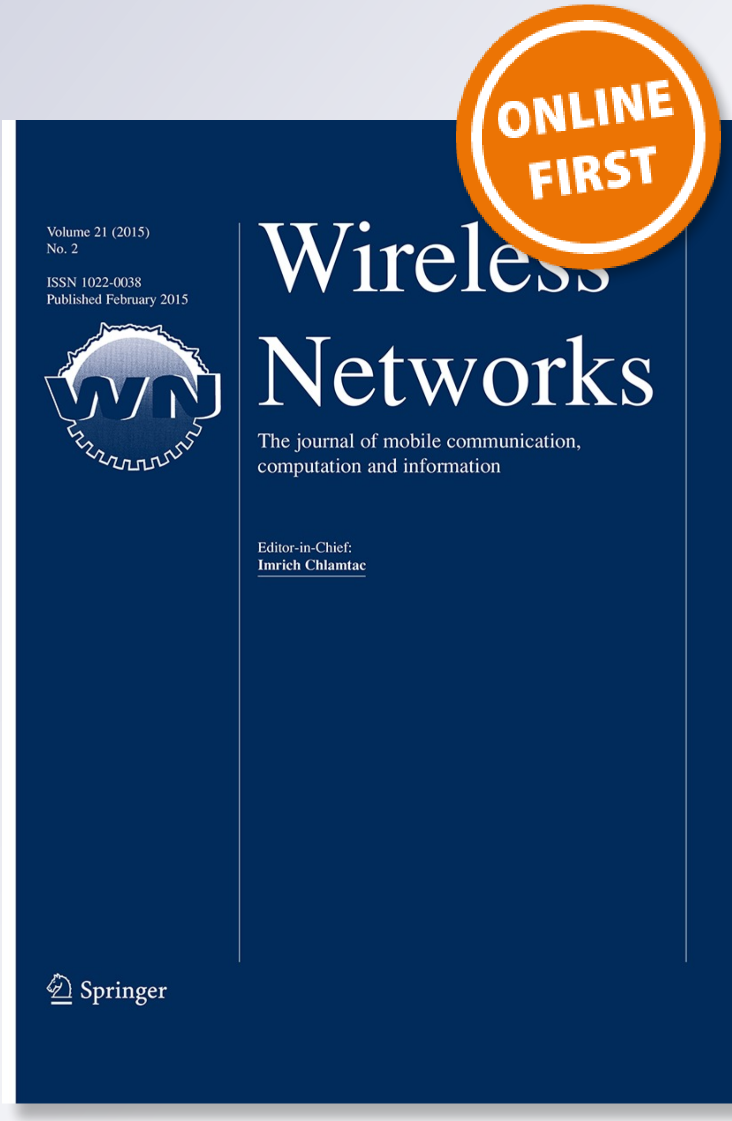
An efficient adaptive method for estimating the distance between mobile sensors

Ruben H. Milocco & Selma Boumerdassi

Wireless Networks
The Journal of Mobile Communication, Computation and Information

ISSN 1022-0038

Wireless Netw
DOI 10.1007/s11276-015-0930-3



Springer

Wireless Netw
DOI 10.1007/s11276-015-0930-3

An efficient adaptive method for estimating the distance between mobile sensors

Ruben H. Milocco[1] · Selma Boumerdassi[2,3]

Abstract The received signal strength (RSS) is a common source of information used for estimating the distance between two wireless nodes, whether these nodes are stationary or mobile. Minimum mean squared error distance estimation methods that use the RSS require prior knowledge of both the variance of the noise and, in the case of mobile sensors, the dynamics of the nodes' mobility. In mobile applications, where low computational complexity is important, pseudo-optimal estimations are preferred, as they do not require such information. In this case, the maximum likelihood estimator (MLE) is often used. In this paper, we propose an efficient pseudo-optimal log-power based distance estimation method using RSS under log-normal shadowing, that improves the MLE. It does not require a priori knowledge either of the movement dynamics or of the variance of the noise. The method is based on adaptively minimizing the variance of the prediction error, using a random walk model with correlated increments. It is analytically demonstrated that the distance estimation error variance of the proposed method improves the MLE in both the static and mobile cases. We use a simulated velocity model example to compare its performance with other algorithms in this group, such as the linear mean square filter and the Gauss–Newton search.

Keywords Adaptive distance estimation · Localization · Wireless network · Received signal strength

1 Introduction

Methods for accurately locating a device on a wireless network are crucial for a wide variety of applications, including tracking mobile devices and routing information between nodes in a wireless sensor network. A number of methods can be used to find the location of both stationary and mobile sensors (see for example [1, 2], and the references therein). All of these methods are based on knowing a set of distances. The most commonly used information sources for estimating distances are the *received signal strength* (RSS), the *time of arrival* (ToA), and the *angle of arrival* (AoA) [3]. Due to their simplicity and low computational cost, the RSS-based methods, where the path loss propagation model (PLPM) is adopted to infer the unknown distance, are often preferred. However, the RSS can be severely affected by the various error sources such as shadow noise [4, 5, 6]. For this reason, it is necessary to use appropriate estimators, in order to obtain the distance from RSS measurements.

In [7] it is shown that maximum likelihood estimation (MLE), using RSS measurements, yields a bias and a minimum mean squared error (MSE) that increases exponentially with the strength of the noise. Various methods have been proposed in order to improve the MLE. For the fixed position case, assuming the PLPM parameters are known, an unbiased estimator is proposed in [7]. In [8],

✉ Ruben H. Milocco
ruben.milocco@fain.uncoma.edu.ar

Selma Boumerdassi
selma.boumerdassi@cnam.fr

1 Grupo Control Automático y Sistemas (GCAyS), Universidad Nacional del Comahue, Buenos Aires 1400, 8300 Neuquén, Argentina

2 INRIA/Hipercom team, Rocquencourt, France

3 CNAM/CEDRIC, Paris, France

Published online: 18 March 2015

using identified PLPM parameters, a reduced-bias MLE estimator is proposed.

For the tracking of mobile node positions, distance estimation is often formulated as a filtering problem. Recursive least squared or extended Kalman filter methods are used in [9–12]. Based on unbiased distance estimations from direct measurements of the RSS, Black et al. [13] propose a two-step sensor location method. In the first step, a stationary initial position is estimated using least-squares or MLE; and in the second step, the mobile node is tracked using a recursive least-squares algorithm.

However, it is important to note that all these methods require a priori knowledge of the noise variance, input variance, and the dynamics of node movement, which are difficult to obtain for each specific case (see [14] for a survey of possible models of movement dynamics). Moreover, their performance greatly depends on the tuning of these parameters. In [15], the authors propose estimating the variance of both noise measurements and moving dynamics using a Kalman filter approach. In [16], an adaptive least squares method for mobile nodes is presented. However, these approaches still require knowledge of the model's dynamics.

In order to overcome these difficulties, different adaptive estimation procedures that do not require knowledge of either the noise variances or the mobility dynamics can be used, such as the linear mean square filter (LMS) [1] or the Gauss–Newton (GN) search approach that we briefly describe for comparison purposes. The main problems with these filters are that their stability is not guaranteed and the difficulty to obtain the optimal tuning of their gains, which may lead to unacceptable errors.

In cases where the nodes, due to their short-range measurement capabilities, are unable to estimate their positions by directly measuring the distances to anchors, multi-hop localization schemes are used, see for example [17] and [18]. In order to avoid large errors, it is important that the distance estimation at each hop be as precise as possible [19].

We believe that, despite their importance, filtering methods that reduce distance estimation uncertainties without prior information about movement dynamics and noise variance have not been sufficiently addressed in the literature. Preliminary results of the one-step-ahead prediction are presented in [20]. The main contribution of this paper is to propose a method for estimating the distance between mobile sensors that outperforms the LMS and GN approaches. We assume that each node has a transceiver. Thus, the distance is estimated between the transmitter of one node and the receiver of the other, or vice versa, by measuring the received power corrupted by log-normal shadowing noise.

The paper is organized as follows: In Sect. 2, the MLE and the unbiased estimation are presented for the PLPM. In Sects. 3 and 4, the pseudo-optimal LMS and the GN minimum error search are derived. In Sect. 5, using the derivations of the GN search, the first order filter with only one parameter is presented. The solution based on the parameter estimation using RPEM is obtained, which constitutes the minimum prediction error filter (MPEF) approach. In order to illustrate the procedure, in Sect. 6 we develop an example of the velocity model driven by increments with Gaussian distribution, which is an important application case in the location of mobile nodes. We compare four methods: the MLE, the LMS, the GN, and our approach, the MPEF. Moreover, the performances are compared with the Cramer–Rao lower bound (CRLB). Simulations show that the proposed method outperforms the LMS and the GN for the various cases. Finally, in Sect. 7, we present our conclusions.

2 The MLE and the unbiased distance estimation

The distance between any emitting source and a receiving device can be determined in terms of the intensity of the received power. The received logarithmic power loss at distance d, from the transmitter, can be represented by the following PLPM [4, 5, 6]:

$$pl(d) = pl(d_0) + 10\gamma \log 10(d/d_0) + \eta, \tag{1}$$

where $pl(d)$ is the power loss, in dB, at distance d between the transmitter and the receiver, $pl(d_0)$ is the logarithmic power loss at a reference distance d_0, which in our case is $d_0 = 1$ m, γ is the path loss distance exponent. The path loss exponent measures the rate at which the RSS decreases with distance, and its value depends on the specific propagation environment. Based on the received power measurements, there are several methods for estimating this parameter [21]. The value of this parameter ranges typically between 1 and 6 for different scenarios. The variable η, in dB, represents the power variations due to different sources of RSS uncertainties including hardware imperfections, movement of persons or any other objects in the surrounding of communicating nodes, random change of electromagnetic field or interference with other wireless networks in the same frequency range, multipath fading, and also the so called shadow noise. It is modeled as a zero mean, normally distributed random variable with standard deviation between 1 and 8 dB (for details, see Section 12.2.10 of [6]). In this paper we consider the path loss exponent is known a priori, but not the variance of the noise η which will be estimated by the proposed novel algorithm.

Thus, the received power at distance d, $p_r(d)$, is equal to the transmitted power pt minus the power loss $pl(d)$, all in dB, such that: $p_r(d) = pt - pl(d)$. Using the logarithmic path loss model (1), the noise-free logarithmic power $p(d)$ and the measured logarithmic power $p_r(d)$ at the receiver are represented by the following model:

$$p = \kappa - 10\gamma \log_{10}(d), \tag{2}$$

$$p_r = p + \eta, \tag{3}$$

where we have renamed $p_r = p_r(d)$, $p = p(d)$, and $\kappa = pt - pl(d_0)$ for the sake of simplicity. Our goal is to obtain the best possible distance estimation from measurements of p_r, using the above model. If we knew the noise-free log-power p, the distance could be obtained from (2) by means of the following simple formula:

$$d = \beta e^{-ap}, \tag{4}$$

where $a = \ln(10)/10\gamma$ and $\beta = 10^{\kappa/10\gamma}$. However, the value of p is unknown. We assume, instead, that a given unbiased estimation of p, denoted $\hat{p}$, is available- we see latter how to obtain different estimations, $\hat{p}$- such that the error, $e = p - \hat{p}$, is zero mean and with Gaussian distribution. The Gaussian assumption has nice stochastic properties since zero mean and minimum variance error means optimality. Thus, if p is replaced by $\hat{p}$, we obtain the following estimator:

$$\hat{d} = \beta e^{-a\hat{p}}, \tag{5}$$

which is in fact the MLE given $\hat{p}$. As a result, by taking $\hat{p}$ equal to the measured received power, p_r, we obtain $\hat{d}(k) = \beta e^{ap_r(k)}$, for $k = 1, 2, \ldots$ is the MLE of distance.

Although the estimated power $\hat{p}$ is unbiased, the estimator of d is not. The bias and MSE of this estimator are developed in detail in [17] and in [7]. The bias is given by

$$d - \mathcal{E}[\beta e^{-a\hat{p}}] = d\left(1 - e^{a^2\sigma_e^2/2}\right), \tag{6}$$

where $\mathcal{E}[\cdot]$ means the expected value, and $\sigma_e^2 = \mathcal{E}[(e - \mathcal{E}[e])^2]$ the variance of the estimated power error. The MSE is given by

$$\begin{aligned}\mathcal{E}[(d - \hat{d})^2] &= \mathcal{E}[(1 - e^{ae})^2]d^2 \\ &= (1 - 2e^{a^2\sigma_e^2/2} + e^{2a^2\sigma_e^2})d^2.\end{aligned} \tag{7}$$

The bias compensation that leads to the unbiased distance estimation, used extensively in [7, 13, 17], is given by:

$$\hat{d}^u = \tilde{\beta} e^{-a\hat{p}}, \quad \text{where } \tilde{\beta} = \beta e^{-a^2\sigma_e^2/2}. \tag{8}$$

Note that for this estimator, it is necessary to have a priori knowledge of the variance of the power error σ_e^2 in order to remove the bias. Using (8), the MSE of the unbiased distance estimator is obtained as

$$\begin{aligned}\mathcal{E}[(d - \hat{d}^u)^2] &= \mathcal{E}[(1 - e^{ae}e^{-a^2\sigma_e^2/2})^2]d^2 \\ &= (e^{a^2\sigma_e^2} - 1)d^2.\end{aligned} \tag{9}$$

Note that the MSE of both estimators, i.e., the MLE (7), and the unbiased estimator, (9), grows exponentially with σ_e^2. For example, when the measured log-power p_r is used as an estimator, even though the MSE of the unbiased estimator in (9) is lesser than the MSE of the biased estimator in (7), both grow exponentially with $\sigma_e^2 = \sigma_\eta^2$. It will be shown in Sect. 5 that it is possible to obtain a power estimate $\hat{p}$, with $\sigma_e^2 \leq \sigma_\eta^2$, improving the error bounds of the MLE and the unbiased estimator, by properly filtering a sequence of sampled measured power. Before that, however, in the next two sections, we introduce two alternative distance-estimation methods where prior knowledge of noise variances or mobility dynamics is also not required: the LMS and the GN. We will calculate the performance of these estimators both for comparative purposes and because they will be used later for developing the proposed algorithm.

In what follows we assume that the received power p_r is converted into digital form by an analog-to-digital converter. The conversion is done at a constant sampling time giving a sequence of numbers equally spaced in time and denoted by $p_r(k)$ for $k = 1, 2, \ldots$. For each sample, the different proposed algorithms produce an estimation of the distance.

3 Linear mean square filter

Assume that the distance between transmitter and recevier changes at each sample k. The LMS filter consists in recursively finding an estimated distance at each sampling time, $\hat{d}(k)$, that minimizes a cost-function by using the steepest descent algorithm (SDA) togheter with the sequence of measured log-powers $p_r(k)$ for $k = 1, 2, \ldots$. Let us assume the following quadratic cost:

$$V(k) = \frac{1}{2}\sum_{i=1}^{k} \lambda^{k-i}\epsilon^2(i), \tag{10}$$

where $\epsilon(k)$ is the prediction error given by

$$\epsilon(k) = p_r(k) - \hat{p}(k-1), \tag{11}$$

$$\hat{p}(k) = -\frac{1}{a}\log\left(\frac{\hat{d}(k)}{\beta}\right). \tag{12}$$

The latter is obtained from (5). λ is a scalar in the interval $(0, 1]$ called the *forgetting factor* which performs an exponential windowing over the previous prediction errors. The width of the exponential window depends on the value of λ. If $\lambda < 1$, previous prediction errors contribute only marginally to the criterion function. The window's width is reduced as λ decreases. In the case where $\lambda = 1$, all past data are equally weighted. Thus, the value of λ determines the memory of the past data, which is a suitable parameter to take into account for time-variant mobility dynamics.

We now apply the SDA in order to estimate the distance which minimizes the cost function $V(k)$. The SDA is the simplest of the gradient methods used in filtering problems [22]. The choice of direction is where the cost $V(k)$ decreases most quickly, which is in the direction opposite to the gradient $V(k)'$. The search starts at an arbitrary point $\hat{d}(0)$ and then slides down the gradient, until the minimum cost is reached. By applying the SDA to the cost $V(k)$ the following recursion is obtained:

$$\hat{d}(k) = \hat{d}(k-1) - \mu V'(k), \tag{13}$$

where μ is the step length and $V'(k)$ means the derivative of the cost with respect to $\hat{d}(k-1)$.

The LMS is a simple algorithm, but the problem is that it can be unstable. Depending on the value chosen for the step length μ, we obtain either a smooth approach to the optimum, or an oscillatory one. A low μ stabilizes the algorithm, but it does so slowly, which means that μ represents a compromise between stability and speed of convergence. A variety of criteria to tune the value of μ are available (see e.g., [22]). However, all of them require knowledge of the signal statistics, which in our case we assume to be unknown. Consequently, the value of the step length should be chosen conservatively.

The gradient of the cost, $V'(k)$, in (13) is obtained as follows:

$$V(k) = \lambda V(k-1) + \frac{1}{2}\epsilon^2(k), \tag{14}$$

$$V'(k) = \lambda V'(k-1) + \epsilon(k)\,\epsilon^{'}(k) = \epsilon(k)\,\epsilon^{'}(k), \tag{15}$$

where the last equation holds because $\hat{d}(k-1)$ is a minimizer of $V(k-1)$. The derivative of the prediction error with respect to $\hat{d}(k-1)$ is given by

$$\epsilon^{'}(k) = 1/a\hat{d}(k-1). \tag{16}$$

Then, by replacing $\epsilon^{'}(k)$ in (15) and in (13), the LMS filter is given by

$$\hat{d}(k) = \hat{d}(k-1) - \mu\epsilon^{'}(k)\epsilon(k). \tag{17}$$

The recursion (17) is in fact a random walk model with correlated increments. This kind of model is defined as a process where the current value of a variable is composed of the past value plus a correlated random increment with a given distribution. In the LMS case, the increments are given by the last term of the right side of (17).

A possible pseudocode of the LMS algorithm is given in Algorithm 1

Parameters: a, β, μ
Input : $p_r(k)$
Output : $\hat{d}(k)$

$\hat{p}(0) \leftarrow p_r(1)$;
$\hat{d}(0) \leftarrow 1$;
$k \leftarrow 1$;
while *1* **do**
  $\bar{d}(k) \leftarrow \beta e^{-a\hat{p}(k-1)}$;
  $\epsilon^{'}(k) \leftarrow 1/(a\bar{d}(k))$;
  $\epsilon(k) \leftarrow p_r(k) - \hat{p}(k-1)$;
  $\hat{d}(k) \leftarrow \hat{d}(k-1) - \mu\epsilon'(k)\epsilon(k)$;
  if $|\hat{d}(k)| < 1$ **then**
    $\hat{d}(k) \leftarrow 1$;
  end
  `deliver` $\hat{d}(k)$;
  $\hat{p}(k) \leftarrow \log(\beta)/a - \log(\hat{d}(k))/a$;
  if *External Interrupt* **then**
    stop
  else
    $k \leftarrow k+1$;
  end
end

Algorithm 1: LMS

4 Gauss–Newton search

The LMS method can be improved by taking into account the second derivative of the cost (10) with respect to $\hat{d}(k-1)$. This minimum search procedure is known as the Newton–Raphson method [23]. The Newton–Raphson method applied to the cost function (10) gives

$$\hat{d}(k) = \hat{d}(k-1) - \frac{V'(k)}{V''(k)}. \tag{18}$$

From (15), we obtain

$$V''(k) = \lambda V''(k-1) + \epsilon^{'2}(k) + \epsilon(k)\,\epsilon^{''}(k). \tag{19}$$

where $\epsilon(k)$ is obtained by using (18) in (11) and (12). Since $V(k)$ is a least-squares criterion, close to the minimum, the error $\epsilon(k)$ is small and can be neglected. Then, the simplified algorithm, called Gauss–Newton, is used. By denoting $m(k) = 1/V''(k)$ and considering small changes between samples, (18) and (19) can be computed recursively by

$$\hat{d}(k) = \hat{d}(k-1) + m(k)\epsilon^{'}(k)\epsilon(k), \tag{20}$$

$$m(k) = \frac{m(k-1)}{\lambda + m(k-1)\epsilon^{'2}(k)}. \tag{21}$$

Note that the resulting distance mobility model obtained for GN in (21) is also a random walk model with correlated increments. Although extensive simulations for different dynamics of movement carried out by the algorithm rapidly converge to a minimum, it is difficult to prove convergence without strong assumptions.

A possible pseudocode of the GN algorithm is given in Algorithm 2

```
Parameters: a, β, λ
Input      : p_r(k)
Output     : d̂(k)
p̂(0) ← p_r(1) ;
m(0) ← 1 ;
d̂(0) ← 1 ;
k ← 1 ;
while 1 do
    d̄(k) ← β e^{-a p̂(k-1)} ;
    ε'(k) ← 1/(a d̄(k)) ;
    ε(k) ← p_r(k) − p̂(k − 1) ;
    m(k) ← m(k − 1)/(λ + ε'^2(k) m(k − 1)) ;
    d̂(k) ← d̂(k − 1) − m(k) ε'(k) ε(k) ;
    if |d̂(k)| < 1 then
        d̂(k) ← 1 ;
    end
    deliver d̂(k) ;
    p̂(k) ← log(β)/a − log(d̂(k))/a ;
    if External Interrupt then
        stop
    else
        k ← k + 1;
    end
end
```

Algorithm 2: GN

5 Minimum prediction error filtering (MPEF)

We now will develop the novel MPEF filter. Assume, as before, that the distance between transmitter and receiver change at each sample k. We propose to smooth the received log-power, p_r, by a unit-gain first-order stable low-pass filter with the following recursion:

$$\hat{p}(k) = p_r(k) - \theta\epsilon(k) \tag{22}$$

$$\epsilon(k) = p_r(k) - \hat{p}(k-1) \tag{23}$$

The filtered signal $\hat{p}(k)$ will be used in (8) to produce the unbiased estimation. Note that, by choosing this simple structure, θ is the sole parameter to tune. In what follows we require $\theta \in [0, 1]$. Note that when $\hat{p}(k) = p(k)$ the prediction error is $\epsilon(k) = \eta(k)$, and $\theta = 1$.

Consequently, the prediction error variance is always greater than or equal to the noise variance, $\sigma_\epsilon^2 \geq \sigma_\eta^2$. Therefore, it makes sense to find the estimator that minimizes the prediction error variance trying to reach the lower bound, σ_η^2. For this purpose we want to tune the parameter θ such that it minimizes the prediction error variance. The value of θ that minimizes σ_ϵ^2 will be obtained recursively by using the well-known RPEM [23]. To this end, the Gauss–Newton recursive algorithm over the cost function (10) is used. The algorithm and its properties are given by the following theorem:

Theorem *Consider the cost function $V(k)$ in (10) to be minimized, with respect to the parameter $\theta(k)$, by the following GN recursion:*

$$m(k) = \frac{m(k-1)}{\lambda + \epsilon'^2(k)m(k-1)}, \tag{24}$$

$$\theta(k) = \theta(k-1) - m(k)\epsilon'(k)\epsilon(k), \tag{25}$$

where the following recursive expressions for the prediction error and its derivative, with respect to θ, are obtained by subtracting $\hat{p}(k-1)$ from $p_r(k)$ in (23), as follows:

$$\epsilon(k) = p_r(k) - p_r(k-1) + \theta\epsilon(k-1), \tag{26}$$

$$\epsilon'(k) = \epsilon(k-1) + \theta\epsilon'(k-1). \tag{27}$$

Then, the following holds:

(i) $\theta(k)$ converges as $k \to \infty$ with probability 1 to one element of the set of minimizers

$$\left\{\theta | \sigma_\epsilon'^2 = 0\right\}; \tag{28}$$

where $\sigma_\epsilon'^2$ is the derivative of the prediction error variance with respect to θ.

(ii) there is a unique minimizer of σ_ϵ^2 in the interval $\theta \in [0, 1]$ for almost all the movement dynamics;

(iii) by using the minimizer obtained by the recursion (24–25) in (22) it holds that $\sigma_e^2 \leq \sigma_\eta^2$ for all correlated or uncorrelated increments $r(k)$. Moreover, in the static case, $r(k) = 0$, the error variance σ_e^2 is zero.

Proof For (i) see [23] and for (ii) and (iii), see the Appendix. □

From (i), it follows that the recursion (24–25) converges to a minimizer of the prediction error variance in the search interval, $\theta(k) \in [0, 1]$, without any other information than the measured power signal. Moreover, from (ii) it follows that the minimizer reaches the global minimum. From (iii), we conclude that the variance of the estimated power error is always less than, or at least equal to, σ_η^2. Thus, using this power estimation in the unbiased estimator (8), the error variance of the distance from that obtained with the MLE is always improved.

Remark 1 Note that the prediction error, $\epsilon(k)$, and its derivative, $\epsilon'(k)$, depend only on the measured log-power $p_r(k)$ and θ. Moreover, by taking expected value in both sides of (26) and considering $|\theta|<1$ and zero mean increments $r(k)$, the prediction error $\epsilon(k)$ and the error $e(k) = p(k) - \hat{p}(k)$, in steady state, are unbiased. □

Remark 2 Note that we have not imposed any constraints with respect to the statistical distribution of the increments. In the particular case where the increments are a sequence of *iid* Gaussian variables, the prediction error method reaches the CRLB. This means that the predictor (22) gives the minimum error variance σ_e^2 achievable by any unbiased estimator. However, in the more general case, where the random walk model has an arbitrary distribution of the increments $r(k)$, correlated or uncorrelated, the method obtains a pseudo optimal estimation, $\hat{p}(k)$, in the sense that the prediction error $\epsilon(k)$ has as small a variance as possible, [23]. □

Finally, the unbiased distance estimate is obtained using $\hat{p}$ in (8) where the variance of the power estimation error is given by

$$\hat{\sigma}_e^2(k) = (1 - 2\theta(k))\hat{\sigma}_\eta^2(k) + \theta^2(k)\hat{\sigma}_\epsilon^2(k), \quad (29)$$

see the Appendix for its derivation. The estimated noise at each sample, the prediction error variance $\hat{\sigma}_\eta^2(k)$, and $\hat{\sigma}_\epsilon^2(k)$ are given by $(p_r(k) - \hat{p}(k))^2$ and $\epsilon^2(k)$, respectively. Note that the MSE, obtained by Eq. (8), is a metric of the absolute distance accuracy.

Routines for recursively estimating the parameter $\theta(k)$ that minimize the prediction error of the structure (22) can be found in the literature (see for example the recursive prediction error method, RPEM [23]). Based on the literature mentioned, a possible pseudocode of the MPFE algorithm is given in Algorithm 3, where in the first four sentences, after the while in MPFE algorithm, the theorem is applied. In the following sentence the parameter is forced to lie within the interval $|\hat{\theta}|<1$ for ensuring stability. After that, the filtered power at the previous step is improved by using the updated value of θ. The same is performed with the error and the error derivative.

```
Parameters: a, β
Input    : p_r(k)
Output   : d̂(k)
p̂(0) ← p_r(1) ;
m(0) ← 1 ;
d̂(0) ← 1 ;
k ← 1 ;
ε(0) ← 0 ;
ε'(0) ← 0 ;
while 1 do
    ε'(k) ← ε(k − 1) + θ̂(k − 1)ε'(k − 1) ;
    m(k) ← m(k − 1)/(0.98 + ε'^2(k)m(k − 1)) ;
    ε(k) ← p_r(k) − p̂(k − 1) ;
    θ̂(k) ← θ̂(k − 1) − m(k)ε(k)ε'(k) ;
    if |θ̂(k)| > 1 then
        θ̂(k) ← |1/θ̂(k)| ;
    else
        θ̂(k) ← |θ̂(k)| ;
    end
    p̂(k − 1) ← p_r(k − 1) − θ̂(k)ε(k − 1) ;
    ε(k) ← p_r(k) − p̂(k − 1) ;
    ε'(k) ← ε(k) + θ̂(k)ε'(k − 1) ;
    p̂(k) ← p_r(k) − θ̂(k)ε(k) ;
    σ̂_e^2(k) ← (1 − 2θ(k))(p_r(k) − p̂(k))^2 + θ^2(k)ε^2(k) ;
    d̂(k) ← β e^(−a^2 σ̂_e^2(k)/2) e^(a p̂(k)) ;
    deliver d̂(k) ;
    if External Interrupt then
        stop
    else
        k ← k + 1;
    end
end
```

Algorithm 3: MPEF

6 Simulation results and discussion

In order to illustrate the performance of the different methods, let us consider the case where the distance is given by the following discrete-time velocity model:

$$v(k+1) = v(k) + hw(k), \quad (30)$$

$$x(k+1) = hv(k) + x(k), \quad (31)$$

$$d(k) = |x(k)|, \quad (32)$$

where h is the sampling interval, $w(k)$ is a zero mean $i.i.d.$ sequence with variance σ_w^2, $v(k)$ and $x(k)$ are the relative velocity and position between both nodes, and $d(k)$ is the distance to be estimated. From (30), the variance of the velocity is proportional to σ_w^2, h^2, and the number of steps N, as follows:

$$\sigma_v^2 = \sigma_w^2 h^2 N. \tag{33}$$

By defining $\tilde{v}(k) = hv(k)$ and using it in (30–31) we obtain the following system:

$$\tilde{v}(k+1) = \tilde{v}(k) + h^2 w(k), \tag{34}$$

$$x(k+1) = \tilde{v}(k) + x(k), \tag{35}$$

$$d(k) = |x(k)|. \tag{36}$$

Thus, the dynamics of the system become independent of the sampling interval. For this reason, in what follows the standard deviation of the speed σ_v and the square root of the distance-MSE (S_d) of the diferent estimators are carried out in the simulations with $h = 1 * \mathrm{s}$. Then, the performance evaluation for different sampling periods can be obtained directly by rescaling the velocity standard deviation by $h\sigma_v$, and the distance error S_d by hS_d.

We have studied the behavior of different distributions of increments $w(k)$, such as Gaussian, zero mean uniform distribution, and the discrete-time sequences of *iid* random variable taking the values 1 and -1 with equal probability. Figure 1 depicts typical trajectories obtained from the different distributions. However, due to that the increments are filtered by a linear model, the statistic of the distance, by the central limit theorem, converges to the Gaussian distribution. Thus, the simulations based on Gaussian or non-Gaussian distributed increments provide very similar results. Thus, only simulation results with Gaussian increments are presented here.

The parameters of the path loss model (2–3) are $\kappa = 10$ and $\gamma = 3$. Since the GN and MPFE filters are based on the Gauss–Newton algorithm and both depend on λ, we will first analyze their comparative performances. In the case where the distance is constant, the value of θ is also constant and theoretically we must have a constant value of $\lambda = 1$ to get convergence to the minimum of the cost $V(k)$,

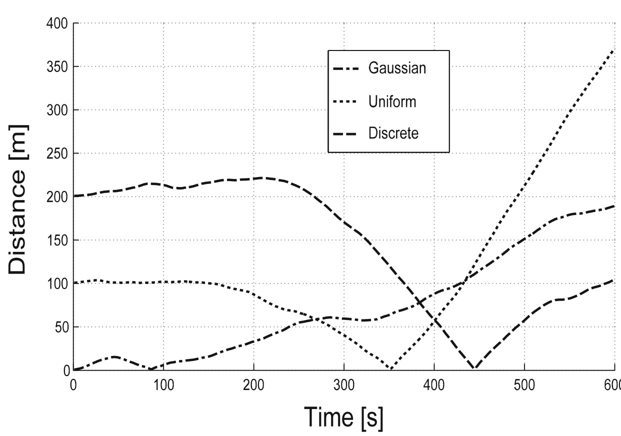


Fig. 1 Typical distance realizations obtained from Gaussian, Uniform, and Discrete distribution of increment

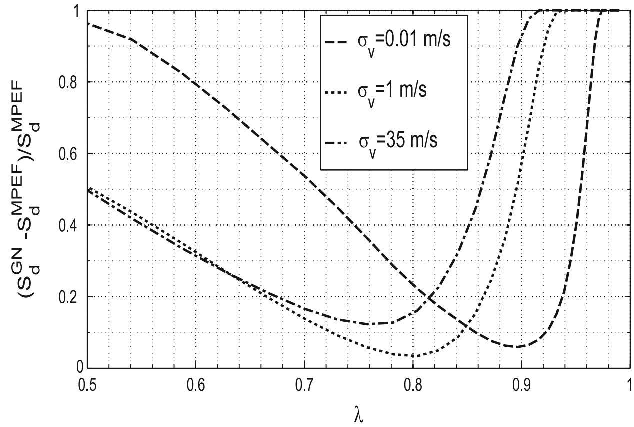


Fig. 2 Relative error between GN and MPFE for $\sigma_\eta = 1$ and different velocity standard deviations

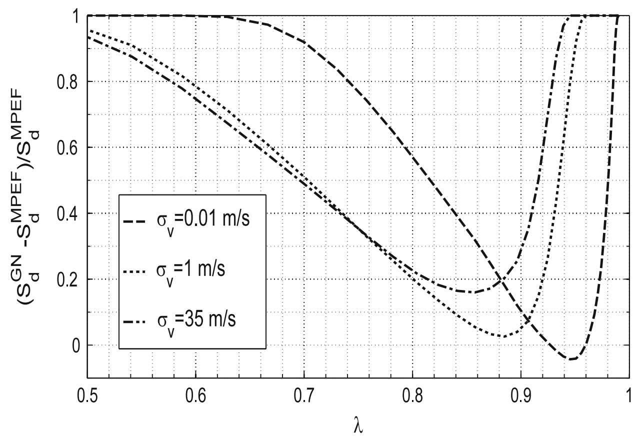


Fig. 3 Relative error between GN and MPFE for $\sigma_\eta = 3$ and different velocity standard deviations

[23]. In the case where the distance changes, also θ changes, and it is preferable that λ be slightly lesser, allowing an adaptive estimation of θ. Therefore, we choose a value of $\lambda = 0.98$ for MPFE. In contrast, the optimum value of λ in the case of GN is not a fix value, so we are interested in knowing how the error of GN behaves at different values of λ in comparisson with MPFE. To this end, let us denote by S_d^{GN} and S_d^{MPFE} the square root of the distance-MSE of GN and MPFE filters. In Figs. 2, 3 and 4 the relative error between both methods, the MPEF and GN with λ variable, are depicted as a funcion of λ for different amplitudes of fading noise, $\sigma_\eta = 1, 3,$ and 6. Although at the minimum the performance of GN is similar to MPFE, from the figures it can be observed that GN requires tuning λ for each speed and noise amplitude. Even when GN behaves slightly better than MPEF—for high noise levels and very low velocity—the performance degrades drastically for small variations of λ, which makes it inappropriate.

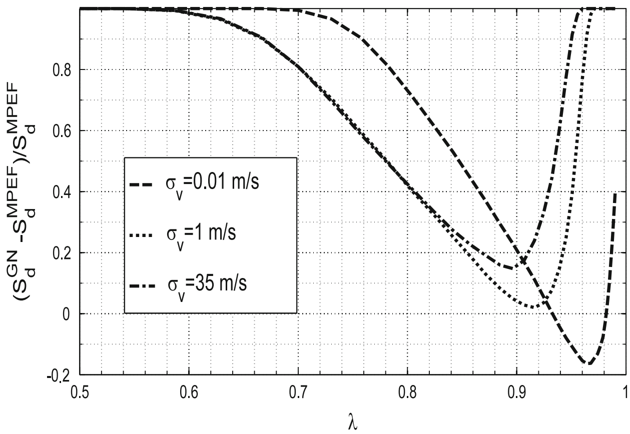


Fig. 4 Relative error between GN and MPFE for $\sigma_\eta = 6$ and different velocity standard deviations

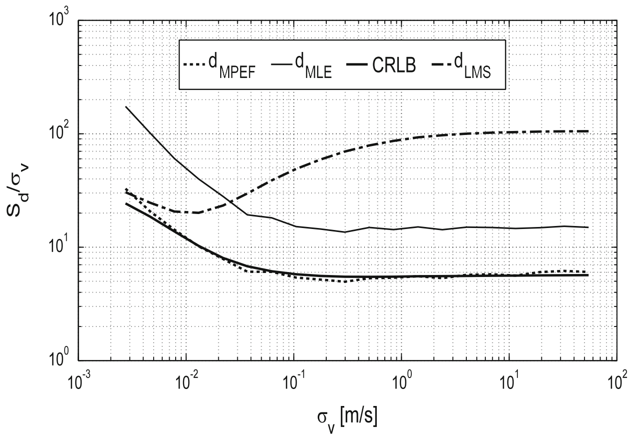


Fig. 5 Relative square root of the distance-MSE with respect to the standard deviation of velocity when $\sigma_\eta = 1$ for MLE, d_{MLE}; Linear Mean Square filter, d_{LMS}; and Minimum Prediction Error Filtering, d_{MPFE}

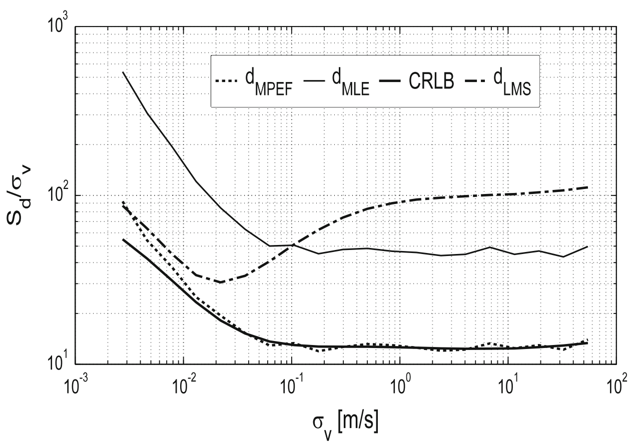


Fig. 6 Relative square root of the distance-MSE with respect to the standard deviation of velocity when $\sigma_\eta = 3$ for MSE, d_{MLE}; Linear Mean Square filter, d_{LMS}; and Minimum Prediction Error Filtering, d_{MPFE}

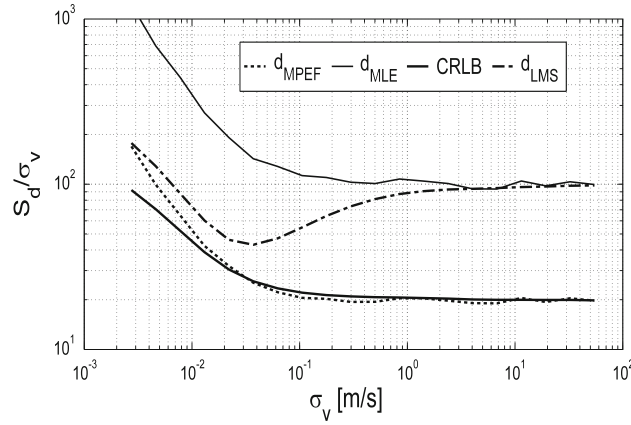


Fig. 7 Relative square root of the distance-MSE with respect to the standard deviation of velocity when $\sigma_\eta = 6$ for MSE, d_{MLE}; Linear Mean Square filter, d_{LMS}; and Minimum Prediction Error Filtering, d_{MPFE}

Figures 5, 6 and 7 show the relative error distance estimation given by the quotient between the square root of the distance-MSE with respect to the standard deviation of velocity for different amplitudes of fading noise, $\sigma_\eta = 1, 3,$ and 6, and for the following estimators: the MSE , d_{MLE}; the LMS, d_{LMS}; and the minimum prediction error filtering, d_{MPFE}.

In the case of the LMS filter, it is impossible to determine, a priori, an optimal value of μ without extra information. As the aim of our simulations is to compare the performance of LMS and LME with the proposed MPEF, we prefer to find the best *a posteriori* value by performing multiple simulations at different velocities and noise levels. The best value of gain for LMS was found to be $\mu = 0.01$.

Since the probability distribution of w is normal distributed, we can compare the performance with the theoretical Cramer–Rao minimum variance lower bound. It must be taken into consideration that the CRLB is an asymptotic bound for unbiased estimators. In [1] and [24], the CRLB for the nonlinear filtering case is analyzed. It consists in running the following recursion until convergence is reached:

$$P(k+1) = \left(\left(AP(k)A^T + B\sigma_w^2 B^T \right)^{-1} + \frac{H^T H}{a^2 d^2(k) \sigma_\eta^2} \right)^{-1}, \tag{37}$$

$$\mathrm{Var}\left[\hat{d}(k) - d(k)\right] \leq H^T P(k) H, \tag{38}$$

Table 1 Comparative computational times

t_{MPFE}/t_{MLE}	t_{GN}/t_{MLE}	t_{LMS}/t_{MLE}
1.47	1.32	1.24

where A, B, and H are obtained from the matrix description of the velocity model with:

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad H = [0,\ 1]. \tag{39}$$

The CRLB is also depicted in the figures.

From the figures, it can be seen that LMS slightly outperforms MPFE estimators in the static case, but in the case of mobile sensors the slow convergence speed of the steepest descent algorithm degrades the performance quickly. However, it is important to note that the improvement of the LMS over the MPEF, in the static case, is negligible since the plotted values are relative to the speed σ_v which is very low. It should also be noted that MPFE is very close to the CRLB. The relative computational time of each method, with respect to the MLE, is shown in Table 1. The price to be paid for the improvements obtained using GN or MPEF is a logical increase in computation time. However, the increments of time with respect to the MLE are relatively small.

7 Conclusions

An efficient adaptive estimation of the distance between sensors in a mobile network was presented. The proposed method outperforms the classical MLE, reaching error values very close to the CRLB for both the static and mobile cases. The computational cost is 50 % greater than the MLE. However, since the MLE is very fast, the extra computational cost is not significant and worthwhile. Three important properties of the algorithm are the following: (i) The algorithm does not need to know the variance of the noises nor the dynamic of the movements; (ii) it was analytically demonstrated that it outperforms the MLE or, in other words, its MSE is less than, or at least equal to, the noise variance; and (iii) it is based on the well-known algorithm RPEM, for which convergence is guaranteed under mild conditions. The method also outperforms the LMS and GN filters.

Acknowledgments This work was supported by the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Agencia Nacional de Promoción Científica y Técnica, Universidad Nacional del Comahue, Argentina, and the Centre d'Etude et de Recherche en Informatique du CNAM, France.

Appendix

We first show that there is a unique minimizer of σ_ϵ^2 in the interval $\theta(k) \in [0, 1]$ for the family of movement dynamics whose autocorrelation can be represented by a Fourier series expansion. Various motion dynamic cases can be represented using this representation, ([14]). By using (26), the prediction error at step $k+1$ can be represented by the following recursion:

$$\epsilon(k+1) = p_r(k+1) - p_r(k) + \theta\epsilon(k) \tag{40}$$

$$= r(k+1) + \eta(k+1) - \eta(k) + \theta\epsilon(k). \tag{41}$$

where $r(k)$ is a zero mean, incremental log-power sequence given by

$$r(k) = p(k) - p(k-1). \tag{42}$$

By taking the expected value of $\epsilon^2(k)$ and considering that $\eta(k)$ is an *i.i.d.* sequence, independent of $r(k)$, we obtain the following relationship for the prediction error variance:

$$\sigma_\epsilon^2 = 2\sigma_\eta^2 + \theta^2\sigma_\epsilon^2 + \sigma_r^2 - 2\theta\mathcal{E}[\eta(k)\epsilon(k)] + 2\theta\mathcal{E}[r(k+1)\epsilon(k)], \tag{43}$$

where $\mathcal{E}[\eta(k+1)\epsilon(k)] = 0$. The expectations of the last two terms are given by

$$\mathcal{E}[\eta(k)\epsilon(k)] = \sigma_\eta^2, \tag{44}$$

$$\begin{aligned} \mathcal{E}[r(k+1)\epsilon(k)] &= \mathcal{E}[r(k+1)r(k) + \theta r(k+1)\epsilon(k-1)] \\ &= \mathcal{E}[r(k+1)r(k) + \theta r(k+1)r(k-1) + \theta^2 r(k+1)\epsilon(k-2)] \\ &\ \vdots \\ &= \sum_{i=1}^{\infty} \theta^{i-1}R(i), \end{aligned} \tag{45}$$

where $R(i) = \mathcal{E}[r(k)r(k-i)]$. By using it in (43) and taking into account that $\sigma_r^2 = R(0)$, the final expression of σ_ϵ^2 is obtained as

$$\sigma_\epsilon^2 = \frac{2(1-\theta)\sigma_\eta^2 - \sigma_r^2 + 2\sum_{i=0}^{\infty}\theta^i R(i)}{1-\theta^2}. \tag{46}$$

In order to analyze the possible minimizers of cost function σ_ϵ^2 with respect to θ, we need to write the correlation function $R(i)/\sigma_r^2$. To this end, let us consider the following complex series expansion representing the autocorrelation:

$$\frac{R(i)}{\sigma_r^2} = \frac{1}{2N}\sum_{n=1}^{N}(\alpha_n^i + \alpha_n^{*i}), \tag{47}$$

where α_n is complex with $|\alpha_n| \le 1$, $(*)$ means compex conjugate, and $R(i)$ is a positive definite function, [25]. Thus, the last term in the numerator of (46) gives

$$\begin{aligned} \frac{\sigma_r^2}{N}\sum_{i=0}^{\infty}\theta^i\sum_{n=1}^{N}(\alpha_n^i + \alpha_n^{*i}) &= \frac{\sigma_r^2}{N}\sum_{n=1}^{N}\sum_{i=0}^{\infty}(\theta\alpha_n)^i + (\theta\alpha_n)^{*i} \\ &= \frac{\sigma_r^2}{N}\sum_{n=1}^{N}\frac{1}{1-\theta\alpha_n} + \frac{1}{1-\theta\alpha_n^*}. \end{aligned} \tag{48}$$

By denoting $\alpha_n = \rho_n e^{j\varphi_n}$ and taking into account that both θ and ρ_n are positive and less than one, a family of possible values for each element of the sum are given by the following parametrization:

$$\frac{1}{1-\theta\alpha_n} + \frac{1}{1-\theta\alpha_n^*} = 2\frac{1-\theta\rho_n\cos(\varphi_n)}{(1-\theta\rho_n e^{j\varphi_n})(1-\theta\rho_n e^{-j\varphi_n})}, \tag{49}$$

where $\rho_n \in [0,1]$ and $\varphi_n \in [0, 2\pi]$. Now, we need the following lemma:

Lemma [26]: Let g and f be defined on a convex set Θ, such that $f(\theta) \neq 0$ for all $\theta \in \Theta$. Then, g/f has only one minimum on Θ (quasi-convex) if both $f(\theta) > 0$ is concave and $g(\theta) \geq 0$ is convex for all $\theta \in \Theta$.

First, from (46) and (49), we can write

$$g(\theta) = 2(1-\theta)\sigma_\eta^2 - \sigma_r^2 + \frac{2\sigma_r^2}{N} \sum_{n=1}^{\infty} \frac{1-\theta\rho_n\cos(\varphi_n)}{(1-\theta\rho_n e^{j\varphi_n})(1-\theta\rho_n e^{-j\varphi_n})} \tag{50}$$

$$f(\theta) = 1-\theta^2. \tag{51}$$

From (51), $f(\theta) > 0$ concave, for $\theta \in (0,1)$, and from (50), $g(\theta)$ is always positive definite. In order to study its convexity we obtain the second derivative with respect to θ which gives

$$\frac{4\cos^2(\varphi_n) - \cos(\varphi_n)[2(\theta\rho_n)^3 + 6\theta\rho_n] + 6(\theta\rho_n)^2 - 2}{[(\theta\rho_n)^2 - 2\theta\rho_n\cos(\varphi_n) + 1]^3}. \tag{52}$$

By calculating derivatives we find that the minimum of the above is given at $\theta\rho_n = 0$ for any possible value of $\cos(\varphi_n)$. Then, for $\cos(\varphi_n) \geq \sqrt{1/2}$, the second derivative is always positive, which restricts the value of φ_n in the interval $[-\pi/4, \pi/4]$. This means that the higher-frequency sinusoidal component of the Fourier Series Expansion of $R(i)$, (47), must contain at least eight samples per period, which is a weak restriction. Thus, we can conclude that, under mild assuptions, there exists only one minimizer for the minimum variance prediction error for θ, within the interval $(0, 1)$.

In order to prove (iii), let us compute the upper bound of σ_ϵ^2. By taking into account that $R(i)/\sigma_r^2$ is an autocorrelation function, from (46) the following inequality holds:

$$\sigma_\epsilon^2 \leq \frac{2(1-\theta)\sigma_\eta^2 + \sigma_r^2\left(2\sum_{i=0}^{\infty}\theta^i - 1\right)}{1-\theta^2} \tag{53}$$

$$= \frac{2(1-\theta)\sigma_\eta^2 + \sigma_r^2\left(\frac{1+\theta}{1-\theta}\right)}{1-\theta^2} = h(\theta). \tag{54}$$

Now, let us define the minimizer of $h(\theta)$ as

$$\bar{\theta} = \arg\min_\theta\{h(\theta)\}. \tag{55}$$

By multiplying both sides of (54) by $1-\theta^2$, within the admissible interval of θ, and performing the derivative, with respect to θ, the following upper bound for the minimum of σ_ϵ^2 is obtained:

$$\sigma_{\epsilon min}^2 \leq \frac{\sigma_\eta^2}{\bar{\theta}} - \frac{\sigma_r^2}{2\bar{\theta}(1-\bar{\theta})^2} \leq \frac{\sigma_\eta^2}{\bar{\theta}}. \tag{56}$$

In order to find the relationship between the variance of $e(k)$ and the variance of the prediction error, we use (22) as follows:

$$\sigma_e^2 = \mathcal{E}[(p(k) - \hat{p}(k))^2] \tag{57}$$

$$= \mathcal{E}[\eta^2(k)] + \theta^2\mathcal{E}[\epsilon^2(k)] - 2\theta\mathcal{E}[\eta(k)\epsilon(k)], \tag{58}$$

$$= (1-2\theta)\sigma_\eta^2 + \theta^2\sigma_\epsilon^2, \tag{59}$$

where the last term of (58) is $\mathcal{E}[\eta(k)\epsilon(k)] = \sigma_\eta^2$. Finally, by replacing the bound (56) in (59) we obtain an upper bound on the pseudo-optimal minimum error attainable for any correlation function as follows:

$$\sigma_e^2 \leq (1-2\bar{\theta})\sigma_\eta^2 + \bar{\theta}^2\sigma_{\epsilon min}^2 \tag{60}$$

$$\leq (1-2\bar{\theta})\sigma_\eta^2 + \bar{\theta}\sigma_\eta^2 \tag{61}$$

$$\leq \sigma_\eta^2. \tag{62}$$

In particular, in the static case, $\sigma_r^2 = 0$, the minimizer is $\bar{\theta} = 1$, which leads to $\sigma_{\epsilon min}^2 = \sigma_\eta^2$ and $\sigma_e^2 = 0$.

References

1. Gustafsson, F., & Gunnarsson, F. (2005). Mobile positioning using wireless networks: Possibilities and fundamental limitations based on available wireless network measurements. *IEEE Signal Processing Magazine*, *22*(4), 41–53.
2. Patwari, N., Ash, J., Kyperountas, S., Hero, I., Moses, A. R., & Correal, N. (2005). Locating the nodes cooperative localization in wireless sensor networks. *IEEE Signal Processing Magazine*, *22*(4), 54–69.
3. Muñoz, D., Bouchereau, F., Vargas, C., & Enriquez-Caldera, R. (2009). *Position location techniques and applications*. Amsterdam: Elsevier.
4. Okumura, Y., Ohmori, E., Kawano, T., & Fukuda, K. (1988). Field strength and its variability in VHF and UHF land-mobile radio service. *Review of the Electrical Communication Lab*, *16*, 9–10.
5. Achutegui, K., Miguez, J., Rodas, J., & Escudero, C. (2012). A multi-model sequential Monte Carlo methodology for indoor tracking: Algorithms and experimental results. *Signal Processing*, *92*, 2594–2613.
6. Su, Shing-Fong. (2007). *The UMTS air-interface in RF engineering*. New York: McGraw-Hill.
7. Chitte, S. D., Dasgupta, S., & Ding, Z. (2009). Distance estimation from received signal strength under log-normal

shadowing: Bias and variance. *IEEE Signal Processing Letter*, *16*, 216–218.
8. Coluccia, A. (2013). Reduced-bias ML-based estimators with low complexity for self-calibrating RSS ranging. *IEEE Transaction on Wireless Communication*, *12*(3), 1220–1230.
9. Liu, T., Bahl, P., & Chlamtac, I. (1998). Mobility modeling, location tracking, and trajectory prediction in wireless ATM networks. *IEEE Journal Selected Areas in Communications*, *16*, 922–936.
10. Zaidi, Z. R., & Mark, B. L. (2005). Real-time mobility tracking algorithms for cellular networks based on Kalman filtering. *IEEE Transaction on Mobile Computing*, *4*, 195–208.
11. Pathirana, P. N., Savkin, A. V., & Jha, S. (2004). Robust extended Kalman filter based technique for location management in PCS networks. *Computer Communications*, *27*, 502–512.
12. Chiou, Yih-Shyh, Wang, Chin-Liang, & Yeh, Sheng-Cheng. (2010). An adaptive location estimator using tracking algorithms for indoor WLANs. *Wireless Networks*, *16*, 1987–2012.
13. Black, T.J., Pathirana, P. N., & Nahavandi, S. (2008). Position estimation and tracking of an autonomous mobile sensor using received signal strength. IEEE International Conference on Intelligent Sensors Sensor Networks and Information Processing, Piscataway, N.J (pp. 19–24).
14. Rong Li, X., & Jilkov, V. P. (2003). Survey of maneuvering target tracking. Part I: Dynamic models. *IEEE Transactions on Aerospace and Electronic Systems*, *39*, 1342–1364.
15. Myers, K. A., & Tapley, B. D. (1976). Adaptive sequential estimation with unknown noise statistics. *IEEE Transaction on Automatic Control*, *21*, 520–523.
16. Zhu, Y. M. (1999). Efficient recursive state estimator for dynamic systems without knowledge of noise covariances. *IEEE Transactions on Aerospace and Electronic Systtems*, *35*, 102–114.
17. Mao, G., Fidan, B., & Anderson, B. D. O. (2007). Wireless sensor network localisation techniques. *Computer Networks*, *51*, 2529–2553.
18. Feng, R. J., Guo, X. L., Wan, J. W., Wu, Y. F., & Yu, N. (2012). Multihop localisation with distance estimation bias for 3D wireless sensor networks. *Electronics letters*, *48*, 884–886.
19. Milocco, R. H., Costantini, H., & Boumerdassi, S. (2014). Improved geographic routing in sensor networks subjected to localization errors. *Ad Hoc Networks*, *13*, 476–486.
20. Milocco, R., & Boumerdassi, S. (2010). Estimation and prediction for tracking trajectories in cellular networks using the recursive prediction error method. In *2010 IEEE International Symposium on a World of Wireless Mobile and Multimedia Networks (WoWMoM)*, pp. 1–7. doi:10.1109/WOWMOM.2010.5534918.
21. Mao, G., Anderson, B. D. O., & Fidan, B. (2007). Path loss exponent estimation for wireless sensor network localization. *Computer Networks*, *51*, 2467–2483.
22. Ramirez Diniz, P. S. (2002). *Adaptive filtering: Algorithms and practical implementation* (2nd ed.). USA: Kluwer Academic Publishers.
23. Ljung, L., & Söderström, T. (1983). *Theory and practice of recursive identification*. Cambridge: MIT Press.
24. Tichavsky, P., Muravchik, C., & Nehorai, A. (1998). Posterior Cramer–Rao bounds for discrete-time nonlinear filtering. *IEEE Transactions Signal Processing*, *46*, 1386–1396.
25. Papoulis, A. (1965). *Probability, Random Variables and Stochastic Process*. New York: McGraw-Hills.
26. Greenberg, H. J., & Pierskalla, W. P. (1971). A review of quasi-convex functions. *Operations Research*, *19*, 1553–1570.

Ruben H. Milocco is Professor at the Department of the Electrical Engineering of the National University of Comahue, Argentina. He is member of the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina. He is research scientist at the Grupo de Control Automático y Sistemas (GCAyS) of the National University of Comahue and his research interests include filtering, estimation and identification theories with applications to communications and energy.

Selma Boumerdassi is an Associate Professor at Conservatoire National des Arts et Métiers, Paris. She received a Ph.D. in Computer Science from University of Versailles in 1998, where she also served as an Assistant Professor from 1998 to 2000. Her research interests include wireless and mobile networks, with a special focus on the impact and use of social networks. Selma Boumerdassi worked on several national projects and served as an expert for the evaluation of French national projects (ANR). She is the author of more than 50 articles and serves as a TPC member for various international journals and conferences.