

# Response of the magnetosphere to perturbation by storms and Alfvén wave trains

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Received: November 1998; accepted: May 6, 1999.

## RESUMEN

El comportamiento de la magnetosfera como un sistema dinámico ha sido estudiado a través del análisis de distintos índices de la actividad geomagnética. Sin embargo aun no se llegó a ninguna respuesta definitiva a este problema. Los estudios iniciales de reconstrucción del espacio de fases a partir de las series temporales del índice AE sugirieron la presencia de un atractor de baja dimensión; estudios posteriores no confirmaron este resultado. El problema radica en que la fuente de excitación de la magnetosfera, el viento solar, es turbulento y por lo tanto es incorrecto considerar a este sistema como autónomo. En el presente trabajo, estudiamos la serie temporal del índice AE con muestreo cada minuto en tres condiciones físicas distintas. Se compara una tormenta geomagnética (intervalo de tiempo durante el cual el índice Dst tiene valores menores de -100 nT), valores de Dst mayores de -20 nT, (actividad geomagnética débil), y actividad auroral continua de alta intensidad y larga duración (evento HILDCAA). El estimador de Takens de la dimensión de correlación se determinó para estas condiciones y un conjunto de datos "surrogated"; los resultados no indicaron concluyentemente un carácter determinista. A continuación se estudió la predictibilidad de las series y sus "surrogated". Se halló que el caso correspondiente al evento HILDCAA es el más predecible y consecuentemente de menor dimensión de correlación. Concluimos que, frente a un evento HILDCAA, la magnetosfera actúa como un sistema excitado en forma estacionaria.

**PALABRAS CLAVE:** Campo magnético, perturbaciones magnéticas.

## ABSTRACT

We investigate AE time series at one-minute sampling intervals under (a) geomagnetic storm Dst index below -100 nT, (b) weak geomagnetic activity  $Dst > -20$  nT, and high-intensity, long-duration continuous auroral activity (HILDCAA). The Takens estimator of correlation dimension and an ensemble of surrogate data sets were calculated for these conditions. No conclusive indication of deterministic behavior was found. The predictability was studied for all series and for surrogated sets. The case corresponding to HILDCAA was the most predictable and the one with the lowest correlation dimension. Thus the magnetosphere during an HILDCAA event may act as a stationary excited system.

**KEY WORDS:** Geomagnetic field, magnetosphere perturbations.

## INTRODUCTION

Temporal variations of amplitude and direction of the geomagnetic field are associated with magnetospheric responses to the solar wind. An unknown small number of state variables may control the evolution over the largest spatial scales, and over sub-storm time scales and beyond. Little is known about the dynamic system. Attempts of estimating the dimension of the dynamic system as reconstructed from geomagnetic index time series have been made (see e.g. Vassiliadis *et al.*, 1990, Prichard and Price, 1992, Sharma *et al.*, 1993), while others have attempted relating solar wind parameters to geomagnetic indices (e.g. Akasofu, 1981).

The low dimensionality of the magnetospheric response obtained by previous phase reconstruction of AE data is still open to discussion. A correlation dimension value of 2.2 to 4.2 was obtained from AE and AL indices (Vassiliadis *et al.*, 1990; Roberts, 1991); however, Prichard and Price, (1992), re-examined these results showing spurious dimension

estimates due to long autocorrelation times, and not to system dynamics.

Several HILDCAA events studied by Tsurutani and Gonzalez (1987); suggested they may be caused by interplanetary Alfvén waves. If the frequency of the Alfvén waves southward magnetic field ( $B_z$ ) is low during a HILDCAA event a peak in the cross-correlation coefficient (0.5) between  $B_z$  and AE is observed. When the frequency of  $B_z$  is high the correlation with AE can be as low as zero (see Tsurutani and González, 1987). This suggests a non-linear response of the magnetosphere to Alfvén-wave excitation. The observed HILDCAA series corresponds to the latter situation.

In the present work we evaluate, under three different physical conditions, whether the magnetosphere is organized in such a way that a few state variables suffice to describe its dynamics. Qualitative nonlinear analysis of temporal variation applied to the AE index contributes to determine

whether the magnetosphere is a stochastic or a low-dimension deterministic chaotic system. In each case we studied three days of one-minute AE data (4096 data points). The first AE time series record starts on August 30, 1978, and corresponds to a HILDCAA event. The second AE time series begins on July 28, 1987, and corresponds to a geomagnetic storm. The third AE begins on January 3, 1987, and corresponds to a quiet day. Figure 1 shows the Dst and AE indices under the three different conditions.

A comparison of results of Takens estimator obtained from the time series for the three physical situations do not confirm the presence of a chaotic attractor, probably because of the presence of noise (see e.g. Klimas *et al.* 1996). Next the predictability of the time series and their surrogate sets

was studied. The case corresponding to HILDCAA was the most predictable and thus had a lower correlation dimension.

### NONLINEAR TIME SERIES ANALYSIS

The embedding technique of phase reconstruction is used to determine the stochastic or deterministic character of the systems. In general, we have a time series of scalar measurements  $x(t_i)$ , that may or may not be a variable of the system. Space reconstruction of time series contains information about unobserved variables that define the state of the system (see e.g. Abarbanel *et al.*, 1993).

The first step is to construct a  $d$ -dimensional delay vector  $X_i = \{x(t_i), x(t_i - \tau), \dots, x(t_i - (d-1)\tau)\}$ , where  $\tau$  is the fixed

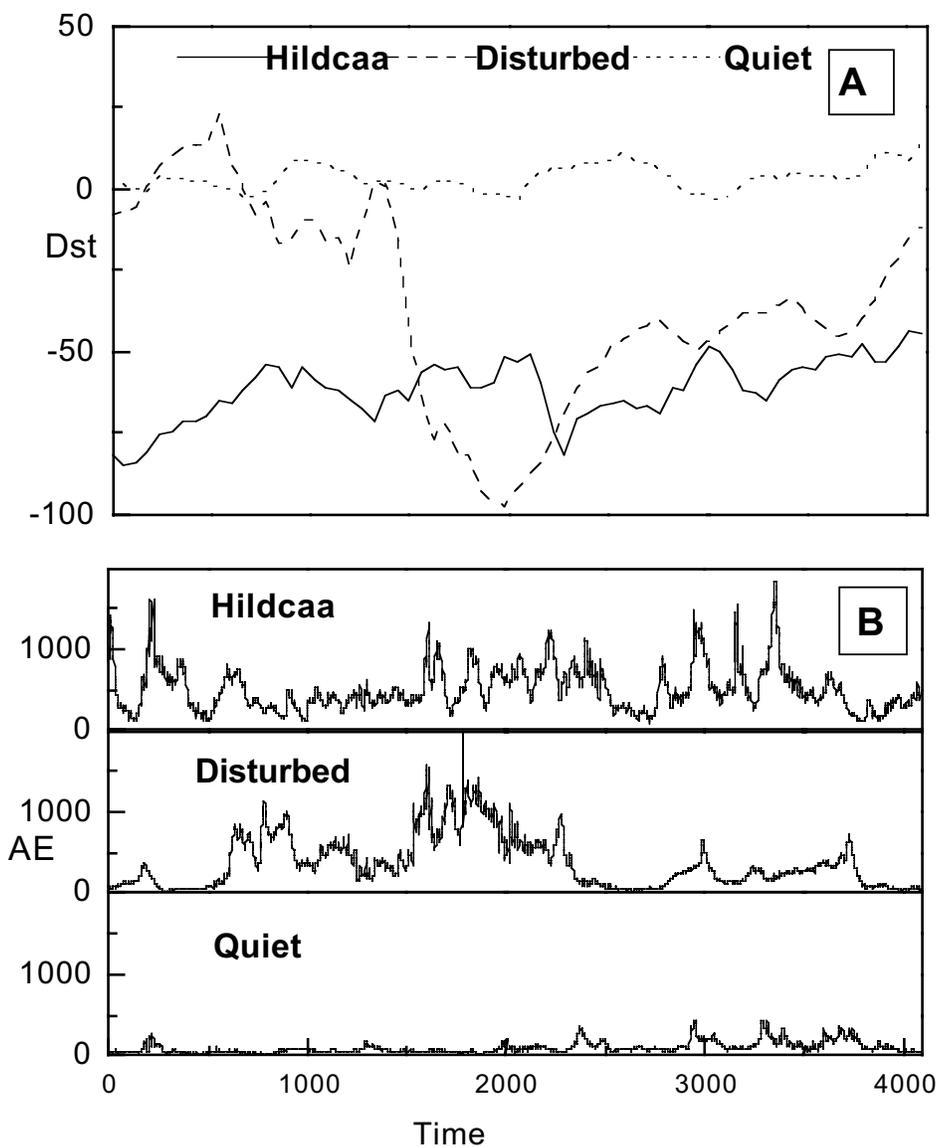


Fig. 1. Dst time series (A) and AE time series (B) under three different magnetospheric perturbations (World Data Center C2 for Geomagnetism data)

time interval between successive elements, chosen large enough to find vector components that are independent, and small enough to preserve the dynamic information about the physical process under study. The choice of  $\tau$  may be made such that the  $X_i$  components be uncorrelated. Thus the estimate for  $\tau$  may be based on the first zero of the linear autocorrelation function, or on the first minimum of the average mutual information (Fraser and Swinney, 1986). The latter permits a better selection of  $\tau$ ; when there is no minimum the value of  $\tau$  should satisfy  $MI(\tau) = MI(0) / 5$  (Abarbanel *et al.*, 1993).

False neighbor statistics makes it possible to find an appropriate dimension  $d$  for the delay vectors to contain the system dynamic properties. When the percentage of false nearest neighbors approaches zero, the desired dimension is obtained.

To discriminate deterministic-stochastic properties of AE time series we use Takens estimator of the correlation dimension (Prichard 1994):

$$D_T = \frac{C(r_0)}{\int_0^{r_0} C(r) / r dr} \quad (1)$$

where  $C(r_0)$  is the correlation integral with an appropriate window to remove autocorrelation effects; and  $r_0$  is an upper cutoff that was set to  $\frac{1}{4}$  the standard deviation of the time series.

A second nonlinear statistics used in this work is the

root mean square error of the predicted time series as obtained from a nonlinear prediction scheme. A simple approach of locally constant dynamics is used to make the forecast (Kantz and Schreiber 1997):

$$\hat{x}(t_i + \Delta t) = \frac{1}{|U_\epsilon(X_i)|} \sum_{X_j \in U_\epsilon(X_i)} x(t_j + \Delta t) \quad (2)$$

where  $\hat{x}(t_i + \Delta t)$  is the data forecast at a time  $\Delta t$  ahead of  $x(t_i)$ ;  $U_\epsilon x(t_i)$  is the set of vectors closer than  $\epsilon$  to  $X_i$ ; and  $|U|$  denotes the number of elements in  $U$ . For  $\epsilon$  we use  $\frac{1}{4}$  of the mean root square of the standard deviation of the time series.

Surrogate data sets are used to compare the true series Takens estimator and the nonlinear prediction error with the stochastic time series. Surrogate data sets are easily obtained by randomizing phases after taking the Fourier Transform of the original data and taking the inverse transform. The surrogate data will have the same power spectrum and autocorrelation function. If the Takens estimator of the true data is within the standard deviation of the surrogate data Taken's estimator, a stochastic process is taking place. The surrogate data nonlinear prediction error is compared to the prediction error of the true measurements; if the latter is smaller than that of the surrogates, possibly a deterministic process is taking place.

## RESULTS AND CONCLUSIONS

Dst data corresponding to a HILDCAA event (Hildcaa), to a geomagnetic storm (Disturbed), and to weak geomagnetic

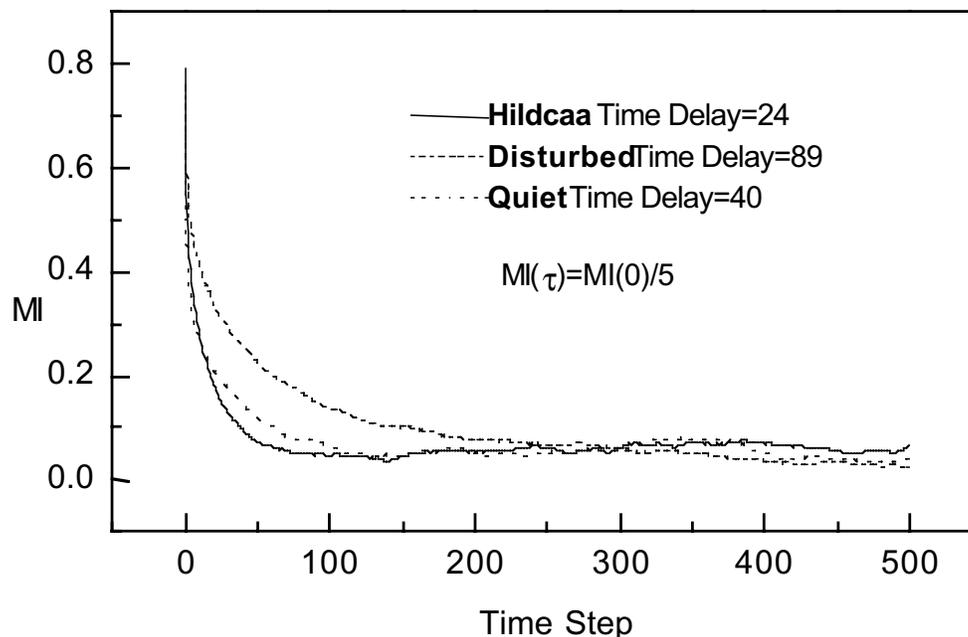


Fig. 2. Mutual Information.

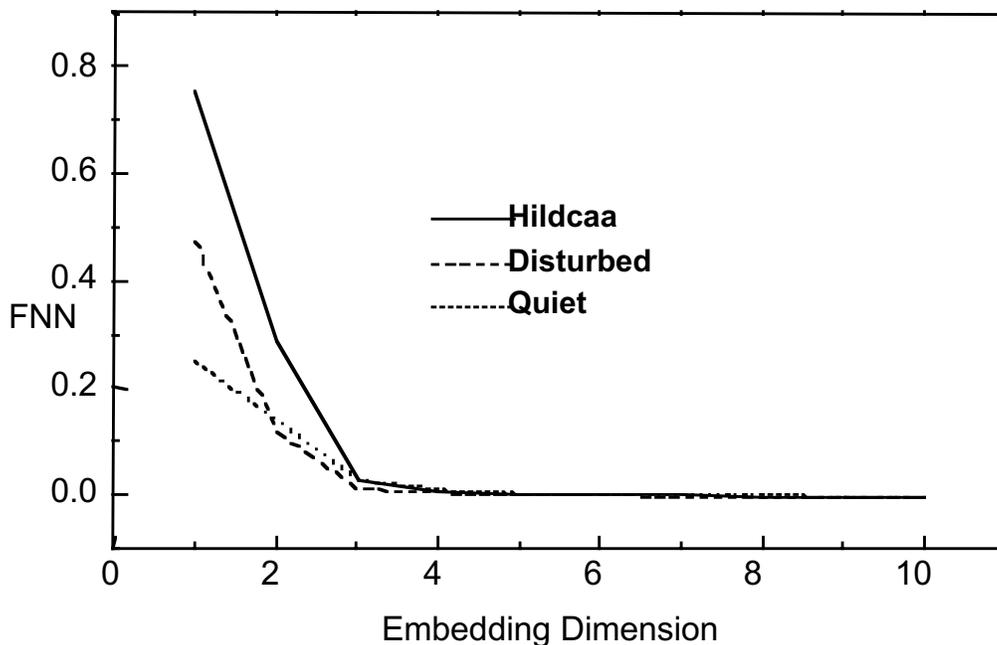


Figure 3: False Nearest Neighbors

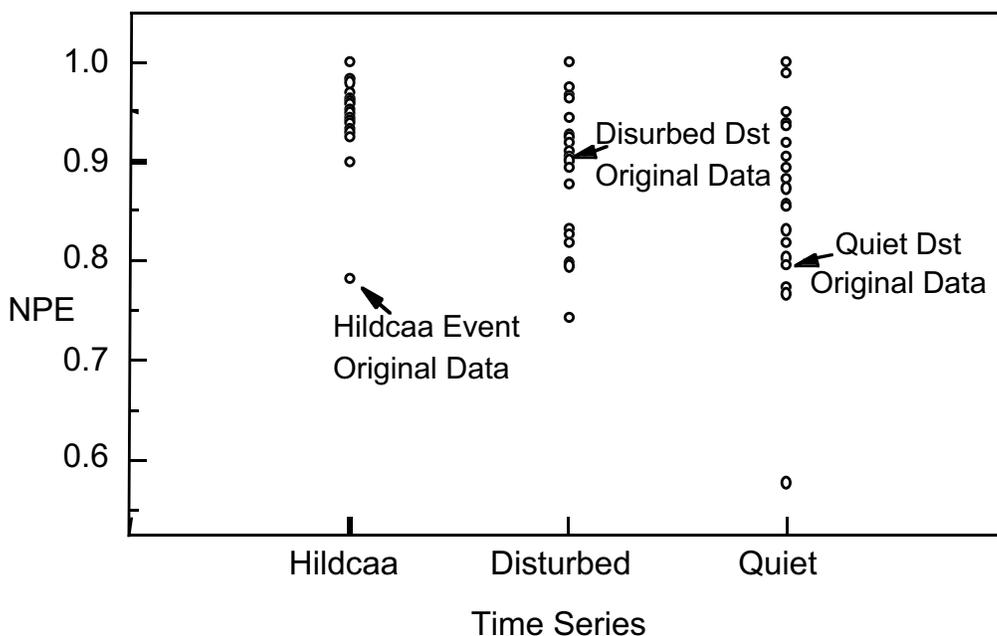


Fig. 4. Non linear prediction error statistic for original data sets and corresponding surrogates.

activity (Quiet), are shown in Figure 1a and the corresponding AE data sets are shown in Figure 1b.

The average mutual information for the three AE index data sets was calculated to find the time delay ( $\tau$ ); the values found using the above criteria are 89, 40 and 24 minutes for series Hildcaa, Disturbed, and Quiet respectively (Figure 2). The percentage of false nearest neighbors is calculated to

determine an appropriate embedding dimension. An embedding dimension of four seems appropriate for all as seen in Figure 3.

The Takens estimator was obtained for the original data sets as well as for 19 surrogate data sets. None of the considered cases seems to converge convincingly to a low-dimension attractor. The main reason for this is noise; the

correlation integral is not stable when we vary slightly the parameters, such as the autocorrelation window or the time delay. To evaluate the influence of noise, we attempt to ascertain whether there is an underlying deterministic process by the simple nonlinear prediction method described above. For each original data set and for 19 sets of surrogate data in each situation, we find the root mean square prediction error. The results are shown in terms of the normalized root mean square prediction errors (Figure 4).

The original data in a HILDCAA event has a lower error than its surrogate data sets. It is over three times lower than the root mean square standard of among the surrogate data sets.

This suggests a difference in the dynamic process involved for each situation. The geomagnetic storm involves a contribution of many random variables (Campbell 1996), and it is reasonable that it be immersed in the surrogate data. The solar wind input for the quiet case is turbulent, so the system cannot be considered autonomous. The predictability of the HILDCAA time series is due to the presence of a stable attractor. This attractor requires the presence of a stationary input. This supports the hypothesis that the high frequency Alfvén wave train that generates the HILDCAA event excites the magnetosphere in a stationary way.

#### ACKNOWLEDGMENTS

This work has been performed under grant PICT0418 from Agencia Nacional de Promoción de la Ciencia y la Tecnología of Argentina.

#### BIBLIOGRAPHY

- ABARBANEL, H. D. I., R. BROWN, J. J. SIDOROWICH and L. TSIMRING, 1993. The analysis of observed chaotic data in physical systems. *Rev. Modern Phys.* **65**, 1331-1392.
- AKASOFU, S. I., 1981. Energy coupling between the solar wind and the magnetosphere. *Space Sci. Rev.* **28**, 121-138.
- CAMPBELL, W. H., 1996. Geomagnetic storms, the Dst ring-current myth, and lognormal distributions. *J. Atmos. Terr. Phys.*, **58**, 1171-1187.
- FRASER, A. M. and H. L. SWINNEY, 1986. Independent coordinates for strange attractors from mutual information. *Phys. Rev. A* **33**, 1134-1140.
- KANTZ, H., T. SCHEIBER, 1997. Nonlinear time series analysis. Cambridge University Press, Cambridge.
- KLIMAS, A. J., D. VASSILIADIS, D. N. BAKER and D. A. ROBERTS, 1996. The organized nonlinear dynamics of the magnetosphere. *J. Geophys. Res.* **A6**, 13089-13113.
- PRICHARD, D., 1994. The correlation dimension of differenced data. *Phys. Lett. A* **191**, 245-250.
- PRICHARD, D. and C. P. PRICE, 1992. Spurious dimension estimates from time series of geomagnetic indices. *Geophys. Res. Lett.* **19**, 1623-1626.
- ROBERTS, D. A., 1991. Is there a strange attractor in the magnetosphere? *J. Geophys. Res.* **96**, 16031-16038.
- SHARMA, A. S., D. V. VASSILIADIS and K. PAPADOPOULOS, 1993. Reconstruction of low-dimensional magnetospheric dynamics by singular spectrum analysis. *Geophys. Res. Lett.* **20**, 335-338.
- TSURUTANI, B. T. and W. D. GONZALEZ, 1987. The cause of high-intensity long duration continuous AE activity (HILDCAAS): Interplanetary Alfvén wave trains. *Planet. Space Sci.* **35**, 405-412.
- VASSILIADIS, D. V., A. S. SHARMA, T. E. EASTMAN and K. PAPADOPOULOS, 1990. Low-dimensional chaos in magnetospheric activity from AE time series. *Geophys. Res. Lett.* **17**, 1841-1844.

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