The principle of causality and the coordination of concepts and spatio-temporal objects in Cassirer's philosophy

Abstract

This paper analyzes the role of the principle of causality in Cassirer's account of the coordination of concepts and spatio-temporal objects. We shall see that, in contradistinction to Kantian schematism, Cassirer maintains that this coordination is not achieved by means of a third element (the schema), which albeit intellectual is nevertheless also sensible. Rather, in Cassirer's view, the coordination will take place through a specification of the concepts that should be sought "*within the domain of concepts itself*." We shall show that the principle of causality is the ultimate condition upon which the possibility of the coordination of concepts and spatio-temporal objects depends.

Keywords

Cassirer, Kant, concept, schema, object, causality

Introduction

It has been argued recently that the key to solving many interpretation problems of the Kantian doctrine of schematism is to determine correctly the object that is to be subsumed under categories by means of schemata. More precisely, it has been maintained that this object is not merely an appearance taken as an object in general, but an appearance considered as a concrete, empirically given object. From this viewpoint, the proper task of schematism is to guarantee the coordination of concepts (in particular categories) and individual spatio-temporal objects.[1] The aim of this paper is to analyze an alternative proposal to the Kantian solution of the coordination problem, which can be found in Ernst Cassirer's philosophy. We shall see that, in contradistinction to Kantian schematism, Cassirer maintains that the coordination of concepts and objects is not achieved by means of a third element (the schema), which albeit intellectual is nevertheless also sensible.[2] Rather, in Cassirer's view, the coordination will take place through a specification of the concepts that should be sought "*within the domain of concepts itself*."[3]

The thread that will enable us to reconstruct Cassirer's argumentation will be his interpretation of the causality principle. We shall show that this principle is the ultimate condition upon which the possibility of the coordination of concepts and spatio-temporal objects depends.

The structure of the paper will be the following. We shall begin by discussing the general framework of Cassirer's investigations on the causality principle. This is the framework provided by the transcendental *method* (§1). Then, we shall analyze the problems that Cassirer finds in the Kantian doctrine of causality, in particular regarding the schematism of that category, once the transcendental method is assumed (§2). Later, we shall discuss the main structure of the system of physical knowledge, for it is within this system that concepts and spatio-temporal objects get coordinated (§3). Finally, we shall investigate the transcendental role that Cassirer assigns to the causality principle, and we shall show how this principle makes the coordination of concepts and objects in the system of physical knowledge first possible (§4).

§1 Cassirer and the transcendental method

The Neo-Kantianism of the Marburg school conceives itself as Kantian not regarding the content of its philosophy but rather regarding the form of its philosophizing. Neo-Kantians stress that they do not accept dogmatically any result of the Kantian doctrine. They just adopt the only real legacy of Kant: the philosophical *method*. This method is the *transcendental* one.[4]

According to the transcendental method, philosophy should take a certain *factum* as the starting point for the reflection and proceed to seek the possibility conditions of that fact. In the case of theoretical philosophy, the fact to be considered is experience. But experience is identified with physico-mathematical science. In this sense, Hermann Cohen maintains that experience is given in mathematics and in pure natural science,[5] and, more precisely, in Newtonian science.[6] The task of transcendental philosophy, as a theory of experience, will be then to determine the conditions of possibility of Newtonian science.[7]

Cassirer adopts this Cohenian conception of the transcendental method.[8] But, while Kant took as a fact the science of his time, Cassirer applies this method to the new *facta* provided by the progress of physico-mathematical sciences, which include, in particular, non-Euclidean geometry and the relativity and quantum theories. In this way, Cassirer thinks that, starting with Kantian presuppositions, it is possible and even necessary to take the philosophical investigation *beyond* the stage reached by Kant himself. This progress, Cassirer remarks, is just the reaffirmation of the spirit of Kantian philosophy, since "the purpose of the *Critique of Pure Reason* was not to ground philosophical knowledge once for all in a fixed dogmatic system of concepts, but to open up for it the 'continuous development of a science' in which there can be only relative, not absolute, stopping points."[9]

Cassirer discusses the role of the causality principle in the framework provided by such an analysis of the possibility conditions of the new scientific facts, according to the transcendental method. In the next section we shall consider the problems that Cassirer finds in the Kantian interpretation of that principle.

§2 Cassirer on Kantian causality

Cassirer maintains: "Of all the various explanations of the causal concept offered in the *Critique of Pure Reason*, perhaps the most precise and most satisfying is the one in which it is said that the concept represents nothing but a direction for the formulation of definite empirical concepts."[10] At this point, Cassirer quotes the following passage of the first critique: "That everything that happens has a cause cannot be inferred merely from the concept of happening in general; on the contrary, it is this fundamental proposition which shows how in regard to that which happens we are in a position to obtain in experience any concept whatsoever that is really determinate."[11] The causality principle is a rule that indicates how we should conceive and form our concepts in order that they can fulfill their transcendental task: that of turning mere appearances into objective knowledge.[12] Thus, the causality principle is a principle about *cognitions* and not about *things* or events.[13] Even though in our everyday use of the principle we identify things as causes and effects, such a use is misleading if we are looking for *scientific* foundation of causality. For what we call *thing* in our everyday experience is a complex of conditions that should be analyzed until the authentic scientific causal judgments are reached.[14] Such a characterization of *thing* as a complex of conditions expresses the core of Kant's doctrine, according to which the concepts of lawlikeness (Gesetzlichkeit) and objectivity are connected in a synthetic *a priori* judgment: only by means of a lawlike ordering can appearances be referred to an object of experience. In this sense, Cassirer indicates: "Objectivity or objective reality, is attained only because and insofar as there is conformity to law –not vice versa."[15] Therefore, we do not cognize *objects*, as if they (logically) preceded their laws, but rather by means of these laws we cognize *objectively*, as far as we establish certain limits and permanent connections in the uniform course of experience.[16]

However, Cassirer maintains, in the deduction of the causality principle carried out in the second analogy of experience, Kant wrongly directs his inquiry to empirical *things* and *events*, instead of exclusively focusing on empirical *knowledge*.[17] According to Cassirer, Kant rightly maintains the logical preeminence of the concept of law upon the concept of object, but the implications of such Copernican inversion are not fully assumed by the Kantian analysis of the causality principle. Here, Kant still struggles against representations of things and substances, as though a causal connection could be established by merely considering successive states of the same thing and determining the earlier as cause of the later.[18] Thus, following the example used by Kant, the objective series of a boat going down the river should not be established, for Cassirer, simply by determining the upstream state of the boat as the cause of its downstream state. Rather, the determination of the objective series requires considering the forces at issue and, more precisely, the physical laws (of gravitation, hydrodynamics and hydrostatics) that govern the movement of the boat. According to Cassirer, "these laws are the real components of the assumed causal connection."[19] In doing this, however, difficulties appear, since the exact formulation of those laws demands the symbolic language of physics, which differs significantly from the language of "things."[20] The determination of the transcendental role of causality as a condition of the possibility of scientific knowledge requires an analysis much more precise than the one carried out by Kant in the second analogy of experience. In particular, it is necessary to give up any consideration of "things and events" and the underlying concept of substance in order to focus on the functional form of experience. For this reason, Cassirer criticizes Kant and declares: "Kant did not follow to the end the road which he took in his solution of the Humean problem."[21]

Cassirer shares the Kantian position concerning the dependence of the concept of object on the concept of law, but he rejects the way in which Kant describes the proper role of the causality principle, for Kant then seems to invert the direction of the dependence. In fact, one might think that Kant initially assumes certain states of an object (like the upstream and downstream states of the boat) which then in a second move are represented in a causal relationship in order to determine their temporal order. However, this reconstruction of Kant's argument would not be correct. Rather, for Kant, it is precisely the causality principle that enables an objective determination according to the concept of law: the principle of causality determines the *objective* series of experience by means of a *lawlike* connection of successive appearances. In contradistinction to what Cassirer suggests, Kant's argument tries to show that *only* the representation of a *law* of my subjective perceptions enables their *objective* reference. Accordingly, the upstream and downstream states of the boat are represented as objective states *only* with the application of the principle of causality. Thus, Kant maintains the dependence of the concept of object on the concept of law in his analysis of causality.

However, according to Cassirer, Kant's position still has another shortcoming. The mere thought of lawlikeness, which for Cassirer expresses the core of the concept of causality, leaves indeterminate *how* this conformity to law is to be empirically realized. Therefore, Cassirer says, Kant stresses that the category of causality should be *specified* in order to be useful and applied to experience.[22] This specification is achieved by means of the *transcendental schematism*. It is precisely this doctrine that explains how categories (in particular causality) may be applied to empirical phenomena. But, according to Cassirer, the validity of Kantian schematism is confined to the framework of Euclidean geometry and Newtonian mechanics. Therefore, Cassirer points out: "it is precisely these schemata which have lost their universal significance through the discovery of non-Euclidean geometry on the hand and the results of the special and general relativity theories on the other."[23] In the same sense, Cassirer maintains that the "crisis of causality" produced by quantum physics is not a crisis of the concept of cause, but rather a crisis of the way in which that concept is empirically applied through schemata. Thus, "such schematization has been definitely limited through the advent of the quantum theory. We can no longer combine causality with space-time description, let alone amalgamate the two in the manner of classical physics."[24]

Nevertheless, Cassirer indicates, Kant himself presents a version of the causality principle in which the latter remains free from the conditions imposed by schematism.[25] This is the A version of the principle, that reads: "Everything that happens, that is begins to be, presupposes something upon which it follows according to a rule."[26] In Cassirer's opinion, this formulation solely demands the possibility of connecting through rules that which happens, without presupposing anything about those rules. Causality just implies the mere conformity of natural events to law.[27] But, in the proof of the causality principle, Kant takes a further step by introducing time through the schema of cause and effect. Finally, Kant relates causality and continuity. The cause does not produce the effect instantaneously, but in a certain time interval $t_b - t_a$, such that a real magnitude b-a increases through all its intermediate degrees from its initial value a in t_a to its final value b in t_b.[28] However, quantum theory rejects this continuity requirement by accepting that certain magnitudes may only have discrete values and vary from an initial to a final value without adopting the intermediate ones. Therefore, the connection between causality and continuity, as Kant understands it, should be abandoned.

Given this criticism of Kantian schemata, one might well expect that Cassirer would search for new transcendental schemata that could perform the task that the Kantian ones, dependent on an earlier stage of science, are no longer able to carry out. However, Cassirer's proposal is much more radical. The new scientific facts, upon which a philosophical investigation carried out according to the transcendental method finally rests, demand a reinterpretation of the whole problem of Kantian schematism. In this sense, Cassirer maintains: "Transcendental logic can thus no longer be connected with or be dependent on transcendental aesthetics, as was the case in Kant's system. The demanded specialization, indispensable for the empirical use of the causal concept, must now be looked for *within the domain of concepts itself*."[29]

In the next section we shall see that the conformity to law required by the principle of causality does not get specified by means of non-conceptual conditions. Rather, Cassirer puts forward a mere logical specification achieved in a system of invariants of experience.[30]

§3 Physical knowledge as a system of invariants

Cassirer conceives physics as a system in which three types of statements are to be distinguished: the statements of *measurement results*, the statements of *laws* and the statements of *principles*.[31]

The statements of measurement results are the first step of the transition from the world of senses to the world of physics. This step is characterized by the conversion of sense data into determinations that may be subsumed under mathematical concepts.[32] That which is perceived is represented in terms of measure and number and the immediate sense apprehension leaves its place to experimental observation.[33]

From the point of view of the *extension* of knowledge, the statements of measurement results constitute a clear progress, because by means of measurement instruments it is possible to go beyond the contingent limits of our senses, as we do when we study, e.g., the lunar surface with a telescope or blood cells with a microscope. However, this extension is not the key point at issue here. In parallel to an expansion of our world image, a *concentration* takes place too. The variety of sensible qualities leaves its place to a few fundamental determinations, from which the richness of the sensible data should be explained.

This concentration makes a crucial modification of our knowledge possible. Multiple perceptions just make up an *aggregate*: the sensible qualities of a perceived thing are merely juxtaposed. Color, smell, flavor and texture of an apple are independent from each other. Any combination of these qualities may contingently take place. Unlike a mere aggregate, the properties of a physical object, such as, e.g., and ideal gas, are organized into a *system*.[34] Thus, the modification of one property entails the modification of the rest of them. In this way, temperature, pressure and volume of the gas are not independent properties, but rather their values are interconnected in a necessary manner.

Such *relationships* between the properties of a physical object are expressed by a different type of statement: the statements of *laws*. Whereas the statements of measurement results are characterized by an unavoidable reference to a "here and now," law statements have the logical form "if, then." [35] Accordingly, a law statement cannot be taken as a mere summary of a number of statements of measurement results. Laws do not connect in a hypothetical manner individual magnitudes to which we may ascribe a spatio-temporal index. Rather, laws relate *classes* of magnitudes. For this reason, the statements of laws are not reached by means of an (always controvertible) inductive inference that, starting from many cases, aims at their totality. To the contrary, in a law statement the "here and know" viewpoint is completely abandoned and the representation of a necessary connection is reached.

However, the transition from statements of measurement results to statements of laws is not the end stage in the process of physical knowledge. For just as the multiplicity of properties of a physical object acquires unity through laws, these laws are in turn unified by means of principles.[36] Such unification is accordingly expressed by a third type of statement: the statements of *principles*. While the statements of measurement results are individual and the statements of laws are general, the statements of principles are universal. These statements do not refer to individual magnitudes or classes of magnitudes, but they connect different domains of physical knowledge, such as optics, mechanics or electrodynamics. The differentiation of these domains is thought of as relative to the higher principle, which therefore grounds the differentiation and, at the same time, unifies the domains.[37] Physics does not stop in front of the multiplicity of its laws, but seeks rules that enable the transition from one law to another. These rules are principles. Examples of them are Carnot's principle, the principle of energy conservation and the principle of least action, to which we shall immediately return.

The different types of statements are *invariants of different order*.[38] The statements of measurement results express values of physical magnitudes that do not depend on the subjectivity of the one carrying out the measurement. For example, in the same place and at the same moment, one observer may be warm, while a second one is cold. In each case, the perception has only subjective validity, varying from observer to observer. To the contrary, the statement of measurement result that expresses the room temperature remains invariant, since it is the same for all observers.

However, even though the temperature value is invariant in this sense, the statement of measurement result contains a spatio-temporal index: temperature has the value T in place x at time t. When knowledge progresses from statements of measurement results to law statements, this index disappears. A law statement does not include the particular temperature value in a certain place at a certain moment, as would be the case if a law were nothing more than a condensed expression of a collection of statements of measurement results. Rather, temperature is present in the law as a magnitude class that gets connected with other classes in a way that is *invariant* regarding the values of those magnitudes at different times and in different places. For example, the laws of Boyle, Mariotte and Gay

Lussac connect temperature, pressure and volume of an ideal gas according to a rule independently of the absolute values of the spatio-temporal coordinates.[39]

But, as we have seen, the multiplicity of laws is to be distinguished from the rule that unifies them. In this case, the rule is contained in a statement of principle. Physics investigates how different laws, in particular those governing different areas of physics, are logically connected to each other. In doing this, the clue is not to be found in the different kinds of facts, but on the equations that express the structure of those areas. Cassirer puts forward the principle of least action as the paradigmatic example. In its application to particular cases, this principle was already known by Heron of Alexandria, who used it to find the reflection laws of light, while Fermat also deduced the law of refraction by means of a more extended and deeper version of the principle. Leibniz made use of the principle in mechanics and Maupertuis even founded a proof of God's existence upon it. Euler gave a rigorous formulation and an exact physical meaning to the principle and Lagrange presented it in a complete and precise manner. Later, Helmholtz enunciated it as a universal physical principle, the validity of which went well beyond mechanics. The principle of least action, under the name of Hamilton's principle, is now a fundamental principle of modern physics, both of relativity and quantum theory.

The essential feature of the principle of least action is that it is not bound to any determinate content, since it is a variational principle. The principle establishes that certain magnitudes should have a minimal value, but these magnitudes can be quite diverse, e.g., the path covered by light (Heron), the required time for light to cover a path (Fermat), the product of velocity and path length (Maupertius), the mean value of potential energy

(Euler) or the difference between kinetic and potential energy (Hamilton). The demand that such magnitudes acquire an extreme value determines the general form of the laws of diverse physical disciplines, providing in this way a heuristic rule for the search and discovery of such laws. These laws will be multiple, but the principle will remain *invariant* against them.

The statements of physics join together according to their invariance degree. The statements of measurement results are invariant against the subjectivity of the observer. Nevertheless, they vary against laws. These laws are in this respect invariant, but they are not invariant against principles. Rather, principles are invariant against laws, and in being so they unify different branches of physics.

We can therefore see a reciprocal conditioning among statements that provides to the whole a systematic character. Neither are laws mere aggregates of measurement results nor are principles mere aggregates of laws. Physical knowledge does not originate from "an sich" elements, which may have sense and meaning independently of their relationship with others and that are accommodated in a kind of knowledge pyramid. Instead, we just find a functional coordination, in which all statements take part: the statements of "lower" type are entailed and presupposed by those of "higher" type. Thus, the right geometrical symbol of the system of physical knowledge would not be a pyramid, but rather a sphere, like the one that Parmenides uses to describe being.[40]

§4 Causality and the coordination of concepts and spatio-temporal objects

Even though the transition between statements of different type amounts to a qualitative jump, since it is in each case a modification of the kind of invariance that the statements express, the consideration of the causality principle entails a much more radical move. The causality principle is neither a metaphysical statement about the world in itself, nor an empirical statement about the sensible world, like those we have discussed so far. The causality principle does not talk about objects, but rather about our *knowledge* of objects and it is in this sense a *transcendental* principle.[41] More precisely, the principle is a statement about our empirical knowledge of objects and, thus, about the statements of measurement results, about the statements of laws and about the statements of principles. According to Cassirer, the causality principle declares that all these statements "can be so related and combined with one another that from this combination there results a system of physical knowledge and not a mere aggregate of isolated observations."[42] In other words, the causality principle states that the conversion of sensible data into measurement results, their ordering according to laws and the unification of the multiplicity of these laws under principles is always possible. Thereby, even though such a process of systematization can never be considered as complete, its achievement should be sought as if an ultimate system were possible, by assuming that natural phenomena do not resist being systematically ordered.

Therefore, Cassirer understands the causality principle in a transcendental sense, as a condition of the possibility of scientific knowledge, but he ascribes to it a meaning that does not coincide with the Kantian one. According to Kant, the causality principle is constitutive for the possibility of experience, in so far as it makes the distinction between the subjective series of perceptions and the objective series of experience first possible. For Cassirer, the causality principle has instead a regulative character,[43] guiding our understanding towards the systematic unity of experience.[44]

Kant distinguishes the necessary task of three different cognitive faculties, which cooperate in knowledge: sensibility, understanding and reason. Sensibility provides us with intuitions. Understanding synthesizes these intuitions by means of concepts and thereby refers them to an object. Reason brings about the systematic unity of such objective cognitions. "Thus," Kant declares, "all human knowledge begins with intuitions, proceeds from thence to concepts, and ends with ideas."[45] From this viewpoint, the subsumption of spatio-temporal objects under concepts is the problem that the theory of schematism deals with. Schemata are precisely those representations that enable the spatio-temporal objects *given* by sensible intuitions to be *thought* by the concepts of understanding.

Cassirer, in contradistinction to Kant, does not pose the problem in terms of cognitive faculties, since in that way the danger of psychologism seems unavoidable.[46] The problem of the coordination between concepts and spatio-temporal objects is not that of the heterogeneity between *intellectual* concepts and *sensible* appearances.[47] Cassirer rather assumes a transcendental perspective from which there is just one single objectifying function. In this regard, Cassirer indicates: "According as the *function* of objectivity, which is unitary in its purpose and nature, is realized in different empirical material, there arise different concepts of physical reality; yet these latter only represent different stages in the fulfillment of the same fundamental demand."[48] The transcendental task of each Kantian faculty is thereby reinterpreted as a different moment of fulfillment of that unitary function.[49] The statements of measurement results provide us with spatio-temporal data that are to be brought under rules expressed by statements of laws, the unity of which is attained by statements of principles.[50]

However, since “in all scientific knowledge laws precede objects,” Cassirer stresses that no object is ‘given’ to us “other than through laws.”[51] Thus, the Kantian distinction between the sensible conditions under which objects are given in intuition and the intellectual conditions under which objects are thought by means of laws cannot be maintained any more. The data for objective knowledge, the “statements of the first level,” cannot be isolated from statements of higher order as if “there would always be the possibility of imagining the higher layers removed without destroying the bottom layer or even altering it essentially.”[52] To the contrary, “everything significantly factual is already theory.”[53]

According to Cassirer, each type of statement expresses a peculiar moment of the conformity to law demanded by the causality principle.[54] More precisely, each moment corresponds to a certain order of logical invariance. In this way, concepts and spatio-temporal objects get coordinated in a system of invariants of experience.

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[1] Caimi (2013).

[2] "Obviously there must be some third thing, which is homogeneous on the one hand with the category, and on the other hand with the appearance, and which thus makes the application of the former to the latter possible. This mediating representation must be pure, that is, void of all empirical content, and yet at the same time, while it must in one respect be *intellectual*, it must in another be *sensible*. Such a representation is the *transcendental schema*." *KrV*, B 177.

[3] Cassirer (1956), p. 166. Our emphasis.

[4] Natorp (1912), p. 194. However, it should be pointed out that the expression "transcendental method" is not to be found in Kant's texts. See: Baum (1980).

[5] Cohen (1877), pp. 24 – 25.

[6] Cohen (1910), p. 32.

[7] Cohen (1918), p. 93.

[8] On Cassirer´s conception of the transcendental method, see: Ferrari (2002). For an analysis of the connections between Cassirer´s philosophy and that of other members of the Marburg school, see: Marx (1975).

[9] Cassirer (1923), p. 355.

[10] Cassirer (1956), p. 127.

[11] *KrV*, A301 = B357.

[12] Cassirer (1956), p. 19. Similarly, Kant declares: “They [the pure concepts of the understanding] serve as it were only to spell out appearances, so that they can be read as experience.” *Prol*, AA IV 312.

[13] Cassirer, (1956), p. 65.

[14] Cassirer, (1956), p. 21.

[15] Cassirer, (1956), p. 132.

[16] Cassirer, (1956), p. 137; Cassirer (1923), p. 303.

[17] Cassirer (1956), p. 60.

[18] Cassirer (1956), pp. 21 - 22.

[19] Cassirer (1956), p. 22.

[20] Cassirer (1956), p. 22.

[21] Cassirer (1956), p. 59.

[22] Cassirer (1956), p. 166.

[23] Cassirer (1956), p. 166.

[24] Cassirer (1956), p. 166. For an overview on Cassirer’s reception of the scientific progress of late 19th century and early 20th century, see: Plümacher (1996). For the specific case of physics, see: Seidengart (1992).

[25] Cassirer (1956), p. 162.

[26] *KrV*, A 189.

[27] This notion of causality may be already found *in nuce* in Cassirer (1994), pp. 478 – 479. However, its full development takes place in Cassirer (1956).

[28] *KrV*, B 253 f.

[29] Cassirer (1956), p. 166. Our emphasis. Nuzzo analyzes the modification of Cassirer’s position on the relationship between logic and time from that maintained in *Substanzsbegriff und Funktionsbegriff* to the one of the *Philosophie der symbolischen Formen.* According to Nuzzo, in a theory of social sciences, a theory of invariants is not possible any more. Rather, logic becomes a kind of Hegelian phenomenology. See: Nuzzo (1996), pp. 76 – 77. To the contrary, Ferrari maintains that Cassirer also seeks for invariants in other cultural forms, even though these invariants may not be reduced to those of scientific knowledge. See Ferrari (2012), 350. Cassirer indicates that the expression “phenomenology” in *The Phenomenology Knowledge*, the subtitle of the third volume of his *Philosophy of Symbolic Forms*, must be understood in the sense of Hegel´s *Phenomenology of the Spirit*. Cassirer (1994), p. VI. On Cassirer´s Hegelianism see: Ferrari (2007).

[30] In a study of Cassirer´s early epistemology, Ryckman claims that the basic principle on which Cassirer bases his account of the relationship between concepts and empirical contents is that of *functional coordination*: Ryckman (1991). However, already in 1910 –albeit not in Cassirer (1907) yet- Cassirer maintains that “the critical theory of experience would constitute the *universal invariant theory of*

experience." Cassirer (1923), 268. Some years later, Cassirer states that the development of modern physics has done nothing but confirm this view: Cassirer (1956), xxiii. In this way, the theory of physical knowledge as system of invariants that we will analyze in the following achieves the goal put forward by Cassirer´s early epistemological writings. Cassirer affirms that the role of invariants in physics, which he established in 1910 only by means of his analysis of classical physics, corresponds to "the program of modern theoretical physics, as presented for instance by P.A. Dirac in his *Principles of Quantum Mechanics*." Cassirer (1956), 138. Also the theory of relativity confirms those early results of a critical theory of experience: Cassirer, (1923), 379. For an analysis of Cassirer´s interpretation of relativity theory, see: Ryckman (2005).

[31] Cassirer (1956), pp. 29 ff. Even though in 1929 [Cassirer (1994)] physics is described as a system, only in 1936 [Cassirer (1956)] does such system get characterized as a system of *invariants*. Falkenburg discusses the relationship between this structure of physical knowledge and the technique as a symbolic form in Falkenburg (2012).

[32] Cf. Cassirer´s reception of Poincaré´s epistemology in Cassirer (1950), p. 110.

[33] Cf. Cassirer (1994), pp. 478f.

[34] See Cassirer (1994), pp. 505 – 507.

[35] "Eine Gesamtheit von Gliedern a, b, c, d …, die zunächst lediglich in dem »Daß«, in der Tatsächlichkeit ihres räumlich-zeitlichen Zusammenseins gegeben sind, sollen als einander »zugehörig« erkannt, sollen durch eine Regel verknüpft werden, auf Grund deren sich das »Hervorgehen« des einen aus dem andern bestimmen und vorhersehen läßt." Cassirer (1994), p. 484. At this point it should be stressed that Cassirer makes a clear distinction between mathematical and physical multiplicities. In both cases, the elements of the multiplicity must be conceived of as members of a series, the general term of which expresses the law of their generation. However, a physical multiplicity is never constructed completely, as it is the case of a mathematical multiplicity. Rather, empirical multiplicities are always conceived of as suitable for an indefinite construction, which is carried out with the progress of experience. Cassirer (1994), p. 493. Cf. also Cassirer (1923), p. 254. Therefore, the generation of physical (and chemical) multiplicities is not, so to say, categorical as in mathematics but hypothetical. Cassirer (1994), p. 519. For a lucid analysis of this issue see: Mormann (2008). See also: Mormann and Katz (2013).

[36] Cf. Cassirer (1950), pp. 110 – 111.

[37] Cassirer (1956), p. 44.

[38] On this issue, see: Ihmig (2001), pp. 81 ff.. Ihmig develops *in extenso* Cassirer's theory of experience's invariants in: Ihmig (1997a). See also Ihmig (1997b). Cassirer makes use of the mathematical notion of invariance regarding a transformation group in order to shed light on the psychology of perception in Cassirer (1938).

[39] Cassirer (1956), p. 42.

[40] Cassirer (1956), p. 35. Cf. also Cassirer´s reception of Duhem´s epistemology in Cassirer (1950), pp. 111f.

[41] Cassirer (1956), p. 58.

[42] Cassirer (1956), p. 60.

[43] At this point, Cassirer agrees with Helmholtz, for whom the law of causality expresses that regulative principle of our thought that requires us to always look for more general laws: Cassirer (1956), pp. 61 ff. See

also: Cassirer (1994), pp. 478 – 479; Cassirer (1950), pp. 85 – 86. On the constitutive/regulative distinction in Cassirer's philosophy, see: Pätzold (1996) and Friedman (2000), p. 117.

[44] Cassirer indicates that, if we wanted to use Kantian terminology, we should call the causality principle a "postulate of empirical thought", since, as a purely methodological principle, it does not concern the content of the different types of statements but only the character of their objective validity. See: Cassirer (1956), p. 60. But, against Cassirer's interpretation of causality it should be pointed out that the causality principle in modern physics does perform the very specific task of guaranteeing the objective character of the series of events, as it can be most clearly seen in relativity theory. On this issue, see: Schmitz-Rigal (2002), pp. 277 ff.

[45] *KrV*, B730.

[46] In fact, according to Cassirer, the true subject of the theory of schematism is the problem of the *psychological* possibility of a general concept. Cassirer (1922), p. 713. See also: Plümacher (1996), p. 119. However, Kant does not seem to aim at explaining here how a concept is formed, but rather how an already formed concept is *applied*. According to Friedman, Cassirer´s decisive rupture with Kant´s theory of schematism takes place when Cassirer separates "the representative function underlying the intuitive world from the purely significative function underlying the theoretical world." Friedman (2000), p. 109. By means of this Leibnizian turn, Cassirer gets close to the position of the logical positivists. Friedman (2000), p. 110.

[47] "Fassen wir den Verstand nicht lediglich als ein Vermögen der abstrakten Gattungsbegriffe, sondern, wie wir es nach der transzendentalen Deduktion der Kategorien tun müssen, als das „Vermögen der Regeln" auf, so hört er in der Tat auf, der Anschauung völlig „ungleichartig" zu sein." Cassirer (1922), p. 716. According to Cassirer, the clear distinction between sensibility and understanding that Kant introduces in the Transcendental Aesthetic is a residue of the *Dissertatio* of 1770, which, in view of the results of the Transcendental Logic, cannot be maintained any longer. On this issue, see: Friedman (2000), p. 89f.

[48] Cassirer (1956), pp. 137 – 138.

[49] "In diesem gesetzmäßigen Aufbau der Erkenntnis, in der Stufenfolge von Anschauung, Verstandesbegriff und Idee wir für uns alle empirische Wirklichkeit erst fassbar." Cassirer (1921), p. 61.

[50] Schmitz-Rigal maintains that the three types of statements are imaginary focal points for our orientation in experience, although they are questionable and alterable in accordance with new empirical data (Schmitz-Rigal, 2009: 82). In this way, Schmitz-Rigal finds the foundations of these statements in the hypothetical use of reason, but she does not establish any parallelism between the specific task of each type of statement and each Kantian faculty of knowledge, as we do here.

[51] Cassirer (1956), p. 143.

[52] Cassirer (1956), p. 35.

[53] Cassirer (1956), p. 35.

[54] Cei and French discuss the relevance of this interpretation of causality for the actual debate on structuralist views on quantum theory in Cei and French (2009).