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Mean and quantile regression Oaxaca-Blinder decompositions with an application to caste discrimination

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Abstract This paper extends the Oaxaca-Blinder decomposition method to the quantile regression random-coefficients framework. Mean-based decompositions are obtained as the integration of the quantile regression decomposition process. This method allows identifying if the observed differences between two groups differ across quantiles, and if so, what is the contribution to the mean-based Oaxaca-Blinder decomposition. The proposed methodology is applied to the analysis of caste discrimination in Nepal. The results indicate that much of the discrimination occurs at the lowest quantiles, which implies that disadvantaged groups are the ones who suffer the most caste discrimination.

Keywords Quantile regression · Oaxaca-Blinder decomposition

1 Introduction

Quantile regression (QR) (see Koenker 2005, for a comprehensive analysis of QR) is a useful way to represent parameter heterogeneity in the response of an outcome variable to the linear effect of certain covariates, in which parameter heterogeneity is presented using conditional quantiles. QR has been used to study inequality. In particular, several papers used

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QR to decompose differences across groups or within a group across time in a Oaxaca-Blinder (OA) decomposition environment (Oaxaca 1973; Blinder 1973; Oaxaca and Ransom 1994), defining an "endowment effect" that captures differences in an outcome variable because of differences in explanatory variables (e.g. differences in education endowments), and a "pricing effect" that corresponds to differences across groups because of differences in the coefficients (e.g. differences in returns to schooling). Machado and Mata (2005) (see also Autor et al. 2005) propose techniques to disentangle the effect of changes in the distribution of covariates from the effect of changes in the distribution of coefficients - or returns- in accounting for inequality changes. Such techniques however, have some limitations in terms of the detailed decompositions of the contributions of each covariate to the total change (see the discussion in Fortin et al. 2011). Firpo et al. (2009) develop a useful framework to account for the particular effect of covariates using the re-centered influence function (RIF) model. This can be applied to different statistics such as quantiles, variance, Gini or Theil coefficients within the OA framework to disentangle endowment and pricing effects. When this is applied to quantiles, the model is called the unconditional QR model.

We formalize OA decompositions at particular quantiles using an alternative representation. Using the QR framework as a special case of a random-coefficients (RC) linear regression model, the mean regression (MR) model can be interpreted as the mean of the QR coefficients because integrating out the quantile coefficients produce the MR coefficients. Thus the mean OA decomposition can be seen as the mean of OA decompositions at different quantiles. Our proposed QR OA decomposition effects are thus the source of the standard OA decomposition at the mean. The proposed analysis does not circumvent the fact that the law of iterated expectations does not hold in the case of quantiles, and thus the conditional QR analysis cannot be used directly to analyze unconditional quantiles (see the discussion in Fortin et al. 2011). The proposed OA decomposition analysis in this paper applies only to the conditional case, that is, where the OA decomposition is implemented at the mean endowment of each group. It does however provide a formal framework to analyze the pricing effect. The proposed methodology is built upon a formal statistical model constructed from QR and, therefore, we are able to derive the asymptotic distribution of these decompositions using the recent results in Bera et al. (2014), where the asymptotic joint distribution of the mean ordinary least-squares (OLS) and QR coefficients is studied.

To illustrate the proposed methodology we apply it to caste wage differentials in Nepal. The QR decomposition allows us to study if discrimination is larger for particular quantiles of the conditional wage distribution, and to explore the causes of those disparities. The results indicate that much of the discrimination occurs at low quantiles, which implies that disadvantaged groups are the ones who suffer the most caste discrimination. Our framework also allows us to identify the decomposition along key covariates such as education, occupation and firm size. In this case, the decomposition for education has the largest effect at low quantiles, but occupation and firm size effects are uniform across quantiles.

This paper is organized as follows. Section 2 analyzes the connection between RC and QR models. Section 3 applies the RC-QR model to the Oaxaca decomposition model. Section 4 reviews the asymptotic distribution of the proposed estimators. Section 5 applies the proposed method to caste wage differentials in Nepal. Section 6 concludes.

2 Random-coefficients and quantile regression

Let Y be a response or outcome variable and X be a $p \times 1$ dimensional vector of covariates. The mean and quantile linear regression models are two well known models to estimate the effect of certain covariates on a response variable.

Mean regression (MR) considers the effect of X on Y through the conditional mean model

$$E(Y|X = x) = x'\beta_M, \tag{1}$$

where β_M is a $p \times 1$ dimensional vector of coefficients.

In QR the conditional quantiles of Y are of interest through the models

$$Q_Y(\tau|X = x) = x'\beta(\tau) \quad for \quad \tau \in (0, 1). \tag{2}$$

Note that Eq. 2 implies that the right-hand side is monotone non-decreasing in τ. In theory, the monotonicity requirement should be satisfied for all realizations of X or for some specified subspace of interest (this is discussed in Koenker 2005, p.59).[1]

As stated in Koenker and Xiao (2006) and (Koenker 2005, section 2.6, pp. 59-62), the monotonicity in the QR model determines that a random-coefficients (RC) notation can be introduced by considering a uniform random variable $U \sim U(0, 1)$ in the role of the fixed τ and writing

$$Y = X'\beta(U). \tag{3}$$

The results in Koenker and Bassett (1982) establish that, under regularity conditions, the estimated conditional quantile function is a strongly consistent estimator of the population quantile function. Thus the process $\{Y, X\}$ can be partially recovered from the marginal distributions, that is, the conditional distribution $Y|X$ can be described by its conditional quantiles based on $\tau \in (0, 1)$. The QR analysis constructs a model $y^* = y(x, \tau)$ in which y^* depends on *endowments* $X = x$ and its location in the conditional distribution given by τ. The linear QR model determines that the coefficients $\beta(\tau)$ are the *prices* of those endowments (this is further developed in the next section). This method has been applied for the analysis of inequality by Autor et al. (2005), Machado and Mata (2005) and others.

A general RC formulation for Y can be obtained by defining a random $p \times 1$ vector $B \in \mathcal{B}$, where $\mathcal{B}$ is the space of $p \times 1$ random real valued vectors. Then,

$$Y = X'B. \tag{4}$$

Model (2) is a special case of Eq. 4 in which $X'B$ is monotone increasing on some common index, i.e. τ. Define the parametric RC family that satisfies the QR monotonicity requirement as $\mathcal{B}_{QR} = \{B \in \mathcal{B} : B = B(\tau), \tau \in (0, 1), x'(B(\tau_2) - B(\tau_1))(\tau_2 - \tau_1) \geq 0, \forall \tau_1, \tau_2 \in (0, 1), \forall x\}$, and note that $\mathcal{B}_{QR} \subseteq \mathcal{B}$.

In order to illustrate the differences between these two models, consider the following data generating processes for $\{Y, X_1, X_2\}$:

$$Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + (\delta_1\omega_1)X_1 + (\delta_2\omega_2)X_2 + (\gamma_0 + \gamma_1 X_1 + \gamma_2 X_2)\epsilon,$$

with $\epsilon \sim IID(0, \sigma^2_\epsilon)$, $\omega_j \sim IID(0, \sigma^2_{\omega_j})$, $j = 1, 2$, $(\epsilon, \omega_1, \omega_2) \perp\!\!\!\perp (X_1, X_2)$, and $(\alpha_0, \alpha_1, \alpha_2, \gamma_0, \gamma_1, \gamma_2, \delta_1, \delta_2)$ fixed parameter values. This can be re-written as

$$Y = (\alpha_0 + \gamma_0\epsilon) + (\alpha_1 + \gamma_1\epsilon + \delta_1\omega_1)X_1 + (\alpha_2 + \gamma_2\epsilon + \delta_2\omega_2)X_2 = B_0 + B_1 X_1 + B_2 X_2,$$

where $B_0 \equiv \alpha_0 + \gamma_0\epsilon$ and $B_j \equiv \alpha_j + \gamma_j\epsilon + \delta_j\omega_j$, $j = 1, 2$.

First, consider the typical linear location-scale model, for which we set $\delta_1 = \delta_2 = 0$. This is a ubiquitous example of a model in which QR is represented. If X_j, $j = 1, 2$,

[1] In practice, however, the monotonicity may not be satisfied for some values of X, a problem known as the quantile crossing problem: the conditional quantile curves $x \longmapsto Q_Y(\tau|x)$ may cross for different values of τ (He 1997). Chernozhukov et al. (2009, 2010) studies this monotonicity requirement and proposes a rearrangement procedure of the estimated quantile curves. He (1997) proposed to impose a location-scale regression model, which naturally satisfies monotonicity.

takes only positive values, the QR model simplifies to $Q_Y(\tau|X = x) = (\alpha_0 + \alpha_1 x_1 + \alpha_2 x_2) + (\gamma_0 + \gamma_1 x_1 + \gamma_2 x_2) Q_\epsilon(\tau)$ and $\frac{\partial Q_Y(\tau|X=x)}{\partial x_j} \equiv \beta_j(\tau) = \alpha_j + \gamma_j Q_\epsilon(\tau)$, $j = 1, 2$, where $Q_\epsilon(\tau)$ corresponds to the τ-quantile of ϵ, while the conditional mean effect is $\frac{\partial E(Y|X=x)}{\partial x_j} = \alpha_j$, $j = 1, 2$. The connection between the RC and QR formulations can be shown by noting that this model implies that $B_j = \alpha_j + \gamma_j Q_\epsilon(U)$, $U \sim U(0, 1)$. Therefore, in a location-scale model, QR implies a RC formulation where the random-coefficients are linear functions of a single random index $Q_\epsilon(\tau)$, $\tau \in (0, 1)$.

Second, consider the case where $\gamma_1 = \gamma_2 = 0$. This is a radically different model because each linear coefficient, B_1 and B_2, is independent of the other. QR models successfully predict the coefficient range, although they implicitly assume that both βs are co-monotonic. That is, the QR model would produce $Q_y(\tau|X = x) = \beta_0(\tau) + \beta_1(\tau)x_1 + \beta_2(\tau)x_2$ for the same common index τ. This picks up parameter variability in B but it presents heterogeneity in an "ordered" way, which may not fully represent parameter heterogeneity. If, for instance, we further assume that $\alpha_1 = \alpha_2 = 0$ and $\delta_1 = \delta_2 = 1$, $\epsilon \sim IID\ U(-1, 1)$, $\omega_j \sim IID\ U(-1, 1)$, $j = 1, 2$, QR correctly shows that B_j, $j = 1, 2$, range from -1 to 1 through the estimated $\beta_j(.)$. But potential realizations for which $sign(B_1) = -sign(B_2)$, that is, cases for which B_1 is positive (negative) but B_2 is negative (positive), are not considered. In fact these realizations are as likely to occur as realizations in which $sign(B_1) = sign(B_2)$. That is, $Prob[sign(B_1) = -sign(B_2)] = Prob[sign(B_1) = sign(B_2)] = 0.5$.

Throughout this paper we consider RC models assuming QR co-monotonicity. As discussed in the next section, the goal of this paper is to propose a general framework to study pricing heterogeneity in endowments. As such, QR models are able to capture the heterogeneity for endowment variables' prices (i.e. a particular B_j) even if the co-monotonicity across prices does not occur.

3 Endowment and pricing effects in the RC-QR model

Note that because of monotonicity of $\beta(\tau)$, $\tau \in (0, 1)$, the order statistics of B correspond to the QR coefficients at the τ-quantile. That is, $Q_B(\tau) = \beta(\tau)$, $\tau \in (0, 1)$. This means that, for instance, the "median" value of B is $\beta(0.5)$. Thus, analyzing the marginal effects of covariates on different quantile of the conditional distribution of $Y|X$ is equivalent to analyzing the quantiles of the linear effects of X on Y.

Moreover, following Koenker and Xiao (2006) we can write

$$Y = \bar{B}_0 + X_1 B_1 + ... + X_p B_p + w, \tag{5}$$

where $\bar{B}_0 = E[B_0]$ and $w = B_0 - \bar{B}_0$. This means that for the MR model we have

$$E(Y|X = x) = \bar{B}_0 + x_1 E[B_1] + ... + x_p E[B_p], \tag{6}$$

and then the mean regression coefficients can be obtained by taking the "mean" of the quantile coefficients. Thus $\beta_M = E(B)$.

Then, combining mean and QR models we could rewrite (4) as

$$Y - x'\beta_M = (X - x)'\beta_M + X'(B - \beta_M). \tag{7}$$
$$\equiv \text{(Endowment effect)} + \text{(Pricing effect)}.$$

This representation is quite useful. Differences across individuals in Y are due to differences in X with respect to some reference value x or to differences in B with respect to the mean of B. Following the traditional OA decomposition analysis (Oaxaca 1973; Blinder 1973; Oaxaca and Ransom 1994), all changes in Y can be expressed as the combined effect of X and B. Then differences in Y due only to X are defined as the *endowment effect*. Changes in Y due only to B are defined as the *pricing effect*. If we further use $x = \overline{x} \equiv E(X)$, then we are in fact comparing any realization of Y with respect to the mean endowment effect valued at the mean price. Thus for instance if Y is wage and X human capital, differences in wages across individuals are either due to differences in human capital or on how the market values those endowments.

As explained in Machado and Mata (2005) in the context of explaining changes in the distribution of wages "the estimated QR coefficients are also quite interesting as they can be interpreted as rates of return (or 'prices') of the labor market skills at different points of the conditional wage distribution"(p.447). If we consider models in $\mathcal{B}_{QR}$, the pricing effect can be analyzed by studying the quantile process $\{\beta(\tau), \tau \in (0, 1)\}$ together with the MR coefficients β_M. Then, we can apply an endowment and pricing decomposition at particular quantiles of the conditional distribution of Y. For different values of x and quantiles τ, we can evaluate

$$\begin{aligned} y(x, \tau) - \overline{x}'\beta_M &= (x - \overline{x})'\beta_M + x'(\beta(\tau) - \beta_M). \\ &\equiv (\text{Endowment effect}) + (\text{Pricing effect}(\tau)). \end{aligned} \tag{8}$$

In this case the difference in pricing at τ with respect to the mean pricing $(\beta(\tau) - \beta_M)$ is of interest. Inference on this object requires the joint consideration of the mean and QR estimators. This decomposition was first proposed by Autor et al. (2005) following the analysis of Machado and Mata (2005), replacing β_M with $\beta(0.50)$, where the second term was referred as *within-group* inequality and *prices*. Equation 8 was thus proposed to evaluate changes in inequality.

This decomposition can be extended for comparing two groups 1 and 2 with realizations $y_1(x_1, \tau)$ and $y_2(x_2, \tau)$. Consider the evaluation at $x_1 = \overline{x}_1$ and $x_2 = \overline{x}_2$. Furthermore assume that groups 1 and 2 are realizations of the same process with differences in endowments and pricings. Then, after some algebra,

$$\begin{aligned} OAQ &\equiv y_1(\overline{x}_1, \tau) - y_2(\overline{x}_2, \tau) = \overline{x}_1\beta_1(\tau) - \overline{x}_2\beta_2(\tau) \\ &= \overline{x}_1'\beta_1(\tau) - \overline{x}_2'\beta_2(\tau) + \overline{x}_1'\beta_2(\tau) - \overline{x}_1'\beta_2(\tau) \\ &= (\overline{x}_1 - \overline{x}_2)'\beta_2(\tau) + \overline{x}_1'(\beta_1(\tau) - \beta_2(\tau)) \\ &\equiv OAQe(\tau) + OAQp(\tau), \end{aligned} \tag{9}$$

where OAQ corresponds to a OA decomposition for a particular quantile τ. $OAQe(\tau)$ is the effect that differences in endowments has on that particular τ quantile. $OAQp(\tau)$ represents differences in prices of that particular quantile, evaluated at $\overline{x}_1$.[2]

Note that if we integrate out τ we obtain the usual OA decomposition at the mean, that is,

$$\begin{aligned} OAM &\equiv E_\tau[y_1(\overline{x}_1, \tau) - y_2(\overline{c}_2, \tau)] = (\overline{x}_1 - \overline{x}_2)'\beta_{M2} + \overline{x}_1(\beta_{M1} - \beta_{M2}) \\ &\equiv OAMe + OAMp, \end{aligned} \tag{10}$$

where β_{Mj} is the average pricing of group $j = 1, 2$, and $OAMe$ and $OAMp$ are the endowment and pricing OA decompositions, respectively. This determines that the OA decomposition can be seen as arising from differences between the groups at different quantiles of the conditional distribution of Y.

[2] As suggested by an anonymous referee alternative decompositions can be constructed depending on what terms are added and subtracted to $\overline{x}_1'\beta_1(\tau) - \overline{x}_2'\beta_2(\tau)$. OAQ might be equivalently written as $(\overline{x}_1 - \overline{x}_2)'\beta_1(\tau) + \overline{x}_2'(\beta_1(\tau) - \beta_2(\tau)) \equiv OAQe'(\tau) + OAQp'(\tau)$.

Furthermore, adding and subtracting $\overline{x}_1'\beta_{M1}, \overline{x}_2'\beta_{M2}, \overline{x}_1'\beta_{M2}$, and rearranging, we obtain

$$\begin{aligned} y_1(\overline{x}_1,\tau) - y_2(\overline{x}_2,\tau) &= OAM + \overline{x}_1'(\beta_1(\tau) - \beta_{M1}) - \overline{x}_2'(\beta_2(\tau) - \beta_{M2}) \\ &\equiv OAM + OAQ_1(\tau) - OAQ_2(\tau). \end{aligned} \quad (11)$$

The first term corresponds to the usual (mean) OA decomposition in Eq. 10, OAM. The second term is the pricing effect of group 1 for that particular quantile, evaluated at the mean endowments of that group ($OAQ_1(\tau)$). The third term is the pricing effect of group 2 for that particular quantile, also evaluated at the mean endowments of that group ($OAQ_2(\tau)$). Thus, differences in Y at a particular τ could either be due to differences in endowments valued at the mean pricings (OAM) or group-specific effects of that particular quantile with respect to the mean pricing.

4 Asymptotic inference

In order to make asymptotic inference on these decompositions we need to review the asymptotic joint distribution of the OLS and QR estimators. Bera et al. (2014) study the interaction of the mean and QR models' coefficients.

The coefficient β_M can be estimated by solving the following minimization problem

$$\hat{\beta}_M = \underset{b\in\mathbb{R}^p}{\arg\min} \sum_{i=1}^{n} (y_i - x_i'b)^2.$$

On the other hand, QR technique, suggested by Koenker and Bassett (1978), formulates that $\beta(\tau)$ can be estimated by solving the following minimization problem

$$\hat{\beta}(\tau) = \underset{b\in\mathbb{R}^p}{\arg\min} \sum_{i=1}^{n} \rho_\tau(y_i - x_i'b),$$

where $\rho_\tau(u) = u(\tau - 1(u < 0))$.

The OA decomposition extension of this paper is based on the comparison of MR and QR, evaluated at a particular quantile, namely, the asymptotic joint distribution of $(\hat{\beta}_M' \ \hat{\beta}(\tau)')'$. To derive the asymptotic joint distribution of the estimators, the following assumptions are imposed.

Assumption 1 $\{(X_i', Y_i), i = 1, ..., n\}$ *are independently but not necessarily identically distributed. The conditional distribution function of Y_i given X_i, F_i, is absolutely continuous, with continuous densities $\{f_i(\xi)\}$ uniformly bounded away from 0 and ∞ at the points $\xi_i(\tau)$, $i = 1, 2, ...$, where $\xi_i(\tau) \equiv F_{Y_i}^{-1}(\tau|X_i)$;*

Assumption 2 *β_M is defined to solve* Eq. 12, *and $\beta(\tau)$ is defined to uniquely solve* Eq. 13 *as*

$$E[X_i(y_i - X_i'\beta_M)] = 0, \quad (12)$$

$$E[X_i\psi_\tau(Y_i - X_i'\beta(\tau))] = 0, \quad (13)$$

where $\psi_\tau(u) = \tau - 1(u < 0)$;

Assumption 3 *Denote X_{ij} to be the jth element of X_i.*

(a): $E|(Y_i - X_i'\beta_M)^2 X_{ij}X_{ih}|^{1+\delta_1} < \Delta_1 < \infty$ *for some* $\delta_1 > 0$, *where* $i = 1, ..., n$ *and* $j, h = 1, ..., p$;

(b): $E|X_{ij}X_{ih}|^{1+\delta_2} < \Delta_2 < \infty$ *for some* $\delta_2 > 0$, *where* $i = 1, ..., n$ *and* $j, h = 1, 2, ..., p$;

(c): *The matrices* $H_n(\tau) := E\left(\frac{1}{n}\sum_{i=1}^{n} X_i X_i' f_i(\xi_i(\tau))\right)$ *and* $\begin{pmatrix} V_n & \Omega_n(\tau) \\ \Omega_n(\tau) & \tau(1-\tau)J_n \end{pmatrix}$ *are uniformly positive definite, where* $V_n := Var\left(\frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i(Y_i - X_i'\beta_M)\right)$, $\Omega_n(\tau) := E\left(\frac{1}{n}\sum_{i=1}^{n} X_i X_i' \rho_\tau(y_i - X_i'\beta(\tau))\right)$, *and* $J_n := E\left(\frac{1}{n}\sum_{i=1}^{n} X_i X_i'\right)$.

Theorem 1 *Under Assumptions 1-3, for a given* $\tau \in (0, 1)$, *we have*

$$D_n(\tau)^{-\frac{1}{2}}\sqrt{n}\left(\begin{pmatrix} \hat{\beta}_M \\ \hat{\beta}(\tau) \end{pmatrix} - \begin{pmatrix} \beta_M \\ \beta(\tau) \end{pmatrix}\right) \xrightarrow{d} N(\mathbf{0}, I),$$

where $D_n(\tau) := \begin{pmatrix} J_n^{-1}V_n J_n^{-1} & J_n^{-1}\Omega_n(\tau)H_n(\tau)^{-1} \\ H_n(\tau)^{-1}\Omega_n(\tau)J_n^{-1} & \tau(1-\tau)H_n(\tau)^{-1}J_n H_n(\tau)^{-1} \end{pmatrix}$.

Proof See Bera et al. (2014) Theorem 1. □

The above theorem shows that the asymptotic joint distribution of the OLS and QR estimators is multivariate normal after being properly centered and scaled. Thus, each of the components derived in the previous section is asymptotically normal. In addition, it reveals an interesting interpretation of the off-diagonal elements of the variance-covariance matrix. The covariance term, $H_n(\tau)^{-1}\Omega_n(\tau)J_n^{-1}$, is composed of three pieces. First, $H_n(\tau)$ is the well known 'bread' element in the simple variance-covariance matrix in the QR literature.

Table 1 Oaxaca mean and quantile aggregate decompositions

OAM	$OAMe$	$OAMp$		
$\overline{x}_1'\beta_{M1} - \overline{x}_2'\beta_{M2}$	$(\overline{x}_1 - \overline{x}_2)'\beta_{M2}$	$\overline{x}_1(\beta_{M1} - \beta_{M2})$		
0.266	0.360	−0.094		
(0.062)	(0.049)	(0.057)		
OAQ	$OAQe$	$OAQp$	OAQ_1	OAQ_2
$y_1(\overline{x}_1, \tau) - y_2(\overline{x}_2, \tau)$	$(\overline{x}_1 - \overline{x}_2)'\beta_2(\tau)$	$\overline{x}_1'(\beta_1(\tau) - \beta_2(\tau))$	$\overline{x}_1'(\beta_1(\tau) - \beta_{M1})$	$\overline{x}_2'(\beta_2(\tau) - \beta_{M2})$
$\tau = 0.25$				
0.376	0.385	−0.009	−0.367	−0.477
(0.078)	(0.060)	(0.077)	(0.026)	(0.044)
$\tau = 0.50$				
0.259	0.350	−0.091	−0.036	−0.029
(0.071)	(0.057)	(0.067)	(0.023)	(0.038)
$\tau = 0.75$				
0.210	0.308	−0.099	0.328	0.384
(0.072)	(0.058)	(0.074)	(0.026)	(0.041)

Notes: Mean and quantile decompositions: $OAM = OAMe + OAMp$; $OAQ(\tau) = OAQe(\tau) + OAQp(\tau)$; $OAQ(\tau) = OAM + OAQ_1(\tau) - OAQ_2(\tau)$. Wild bootstrap standard errors in parentheses based on 200 bootstrap samples

Second, the element J_n is the usual 'bread' element in the variance-covariance matrix for least squares estimator. The final element, $\Omega_n(\tau)$, is novel, and it is simply the weighted QR loss function, where the weight is given by the matrix of regressors $X_i X_i'$.

In order to implement the tests we need to estimate $D_n(\tau)$ consistently. To guarantee the consistency, we further require the following assumption.

Assumption 4 *$E(X_{im}X_{ih}X_{ij}^2X_{il}^2) < \Delta_3 < \infty$ and $E((Y_i - X_i'\beta_M)^2 X_{ij}^2 X_{il}^2) < \Delta_4 < \infty$ for all i and $m, h, j, l = 1, 2, ..., p$.*

In practice $D_n(\tau)$ can be implemented by wild bootstrap.

5 Empirical application: caste discrimination in Nepal

Labor market discrimination is defined as a situation in which a person who provides labor market services and is equally productive in a physical and material sense is paid less in a way that is related to gender, race, caste or ethnicity (Altonji and Blank 1999). While considerable attention has been paid to labor market discrimination based on race and gender, less attention has been paid to caste even though caste-based discrimination might be more powerful and persistent than racial discrimination. A caste system allocates social labor on the basis of a hierarchy of caste classifications and this restricts occupational mobility (Banerjee and Knight 1985). A caste-based division of labor can perpetuate itself through the inter-generational transmission of low educational and occupational status from one generation to the next even once discrimination *per se* is abolished (Borjas 1994; Darity and Mason 1998).

Table 2 Oaxaca mean and quantile education decompositions

OAM	$OAMe$	$OAMp$		
$\overline{x}_1'\beta_{M1} - \overline{x}_2'\beta_{M2}$	$(\overline{x}_1 - \overline{x}_2)'\beta_{M2}$	$\overline{x}_1(\beta_{M1} - \beta_{M2})$		
0.445	0.192	0.253		
(0.136)	(0.038)	(0.155)		
OAQ	$OAQe$	$OAQp$	OAQ_1	OAQ_2
$y_1(\overline{x}_1, \tau) - y_2(\overline{x}_2, \tau)$	$(\overline{x}_1 - \overline{x}_2)'\beta_2(\tau)$	$\overline{x}_1'(\beta_1(\tau) - \beta_2(\tau))$	$\overline{x}_1'(\beta_1(\tau) - \beta_{M1})$	$\overline{x}_2'(\beta_2(\tau) - \beta_{M2})$
$\tau = 0.25$				
0.519	0.207	0.312	0.111	0.037
(0.189)	(0.057)	(0.227)	(0.112)	(0.111)
$\tau = 0.50$				
0.466	0.211	0.255	0.067	0.046
(0.184)	(0.056)	(0.220)	(0.104)	(0.096)
$\tau = 0.75$				
0.389	0.162	0.227	−0.128	−0.073
(0.179)	(0.054)	(0.219)	(0.089)	(0.093)

Notes: Mean and quantile decompositions: $OAM = OAMe + OAMp$; $OAQ(\tau) = OAQe(\tau) + OAQp(\tau)$; $OAQ(\tau) = OAM + OAQ_1(\tau) - OAQ_2(\tau)$. Wild bootstrap standard errors in parentheses based on 200 bootstrap samples

Nepal, along with other countries, had a caste-based social division of labor. This was however abolished in 1963 and declared illegal after the promulgation of the new constitution in 1990. Several studies (see the literature review in Mainali et al. (2017)) concluded however that discrimination persists in the labor market. We apply the Oaxaca-Blinder decomposition framework developed in this paper to study this issue. We aggregate caste-ethnic groups into two broad categories, namely *Tagadhari* and *Inferior* = *Matwali & Pani Nachalne*. We use data from the 2010 National Living Standard Survey (NLSS). A detailed description of the data and caste classification appears in Mainali et al. (2017).

Table 1 reports the aggregate wage differences between castes. Overall, the OAM calculation reveals that *Tagadhari* (group 1) individuals earn 26.6% more than *Inferior* castes (group 2). Most of this difference arises because of differences in endowments ($OAMe$), while pricing effect seems to benefit the inferior castes ($OAMp$). The aggregated wage differential (OAQ) is almost double for $\tau = 0.25$ than for $\tau = 0.75$. This determines that much of the discrimination occurs at the lowest quantiles. Moreover this implies that disadvantaged groups are the ones who suffer the most caste discrimination. In the same line, studying OAQ_1 and OAQ_2 reveals that, while OAQ_1 is rather constant across quantiles, OAQ_2 has significant differences comparing low and high quantiles, and in particular the discrimination is partially explained by differences in pricings among the *Inferior* individuals. This could be due to the fact that while a fraction of the *Inferior* caste is able to eliminate the caste discrimination, the rest is still suffering from the age-old discrimination.

Next, we evaluate the Oaxaca decomposition in terms of education only in Table 2. In this case, the differences between castes is 44.5% comprising the most important contribution to the aggregate difference. This difference is composed of both endowment ($OAMe$) and pricing ($OAMp$) components, the latter being marginally statistically significant. In a

Table 3 Oaxaca mean and quantile firm size decompositions

OAM	$OAMe$	$OAMp$		
$\overline{x}_1'\beta_{M1} - \overline{x}_2'\beta_{M2}$	$(\overline{x}_1 - \overline{x}_2)'\beta_{M2}$	$\overline{x}_1(\beta_{M1} - \beta_{M2})$		
0.127	0.065	0.062		
(0.269)	(0.022)	(0.274)		
OAQ	$OAQe$	$OAQp$	OAQ_1	OAQ_2
$y_1(\overline{x}_1, \tau) - y_2(\overline{x}_2, \tau)$	$(\overline{x}_1 - \overline{x}_2)'\beta_2(\tau)$	$\overline{x}_1'(\beta_1(\tau) - \beta_2(\tau))$	$\overline{x}_1'(\beta_1(\tau) - \beta_{M1})$	$\overline{x}_2'(\beta_2(\tau) - \beta_{M2})$
$\tau = 0.25$				
0.430	0.060	0.371	0.170	−0.132
(0.312)	(0.029)	(0.320)	(0.201)	(0.166)
$\tau = 0.50$				
0.218	0.062	0.156	0.061	−0.029
(0.383)	(0.027)	(0.388)	(0.225)	(0.133)
$\tau = 0.75$				
−0.426	0.069	−0.496	−0.435	0.119
(0.328)	(0.027)	(0.335)	(0.184)	(0.179)

Notes: Mean and quantile decompositions: $OAM = OAMe + OAMp$; $OAQ(\tau) = OAQe(\tau) + OAQp(\tau)$; $OAQ(\tau) = OAM + OAQ_1(\tau) - OAQ_2(\tau)$. Wild bootstrap standard errors in parentheses based on 200 bootstrap samples

Table 4 Oaxaca mean and quantile occupation decompositions

OAM	$OAMe$	$OAMp$		
$\overline{x}_1'\beta_{M1} - \overline{x}_2'\beta_{M2}$	$(\overline{x}_1 - \overline{x}_2)'\beta_{M2}$	$\overline{x}_1(\beta_{M1} - \beta_{M2})$		
−0.135	0.058	−0.193		
(0.171)	(0.035)	(0.179)		
OAQ	$OAQe$	$OAQp$	OAQ_1	OAQ_2
$y_1(\overline{x}_1, \tau) - y_2(\overline{x}_2, \tau)$	$(\overline{x}_1 - \overline{x}_2)'\beta_2(\tau)$	$\overline{x}_1'(\beta_1(\tau) - \beta_2(\tau))$	$\overline{x}_1'(\beta_1(\tau) - \beta_{M1})$	$\overline{x}_2'(\beta_2(\tau) - \beta_{M2})$
$\tau = 0.25$				
−0.106	0.067	−0.174	0.044	0.015
(0.192)	(0.049)	(0.203)	(0.119)	(0.105)
$\tau = 0.50$				
−0.251	0.048	−0.299	0.074	0.190
(0.283)	(0.042)	(0.295)	(0.102)	(0.168)
$\tau = 0.75$				
−0.091	0.035	−0.127	−0.068	−0.112
(0.226)	(0.040)	(0.235)	(0.10)	(0.121)

Notes: Mean and quantile decompositions: $OAM = OAMe + OAMp$; $OAQ(\tau) = OAQe(\tau) + OAQp(\tau)$; $OAQ(\tau) = OAM + OAQ_1(\tau) - OAQ_2(\tau)$. Wild bootstrap standard errors in parentheses based on 200 bootstrap samples

similar fashion to the aggregate analysis, lower quantiles contribute the most to wage differentials and the $OAQp$ component arises as the most important in terms of the interquartile variation. In particular, differences for $\tau = 0.25$ are the largest. OAQ_1 and OAQ_2 reveals that intra-caste variation in returns to schooling does not contribute to explain wage differentials.

Following Banerjee and Knight (1985) and Mainali et al. (2017) we also apply the decomposition analysis to firm size and occupations to proxy access to better paid jobs by castes (Tables 3 and 4). The firm size decomposition reveals a contribution of access to larger firms and better paid occupations by the dominant caste. Quantile analysis shows that this effect is uniform across quantiles.

6 Conclusion

This paper extends the mean-based Oaxaca-Blinder decomposition to quantile regression. The random-coefficients structure allows for simple formal statistical framework to analyze pricing effects. This paper provides a simple formalization of the decomposition methods, and proposes the application of the appropriate asymptotic method. Further extensions of the model should formalize the Oaxaca-Blinder methodology and the approach developed here to compute unconditional quantiles to compare distributional effects.

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