

Inflation without inflaton: A model for dark energyH. Falomir^{*}*Departamento de Física, Universidad Nacional de La Plata, 1900 La Plata, Argentina*J. Gamboa[†] and F. Méndez[‡]*Departamento de Física, Universidad de Santiago de Chile, Casilla 307, Santiago, Chile*P. Gondolo[§]*Department of Physics, University of Utah, Salt Lake City, Utah 84112, USA*

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The interaction between two initially causally disconnected regions of the Universe is studied using analogies of noncommutative quantum mechanics and the deformation of Poisson manifolds. These causally disconnect regions are governed by two independent Friedmann-Lemaître-Robertson-Walker (FLRW) metrics with scale factors a and b and cosmological constants Λ_a and Λ_b , respectively. The causality is turned on by positing a nontrivial Poisson bracket $[\mathcal{P}_\alpha, \mathcal{P}_\beta] = \epsilon_{\alpha\beta} \frac{\kappa}{G}$, where G is Newton's gravitational constant and κ is a dimensionless parameter. The posited deformed Poisson bracket has an interpretation in terms of 3-cocycles, anomalies, and Poissonian manifolds. The modified FLRW equations acquire an energy-momentum tensor from which we explicitly obtain the equation of state parameter. The modified FLRW equations are solved numerically and the solutions are inflationary or oscillating depending on the values of κ . In this model, the accelerating and decelerating regime may be periodic. The analysis of the equation of state clearly shows the presence of dark energy. By completeness, the perturbative solution for $\kappa \ll 1$ is also studied.

DOI: [10.1103/PhysRevD.96.083534](https://doi.org/10.1103/PhysRevD.96.083534)**I. MOTIVATING THE PROBLEM**

Understanding the origin and behavior of dark matter and dark energy poses one of the most important challenges of today's physics, and its solution could require new radical ideas.

Standard cosmology rests on the cosmological principle, the assumption that the Universe is homogeneous and isotropic on large scales. However in the big bang era, approximately 13.8 billion years ago, when the Universe violently expanded from a very high density and temperature state, the cosmological principle conditions were not fulfilled because of the extraordinarily nonhomogeneous and anisotropic nature of this expansion. The released energy, then, was redistributed in such a way that causally disconnected sectors were formed [1].

After this extremely short period of time, our known laws of physics apply and one can speculate, for example, about the formation of topological defects which break the large-scale homogeneity [2–4], as domain walls, cosmic strings, or monopoles, of which no visible sign has been found.

This led to the assumption of a period of *cosmic inflation* [5–14] during which the Universe grew exponentially, smoothing out inhomogeneities inside the cosmological

horizon, the boundary of our observable causal patch of the Universe. Inflation ends in a reheating phase where the standard model particles are produced and, as temperature decreases, quantum fluctuations explain the formation of galaxies and the current large-scale structure of the observable universe.

After the cosmic inflation, most of the evolution of the Universe has been dominated by matter and radiation. But evidence coming from the red shift of Type Ia supernovae [15] and from the fluctuations in the cosmic microwave background [16] suggests that our Universe is presently in a phase of accelerated expansion. Lambda cold dark matter (Λ CDM) is the standard cosmological model describing this situation, with approximately 4.9% of ordinary (baryonic) matter, 26.8% of (cold) *dark matter*, and 68.3% of *dark energy* (Λ stands for the *cosmological constant*), compatible with a flat space with a critical total density $\rho_{\text{tot}} = \rho_{\text{matt}} + \Lambda/8\pi G \approx 3H^2/8\pi G$ (where H the Hubble parameter). The origin of dark matter and dark energy is still unknown.

The universe in this model is described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a(t)^2 R_0^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right\}, \quad (1)$$

where $a(t)$ is the (dimensionless) time-dependent *scale factor* of spatial sections, R_0 is the present length scale (if $a(0) = 1$

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at the present time $t = 0$), r is the (dimensionless) radial coordinate, and k is the curvature of spatial sections, being $k = 0$ for a flat space. For this geometry, Einstein's equations reduce to the two Friedmann's equations [17]

$$\begin{aligned} H(t)^2 &:= \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{R_0^2 a(t)^2}, \\ \frac{\ddot{a}(t)}{a(t)} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \end{aligned} \quad (2)$$

where *dots* refer to time derivatives. The role of Λ is clear from the previous equations. Indeed, for a Λ -dominated era—the dark energy dominated epoch—we have that asymptotically $a(t) \sim e^{t\sqrt{\Lambda/3}}$. The physical origin of the cosmological constant is troublesome, on the other hand. For example, the identification of Λ with the vacuum energy of the various species of particles of the standard model, evaluated with a cutoff of the order of the Planck scale, leads to a mismatch of around 120 orders of magnitude when compared with observable data, while an exact supersymmetric field theory predicts a vanishing result. If SUSY is broken—a natural way out to the problem—a *fine-tuning* is necessary to approximate the experimental value of Λ .

This is the *cosmological constant problem* [18–20], one of the most significant open problems in fundamental physics. Models to solve this puzzle have been formulated in which Λ is related with the vacuum expectation value of the energy density of dynamical light scalar fields [21] with local minima in the potential energy, which would produce *phase transitions* as the temperature decreases, or with a sufficiently small slope to produce a slow roll down to the minimum of the potential.

Different scenarios explore the possibility that cosmic acceleration could be described by higher-dimensional theories [22]. It has also been argued [23] that quantum mechanics combined with Einstein's theory would require a kind of nontrivial uncertainty relation at the Planck scale, which impose effective short distance or large momentum cutoffs, since the attempt to localize an event with extreme precision would demand an energy that would lead to a gravitational collapse.

In the present work, we consider a model where patches of the Universe—causally disconnected after the inflation era—have independently evolved to a dark energy dominated era according to Einstein's equations. A tiny effective interaction is considered, which can be interpreted as a relic of a primordial nontrivial uncertainty relation.

Technically, a mechanism that might explain how our Universe evolved to the conditions we know today—an accelerated expansion state—is based on a deformed Poisson-bracket structure for the metrics describing the patches of initially causally disconnected components.

Indeed, deformed Poisson-bracket algebras, which describe phase-space noncommutative geometries [24–30],

usually imply nontrivial interactions [31]. As an example, consider the Hamiltonian $\mathcal{H}_0 = \frac{1}{2}\pi_i\pi_i$ (sum is implied) in two dimensions, with phase space coordinates $\{x^i, \pi_j\}_{\{i,j\} \in \{1,2\}}$ satisfying the noncanonical Poisson brackets

$$\{x^i, x^j\} = 0, \quad \{x^i, \pi_j\} = \delta_j^i, \quad \{\pi_i, \pi_j\} = B\epsilon_{ij}. \quad (3)$$

Here B is a constant and ϵ_{ij} is the totally antisymmetric tensor.

This system describes the Landau model as can be seen by performing the change of variables in the momentum sector $\pi_j = p_j + \frac{B}{2}\epsilon_{jk}x^k$ (the so called *Bopp's shift*). The system is now described by the Hamiltonian

$$\mathcal{H} = \frac{1}{2} \left(p_i + \frac{B}{2}\epsilon_{ik}x^k \right)^2, \quad (4)$$

while the Poisson-bracket structure is the canonical one, namely,

$$\{x^i, x^j\} = 0, \quad \{x^i, p_j\} = \delta_j^i, \quad \{p_i, p_j\} = 0. \quad (5)$$

Let us just mention that performing a linear Bopp's shift is equivalent to using the \star -Moyal product when the deformation parameters are constant. Otherwise, the more general \star -Kontsevich product or an \hbar expansion should be employed [29].

This paper is organized as follows. In Sec. II, we implement the main idea by extending the FLRW metric to a universe with two FLRW metrics—that is with two scale factors $a(t)$ and $b(t)$ —coupled by rules resembling the case of the Landau problem, which will be explained below. Basically, the idea is to consider the standard cosmological model as a Hamiltonian system formally similar to classical mechanics with the second metric as a new degree of freedom coupled through a Landau-like mechanism. In Sec. III, the coupled FLRW equations are solved, first by using numerical methods in Sec. III and then through a first-order perturbative expansion in Sec. III B. These solutions show inflation at early times but another behavior emerges at late times. The inflation at early times is an effect that becomes manifest assuming that cosmological constants in the two different patches satisfy a relation $\Lambda_a \ll \Lambda_b$. In Sec. IV, the interpretation in terms of dark energy is given for the solutions previously found, and the last section is devoted to discussion and conclusions.

II. THE MODEL

Following the analogy with the Poisson manifold deformation (Landau problem) described above, we consider a universe with two scale factors $a(t)$ and $b(t)$ and we *posit* the following deformed Poisson bracket for the conjugate momenta π_a and π_b of a and b :

$$\{\pi_\alpha, \pi_\beta\} = \epsilon_{\alpha\beta}\theta. \quad (6)$$

Here, indices $\alpha, \beta \in \{1, 2\}$ label scale factors—that is, $a_1 \equiv a$, $a_2 \equiv b$, $\pi_1 \equiv \pi_a$, $\pi_2 \equiv \pi_b$ —while $\epsilon_{\alpha\beta}$ is the two-dimensional Levi-Civita tensor.

The θ parameter can be chosen with dimensions of (energy)² if a_α has dimensions of (energy)⁻¹ and π_α dimensions of energy (see the Appendix). However, we must note that, contrary to the Landau problem, θ should not be identified with an external magnetic field and its value should be fixed using different physical arguments.

The remaining Poisson brackets are the standard ones, i.e.,

$$\{a_\alpha, a_\beta\} = 0, \quad \{a_\alpha, \pi_\beta\} = \delta_{\alpha\beta}. \quad (7)$$

Before continuing with the technical discussion, let us introduce a useful parametrization for θ . Indeed, since it is a constant for the two metrics under consideration in this universe, it seems natural to define $\theta = \frac{\kappa}{G}$, where κ is a dimensionless parameter and G is Newton's constant. From this point of view, the $G \rightarrow \infty$ limit with fixed κ (formally equivalent to $\kappa \rightarrow 0$ with fixed G) would correspond to a universe with two causally disconnected patches.

In fact, the $G \rightarrow \infty$ limit closely resembles the tensionless limit in string theory [32]. In string theory the tension of the string T and the Regge slope α' are related through $T \propto \frac{1}{\alpha'}$, the tensionless limit corresponds to $\alpha' \rightarrow \infty$, generating the so called ultralocal limit, where every point in the string evolves in a causally disconnected manner from the rest of the points on the same string. (Parenthetically, many efforts were devoted to the study of such scenario as can be seen, for example, in [33]. See also [34]) However we stress the difference with our approach where no relic of spatial dependence appears in the metric, as a consequence of the cosmological principle.

For the total Hamiltonian of the model presented here, under the conditions set out above, we take¹

¹We are taking the same time coordinate for both sectors. We recall that Friedmann's equations can be derived from the effective Lagrangian

$$L[a, \dot{a}, N] := \frac{1}{G} \left[\frac{a\dot{a}^2}{2N} + \frac{\Lambda}{6} Na^3 - \frac{k}{2} Na \right],$$

where $N(t)$ is an auxiliary variable which ensures the time-reparametrization invariance of the action. Indeed, the second line in (2) is the equation of motion for $a(t)$ if we choose $N(t) \equiv 1$, while the equation for $N(t)$ imposes the first line in (2) as a constraint on the system. The corresponding Hamiltonian, with $p_a = a\dot{a}/GN$ and $p_N = 0$, reads

$$H(p_a, a, p_N, N) = N\mathcal{H}(p_a, a),$$

where

$$\mathcal{H}(p_a, a) = \frac{1}{2} \left[G \frac{p_a^2}{a} - \frac{\Lambda}{3G} a^3 + \frac{k}{G} a \right],$$

and the constraint implies $\mathcal{H} = 0$ on the physically acceptable trajectories.

$$H = N \left[\frac{G\pi_a^2}{2a} + \frac{G\pi_b^2}{2b} + \frac{1}{2G} \left(k_a a - \frac{\Lambda_a}{3} a^3 + k_b b - \frac{\Lambda_b}{3} b^3 \right) \right], \quad (8)$$

where k_a and k_b are the spatial curvatures of the patches described by the scales $a(t)$ and $b(t)$, respectively.

The equations of motion derived from this Hamiltonian with the Poisson-bracket structure defined in (6) and (7) turn out to be

$$\dot{a} = G \frac{\pi_a}{a}, \quad (9)$$

$$\dot{b} = G \frac{\pi_b}{b}, \quad (10)$$

$$\dot{\pi}_a = G \frac{\pi_a^2}{2a^2} + \frac{\Lambda_a a^2 - k_a}{2G} + \kappa \frac{\pi_b}{b}, \quad (11)$$

$$\dot{\pi}_b = G \frac{\pi_b^2}{2b^2} + \frac{\Lambda_b b^2 - k_b}{2G} - \kappa \frac{\pi_a}{a}, \quad (12)$$

where we have used the reparametrization $Ndt \rightarrow dt$ or, equivalently, we have taken $N = 1$ at the end of the derivation.

The constraint $\dot{p}_N = 0$ derived from this Hamiltonian (a consequence of time-reparametrization invariance of the effective action) turns out to be

$$\frac{\pi_a^2}{a} + \frac{\pi_b^2}{b} + \frac{1}{G^2} \left(k_a a - \frac{\Lambda_a}{3} a^3 + k_b b - \frac{\Lambda_b}{3} b^3 \right) = 0. \quad (13)$$

Notice that this constraint is independent of κ and so it applies to our model even in the canonical Poisson-brackets case.

The equations of motion (9) and (10) can be used to write the momenta equations (11) and (12) as second order differential equations, and also to bring Eq. (13) to the *standard* form. In so doing we find the following set of equations:

$$2 \frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 = \Lambda_a - \frac{k_a}{a^2} + 2\kappa \frac{\dot{b}}{a^2}, \quad (14)$$

$$2 \frac{\ddot{b}}{b} + \left(\frac{\dot{b}}{b} \right)^2 = \Lambda_b - \frac{k_b}{b^2} - 2\kappa \frac{\dot{a}}{b^2}, \quad (15)$$

$$a\dot{a}^2 + b\dot{b}^2 = \frac{\Lambda_a}{3} a^3 - k_a a + \frac{\Lambda_b}{3} b^3 - k_b b. \quad (16)$$

They contain all the dynamical information about the model and show that the evolution of the scale factor of one patch is modified by the behavior of the scale factor of the other patch. We can venture an interpretation here in terms of gravitational bubbles. Indeed, each scale factor

describes one of the bubbles and they evolve under a sort of interaction induced by (6).

The previous system of differential equations can also be derived from a canonical Poisson structure by performing a Bopp's shift in the momenta π_α , in complete analogy with noncommutative quantum mechanics. That is, we can perform a change of variables in the form

$$\pi_\alpha = p_\alpha + \frac{\theta}{2}\epsilon_{\alpha\beta}a_\beta = p_\alpha + \frac{\kappa}{2G}\epsilon_{\alpha\beta}a_\beta,$$

where p_α are the canonical momenta ($\{a_\alpha, p_\beta\} = \delta_{\alpha\beta}$).

The Hamiltonian in these variables turns out to be

$$H = N \left[\frac{G}{2a} \left(p_a + \frac{\kappa}{2G} b \right)^2 + \frac{G}{2b} \left(p_b - \frac{\kappa}{2G} a \right)^2 + \frac{1}{G} \left(k_a a - \frac{\Lambda_a}{3} a^3 + k_b b - \frac{\Lambda_b}{3} b^3 \right) \right]. \quad (17)$$

The corresponding Hamilton's equations of motion are

$$\dot{a} = \frac{G}{a} \left(p_a + \frac{\kappa}{2G} b \right), \quad (18)$$

$$\dot{b} = \frac{G}{b} \left(p_b - \frac{\kappa}{2G} a \right), \quad (19)$$

$$\dot{p}_a = \frac{G}{2a^2} \left(p_a + \frac{\kappa}{2G} b \right)^2 + \frac{\Lambda_a a^2 - k_a}{2G} + \frac{\kappa}{2b} \left(p_b - \frac{\kappa}{2G} a \right), \quad (20)$$

$$\dot{p}_b = \frac{G}{2b^2} \left(p_b - \frac{\kappa}{2G} a \right)^2 + \frac{\Lambda_b b^2 - k_b}{2G} - \frac{\kappa}{2a} \left(p_a + \frac{\kappa}{2G} b \right), \quad (21)$$

where the gauge $N = 1$ has been chosen again. Of course, the constrained system of second order differential equations derived from here is also given by (14), (15), and (16).

Finally, note that it is possible to identify the right-hand sides of Eqs. (14) and (15) with an energy-momentum tensor that is covariantly conserved. Indeed, let us consider the a sector of the model, that is, the patch of the Universe described by the scale factor a . The FLRW Einstein tensor $G_{\mu\nu}$ for the a patch has time component $G_0^0 = -3(\dot{a}^2 + k_a)/a^2$ and space components $G_1^1 = G_2^2 = G_3^3 = -(2a\ddot{a} + \dot{a}^2 + k_a)/a^2$. Then Eqs. (14) and (16) can be written as the Einstein equations for the a patch

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (22)$$

provided

$$T_0^0 = \frac{\Lambda_a}{8\pi G} + \frac{b^3}{a^3} \left[\Lambda_b - \frac{3(\dot{b}^2 + k_b)}{b^2} \right],$$

$$T_1^1 = T_2^2 = T_3^3 = -\frac{\Lambda_a}{8\pi G} - \frac{\theta}{4\pi} \frac{\dot{b}}{a^2}. \quad (23)$$

Notice that the term in square brackets in T_0^0 is $G_{(b)0}^0 + \Lambda_b \delta_0^0$, with $G_{(b)0}^0 = -3(\dot{b}^2 + k_b)/b^2$, and thus vanishes when for $\theta = 0$ the a and b patches evolve independently. Energy-momentum covariant conservation then implies that

$$T^\mu{}_{\nu;\mu} = 0 \Rightarrow \frac{3\dot{b}}{a^3} (2b\ddot{b} + \dot{b}^2 + k_b - \Lambda_b b^2 + 2\kappa\dot{a}) = 0. \quad (24)$$

This condition is just the equation of motion for $b(t)$, Eq. (15). This is a self-consistency property, intrinsic to the model proposed here. Notice that, in this way, each sector appears as a kind of *local* source of the other. Moreover, the effective density, which does not depend on κ , is induced by the time-reparametrization invariance through the constraint in Eq. (13), while the effective pressure, proportional to κ , is a consequence of the assumed noncommutativity, Eq. (6). Notice also that the sign of the effective pressure depends on the behavior of the scale $b(t)$ (expansion or contraction) of the second sector.

The realization of our model in a more fundamental theory is not unique and deserves a careful analysis. A possible picture of the model could be the effective description of two regions of the Universe, originally disconnected (as, for example, regions separated by domain walls as mentioned in the Introduction). It might also be possible to have an interpretation in terms of two universes,² albeit initially disconnected, such that this disconnection is broken at a certain time through the modification of the canonical brackets. Another interpretation is simply in terms of a universe with two metrics, namely, a bimetric theory, which does not suffer from the usual instabilities and in this sense our model resembles the results presented by Freidel *et al.* [35,36].

Finally, and this is the idea we explore now, the possibility that the effect of the *dark energy* responsible for the accelerated expansion of our Universe at the present time could be encoded in a second metric interacting in the way we have explored here is very attractive.

III. SOLUTIONS OF THE GENERALIZED FLRW EQUATION

In order to study the properties of the model proposed here, we will analyze the solutions of the equations of motion in different regimes. The perturbative regime is

²This point of view has been defended vigorously by Linde; see [1] and references therein.

defined by $\kappa \ll 1$. We will show that this analysis can be done consistently only at early times in the evolution of the Universe.

For late times, instead, numerical solutions of the equations of motion will be useful. For this case, instead of using the second order set of equations, (14) and (15), it will be convenient to solve directly the Hamiltonian system defined in Eqs. (9) to (12).

For the analysis, it is convenient to define a scale μ with dimensions of energy, that is $[\mu] = +1$, such that the quantities $\bar{t} \equiv \mu t$, $\bar{a} \equiv \mu a$, $\bar{b} \equiv \mu b$ are dimensionless. For the momenta we define $\bar{\pi}_\alpha \equiv \mu G \pi_\alpha$ (or in terms of canonical variables $\bar{p}_\alpha \equiv \mu G p_\alpha$). Finally, cosmological constants can be rescaled also and we define λ such that

$$\sin \lambda \equiv \frac{\Lambda_b}{\mu^2}, \quad \cos \lambda \equiv \frac{\Lambda_a}{\mu^2},$$

so $\mu = (\Lambda_a^2 + \Lambda_b^2)^{\frac{1}{4}}$.

In terms of these dimensionless quantities, the set of first-order dynamical equations reads

$$\frac{d\bar{a}}{d\bar{t}} = \frac{\bar{\pi}_a}{\bar{a}}, \quad (25)$$

$$\frac{d\bar{b}}{d\bar{t}} = \frac{\bar{\pi}_b}{\bar{b}}, \quad (26)$$

$$\frac{d\bar{\pi}_a}{d\bar{t}} = \frac{\bar{\pi}_a^2}{2\bar{a}^2} + \frac{\bar{a}^2 \cos \lambda - k_a}{2} + \frac{\kappa \bar{\pi}_b}{\bar{b}}, \quad (27)$$

$$\frac{d\bar{\pi}_b}{d\bar{t}} = \frac{\bar{\pi}_b^2}{2\bar{b}^2} + \frac{\bar{b}^2 \sin \lambda - k_b}{2} - \frac{\kappa \bar{\pi}_a}{\bar{a}}, \quad (28)$$

while the second order dimensionless system is

$$2\bar{a} \frac{d^2 \bar{a}}{d\bar{t}^2} + \left(\frac{d\bar{a}}{d\bar{t}} \right)^2 = \bar{a}^2 \cos \lambda - k_a + 2\kappa \frac{d\bar{b}}{d\bar{t}}, \quad (29)$$

$$2\bar{b} \frac{d^2 \bar{b}}{d\bar{t}^2} + \left(\frac{d\bar{b}}{d\bar{t}} \right)^2 = \bar{b}^2 \sin \lambda - k_b - 2\kappa \frac{d\bar{a}}{d\bar{t}}. \quad (30)$$

Finally, the dimensionless form of the constraint is

$$\bar{a} \left(\frac{d\bar{a}}{d\bar{t}} \right)^2 + \bar{b} \left(\frac{d\bar{b}}{d\bar{t}} \right)^2 = \bar{a}^3 \frac{\cos \lambda}{3} + \bar{b}^3 \frac{\sin \lambda}{3} - \bar{a} k_a - \bar{b} k_b. \quad (31)$$

In what follows we will take the case $k_a = 0 = k_b$.

A. Nonperturbative solutions: Numerical analysis and late times

In order to extract qualitative physical information from the model, in the present section we will perform a study of the behavior of $\bar{a}(\bar{t})$ and $\bar{b}(\bar{t})$ for different regimes of the parameter κ and also for different values of λ .

For the nonperturbative case, it is much better to consider the set of Hamiltonian equations, (25) to (28), with $k_a = 0 = k_b$. Initial conditions for the system are $\bar{a}(0) = r_a$, $\bar{b}(0) = r_b$, and for numerical solutions we use $r_a = 1 = r_b$. This symmetric condition just encodes the fact that patches of the Universe are distinguished at initial time only due to the content of cosmological constant. Note also that these initial conditions translate to the functions $a(t)$, $b(t)$ as $a(0) = (\Lambda_a^2 + \Lambda_b^2)^{-1/4} = b(0)$ so that the case $\Lambda_a = 0 = \Lambda_b$ is not included in the rest of the discussion.

On the other hand, the initial conditions for the momenta π_a and π_b are restricted by the constraint (31) which can be written also as

$$\frac{\bar{\pi}_a^2}{\bar{a}} + \frac{\bar{\pi}_b^2}{\bar{b}} = \frac{\cos \lambda}{3} \bar{a}^3 + \frac{\sin \lambda}{3} \bar{b}^3. \quad (32)$$

For the numerical solutions we choose

$$\pi_a(0) = r_a^2 \sqrt{\frac{\cos \lambda}{3}}, \quad \pi_b(0) = r_b^2 \sqrt{\frac{\sin \lambda}{3}}. \quad (33)$$

This symmetric choice of initial conditions fulfills the constraint (31) at $t = 0$ and, since the Hamiltonian is preserved during the evolution of system, it is satisfied at any time.

In our numerical study we are interested in the behavior of four quantities in the patch described by a . Namely, the scale factor a , the velocity of the expansions \dot{a} , the Hubble parameter H_a , and the deceleration parameter q_a . The last two are defined as follows:

$$H_a = \frac{\dot{a}}{a} = \mu \frac{\dot{\bar{a}}}{\bar{a}} \equiv \mu \bar{H}_a, \quad q_a = -\frac{\ddot{a}}{\dot{a}^2} a = -\frac{\ddot{\bar{a}}}{\dot{\bar{a}}^2} \bar{a}, \quad (34)$$

where time derivatives of quantities with a bar are taken with respect to \bar{t} . Similar definitions hold for the scale factor \bar{b} . Note finally that q_a is independent of the scale μ .

It is interesting to explore the cases $\Lambda_a \ll \Lambda_b$ and $\Lambda_a \sim \Lambda_b$ separately. The case $\Lambda_a \gg \Lambda_b$ is contained in the first, due to the symmetry $a \rightarrow b$ and $\kappa \rightarrow -\kappa$. Then we will study the quantities of interest in such limits for different values of κ .

1. The case $\Lambda_a \ll \Lambda_b$

We first examine the case $\Lambda_a \ll \Lambda_b$, which we illustrate by $\Lambda_a = \mu^2 \sin \epsilon \approx \epsilon \mu^2$, $\Lambda_b = \mu^2 \cos \epsilon \approx (1 - \epsilon^2/2) \mu^2$, $\epsilon = 10^{-4}$. The behavior of the scale factors a and b for $\kappa = 0.1$, $\kappa = 1$, $\kappa = 2$ and $\kappa = 4$ is shown in Fig. 1.

We observe here that as κ increases, the scale factors \bar{a} and \bar{b} start an exponential growth. Indeed, while in panels 1(a) and 1(b) we see that $\bar{b}(\bar{t}) > \bar{a}(\bar{t})$, the situation is reversed in panels 1(c) and 1(d) for $\bar{t} \gtrsim 1$.

On the other hand, as κ increases further, the system exhibits a quasiperiodic behavior as can be checked in

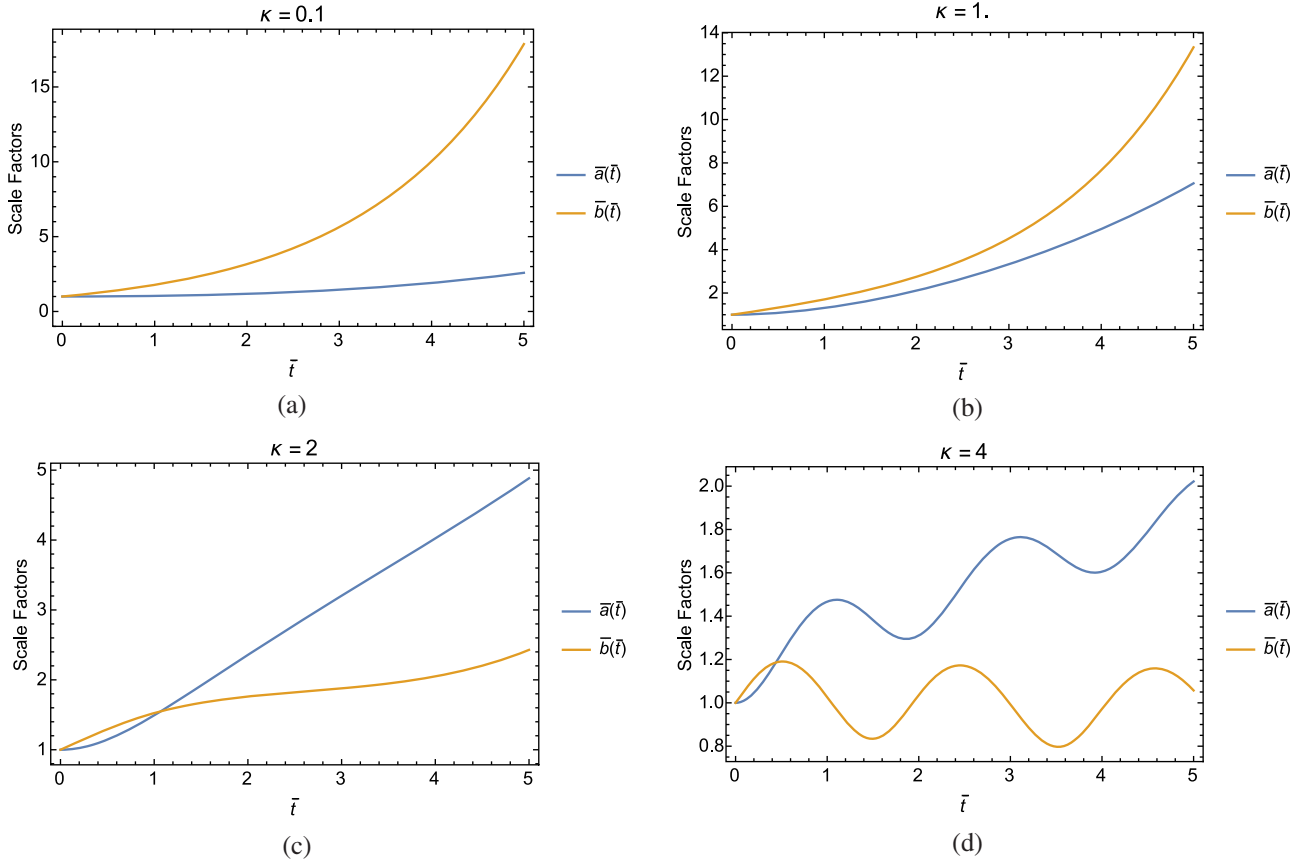


FIG. 1. Scale factors for different values of κ and for $\Lambda_a = \mu^2 \sin \epsilon$, $\Lambda_b = \mu^2 \cos \epsilon$, $\epsilon = 10^{-4}$.

panels 1(c) and 1(d) in the same figure. The oscillation pattern also shows how \bar{a} grows at the expenses of \bar{b} .

Figure 2 shows the behavior of the scale factors for negative κ and $\Lambda_a \leftrightarrow \Lambda_b$. We show only the cases $\kappa = -0.1$ and $\kappa = -4$. We verify here our statement that the case $\Lambda_b \ll \Lambda_a$ is already contained in the present discussion.

From here on we plot only the values of κ that exhibit the main features we would like to highlight.

The Hubble parameter H_a defined in (34) also has an interesting behavior. Figure 3 shows the evolution of H_a and H_b as a function of their respective scale factor. For the scale factor \bar{a} , we observe a different behavior of the Hubble parameter for $\kappa = 0.1$ in panel 3(a) compared with the case $\kappa = 2$ in panel 3(b). It is similar for the scale factor \bar{b} shown in panels 3(c) and 3(d) for same values of κ .

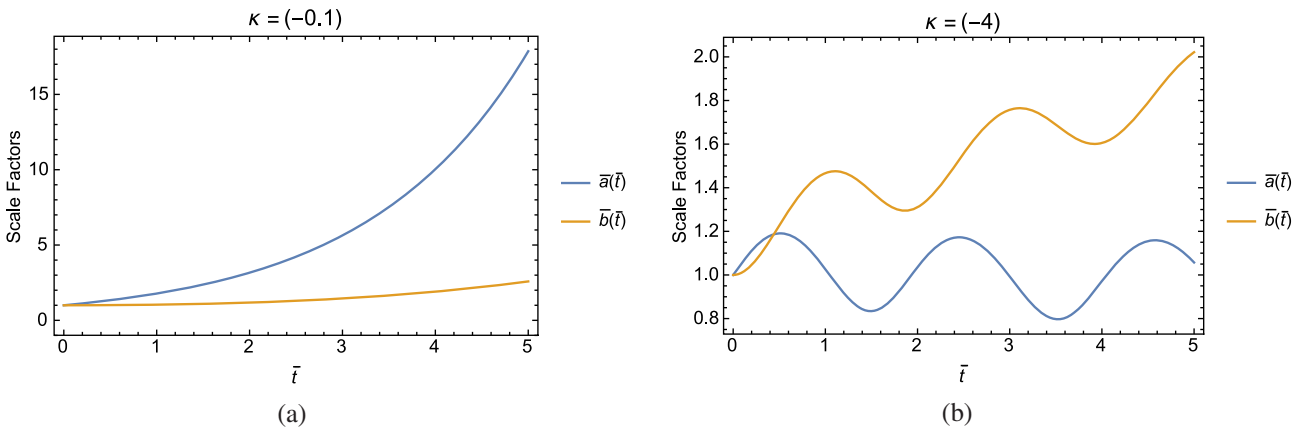


FIG. 2. Scale factors for negative values of κ and for $\Lambda_b = \mu^2 \sin \epsilon \approx \epsilon \mu^2$, $\Lambda_a = \mu^2 \cos \epsilon \approx (1 - \epsilon^2/2)\mu^2$, $\epsilon = 10^{-4}$. The plots are the same as in Figs. 1(a) and 1(d) after changing $\bar{a} \leftrightarrow \bar{b}$.

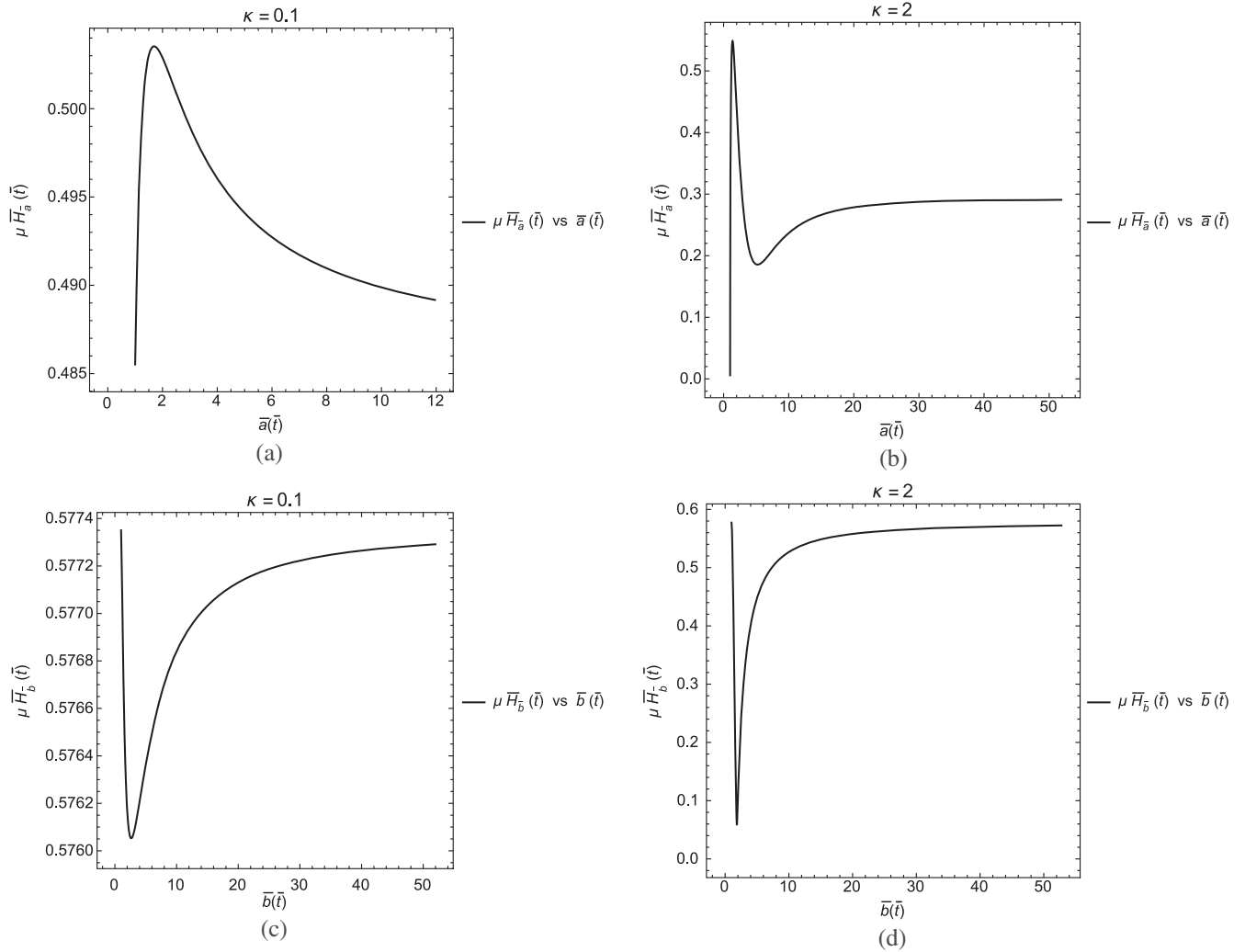


FIG. 3. Parametric plot of the Hubble parameter as a function of the scale factor for different values of κ and for $\Lambda_a \ll \Lambda_b$. On the horizontal axes we can read the value of the scale factor at different times.

It is interesting again to note the *complementary* behavior of \bar{a} and \bar{b} , in the sense that the increase in one of the Hubble parameters is accompanied by the decrease of the Hubble parameter of the other scale factor.

Indeed, for $\lambda = \pi/2$ and $\kappa = 0$, $\bar{H}_a = 0$ since the solution of the equations of motion in such a situation is $\bar{a}(\bar{t}) = \text{constant}$. The Hubble parameter for $\bar{b}(\bar{t})$ in such case is a nonzero constant ($\mu^2 \bar{H}_b = 3^{-1/2} = 0.57735$). Panel 3(a) shows how the Hubble parameter for \bar{a} going from 0 to ≈ 0.31 in $\Delta \bar{a} \approx 2$ while in panel 3(c) the Hubble parameter for \bar{b} diminishes from ≈ 0.57735 to ≈ 0.576005 in $\Delta \bar{b} \approx 2$.

This kind of complementarity is also observed in panels 3(c) and 3(d), where the increase-decrease process occurs for $\Delta \bar{a} \approx 10 \approx \Delta \bar{b}$.

Figure 4 shows velocities and deceleration parameters of \bar{a} and \bar{b} for two different values of κ . For $\kappa = 2.07$ we observe the imprints of the quasiperiodic behavior of the scale factor. In panel 4(b) we appreciate an increasing

deceleration parameter, which starts to decrease at $\bar{t} \sim 6$ and for $\bar{t} \gtrsim 12$ starts to stabilize to zero.

The increase of the velocity expansion of \bar{a} due to the interaction with \bar{b} can be appreciated as κ increases. While the velocity of the \bar{a} patch increases, the velocity of the \bar{b} patch decreases.

2. The case $\Lambda_a \sim \Lambda_b$

Let us consider the case $\Lambda_a \sim \Lambda_b$. For illustration, we have taken $\lambda = (\pi/4) - \epsilon$ with $\epsilon = 10^{-4}$. The scale factor behavior can be appreciated in Fig. 5. We note that the effect of κ is to increase the exponential growth of the scale factor \bar{a} . The cosmological constants satisfy $\Lambda_a \approx (1 + \epsilon)\Lambda_b$ and we expect the scale factor \bar{a} to increase faster than \bar{b} . This is what panel 5(a) shows. In the rest of the panels we see how the faster increase of \bar{a} becomes more pronounced as κ grows. The quasiperiodic behavior of \bar{a} and \bar{b} can be appreciated in panels 5(c) and 5(d).

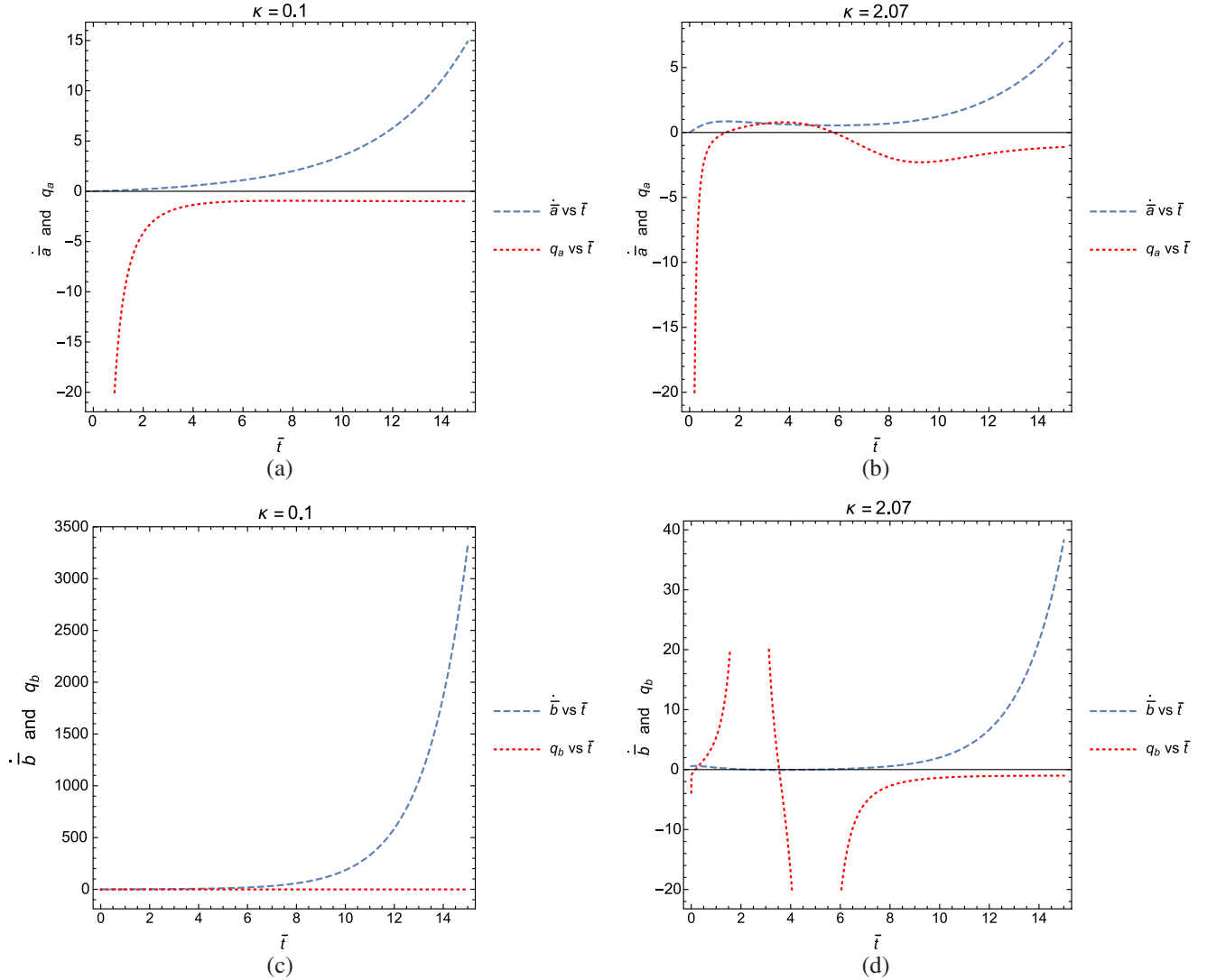


FIG. 4. Velocity expansion and deceleration parameters for the scale factors \bar{a} and \bar{b} as functions of \bar{t} in the regime $\Lambda_a \ll \Lambda_b$ for two different values of κ . The panels on the right show imprints of the quasiperiodic behavior of the scale factors.

The Hubble constant behavior can be appreciated in Fig. 6. Panel 6(a) shows how the expansion rate of \bar{a} increases, and panel 6(c) shows the decrease of the expansion rate of \bar{b} , both cases for the same value of $\kappa = 0.1$. This is in agreement with the behavior of \bar{a} and \bar{b} previously shown in panel 5(a). The situation changes as κ increases. The scale factor \bar{b} decreases from its initial value $\bar{b}(0) = 1$ and \bar{a} starts to grow [see panels 5(b) and 5(c) in Fig. 5, and note that in this figure we take $\kappa = 1.5$]. The expansion rate of \bar{a} , however, is greater than the expansion rate of \bar{b} , but in any case always decreases.

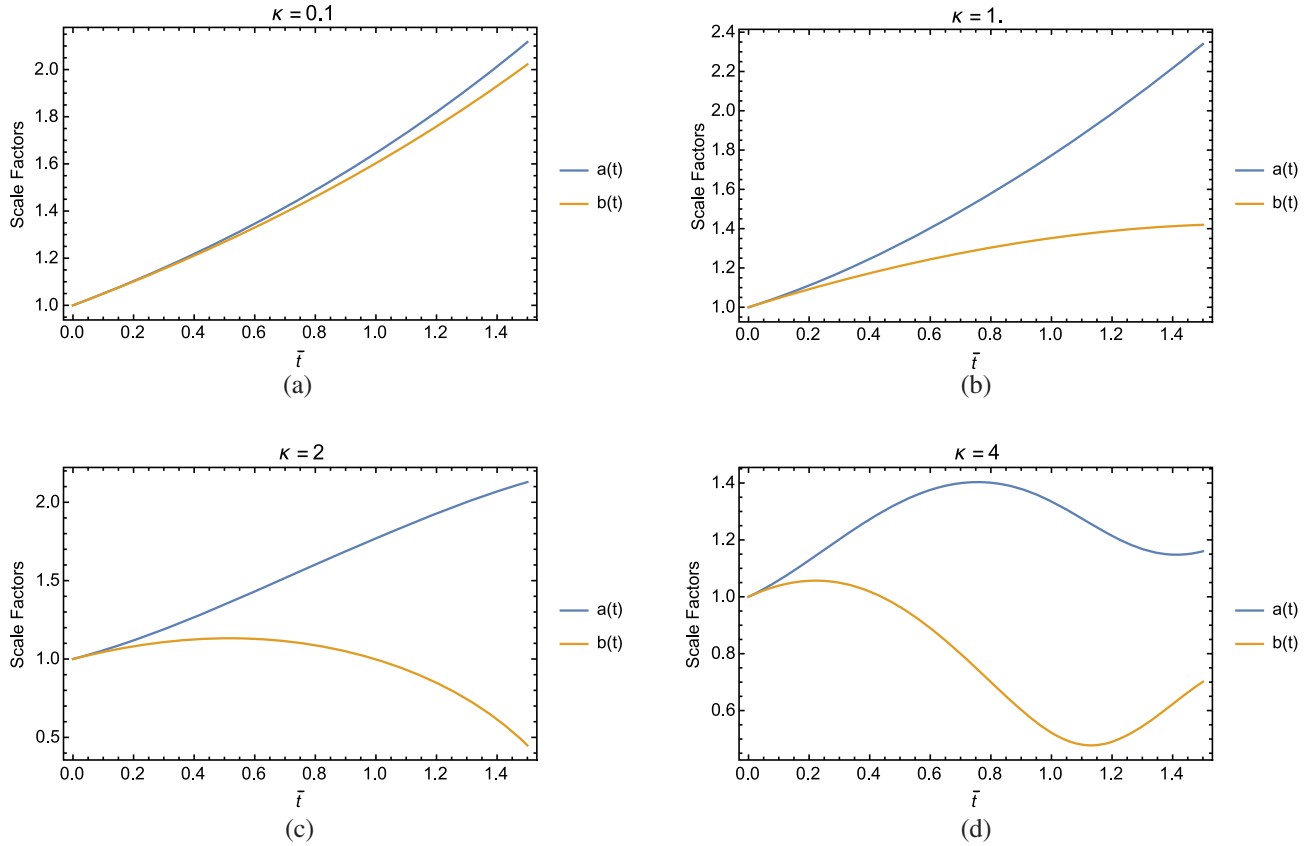
The velocity of the expansion and the deceleration parameter are shown in Fig. 7 for the \bar{a} patch [panels 7(a) and 7(b)] and for the \bar{b} patch [panels 7(c) and 7(d)].

For $\kappa = 0.1$, the expansion velocities $\dot{\bar{a}}$ and $\dot{\bar{b}}$, and the deceleration parameters q_a and q_b , show a similar behavior,

as expected for $\Lambda_a \approx \Lambda_b$ and initial conditions that are symmetric under the change $\bar{a} \leftrightarrow \bar{b}$.

For a higher value of κ ($\kappa = 3$), the quasiperiodic structure is present although the behavior of velocity and deceleration is not symmetric [panels 7(b) and 7(d)]. Indeed, in the range $1.0 \lesssim \bar{t} \lesssim 2.3$ the scale factor \bar{a} shows two zeroes while the scale factor \bar{b} shows three zeroes (the instants for which q_a and q_b go to infinity). The fact that κ appears with a different sign in the equations of motion for \bar{a} and for \bar{b} is responsible for this asymmetry. Figure 8 shows the plots for $\kappa = -3$, clearly illustrating the change $\bar{a} \leftrightarrow \bar{b}$.

As a conclusion we would like to stress that the interaction induced through (6) substantially modifies the evolution of the patches of the Universe. For a patch with a small cosmological constant compared with the


 FIG. 5. Scale factors \bar{a} and \bar{b} for different values of κ and for $\Lambda_a \approx \Lambda_b$.

cosmological constant of a second patch, interacting in the way described before, an accelerated expansion rate is observed even for small values of κ . Such expansion occurs at the expense of the expansion of the second patch, which is consistent with the fact that the interaction term can be thought of as an energy-momentum source for the first patch, as can be seen in (23).

B. Perturbative solutions for $\kappa \ll 1$

In this section we explore the solutions of the full system of equations to first order in a perturbative expansion in the dimensionless parameter κ . However, as for the case of the Landau problem which is formally related to the model proposed here in view of the modified Poisson bracket in (6), the limit $\kappa \ll 1$ might be subtle.

Indeed, for the Landau problem the magnetic length $\ell = 1/\sqrt{B}$ is ill defined for a magnetic field $B \rightarrow 0$. Even though this effect is quantum in nature, our model contains the dimensional parameter $\ell_g = \sqrt{G/\kappa}$, which does not admit the limit $\kappa \rightarrow 0$. We will show that this fact is linked to the inflationary behavior of the unperturbed solution and, therefore, the perturbation theory works well only for times close to the initial time. The length ℓ_g is a clear example where the nonperturbative behavior becomes important.

We look for solutions of the second order set of Eqs. (29) and (30) of the form

$$\begin{aligned}\bar{a}(\bar{t}) &= \bar{a}_0(\bar{t}) + \kappa \bar{a}_1(\bar{t}) + \mathcal{O}(\kappa^2), \\ \bar{b}(\bar{t}) &= \bar{b}_0(\bar{t}) + \kappa \bar{b}_1(\bar{t}) + \mathcal{O}(\kappa^2),\end{aligned}\quad (35)$$

with initial condition $\bar{a}(0) = r_a, \bar{b}(0) = r_b$.

The zeroth order solutions, $\bar{a}_0(\bar{t})$ and $\bar{b}_0(\bar{t})$, are superpositions of functions that contract and expand exponentially in time. We choose the expanding solutions

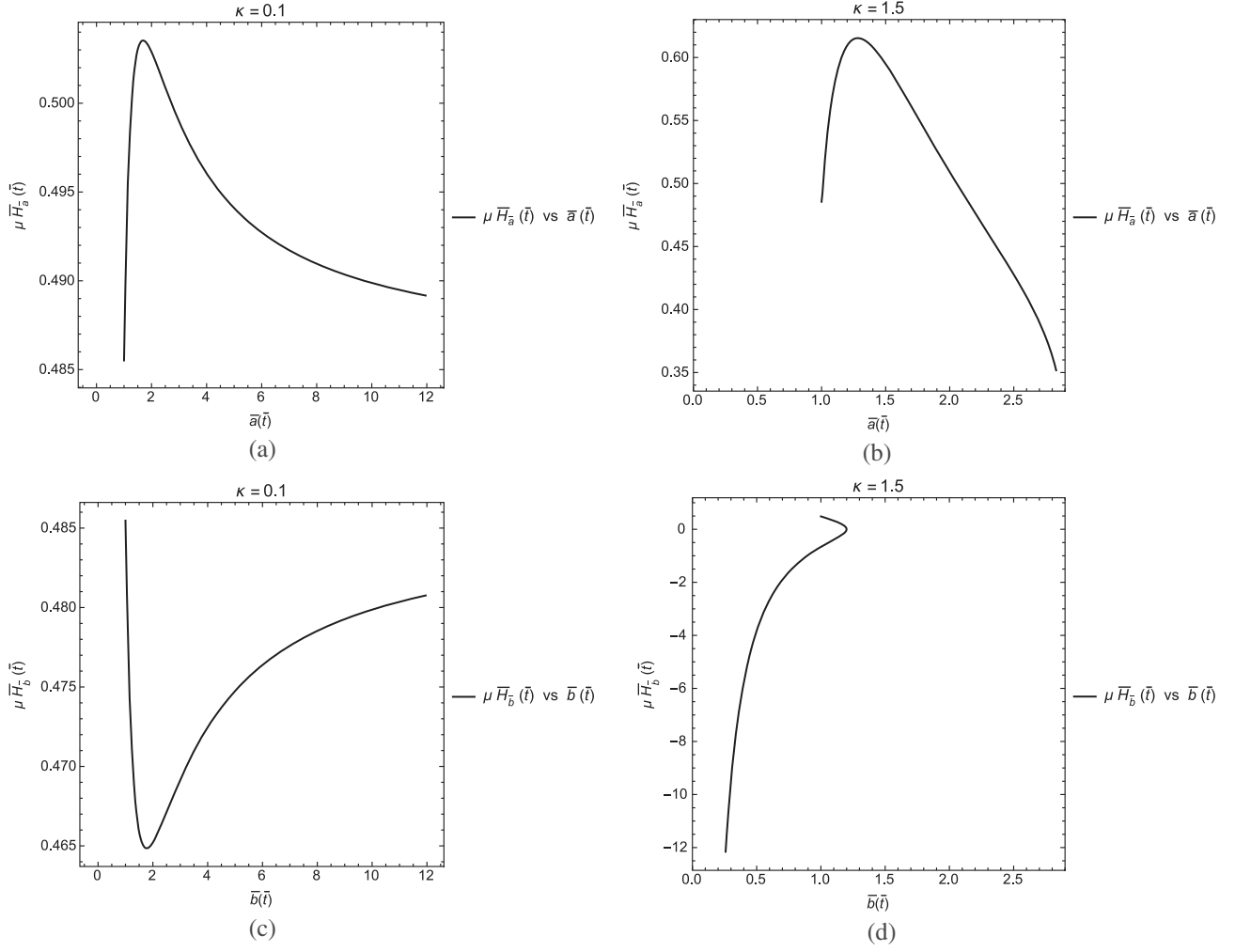
$$\bar{a}_0(\bar{t}) = r_a e^{\bar{t} \sqrt{\frac{\cos \lambda}{3}}}, \quad \bar{b}_0(\bar{t}) = r_b e^{\bar{t} \sqrt{\frac{\sin \lambda}{3}}}. \quad (36)$$

These solutions satisfy the constraint (31) (with $k_a = 0 = k_b$) and for the given initial conditions one also has

$$\dot{\bar{a}}_0(0) = r_a \sqrt{\frac{\cos \lambda}{3}}, \quad \dot{\bar{b}}_0(0) = r_b \sqrt{\frac{\sin \lambda}{3}}. \quad (37)$$

The equations at first order in κ read

$$\ddot{\bar{a}}_1 + \sqrt{\frac{\cos \lambda}{3}} \dot{\bar{a}}_1 - \frac{2}{3} \cos \lambda \bar{a}_1 = \sqrt{\frac{\sin \lambda}{3}} \frac{r_b}{r_a} e^{\bar{t}(\sqrt{\frac{\sin \lambda}{3}} - \sqrt{\frac{\cos \lambda}{3}})}, \quad (38)$$


 FIG. 6. Hubble parameters vs scale factors for different values of κ and for $\Lambda_a \approx \Lambda_b$.

$$\ddot{b}_1 + \sqrt{\frac{\sin \lambda}{3}} \dot{b}_1 - \frac{2}{3} \sin \lambda \bar{b}_1 = -\sqrt{\frac{\cos \lambda}{3}} \frac{r_a}{r_b} e^{-\bar{t}(\sqrt{\frac{\sin \lambda}{3}} - \sqrt{\frac{\cos \lambda}{3}})}, \quad (39)$$

$$\begin{aligned} r_a^2 \sqrt{\cos \lambda} e^{2\bar{t} \sqrt{\frac{\cos \lambda}{3}}} \left[\dot{a}_1 - \sqrt{\frac{\cos \lambda}{3}} \bar{a}_1 \right] \\ = -r_b^2 \sqrt{\sin \lambda} e^{2\bar{t} \sqrt{\frac{\sin \lambda}{3}}} \left[\dot{b}_1 - \sqrt{\frac{\sin \lambda}{3}} \bar{b}_1 \right]. \end{aligned} \quad (40)$$

The solutions of the set of Eqs. (38) and (39) depend on two arbitrary constants when the conditions $\dot{a}_1(0) = 0 = \dot{b}_1(0)$ are imposed. Initial conditions for the velocities are chosen in agreement with (33), which guarantees the constraint and then $\dot{a}(0) = 0 = \dot{b}(0)$. The integration constants are now fixed and the solutions to order κ are

$$\begin{aligned} \bar{a}(\bar{t}) &= e^{\bar{t} \sqrt{\frac{\cos \lambda}{3}}} r_a + e^{-2\bar{t} \sqrt{\frac{\cos \lambda}{3}}} \left(\frac{\kappa r_b}{r_a} \right) \\ &\times \sqrt{\frac{\sin \lambda}{3}} \left(\frac{\sec \lambda}{2 + \sqrt{\tan \lambda} - \tan \lambda} \right) [2 + e^{\bar{t} \sqrt{3 \cos \lambda}} \\ &- 3e^{\bar{t} \sqrt{\frac{\cos \lambda}{3}}} (1 + \sqrt{\tan \lambda}) + \sqrt{\tan \lambda} (e^{\bar{t} \sqrt{3 \cos \lambda}} - 1)], \end{aligned} \quad (41)$$

$$\begin{aligned} \bar{b}(\bar{t}) &= e^{\bar{t} \sqrt{\frac{\sin \lambda}{3}}} r_b - e^{-2\bar{t} \sqrt{\frac{\sin \lambda}{3}}} \left(\frac{\kappa r_a}{r_b} \right) \\ &\times \frac{1}{\sqrt{3 \sin \lambda}} \left(\frac{1}{2 \tan \lambda + \sqrt{\tan \lambda} - 1} \right) [e^{\bar{t} \sqrt{3 \cos \lambda}} - 1 \\ &+ (2 + e^{\bar{t} \sqrt{3 \sin \lambda}} - 3e^{\bar{t} \sqrt{\frac{\cos \lambda}{3}}} (1 + \sqrt{\tan \lambda})) \sqrt{\tan \lambda}]. \end{aligned} \quad (42)$$

These are the scale factors of the two patches when a kind of interaction is introduced by a modification of the Poisson brackets of momenta in the $\kappa \ll 1$ limit. These solutions satisfy the initial conditions $\bar{a}(0) = r_a, \bar{b}(0) = r_b, \dot{\bar{a}}(0) \neq 0, \dot{\bar{b}}(0) \neq 0$.

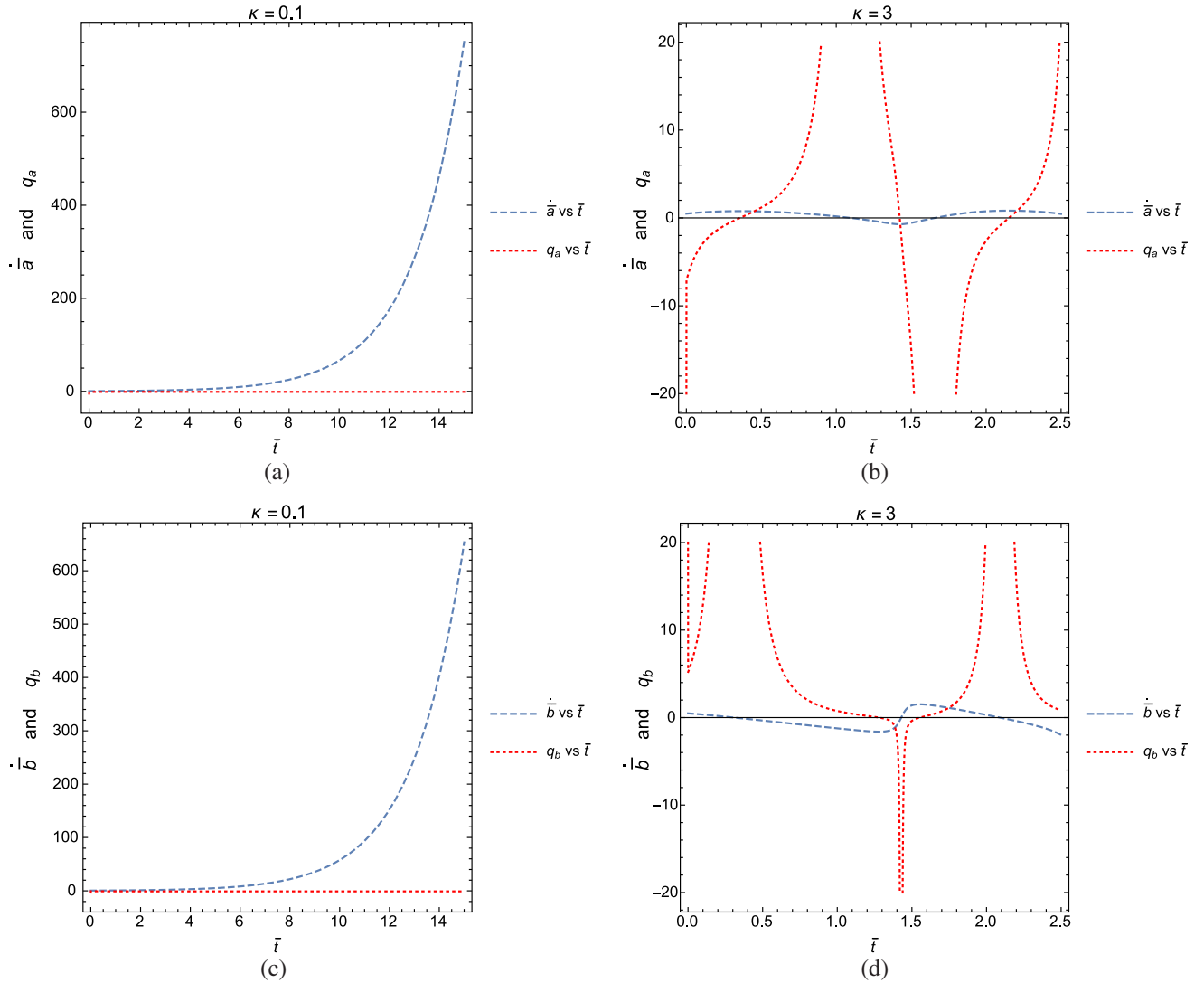


FIG. 7. Expansion velocities and deceleration parameters for the scale factors \bar{a} and \bar{b} as functions of \bar{t} in the regime $\Lambda_a \approx \Lambda_b$ for two different values of κ . The panels on the right show imprints of the quasicontinuous behavior of scale factors.

Let us explore the case $\Lambda_a \sim 0$ (or $\lambda \sim \pi/2$). In Fig. 9 we observe the scale factors for $\kappa = 10^{-2}$ and $\lambda = \pi/2 - 10^{-4}$.

It can be seen in (41) and (42) that perturbative terms contain an exponential dependence in time and therefore the perturbative approach is valid only for early times. Indeed, in Fig. 10 we can compare the scale factors with $\kappa = 0$, with the behavior of the perturbative term $\Delta\bar{a} \equiv \bar{a}(\bar{t}) - \bar{a}_0(\bar{t})$ (and the analogous definition for $\Delta\bar{b}$). Panel 10(a) shows the case of \bar{a} , while panel 10(b) shows the case of \bar{b} .

The perturbative approach in the present case (see Fig. 10) is valid for times $\bar{t} < 8$. But we observe that, already for $\bar{t} \approx 4$, the scale factor \bar{a} starts growing. In this sense, we say that the perturbation solution is valid at early times. The precise value of the time cutoff depends on κ , as well as λ .

The second case of interest is $\Lambda_a \sim \Lambda_b$. Scale factors are shown in Fig. 11 and one can observe that they are almost equal, as expected.

To check the validity of the perturbation expansion, we plot the scale factors with $\kappa = 0$ (unperturbed solutions) and compare them with the perturbed solutions. Figure 12 shows the results. We observe that for the present case, no early time restrictions are present and therefore the perturbation expansion can be safely applied for $\kappa \ll 1$.

In any case, we observe that the \bar{b} scale factor induces a cosmological constant on the Universe described by the \bar{a} scale factor, i.e., a sort of dark energy [17] coming from another sector (or patch) of the Universe.

The idea that causally disconnected regions influence the evolution of some part of the Universe by assuming a small interaction between the regions is an interesting proposal in itself. The solutions discussed in this paper show that the Universe would experience inflation faster than in the cosmological standard model (or slower, depending on the initial conditions). More importantly, even if the cosmological constant Λ_a of our patch of the Universe

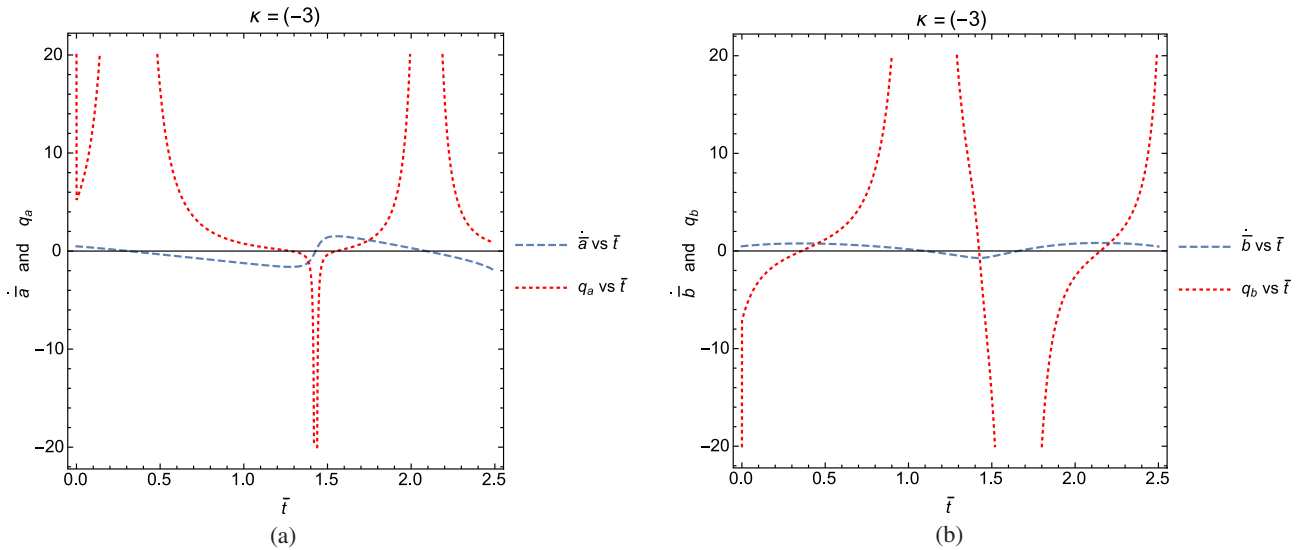


FIG. 8. Expansion velocities and deceleration parameters for the scale factors \bar{a} and \bar{b} as functions of \bar{t} in the regime $\Lambda_a \approx \Lambda_b$ for two different values of $\kappa < 0$. We observe that panel 7(b) in Fig. 7 goes over to panel 8(b) in the present figure and panel 7(d) goes over to panel 8(a). The asymmetry mentioned in the text is then explained as due to the sign of κ .

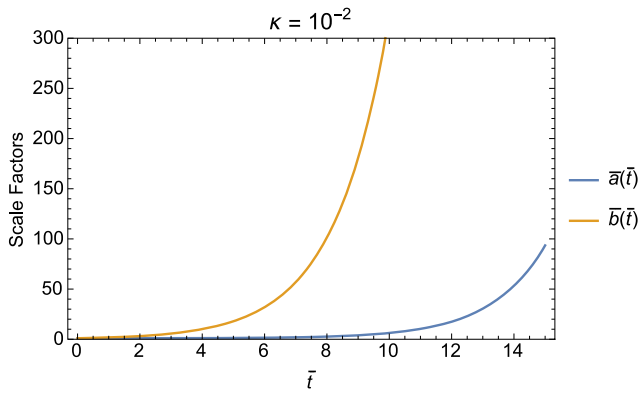


FIG. 9. Scale factors $\bar{a}(\bar{t})$ and $\bar{b}(\bar{t})$ for $\kappa = 10^{-2}$ and $\lambda = \pi/2 - 10^{-4}$. One can observe the exponential growth of \bar{a} .

were very small at the present epoch, our patch may actually accelerate [37,38] by the presence of a second patch with nonzero cosmological constant Λ_b under the Poisson-bracket interaction proposed here.

IV. DARK ENERGY

As discussed in Sec. II, the coupling between two causally disconnected regions induces interactions that are consistent with energy-momentum conservation. An observer in the a patch would measure an effective density $\rho^{(a)}$ and an effective pressure $p^{(a)}$ as given in Eq. (23).

Knowing the solutions for $a(t)$ and $b(t)$, this model can predict whether there is dark energy effect or not. We can numerically compute the pressure $p^{(a)}$, the density $\rho^{(a)}$, and evaluate from the equation of state the parameter w .

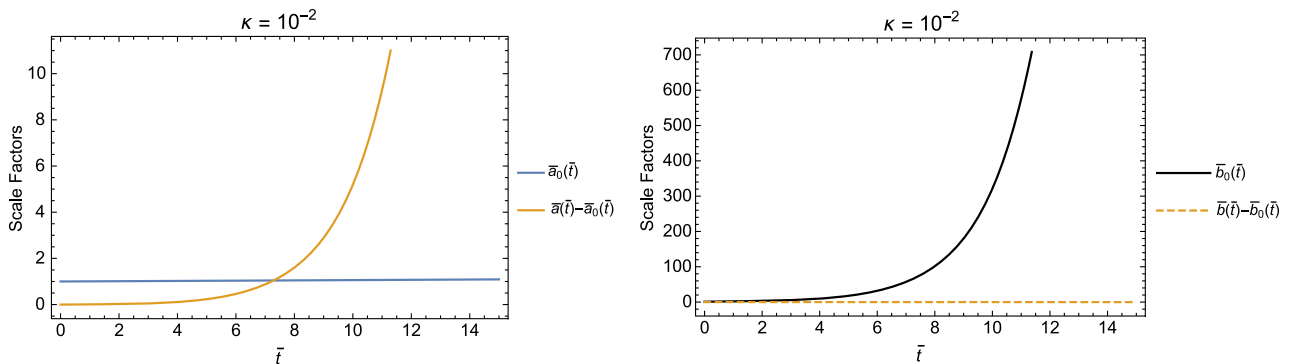


FIG. 10. The perturbative terms grow exponentially for the scale factor \bar{a} for $\kappa \ll 1$. The scale factor \bar{b} does not show the same behavior. The perturbative approach is valid, therefore, only at early stages in the evolution of the system.

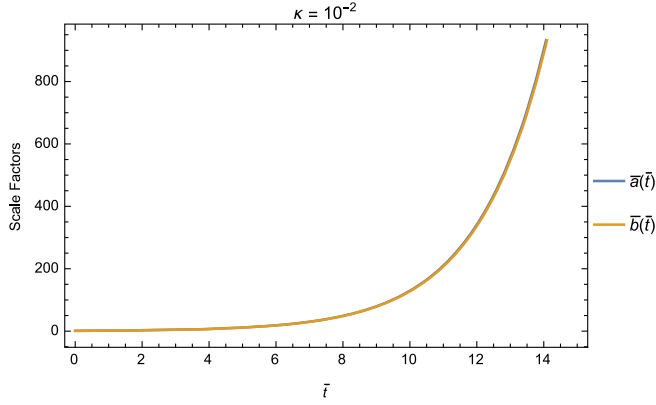


FIG. 11. Comparison of scale factors $\bar{a}(\bar{t})$ and $\bar{b}(\bar{t})$ for $\kappa = 10^{-2}$ and $\lambda = \pi/4 - 10^{-4}$.

In particular, it follows that

$$p^{(a)} + \rho^{(a)} = \frac{1}{8\pi G} \left[\Lambda_b \left(\frac{b}{a} \right)^3 - 3 \frac{b}{a^3} (\dot{b}^2 + k_b) - 2\kappa \frac{\dot{b}}{a^2} \right]. \quad (43)$$

For dark energy, one has empirically that $p + \rho \approx 0$. Therefore, in the context we have been discussing in the present work, if the solutions of the Einstein equations become quasiperiodic, they acquire a behavior compatible with a dark energy component only during some specific periods of the evolution.

In other words, the quasioscillatory nature of the cosmological solutions we have found eventually stops the accelerated expansion of the Universe and, from this point of view, the concept of dark energy as a particular component of the Universe could become unnecessary.

V. COCYCLES

The mathematics of Poisson bracket deformation is well known [24–29] and as was previously mentioned, the deformed Poisson bracket (6) is reminiscent of the

magnetic translations group in the quantum Hall effect. A magnetic translation is an operator $\hat{T}[\theta]$ defined by [39]

$$\hat{T}[\theta] = e^{i\pi_a \theta^a}, \quad (44)$$

where π_a are the momenta defined in Sec. I and θ^a are real parameters. In terms of this notation, the internal composition law of the magnetic group is very unconventional because it satisfies

$$\hat{T}[\theta]\hat{T}[\tau] = e^{\frac{i\kappa}{2}\epsilon_{ab}\theta^a\tau^b} \hat{T}(\theta + \tau). \quad (45)$$

The phase in (45) is a 3-cocycle implying that the generators $\hat{T}[\theta]$ are a ray representation of the group of magnetic translations [40]. A good example of this are the Gauss's anomalies terms of cocycles found by Faddeev in 1984 [41,42].

From a conceptual point of view the deformation of the Poisson bracket includes not only a crucially important change (namely, the appearance of quasiperiodicity) but also makes explicit the presence of 3-cocycles which were not obvious *a priori*.

VI. SUMMARY AND DISCUSSION

In this paper we have studied an extension of the FLRW model with two metrics assuming that two patches of the Universe (causally disconnected in principle) interact through a modification of the Poisson bracket of the momenta of the two metrics, one on each patch. This modification is inspired by an analogy with the quantum Hall effect. Following this analogy, the deformation parameter of the Poisson bracket might be interpreted as a minimum distance between two neighboring regions that rotate with respect to a plane of the target two-metric space or, in other words, this assumption is equivalent to the analog of the lowest Landau level in cosmology.

However, the assumption explained above heuristically is not exempt from mathematical subtleties; the first, as explained above, assumes that the deformation (6) corresponds to a change of algebraic structure known as a group

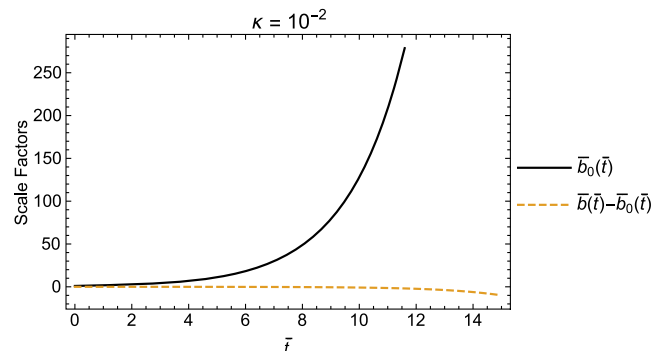
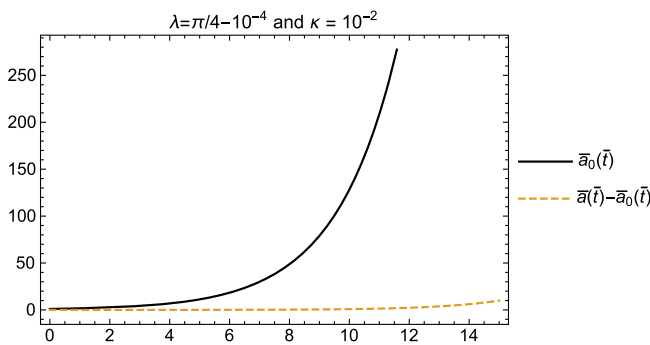


FIG. 12. For $\Lambda_a \sim \Lambda_b$, perturbations are always smaller than the unperturbed solution. Here we show $\lambda = \pi/4 - 10^{-4}$ and $\kappa = 10^{-2}$.

of magnetic translations which contain an internal composition rule containing a 3-cocycle. This 3-cocycle is responsible for the oscillatory behavior of the equations of FLRW we have found in this paper.

Finally, we would like to insist on the fact that the quasiperiodic structure of the solutions of the extended FLRW equations contain both acceleration and deceleration epochs, and therefore if these quasiperiodic solutions are used for interpreting dark energy, it would mean that the observations of the current universe [37,38] are only a snapshot of the Universe evolution at a particular (accelerating) time.

However, there is a highly nontrivial phenomenon in the results presented here. In the theory discussed in this paper there are inflationary solutions with a negligible cosmological constant in our patch of the Universe ($\Lambda_a \ll \Lambda_b$). This fact is a consequence of the causality breaking produced by the deformation of the momenta of the metric.

ACKNOWLEDGMENTS

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APPENDIX: DIMENSIONS

We consider the Hilbert-Einstein action with cosmological constant

$$S = \frac{1}{16\pi G} \int \sqrt{-g}(R + \Lambda)d^4x. \quad (\text{A1})$$

In natural units, the canonical dimensions of the different elements composing the action are $[G] = -2$, $[g] = 0$, $[R] = +2$, $[d^4x] = -4$, $[\Lambda] = 2$, so that the action has canonical dimension zero.

For the FLRW metric, the scale factor a and the lapse function N can be chosen as dimensionless. Since the scale factor depends on time only, the action (A1) reduces to

$$S = \frac{V}{16\pi G} \int \sqrt{-g(t)}(R(t) + \Lambda)dt \quad (\text{A2})$$

where V is a space volume. The quantity V/G has canonical dimension -1 . We define a quantity $v = V/(16\pi G)$ and write the action as

$$S = v \int \mathcal{L}(a, \dot{a})dt. \quad (\text{A3})$$

The canonical dimension of $v\mathcal{L}$ is $+1$. In order to have a close analogy with the case of the Landau problem (which is defined for particles rather than fields) we consider the Lagrangian $L \equiv v\mathcal{L}$ and the action

$$S = \int Ldt.$$

Since a is dimensionless and L has canonical dimension $+1$, the canonical momentum $p_a = \partial L/\partial \dot{a}$ is also dimensionless. The Poisson bracket between coordinates and momenta—inherited from the gravity theory as a field theory—is also dimensionless as it should be

$$\{a, p_a\} = 1.$$

For the case of particles, the coordinates and canonical momenta have inverse dimensions to each other, and while the Poisson bracket between coordinates and momenta is dimensionless, the modified Poisson bracket between momenta has canonical dimension $+2$.

In our case one can make a similar choice. Indeed, let us define a new variable $\tilde{a} = a\sqrt{G}$ with canonical dimension -1 (as the spatial coordinates in a particle theory), then the canonical momentum will have dimension $+1$.

Since \mathcal{L} is a homogeneous function of a of degree 3

$$L = v\mathcal{L}(a, \dot{a}) = \frac{v}{G^{3/2}}\mathcal{L}(\tilde{a}, \dot{\tilde{a}}) = \frac{V}{16\pi G^{3/2}}\frac{1}{G}\mathcal{L}(\tilde{a}, \dot{\tilde{a}}).$$

Finally, the quantity $V/(16\pi G^{3/2})$ is a dimensionless constant, and therefore we can take as the Lagrangian for our model

$$L \equiv \frac{1}{G}\mathcal{L}(\tilde{a}, \dot{\tilde{a}}). \quad (\text{A4})$$

The action is thus redefined up to a dimensionless parameter. The Lagrangian so redefined has canonical dimension $+1$, \tilde{a} has dimension -1 , and \tilde{p}_a has dimension $+1$. The deformed Poisson bracket then has dimension $+2$, as desired.

In the text we use a instead of \tilde{a} for simplicity.

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