



Phase transitions and damage spreading in a nonequilibrium lattice gas model with mixed dynamic rules

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HIGHLIGHTS

- Nonconservative dynamics included in DLG model leads to Ising-like phase transitions.
- Short-time dynamics allows us to determine the order of the phase transitions.
- A crossover between a DLG and Ising-like behaviors is observed.
- Damage always spreads, and reaches a saturation value.
- Damage spreading is sensitive to phase transitions in the present model.

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ABSTRACT

Phase transitions and damage spreading for a lattice gas model with mixed driven lattice gas (DLG)–Glauber dynamics are studied by means of Monte Carlo simulations. In order to control the number of sites updated according to the nonconservative Glauber dynamics, a parameter $p \in [0, 1]$ is defined. In this way, for $p = 0$ the system corresponds to the DLG model with biased Kawasaki conservative dynamics, while for $p = 1$ it corresponds to the Ising model with Glauber dynamics. The results obtained show that the introduction of nonconservative dynamics dramatically affects the behavior of the DLG model, leading to the existence of Ising-like phase transitions from fully occupied to disordered states. The short-time dynamics results suggest that this transition is second order for values of $p = 0.1$ and $p > 0.6$ and first order for $0.1 < p \leq 0.6$. On the other hand, damage always spreads within the investigated temperature range and reaches a saturation value D_{sat} that depends on the system size, the temperature, and p . The value of D_{sat} in the thermodynamic limit is estimated by performing a finite-size analysis. For $p < 0.6$ the results show a change in the behavior of D_{sat} with temperature, similar to those reported for the pure ($p = 0$) DLG model. However, for $p \geq 0.6$ the data remind us of the Ising ($p = 1$) curves. In each case, a damage temperature $T_D(p)$ can be defined as the value where either D_{sat} reaches a maximum or it becomes nonzero. This temperature is, within error bars, similar to the reported values of the temperatures that characterize the mentioned phase transitions.

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1. Introduction

In nature, most phenomena take place under far-from-equilibrium conditions where an open system is coupled to external reservoirs and fields, in such a way that they can exchange energy, particles or other quantities with it, by generating

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net currents of energy, particles, etc. through the system that govern its dynamic evolution. In contrast with equilibrium systems where a complete theoretical description is well-established [1,2], nonequilibrium systems do not have an overall theoretical background yet [3,4]. However, many attempts have been made, such as the formulation and study of models that are capable of capturing the essential nonequilibrium behavior. These models can be classified into two categories: (i) dynamic extensions of static universality classes and (ii) intrinsically out-of-equilibrium models defined without a Hermitian Hamiltonian with transition rates that violate the principle of detailed balance. The latter category includes models with competing dynamics or external currents. Paradigmatic examples of both categories are the well-known kinetic Ising and the driven lattice gas (DLG) models [5], respectively.

The DLG model consists of a set of particles located in a two-dimensional square lattice in contact with a thermal reservoir. Particles can exchange places with nearest-neighbor empty sites according to conserved spin exchange, i.e., the Kawasaki dynamics. Also, an external drive \mathbf{E} is applied, causing the system to exhibit nonequilibrium stationary states (NESS) in the limit of large evolution times. For low enough temperatures, the DLG model develops an ordered phase characterized by stripes of high particle density running along the driving direction. However, by increasing the temperature a nonequilibrium phase transition into a disordered (gas-like) phase takes place (see Ref. [6] and references therein). For all values of the density first-order phase transition are observed, except in the case of half-filled lattices where it is of second-order. The critical behavior of the DLG model has been studied by using many different techniques, such as field theoretical calculations [3], Monte Carlo simulations [3,7], finite-size scaling methods [8,9], and short-time dynamic scaling [10–12], but the complete understanding of this model is still open and has originated a long-standing controversy (for more details see Ref. [6] and references therein).

In particular, the short-time dynamics (STD) used in this work allows us to characterize the phase transitions present in the system. This method is based on the fact that the spatial correlation length is smaller than the system lattice size at the early stages of its evolution [12,13]. In order to implement it, the early-time evolutions of one or several observables, e.g., the order parameter, its cumulants, etc., are monitored when the system is started from two specifically selected initial configurations. Then, two possible scenarios are present. On the one hand, if the evolution of the observables exhibits power-law behaviors at two different values of the system control parameter X , let us say X_1 for the first initial condition and X_2 for the second one, with $X_1 \neq X_2$, a first-order phase transition takes place in the system. These X_1 and X_2 values have been identified as metastability limits for several equilibrium and nonequilibrium models [12,14], and are called pseudocritical points because the order parameter fluctuations diverge. For the STD method [12,14,15], the “strength” of the transition is defined as the difference $\Delta X = |X_2 - X_1|$. On the other hand, if the observables show power-law behaviors at $X^* = X_1 = X_2$, the phase transition is of second-order [12,13], and X^* is identified as the critical point of the transition. Furthermore, the STD exponents of the power laws are linear combinations of the equilibrium or steady-state critical exponents of the transition [12,13], so they can be combined to estimate the latter. In both cases, second- and first-order transitions, deviations from the power-law behavior can be observed when the system is not tuned at the critical or pseudocritical points, respectively. These deviations are important, since they can be compared in order to find the time evolution that shows the best power-law behavior of the observables (for more details about this technique, see the review [12] and references therein).

From the theoretical point of view, it is interesting and challenging to study the effects of a very small perturbation on the dynamics of a nonequilibrium system. One way to do this is to apply the concept of *damage spreading* (DS). Basically, the DS method consists in a point-to-point comparison between two initially identical copies of a given system, each one running with the same dynamic rules and the same thermal noise, after the insertion of a small perturbation (called damage) in one of them. Originally, this technique was introduced to study biological systems [16], but DS has also been extensively applied to understand the dynamic properties of several statistical models, such as the Ising, Potts, XY and Heisenberg magnetic models; two-dimensional trivalent cellular structures; biological evolution; opinion dynamics; the ZGB model; small world and scale-free networks, and recently to the DLG model (see e.g., Ref. [17] and references therein, and for more recent results see Refs. [12,18–27]).

In this work, phase transitions and damage spreading are investigated in a nonequilibrium model based on the DLG model, where its evolution is governed by a mixture of nonconservative (Glauber) and conservative DLG (biased Kawasaki) dynamics. In this sense, a parameter p is introduced in order to determine the fixed fraction of sites that do not take part in the DLG dynamics, but their states are updated by following Glauber dynamics at each time step. Wang et al. [28] studied the same mixed model for the cases $p = 0.1$ and $p = 0.5$ by means of Monte Carlo simulations, aiming to determine its critical behavior. They analyzed the magnetization, susceptibility, and correlation functions of the system. Unlike this previous work, in the present case the effects on the particle density produced by the introduction of the Glauber dynamics are considered, i.e., the shift of the particle density from $\rho = 0.5$ where the second-order phase transition takes place [29,15]. This fact allowed us to obtain a rich phase diagram as a function of p for the mixed DLG model, where both first- and second-order phase transitions, and a new Ising-like ordered phase are present.

Finally, it is important to remark that further studies of DS have been performed on mixed dynamic models [30–32], but all of them were based on the Ising model. To the best of our knowledge, this is the first time that DS has been studied in a mixed-dynamic nonequilibrium model based on the DLG model.

The manuscript is organized as follows: in 2 we introduce the main ideas of the mixed DLG-Ising model, 3 describes the damage spreading technique, 4 gives simulation details and a summary of the STD technique, the results are presented in 5. Finally, the conclusions are reported in 6.

2. Mixed DLG model

In the past decades, the study of mixed conservative–nonconservative dynamics has been carried on different systems, e.g., the Ising [30,33,34] and DLG models [28,35], also including damage spreading [30–32].

As was mentioned in the Introduction, a mixed DLG–Ising model was studied, where a fraction p of randomly selected sites are updated by nonconservative Glauber dynamics at each time step, while the remaining fraction $(1 - p)$ is updated with the conservative DLG dynamics. It is important to remark that the number of sites updated by nonconservative rules (i.e., $p \cdot N$, where N is the size of the lattice) is fixed, but the sites are randomly selected at each time step, so it is not always the same set of sites that are updated. With this definition, the case $p = 0$ corresponds to the DLG model, while for $p = 1$ becomes the kinetic Ising model with Glauber dynamics in the absence of an external magnetic field. For this reason, in the present section both archetypical models are briefly described, and the main characteristics of the mixed system are included at the end.

The Ising model [36] is a very well-known and studied model that describes the main characteristics of a system of interacting spins with high anisotropy. The Hamiltonian of this system can be written as

$$\mathcal{H} = -J \sum_{\langle ij, i'j' \rangle} \eta_{ij} \eta_{i'j'} - H \sum_{\langle ij \rangle} \eta_{ij}, \quad (1)$$

where η_{ij} is the spin variable that can assume two different values $\eta_{ij} = \pm 1$; the indexes $1 \leq i, j \leq N$ are used to label the sites, $N = L_x \times L_y$ being the size of the sample. Also, $J > 0$ is the ferromagnetic coupling constant, H is the external magnetic field and $\langle \cdot \rangle$ means that the summation is made over nearest-neighbor sites only. In the absence of an external magnetic field ($H = 0$) and at low temperature, the system is, for more than one dimension, in the ferromagnetic phase and on average, most spins are pointing in the same direction. In contrast, at high temperature the system maximizes the entropy, thermal fluctuations break the order, and the system is in the paramagnetic phase. In this way, a second-order ferromagnetic–paramagnetic phase transition takes place, with the magnetization M (defined as $M = \frac{1}{N} \sum_{ij=1}^N \eta_{ij}$) as the order parameter, and it occurs at a well-defined critical temperature. In the two-dimensional case, one has exactly $k_B T_c^{\text{Ising}}/J = 2/\ln(1 + \sqrt{2}) = 2.269 \dots$, where k_B is the Boltzmann constant. In the kinetic Ising model with Glauber dynamics, a randomly selected spin is flipped with probability W^G given by

$$W^G = \frac{e^{-\Delta\mathcal{H}/k_B T}}{1 + e^{-\Delta\mathcal{H}/k_B T}}, \quad (2)$$

where T is given in units of J/k_B and $\Delta\mathcal{H}$ is the energy difference between the would-be new and the initial configurations.

On the other hand, the DLG model [5] is defined as a lattice gas on a two-dimensional square lattice of size $L_x \times L_y$ with periodic boundary conditions applied along both directions. The external driving field \mathbf{E} is applied along the L_x -direction. Each lattice site label (i, j) has two possible states, empty or occupied by a particle, where the occupation variable takes values $\sigma_{i,j} = 0$ or 1, respectively. By assuming that particles have an attractive interaction ($J > 0$) between first nearest-neighbors and in the absence of an external field, the Hamiltonian \mathcal{H} of the system is given by

$$\mathcal{H} = -4J \sum_{\langle ij, i'j' \rangle} \sigma_{ij} \sigma_{i'j'}, \quad (3)$$

where $\langle \cdot \rangle$ means that the summation is made over nearest-neighbor sites only. It is worth mentioning that the Hamiltonians given by Eq. (1) (for $H = 0$) and (3) are equivalent under the transformation $\eta_{ij} = (2\sigma_{ij} - 1)$. The transition probability of a randomly selected particle to jump into one of its four possible empty near-neighbors, W^{DLG} , corresponds to Kawasaki dynamics with Metropolis rates [37] modified by the presence of the external field \mathbf{E}

$$W^{\text{DLG}} = \min[1, e^{-[\Delta\mathcal{H} - \varepsilon \cdot \mathbf{E}]/k_B T}], \quad (4)$$

where $\Delta\mathcal{H}$ is the energy change after the particle–hole exchange, and $\varepsilon = (1, 0, -1)$ assumes these values when the jump direction of the particle is against, orthogonal or along the driving field \mathbf{E} , respectively. Also, the magnitude E of the external field \mathbf{E} is measured in units of J and the temperature T is given in units of J/k_B . The dynamics imposed on the system does not allow the removal of particles, so that the amount of particles – and therefore the particle density (analog to the magnetization in the Ising model) – is a conserved quantity. As was mentioned above, for a half-filled two-dimensional system, i.e., initial density $\rho_0 = 0.5$, the DLG model exhibits second-order nonequilibrium phase transition between an ordered phase (characterized by stripes of high particle density running along the driving direction) and a disordered (gas-like) phase [5,29,6]. The critical temperature (T_c^{DLG}) depends on the value of the driving field and it increases monotonically with \mathbf{E} . In the limit of $E \rightarrow \infty$, one has $T_c^{\text{DLG}}(E = \infty) \simeq 1.414 T_c^{\text{Ising}} \simeq 3.20$ in J/k_B units [29].

Finally, the mixed model dynamics W^{mix} is defined by the addition of a parameter p , in the following way:

$$W^{\text{mix}} = pW^G + (1 - p)W^{\text{DLG}}, \quad (5)$$

where W^G and W^{DLG} are given by Eqs. (2) and (4), respectively. Of course, the changes introduced in Eq. (5) mean that the mixed DLG model does not conserve the number of particles (and therefore the magnetization).

3. The damage spreading method

The DS method was initially introduced [16,38–40] to investigate the effects of tiny perturbations in the initial condition of physical systems on their final stationary or equilibrium states. In order to implement the DS method in computational simulations [41,42], two configurations or samples S and S' of a given model are allowed to evolve simultaneously. Initially, both samples differ only in the state of a small number of sites. Then, the difference between S and S' can be considered as a small initial perturbation. In order to give a quantitative measure of the evolution of the perturbation, the “Hamming” distance or damage $D(t)$ is defined as

$$D(t) = \frac{1}{N} \sum_{i,j}^N 1 - \delta_{\eta_{ij}(t), \eta'_{ij}(t)}, \quad (6)$$

where $N = L_x \times L_y$ is the total number of sites in the lattices, $\eta'_{ij}(t)$ ($\eta_{ij}(t)$) is the occupation number of site (i, j) in the sample S' (S), and $\delta_{\eta_{ij}(t), \eta'_{ij}(t)}$ is the Kronecker delta function. The sum runs over all sites, so $0 \leq D(t) \leq 1$. The main question is whether the damage $D(t = 0) \sim O(1/N)$ in a given model survives or vanishes after some time $t \rightarrow \infty$ in the thermodynamic limit. In several cases, there is a transition between a state where damage heals and a state where the perturbation propagates throughout the system. Often, this is an irreversible critical transition that is named the DS transition (for a detailed discussion of the DS method, see the review [17] and references therein). The universality class of the DS transition is still an open question and in some cases it has been demonstrated numerically that the DS transition can be related to the directed percolation class [43,44]. Moreover, the DS transition depends on the dynamic rules used to implement Monte Carlo simulations [44–46] even if the damaged system verifies the principle of detailed balance. It is necessary to remark that this principle only assures that the system will arrive at an equilibrium state, but it does not establish a unique dynamic evolution [42,44]. For example, for the kinetic Ising model with Glauber dynamics, the initial damage goes to zero below a damage temperature $T_{DS} \lesssim T_c^{Ising}$ [17,45], and it spreads above T_{DS} , while the opposite scenario ($D > 0$ for $T < T_{DS}$, and $D = 0$ for $T > T_{DS}$) is observed when using the heat-bath dynamics [40,17,46]. On the other hand, in the case of the kinetic Ising model with Kawasaki dynamics the results indicate that damage always spreads [47,48].

Recently, the damage spreading behavior has been studied in the DLG model [27] as a function of the temperature T , the magnitude of the external driving field \mathbf{E} , and the lattice size. The authors found that damage always spreads for all the investigated temperatures and it reaches a saturation value D_{sat} that depends only on T . D_{sat} increases for $T < T_{DLG}(E = \infty)$, decreases for $T > T_{DLG}(E = \infty)$, and it is free of finite-size effects. This behavior is explained as a consequence of the existence of interfaces between the high-density stripes and the lattice-gas-like phase whose roughness depends on T .

4. Simulation details

Monte Carlo simulations were performed in lattices of size $L_x \times L_y$, where $L_y = 2L_x$, and $L_x = 120 \rightarrow 480$, with periodic boundary conditions in all directions. The external field \mathbf{E} was applied along the horizontal direction, i.e., along L_x , therefore, it can be written as $\mathbf{E} = (E, 0)$, and the magnitude of the vector was fixed at $E = 50$ (notice that according to the transition rule given by Eq. (4), $E = 50$ is in practice equivalent to $E \equiv \infty$). The temperatures of the thermal bath were taken in the range $T = 2.0 \rightarrow 4.0$, given in units of J/k_B , and p was varied within the range $0 < p < 1$. The time was measured in Monte Carlo time steps (MCS), where one time unit corresponds to $(1 - p)(L_x \times L_y)$ attempts of particle jumps to empty neighbors with W^{DLG} transition rates and $p(L_x \times L_y)$ attempts at removing or placing a particle in that site with W^G transition rates. The physical observables were averaged over 10^3 different realizations of the thermal noise.

In order to show the results clearly, the next section was divided into two parts: (i) the short-time Dynamics study; and (ii) the damage spreading analysis. In the first part, STD was used to determine the phase transitions as a function of p . For reasons that will be clarified below, the control and order parameters were identified as the temperature $X = T$ and the magnetization M , respectively. By performing the linear transformation $\eta_{ij} = (2\sigma_{ij} - 1)$ the magnetization M and its second moment M^2 were evaluated from ordered ($T = 0$) and disordered ($T = \infty$) initial configurations. On the other hand, the second part was dedicated to studying the damage spreading behavior in the mixed DLG model. In this case, the initial configuration was the disordered one described above.

5. Results and discussion

5.1. Short-time dynamics study of the mixed DLG model

In order to understand the behavior of the proposed model, the NESS configurations were obtained for different values of the parameter p and temperature T . As a consequence of the nonconservative dynamics, the configurations at low temperatures correspond to almost full occupancy of the lattice sites when p belongs to the interval $[0.1, 0.9]$ (ordered state). On the other hand, at high temperatures a random occupation is obtained with density $\rho = 0.5$ (disordered state). This happens for all values of p in the interval $[0.1, 0.9]$. Fig. 1 shows the NESS configurations for longitudinal lattice size $L_x = 120$

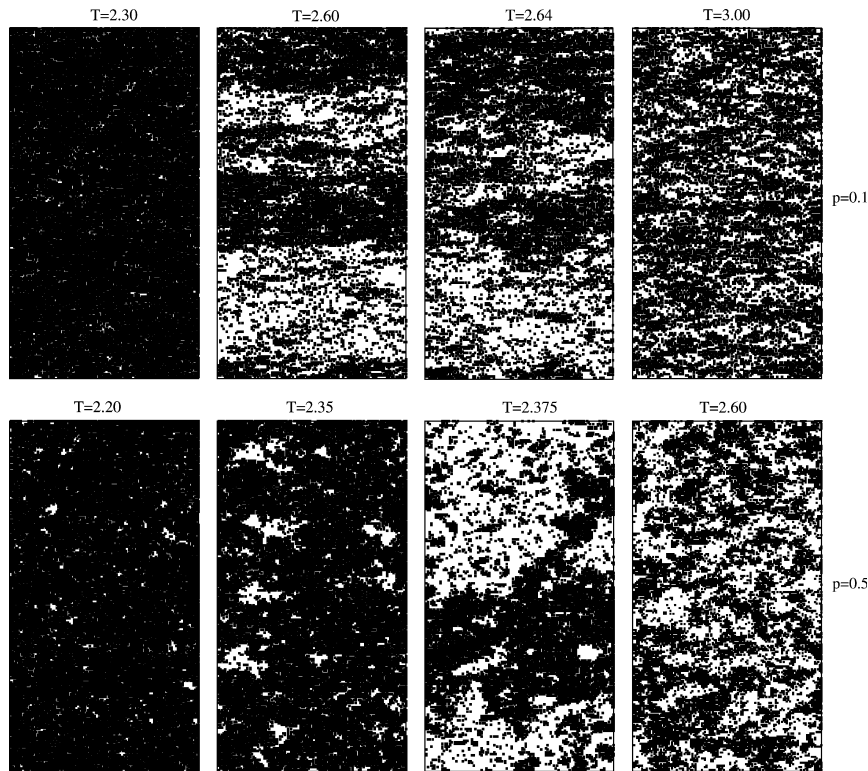


Fig. 1. NESS configurations of the mixed DLG model obtained after 4×10^6 MCS for a system of size $L_x = 120$, and at different values of temperature T and parameter p (as indicated).

at different temperatures for two selected values: $p = 0.1$ close to DLG dynamics, and $p = 0.5$, where both conservative and nonconservative dynamics are applied with the same probability. As can be observed, the NESS configurations are consistent with a transition from ordered to disordered states similar to those observed in the ferromagnetic–paramagnetic phase transition of the Ising model. For $p = 0.1$ a subtle influence of the DLG conservative dynamics can be inferred from the speckle-like bands along the external field direction, so the appearance of the system is more DLG-like, than Ising-like. These bands disappear at higher values of p , as is shown for the case of $p = 0.5$ in Fig. 1.

The described results allow us to define the order parameter as the magnetization $M(t)$ and to use the STD method to study the nonequilibrium order–disorder phase transition. Fig. 2(a) shows the time evolution of $M(t)$ from the fully occupied ordered state ($M(0) = 1$ corresponding to $T = 0$), for $p = 0.2$ and different annealing temperatures. The temperature T^* corresponds to the best power law obtained and the error bars were estimated by using the closest temperatures that show small but noticeable departures from it. The temperature T^{**} was obtained in the same way from the time evolution of the second moment of the magnetization $M^2(t)$ when the initial configuration is the disordered state ($M(0) = 0$ at $T = \infty$), as is shown in Fig. 2(b). It is worth mentioning that for the determination of T^{**} it is useful to study the time evolution of $M^2(t)$ instead of $M(t)$. This is due to the fact that the average of the magnetization over different realizations is zero. However if $M(t)$ were employed to study the phase transition, the simulations must be performed by imposing a very small, positive initial magnetization m_0 as an initial configuration, in order to have all the time series with the same magnetization sign. In this way, the dynamical behavior of the magnetization should be obtained as the limit $m_0 \rightarrow 0$ (for a detailed discussion see Refs. [12,13]).

Fig. 3 exhibits the temperatures T^* and T^{**} that were determined for different values of p by following the procedure described above. As was already mentioned in Section 1, the correlation length remains smaller than the system size, so the STD method will allow us to obtain the critical and pseudocritical temperatures of the model in the thermodynamic limit. Consequently, the results are free of finite-size effects within its validity range.

Due to the fact that $T^* = T^{**}$, within error bars, at $p = 0.1$ and $p > 0.6$, our results suggest the existence of Ising-like second-order phase transitions at these values of p . The critical temperatures for $p > 0.6$ are close to those corresponding to the Ising model, within error bars. In the interval $0.1 < p \leq 0.6$, the difference between T^* and T^{**} becomes clear and it would be a signature of a first-order phase transition whose maximum strength occurs at $p \simeq 0.45$. This assumption is also supported by the behavior observed in Fig. 4, for representative values of the parameter p ($p = 0.1$ and 0.5). Fig. 4(a) shows the time evolution of the squared-density fluctuations for $p = 0.1$ corresponding to three temperatures near the reported transition temperature $T^* = 2.643$ (see Table 1). As can be observed, the fluctuations exhibit two clear regimes: a first growing one, followed by a second regime where they tend to saturate. In this last regime the fluctuations depend on T ,

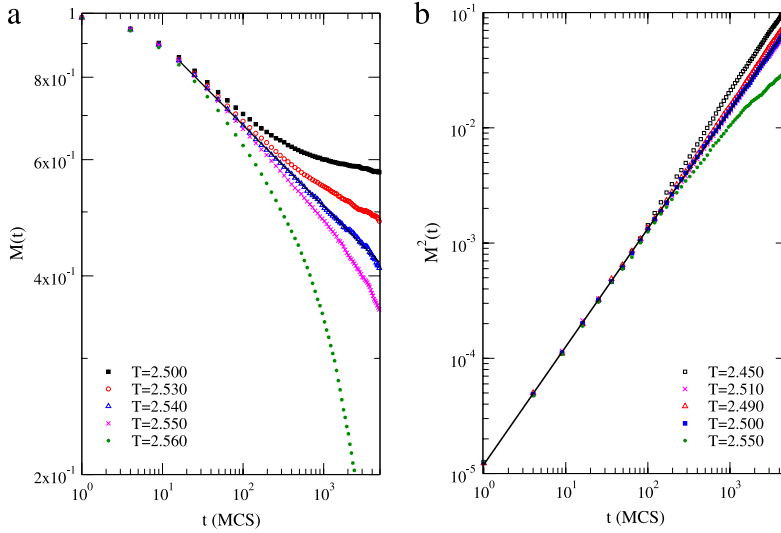


Fig. 2. (Color online) Log–log plots of the time evolution of (a) magnetization $M(t)$ from the initial ordered state corresponding to $T = 0$, and (b) second moment of magnetization $M^2(t)$ from initial disordered states corresponding to $T = \infty$, for $p = 0.2$ and $L_x = 120$ at the indicated temperatures. The data were averaged over 10^3 realizations. The best fitted power laws are also indicated with a solid line. More details in the text.

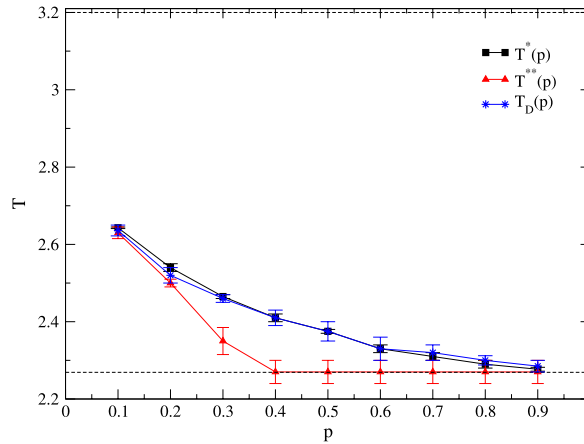


Fig. 3. (Color online) Temperatures T^* and T^{**} obtained from the best power-law behavior of the time evolution of $M(t)$ and $M^2(t)$ as a function of p , when the system was started from ordered and disordered initial configurations, respectively. Temperatures $T_D(p)$ were obtained from the damage spreading, as will be described in the following section. The critical temperatures of the DLG and Ising models are also indicated with dashed lines. More details in the text.

Table 1

Temperatures $T^*(p)$ and $T^{**}(p)$ determined by applying STD to the mixed DLG model and temperature $T_D(p)$ obtained from the behavior of the saturation damage (D_{sat}), in the thermodynamic limit, as a function of temperature.

p	$T^*(p)$	$T^{**}(p)$	$T_D(p)$
0.1	2.643(2)	2.630(15)	2.636(14)
0.2	2.54(1)	2.50(1)	2.52(2)
0.3	2.465(5)	2.350(35)	2.46(1)
0.4	2.41(1)	2.27(3)	2.41(2)
0.5	2.375(5)	2.27(3)	2.375(25)
0.6	2.33(1)	2.27(3)	2.33(3)
0.7	2.31(1)	2.27(3)	2.32(2)
0.8	2.29(1)	2.27(3)	2.300(12)
0.9	2.277(5)	2.27(3)	2.285(15)

and become less important when T moves away from the transition temperature into the disordered phase, as the evolution for $T = 2.680$ shows. This is consistent with the fact that in both DLG and Ising models keep $\rho(t) = 0.5$ or an average zero magnetization in the disordered or paramagnetic phases, respectively. Fig. 4(b) exhibits the same observable for $p = 0.5$, at

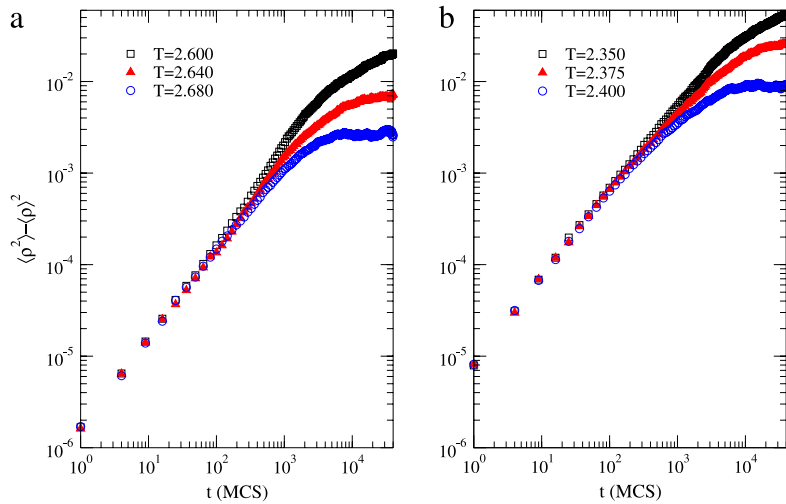


Fig. 4. (Color online) Time evolution of the density fluctuations for different values of T (as indicated), and (a) $p = 0.1$ and (b) $p = 0.5$. The data correspond to a system size of $L_x = 120$ and initial disordered configurations. More details in the text.

temperatures that are in the coexistence zone (see Fig. 3 and Table 1). Qualitatively, the same behavior occurs, but the values of the fluctuations are about one order of magnitude higher than those found for $p = 0.1$. This difference suggests that the density fluctuations are enough to produce $\rho(t) \neq 0.5$ for each realization and to induce a first-order phase transition in the mixed model.

In this way, the introduction of mixed dynamics on the one hand shifts the density to values where the DLG model exhibits first-order phase transitions [15] and on the other hand, it leads to phase transition temperatures higher than the Ising critical temperature for $p < 0.9$. This can be explained by the fact that the conservative dynamics keeps a nonzero magnetization at temperatures where the Ising model is in the paramagnetic phase. Similar results were reported by Acharyya et al. [35] for a DLG model where the nonconservative dynamics was introduced by means of a parameter that is proportional to H^2 (the external magnetic field in the Ising Hamiltonian, see Eq. (1)) and taking the limit $H \rightarrow 0$. More careful studies should be performed in order to understand the mechanism of the phase transitions, so this open subject will be left for future work.

5.2. Damage spreading study in mixed DLG model

Fig. 5 shows the time evolution of damage $D(t)$ Eq. (6) measured at different temperatures and values of the parameter p , for the lattice size $L_x = 480$. As can be observed, damage increases and reaches a saturation value $D_{sat}(L_x = 480)$ that depends on both the temperature and p . At low temperatures, as is shown in Fig. 5(a), for $T = 2$, $D_{sat}(L_x = 480)$ decreases with increasing p . This result is consistent with the fact that the temperature of the system is lower than the temperature of the DS transition in the Ising model with Glauber dynamics, T_{DS} , below which $D_{sat} = 0$ in the thermodynamic limit. Furthermore, Fig. 5(b) shows that at $T = 2.30$ the dependence of saturation damage on p becomes weaker than in the previous cases. For $T = T_c^{DLG} = 3.20$, see Fig. 5(c), the saturation values are the same, within error bars, for all investigated p values. As was previously mentioned for both extreme cases, i.e., $p = 0$ and $p = 1$, the damage spreads at $T = 3.2$. Moreover, the present results indicate that the system is in the disordered phase, which favors the spreading of damage.

In order to determine whether the mixed DLG model exhibits a DS transition, finite-size effects were studied by measuring the damage $D(t)$ for different lattice sizes (L_x). The results obtained for $p = 0.1$ and $T = 2.3$ (see Fig. 6(a)) show that $D(t)$ presents a maximum followed by a dip of the damage before reaching a saturation value, $D_{sat}(L_x)$. This maximum shifts to larger times and the dip flattens with the increase in the lattice size, which indicates that the dip is a finite-size effect. On the other hand, the dip disappears at higher temperatures $T \geq 2.6$ and with increasing p , as can be observed in Fig. 6(c) and (d).

Fig. 7 shows the saturation damage $D_{sat}(L_x)$ as a function of $1/L_x$ for different temperatures in the interval [2.00, 4.00]. In contrast with the DLG model where finite-size effects were practically not present [27], $D_{sat}(L_x)$ decreases with increasing L_x until it reaches a lower bound, which occurs at low temperatures. This bound can be defined as the value of D_{sat} in the thermodynamic limit. For $T \geq 2.60$, the data indicate that $D_{sat}(L_x)$ is independent of L_x . In these cases, the limit $D_{sat}(L_x \rightarrow \infty) \equiv D_{sat}$ was determined by means of a linear fit. Similar results were obtained for the other values of p (not shown here for the sake of space) and the temperatures where $D_{sat}(L_x)$ is independent of the lattice size were found to be closer to the pseudocritical point $T^*(p)$.

Fig. 8 shows D_{sat} versus T for the studied values of p . For $p < 0.6$, damage always spreads, which may suggest that the damage spreading transition occurs at zero temperature ($T_{DS} = 0$). Also, two regimes can be clearly distinguished, i.e., D_{sat}

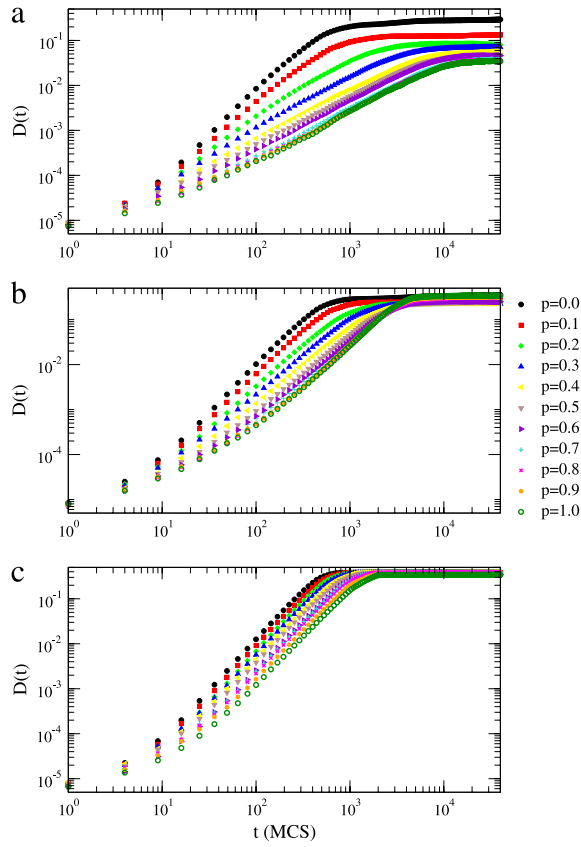


Fig. 5. (Color online) Log–log plot of damage $D(t)$ obtained for a system of size $L_x = 480$ and for all investigated values of p , as indicated in the legend: (a) $T = 2.0$, (b) $T = 2.3$, and (c) $T = 3.2$.

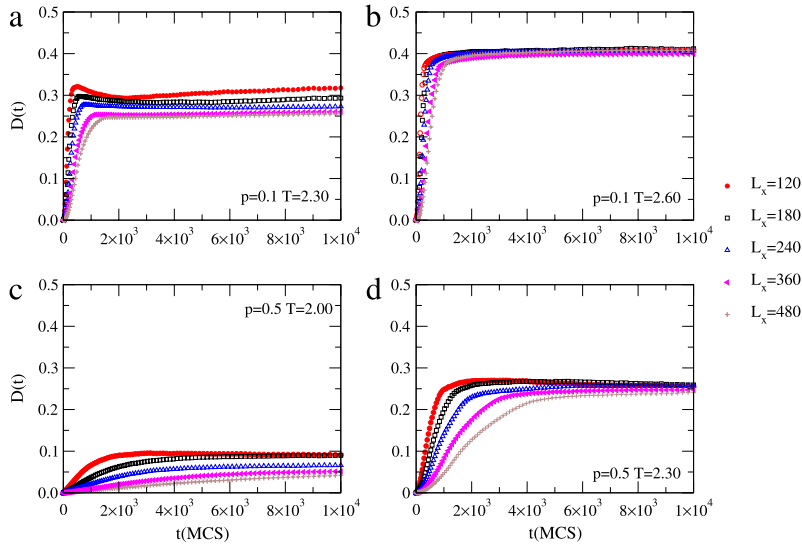


Fig. 6. (Color online) $D(t)$ obtained for the indicated system sizes and for different temperatures and values of parameter p : (a) $p = 0.1$, $T = 2.3$; (b) $p = 0.1$, $T = 2.6$; (c) $p = 0.5$, $T = 2.0$; and (d) $p = 0.5$, $T = 2.3$.

grows up to a given value D_{sat}^{max} , and after that it presents a tiny decrease (see Fig. 8(a)). Since in this regime of p there is a majority of (nonequilibrium) conservative DLG dynamics, it is not surprising that the shape of the curves becomes similar to that reported for the pure DLG model [27], where D_{sat}^{max} is a relevant point whose temperature matches T_c^{DLG} . Here, the temperature corresponding to D_{sat}^{max} , referred to as $T_D(p)$, decreases as long as p increases when more nonconservative events

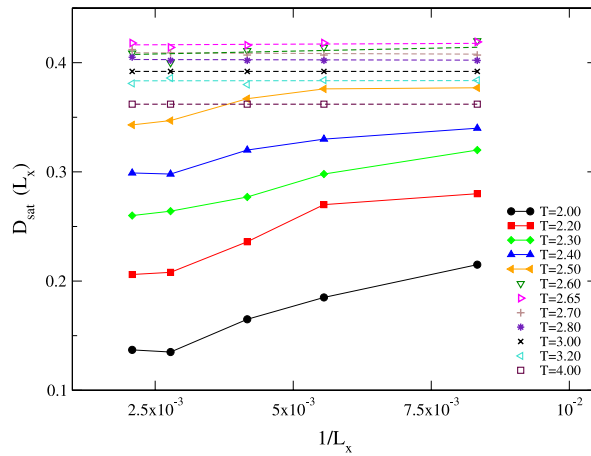


Fig. 7. (Color online) $D_{sat}(L_x)$ as a function of $1/L_x$ for $p = 0.1$ and the indicated temperatures. Dashed lines corresponds to the linear extrapolation to $1/L_x \rightarrow 0$, and the solid lines are only for guiding the eye. More details in the text.

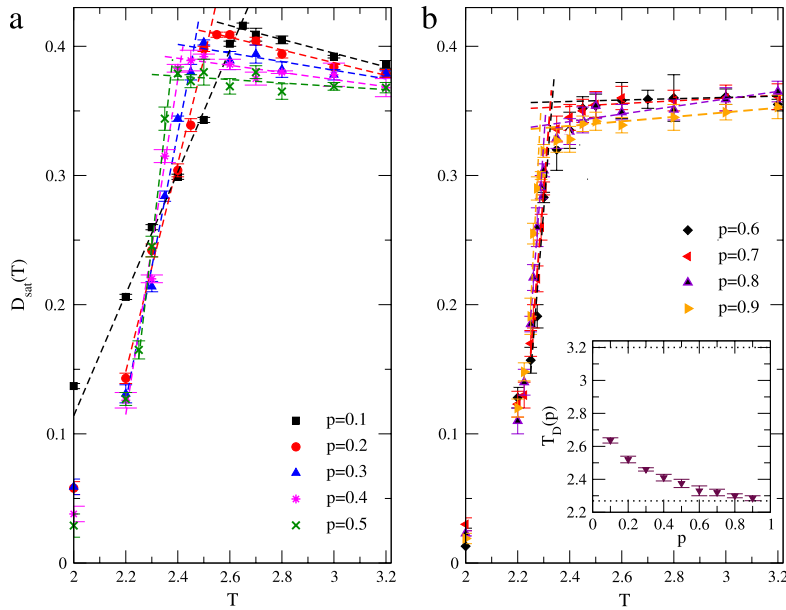


Fig. 8. (Color online) Saturation value of damage (D_{sat}) in the thermodynamic limit as a function of temperature for (a) $0.1 \leq p \leq 0.5$ and (b) $p \geq 0.6$. The dashed lines correspond to linear fits. The inset presents the temperature $T_D(p)$ versus p obtained from the intersection between the linear fits performed for $T < T_D(p)$ and $T > T_D(p)$, respectively. More details in the text.

are included in the dynamics. As happens in the pure (i.e., $p = 0$) DLG model, this behavior can also be associated with the existence of high density regions where the damage can spread mostly through its interfaces, as it is shown in the snapshots of the system configurations and damaged sites in Fig. 9(a). At $p \geq 0.6$ (see Fig. 8(b)) the plot resembles the diagram D_{sat} versus T for the Ising model with Glauber dynamics [17,45].

The temperature $T_D(p)$ was estimated by means of the intersection between the two linear fits indicated with dashed lines in Fig. 8(a) and (b). It is important to remind that $T_D(p)$, defined above, is different from the damage spreading critical temperature T_{DS} , defined in Section 3. The inset of Fig. 8(b) shows a decreasing behavior of $T_D(p)$ with p between both boundary values, $T_D(p = 0) = 3.20$ and $T_D(p = 1) \simeq 2.269$, corresponding to the DLG [27] and Ising model critical temperatures, respectively. Furthermore, the results indicate that $T_D(p) \approx T^*(p)$, within error bars, (see Table 1 and Fig. 3). This behavior is not strange, since damage is sensitive to the presence of phase transitions [17] in both models. In this way, $T_D(p)$ could be associated with a change in the system configuration related to the existence of a phase transition.

Fig. 9 shows typical snapshot configurations of the system (left panel) and the damaged sites (right panel) during the evolution to the NESS for the lattice size $L_x = 120$. Fig. 9(a) presents the configurations corresponding to $p = 0.1$, $T = 2.30$ at the time when the system reaches the maximum value of damage, $t = 484\text{MCS}$ (see Fig. 6(a)). As can be observed, the damage spreads close to the rough interfaces of high density regions. A similar situation is observed at $T = 2.60$ (Fig. 9(b)),

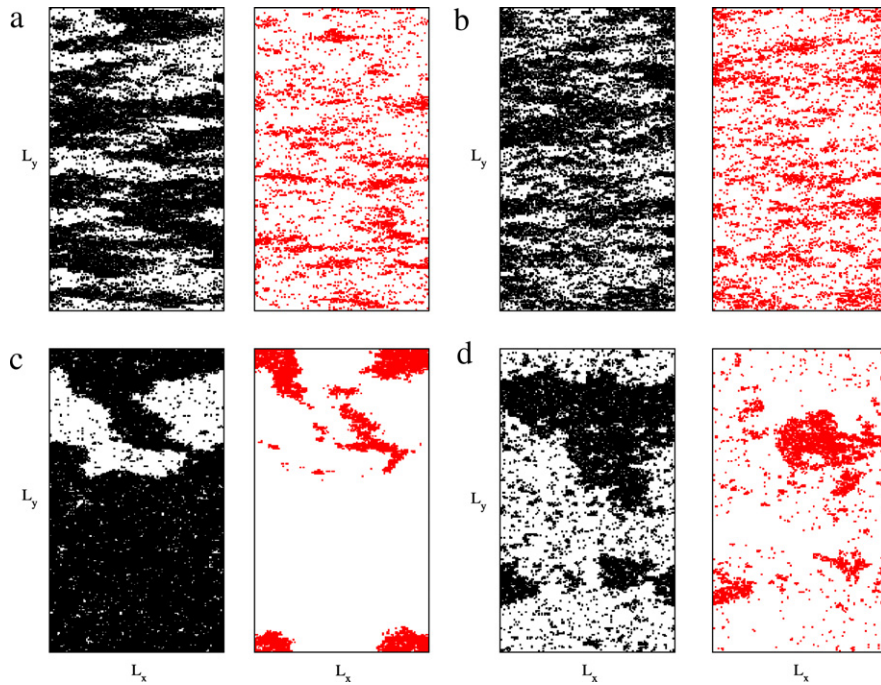


Fig. 9. (Color online) Snapshots of the spin configurations (left panel) and the damaged sites (right panel) for a lattice of size $L_x = 120$ and different values of p and T : (a) $p = 0.1$, $T = 2.30$; (b) $p = 0.1$, $T = 2.60$; (c) $p = 0.5$, $T = 2.00$ and (d) $p = 0.5$, $T = 2.3$. The data correspond to times of $t = 484$ and 1444MCS for $p = 0.1$ and $p = 0.5$, respectively.

near $T_D(p = 0.1) = T^*(p = 0.1)$ where the order–disorder phase transition occurs. In this case the increase of the interface density enhances damage spreading. The increment of the sites updated by Glauber dynamics causes the damage to spread also to the high density regions, as Fig. 9(c) and (d) shows for $p = 0.5$. At low temperatures when the system has reached damage saturation, high density regions are more compact and the interface density becomes smaller than in the previous case. This fact hinders damage spreading as can be inferred from Fig. 9(c) for $T = 2.00$ and $t = 1444\text{MCS}$. If the temperature is raised to reach the phase coexistence, $T = 2.30$ (see Fig. 9(d)), the damage can spread to high density regions as well as in the disordered phase. This behavior has been observed in both Ising [17] and DLG [27] models, and can be explained in terms of the magnetization or density fluctuations in the region near the interfaces, respectively. In fact, around these regions one has the largest fluctuations that enhance the propagation of the perturbation, i.e. the damage. In contrast, within a high density region, the propagation of damage is energetically unfavorable because the short-range interactions promote that all the nearest-neighbors of a given particle to be of the same kind.

6. Conclusions

In this work the effects of introducing Glauber dynamics in the DLG model were investigated. With this aim the parameter p , which regulates the fraction of particles that are updated with Glauber or DLG dynamics, respectively, was defined. The influence of Glauber dynamics becomes fundamental for the behavior of the mixed DLG model, since it breaks the density conservation rule. In this way, the existence of a phase transition from a configuration with full occupation at low T 's to a random one at higher temperatures was demonstrated by studying the NESS configurations and using the STD method. Moreover, STD results indicated that the transition would be of second order at $p = 0.1$ and $p \geq 0.7$. For the rest of the interval $0.1 < p \leq 0.6$, the difference between the temperatures T^* and T^{**} becomes relevant, T^* and T^{**} being the points where the dynamic observables present power-law behavior. This is the signaling of a first-order phase transition. Furthermore, both T^* and T^{**} decrease with p , and $(T^*, T^{**}) \rightarrow T_c^{\text{Ising}}$, which is consistent with the fact that the mixed DLG model becomes the Ising model in the limit $p \rightarrow 1$. In this way, the presence of the DLG dynamics keeps a nonzero magnetization at a temperature at which the pure Ising model is on the disordered phase. This fact allows us to explain the shift in the described phase transition temperatures.

On the other hand, in order to study the damage transition, the saturation damage in the thermodynamic limit was obtained in the temperature range $[2.00, 4.00]$ from studies of damage spreading and finite-size effects. The temperature $T_D(p)$ that characterizes the point where the behavior of the damage changes for each p was obtained. The fact that $T_D(p) \approx T^*(p)$ indicates that $T_D(p)$ is related to the phase transition, in analogy with previous works that reported the same behavior for DLG and Ising models.

In summary, a nonequilibrium model with mixed conservative and nonconservative biased dynamics was studied and a novel and rich behavior related to the existence of phase transitions of first and second order was obtained.

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