level l_4 is obtained. It is defined by: $R_{l_4} = \{(x, y) \in HxT / \mu_R(x, y) \geq l_4\}$ (Figure 2).

$$\begin{array}{c|c} R_{l_4} & \begin{array}{ccccc} t_1 & t_2 & t_3 & t_4 & t_5 \end{array} \\ \begin{array}{c} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{array} & \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Figure 2. Boolean matrix at level l_4

iii) We obtain the power set of H , $\#(\wp(H)) = 32$.

$\wp(H)$ = { $\varnothing$, $\{h_1\}$,$\{h_2\}$, $\{h_3\}$, $\{h_4\}$, $\{h_5\}$, $\{h_1, h_2\}$, $\{h_1, h_3\}$, $\{h_1, h_4\}$, $\{h_1, h_5\}$, $\{h_2, h_3\}$, $\{h_2, h_4\}$, $\{h_2, h_5\}$, $\{h_3,h_4\}$, $\{h_3, h_5\}$, $\{h_4, h_5\}$, $\{h_1, h_3, h_5\}$, $\{h_1, h_3, h_4\}$, $\{h_1, h_2, h_3\}$, $\{h_1, h_2,h_4\}$, $\{h_1, h_2, h_5\}$, $\{h_2, h_3, h_4\}$, $\{h_2, h_3, h_5\}$, $\{h_3, h_4, h_5\}$, $\{h_1, h_4, h_5\}$, $\{h_2, h_4, h_5\}$, $\{h_1, h_2, h_3, h_4\}$, $\{h_1, h_2, h_3, h_5\}$, $\{h_2, h_3, h_4, h_5\}$, $\{h_1, h_3, h_4, h_5\}$, $\{h_1, h_2, h_4, h_5\}$, $\{h_1, h_2, h_3, h_4, h_5\}$ }.

iv) Each element of $\wp(H)$ is correlated with the elements of the set *T* which are associated at a level greater than or equal to "high" (Table 3).

Table 3. $\wp(H)$ and its correlation at a level greater than or equal to “high”

$\varnothing$	$\{t_1, t_2, t_3, t_4, t_5\}$	$\{h_1, h_3, h_5\}$	$\varnothing$
$\{h_1\}$	$\{t_2, t_3, t_5)$	$\{h_1, h_3, h_4\}$	$\varnothing$
$\{h_2\}$	$\{t_2, t_5\}$	$\{h_1, h_2, h_3\}$	$\{t_5\}$
$\{h_3\}$	$\{t_5\}$	$\{h_1, h_2, h_4\}$	$\varnothing$
$\{h_4\}$	$\{t_3\}$	$\{h_1, h_2, h_5\}$	$\varnothing$
$\{h_5\}$	$\varnothing$	$\{h_2, h_3, h_4\}$	$\varnothing$
$\{h_1, h_2\}$	$\{t_2, t_5\}$	$\{h_2, h_3, h_5\}$	$\varnothing$
$\{h_1, h_3\}$	$\{t_5\}$	$\{h_3, h_4, h_5\}$	$\varnothing$
$\{h_1, h_4\}$	$\{t_3\}$	$\{h_1, h_4, h_5\}$	$\varnothing$
$\{h_1, h_5\}$	$\varnothing$	$\{h_2, h_4, h_5\}$	$\varnothing$
$\{h_2, h_3\}$	$\{t_5\}$	$\{h_1, h_2, h_3, h_4\}$	$\varnothing$
$\{h_2, h_4\}$	$\varnothing$	$\{h_1, h_2, h_3, h_5\}$	$\varnothing$
$\{h_2, h_5\}$	$\varnothing$	$\{h_2, h_3, h_4, h_5\}$	$\varnothing$
$\{h_3, h_4\}$	$\varnothing$	$\{h_1, h_3, h_4, h_5\}$	$\varnothing$
$\{h_3, h_5\}$	$\varnothing$	$\{h_1, h_2, h_4, h_5\}$	$\varnothing$
$\{h_4, h_5\}$	$\varnothing$	$\{h_1, h_2, h_3, h_4, h_5\}$	$\varnothing$

v) The empty subsets and those included into another are discarded, and we obtain the maximum subrelations, also called “Affinity subrelations” (Figure 3).

$$\begin{array}{cc} & \begin{array}{ccc} t_2 & t_3 & t_5 \end{array} \\ h_1 & \begin{pmatrix} l_6 & l_5 & l_5 \end{pmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} t_2 & t_5 \end{array} \\ \begin{array}{c} h_1 \\ h_2 \end{array} & \begin{pmatrix} l_6 & l_5 \\ l_4 & l_5 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} & t_3 \\ \begin{array}{c} h_1 \\ h_4 \end{array} & \begin{pmatrix} l_5 \\ l_4 \end{pmatrix} \end{array} \qquad \begin{array}{cc} & t_5 \\ \begin{array}{c} h_1 \\ h_2 \\ h_3 \end{array} & \begin{pmatrix} l_5 \\ l_5 \\ l_4 \end{pmatrix} \end{array}$$

Figure 3. Affinity subrelations

The affinities obtained at level l_4 show that:

- Household h_1 has the needs t_2, t_3 and t_5;
- Households h_1 and h_2 have the needs t_2 and t_5;
- Households h_1 and h_4 have the need t_3;
- Households h_1, h_2 and h_3, need t_5.

The correspondent Galois lattice is represented at Figure 4.

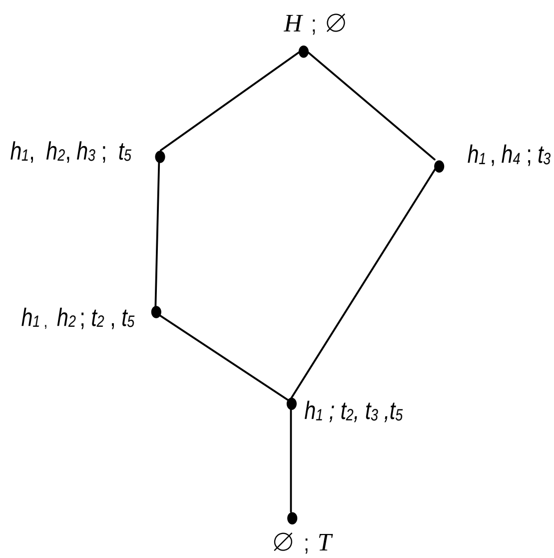

Figure 4. Galois lattice

This reticular structure reveals in a visual way the affinities among the households in relation to their UBN. It is clear that as more dissatisfied BN at a same level α are required, the number of households decreases, and vice versa: the requirement for fewer UBN groups a larger quantity of households.

4.4. PUBLIC POLICIES - BASIC NEEDS MATRIX

At this stage first it is analyzed what kind of actions can be taken to diminish the households` considered poor by means of UBN, leading to a set of public policies (PP).

Later we build a fuzzy linguistic binary relation M defined from the set PP to NB, to evaluate the impact of existing public policies to be taken to the diminish of the UBN. It can be obtained through discussion with experts (for example, using Fuzzy - Delphi methodology).

Additionally M may be subjected to the technique for recovering forgotten effects to check their coherence and identifying potential intermediate incidences (Kaufmann & Gil Aluja, 1989).

To continue with the case presented at 4.2, let $P = \{p_1, p_2, p_3, p_4, p_5\}$ be a set of public policies, where:

p_1: Scholarships for children at school age.

p_2: Education and training for the head of household.

p_3: Social housing plan.

p_4: Sewerage network and drinking water network expansion.

p_5: Delivery of building materials subsidized.

And let $T = \{t_1, t_2, t_3, t_4, t_5\}$ be the BN set given in Table 2.

The associated matrix to the fuzzy linguistic binary relation $M : P \times T \to L$ is given at Figure 5.

$$\begin{array}{c c} M & \begin{array}{ccccc} t_1 & t_2 & t_3 & t_4 & t_5 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{array} & \begin{pmatrix} l_1 & l_3 & l_0 & l_6 & l_2 \\ l_3 & l_2 & l_4 & l_5 & l_6 \\ l_6 & l_6 & l_5 & l_3 & l_0 \\ l_1 & l_5 & l_6 & l_1 & l_0 \\ l_6 & l_6 & l_4 & l_4 & l_0 \end{pmatrix} \end{array}$$

Figure 5. Public Policies - Basic Needs Matrix

An example will explain the significance of this matrix: the values of the first row indicate that the policy of scholarships for children at school age impacts moderately in diminishing the housing conditions, it does not affect the sanitary conditions and affects in a very high degree school attendance.

4.5. HOUSEHOLDS – PUBLIC POLICIES MATRIX

To obtain a relationship that indicate the degree at which every household has the need of a public policy of set *P* to diminish their UBN the max-min composition of linguistic relations *R* and M^t is done. Where $M^t : T \times P \to L$ is the transposed matrix of M , which is written S (Figure 6).

$$\begin{array}{c c} S & \begin{array}{ccccc} p_1 & p_2 & p_3 & p_4 & p_5 \end{array} \\ \begin{array}{c} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{array} & \begin{pmatrix} l_1 & l_3 & l_6 & l_1 & l_6 \\ l_3 & l_2 & l_6 & l_5 & l_6 \\ l_0 & l_4 & l_5 & l_6 & l_4 \\ l_6 & l_5 & l_3 & l_1 & l_4 \\ l_2 & l_6 & l_0 & l_0 & l_0 \end{pmatrix} \end{array}$$

Figure 6. M^t Matrix

As *L* is a set outfitted with a full order, it is possible to generalize the definition of max-min composition of fuzzy binary relations to compose fuzzy linguistic binary relations, as follows:

Definition 2. Let $R : H \times T \to L$ be and $S : T \times P \to L$ be fuzzy linguistic binary relations, we define its composition as:

$R \circ S : H \times P \to L / \forall x \in H, \forall y \in T, \forall z \in P :$

$\mu_{R \circ S}(x, z) = \max_y \{\min[\mu_R(x, y); \mu_S(y, z)]\}$.

$\mu_{R \circ S}(x, z) \in L$, $R \circ S$ is a fuzzy linguistic binary relation.

In this case the rows of $R \circ S$ are the households of the set *H* and the columns the public policies of set *P*.

We use software specially designed for this composition (Lazzari *et al.*, 2008).

$$\begin{array}{c c} R \circ S & \begin{array}{ccccc} p_1 & p_2 & p_3 & p_4 & p_5 \end{array} \\ \begin{array}{c} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{array} & \begin{pmatrix} l_3 & l_5 & l_6 & l_5 & l_6 \\ l_3 & l_5 & l_4 & l_4 & l_4 \\ l_2 & l_4 & l_3 & l_3 & l_3 \\ l_3 & l_4 & l_4 & l_4 & l_4 \\ l_1 & l_1 & l_1 & l_1 & l_1 \end{pmatrix} \end{array}$$

Figure 7. Households – Public Policies Matrix

The values obtained in the matrix $R \circ S$ (Figure 7) indicate the needs of every household in being the beneficiary of each public policy. For example row three and column three states that the household h_3 needs moderately to be the beneficiary of social housing; the value in row five and column five shows that the household h_5 needs in a very low way to be the receiver of building materials subsidized.

4.6. Households Grouping Due to the Need Of Implementation Of Public Policies

We will explore again the affinities to segment households by grouping them, at this instance, by the need of implementation of public policies.

We apply the corresponding algorithm:

i) It is considered again, like in the case of matrix R, the affinities at a level greater than or equal to l_4, even though it is possible to study them at different levels.

ii) It is obtained the crisp relation at level "high" (l_4) associated to $R \circ S$ (Figure 8).

$$\begin{array}{cc} (R \circ S)_{l_4} & \begin{array}{ccccc} p_1 & p_2 & p_3 & p_4 & p_5 \end{array} \\ \begin{array}{c} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Figure 8. Boolean Matrix of level l_4

iii) It is considered the set $\wp(H)$ obtained at 4.3 iii).

iv) Each element of $\wp(H)$ is correlated with the elements of the set P which are associated at a level greater than or equal to "high" (Table 4) according to the values of $(R \circ S)_{l_4}$ (Figure 8).

Table 4. $\wp(H)$ and its correlation at a level greater than or equal to "high"

$\varnothing$	$\{p_1, p_2, p_3, p_4, p_5)$	$\{h_1, h_3, h_5\}$	$\varnothing$
$\{h_1\}$	$\{p_2, p_3, p_4, p_5)$	$\{h_1, h_3, h_4\}$	$\{p_2\}$
$\{h_2\}$	$\{p_2, p_3, p_4, p_5)$	$\{h_1, h_2, h_3\}$	$\{p_2\}$
$\{h_3\}$	$\{p_2\}$	$\{h_1, h_2, h_4\}$	$\{p_2, p_3, p_4, p_5)$
$\{h_4\}$	$\{p_2, p_3, p_4, p_5)$	$\{h_1, h_2, h_5\}$	$\varnothing$
$\{h_5\}$	$\varnothing$	$\{h_2, h_3, h_4\}$	$\{p_2\}$
$\{h_1, h_2\}$	$\{p_2, p_3, p_4, p_5)$	$\{h_2, h_3, h_5\}$	$\varnothing$
$\{h_1, h_3\}$	$\{p_2\}$	$\{h_3, h_4, h_5\}$	$\varnothing$
$\{h_1, h_4\}$	$\{p_2, p_3, p_4, p_5)$	$\{h_1, h_4, h_5\}$	$\varnothing$

$\{h_1, h_5\}$	$\varnothing$	$\{h_2, h_4, h_5\}$	$\varnothing$
$\{h_2, h_3\}$	$\{p_2\}$	$\{h_1, h_2, h_3, h_4\}$	$\{p_2\}$
$\{h_2, h_4\}$	$\{p_2, p_3, p_4, p_5)$	$\{h_1, h_2, h_3, h_5\}$	$\varnothing$
$\{h_2, h_5\}$	$\varnothing$	$\{h_2, h_3, h_4, h_5\}$	$\varnothing$
$\{h_3, h_4\}$	$\{p_2\}$	$\{h_1, h_3, h_4, h_5\}$	$\varnothing$
$\{h_3, h_5\}$	$\varnothing$	$\{h_1, h_2, h_4, h_5\}$	$\varnothing$
$\{h_4, h_5\}$	$\varnothing$	$\{h_1, h_2, h_3, h_4, h_5\}$	$\varnothing$

v) The empty subsets and those included into another are discarded, and we obtain the affinity subrelations (Figure 9).

$$\begin{array}{c} \\ h_1 \\ h_2 \\ h_4 \end{array} \begin{array}{c} \begin{array}{cccc} p_2 & p_3 & p_4 & p_5 \end{array} \\ \begin{pmatrix} l_5 & l_6 & l_5 & l_6 \\ l_5 & l_4 & l_4 & l_4 \\ l_4 & l_4 & l_4 & l_4 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} \begin{array}{c} p_2 \\ \begin{pmatrix} l_5 \\ l_5 \\ l_4 \\ l_4 \end{pmatrix} \end{array}$$

Figure 9. Affinity subrelations

Affinities obtained at greater than or equal to *high* that are visualized on Figure 10 indicate that:

- Households h_1, h_2, h_4 need policies p_2, p_3, p_4 and p_5.
- Household h_3 needs policy p_2.
- Household h_5 doesn´t need any policy of set P at a degree greater than high, then it could be considered a non-poor household having into account the dissatisfaction of the BN at Table 2 at the level equal to or greater than "high".

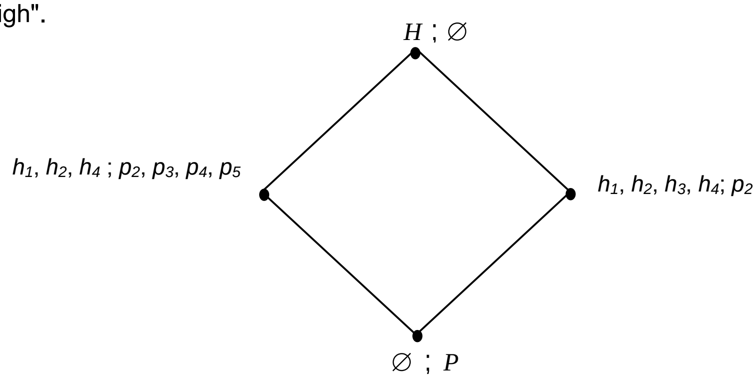

Figure 10. Galois lattice

We have software specially designed to obtain the affinities available (Lazzari, *et al.,* 2008).

5. CONCLUDING REMARKS

The development of the affinity theory has solved many problems presented by the complex present realities. This tool has a high degree of adaptability and flexibility to perform under different approaches or to set dissimilar criteria when trying to solve a problem.

The level of homogeneity can vary according to the case under review, and it is also possible to analyze the affinities at a range of levels.

The direct method or the Unsatisfied Basic Needs measures, in some way, the population's standard of living. These needs shall differ by the region, the country and the period that is being studied. Therefore, it is appropriate to carry out multidisciplinary studies to determine the components and indicators to reflect the reality that is tried to be measured.

The linguistic model for the study of poverty presented in this paper was applied to a small case in Argentina but can be generalized to the entire country as well as different states or regions of the world, particularly in the poorest areas of the planet.

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