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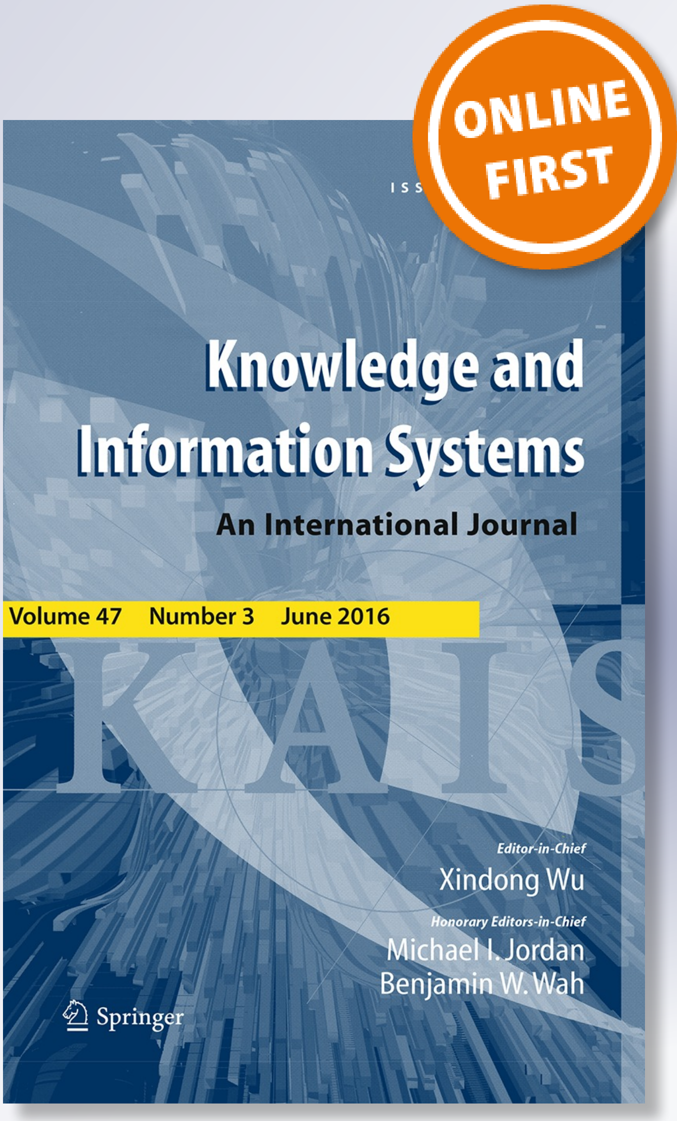

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REGULAR PAPER

Sharing beliefs among agents with different degrees of credibility

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Abstract This paper introduces an approach for sharing beliefs in collaborative multi-agent application domains where some agents can be more credible than others. In this context, we propose a formalization where every agent has its own partial order among its peers representing the credibility the agent assigns to its informants; each agent will also have a belief base where each sentence is attached with an agent identifier which represents the credibility of that sentence. We define four different forwarding criteria for computing the credibility information for a belief to be forwarded, and for determining how the receiver should handle the incoming information; the proposal considers both the sender's and the receiver's points of view with respect to the credibility of the source of the information.

Keywords Forwarding · Trust · Multi-agent system · Credibility orders

1 Introduction

The focus of this research is on collaborative multi-agent application domains where deliberative agents communicate with each other sharing their beliefs. Each agent can play the role of source of information (informant) to other agents in the system and receives information from multiple sources. In various proposals, all the participants are considered to have the same credibility; nevertheless, in this paper, we center on a more general approach

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where some agents can be more credible than others. Hence, we present a formalization where every agent imposes a partial order on the set of agents, reflecting its own perception of how credible each agent is. Each agent also has a consistent belief base where each sentence is labeled with an agent's identifier; that label represents the credibility of that sentence.

The main contribution of this work is the formalization and characterization of an approach for sharing beliefs that encompasses both ends of the process, i.e., considers the sender's and the receiver's points of view with respect to the credibility of the source of the information. From the sender's standpoint, we introduce four different forwarding criteria that a sender agent can use for computing the associated credibility information for the belief to be forwarded; from the receiver's perspective, we present an approach that is applied when an agent receives a forwarded object which was computed using any of the four proposed sender's criteria. Upon receipt of a forwarded belief, the receiver agent uses its own credibility order to determine which information prevails. As part of our contribution, we have included theoretical results and an empirical analysis that will help to decide which forwarding criteria can be used in the interaction.

The importance of defining a trust model has been emphasized in the literature, and its usefulness has been widely recognized. As stated in [1], the introduction of the multi-agent system paradigm and the evolution of e-commerce have contributed to substantially increase the interest on representing trust, and the work on trust has produced many applications in information and communication technologies [2–4]. Clearly, some form of trust model is needed in any problem where the adoption of a critical decision depends on the credibility (informational trust) assigned to the information received from other agents; thus, we will favor the use of the word credibility to refer to this characteristic of informant agents as this particular word carries an intuitive sense that helps to understand the related problems. Agent's credibility will be compared establishing a strict partial order; this type of order will provide the capability of representing cases where the credibility of two informants cannot be compared because that relation has not been established.

Similarly, to [5–9], we will adopt a symbolic approach for representing a belief's credibility. As we discussed above, every agent holds a strict partial order of informants, and if an agent believes that informant A_1 is less credible than informant A_2, then when this agent receives conflictive information from them, the information received from A_2 will be preferred over the information received from A_1; that is, the trust recognized to A_2's information is higher than the trust assigned to A_1's. In [8], a preliminary approach that considers forwarding information was proposed. However, contrasting with the present proposal, that work presupposes a total order over informants; furthermore, unlike here, in [8] the receiver's point of view was not taken into account. Later, in the section dedicated to the related work, a more detailed comparison will be provided.

The choice of what credibility information is to be sent with a forwarded belief is significant because it will affect the decision of the receiver regarding whether to accept a received belief. As we will discuss in the related work section, a simple alternative found in the literature is to send the sender's identifier making that sender accountable for the forwarded belief; thus, the receiver can consider that the belief's credibility corresponds to the last agent that retransmits the information. However, taking a different stance, we will introduce other alternatives that consider not only the sender but all the sources of the related beliefs used for entailing a belief to be forwarded. Furthermore, these forwarding criteria will also consider both the sender's and receiver's partial orders over the informant agents.

As outlined in Fig. 1, each agent will have its own belief base and its strict partial order over the set of agents, called credibility order. A sender agent builds a message containing the

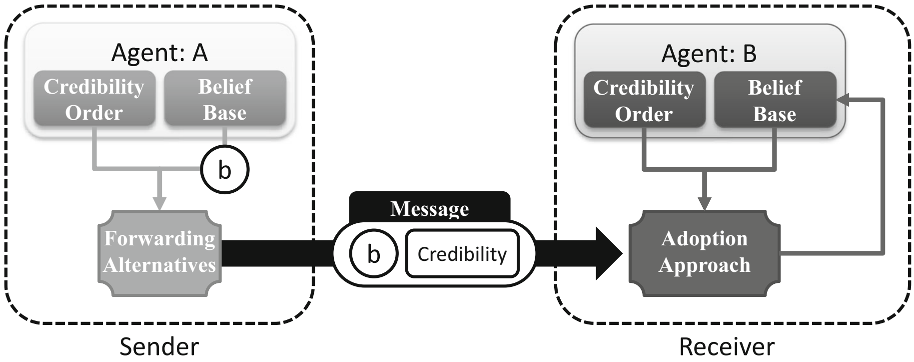


Fig. 1 Belief forwarding outline

belief about to be forwarded using its own credibility order and its belief base; the receiver agent will use its own credibility order and belief base to process the message and decide whether the incoming belief is accepted.

Our proposal is based on the following principle: Since we consider collaborative multi-agent application domains, then the credibility of forwarded information should not diminish. Hence, reflecting this rationale, one property that we pursue is that in any individual retransmission, if the receiver accepts (i.e., does not reject) the new information, then from its point of view, the credibility of the accepted information should not be degraded. We will present formal results showing that the proposed framework follows this principle.

The paper is organized as follows. The formalization of a belief base and a credibility order will be introduced in Sect. 2. In Sect. 3, we will propose four alternatives that can be used for computing the credibility information to be forwarded. Then, in Sect. 4, we will introduce an approach that can be applied by an agent that receives a forwarded object, and we also include theoretical results and an empirical analysis of our proposal. The related work will be analyzed in Sect. 5, and finally, we will present our conclusions in Sect. 6. We have included an appendix which contains all the proofs for the corresponding propositions.

2 Preliminaries

The assumption of the existence of a total order over informants is not quite realistic in many multi-agent application domains, and a similar observation applies to the existence of a global order shared by all agents. With this observation in mind, the approach to be introduced below will consider that every agent has its own partial order defined among its informants representing the credibility that an agent assigns to its informants. Each agent has a belief base where each sentence is attached with an agent identifier corresponding to the credibility of that sentence; moreover, agents can communicate with their peers to obtain new information or for sharing their beliefs. The credibility order among informants can be used to decide which information prevails. The details of the formalism are introduced next.

We assume a finite set $\mathbb{A}$ of identifiers for naming informant agents that is shared by all agents. Agents' identifiers will be denoted with uppercase italic letters that can have natural numbers as subscripts (i.e., $\mathbb{A} = \{A, B, \ldots, Z, A_1, \ldots, Z_n\}$); each identifier will represent a unique agent.

Each agent will have its own *credibility order* defined over the set of agents that will be represented by an irreflexive, asymmetric, and transitive binary relation over $\mathbb{A}$ denoted $<^X_{co}$, i.e., $<_{co}^{X}$ is a strict partial order over $\mathbb{A}$, where the superscript $X \in \mathbb{A}$ stands for the agent identifier this order belongs to; for instance, $<_{co}^{A}$ is the credibility order the agent identified as A. In this paper, we will assume that this credibility order is for a single topic; multi-topic or multi-context credibility orders are left as future work. The notation $B <_{co}^{A} C$ means that for agent A, agent B is less credible than C (or equivalently C is more credible than B). The notation $B \not<_{co}^{A} C$ is used to express that for agent A, agent B is not less credible than C. If $B \not<_{co}^{A} C$ and $C \not<_{co}^{A} B$, then for agent A, agents B and C are incomparable, and this is denoted as $B \sim_{co}^{A} C$. This order's dynamics is out of the scope of this paper; however, the formalism proposed in [10] could be used as a complement of our proposal which introduces a way of handling the dynamics of the credibility order.

In this approach, we consider a propositional logic language $\mathcal{L}$ and a classical consequence operator Cn over that language. Besides its credibility order, an agent A has a consistent *belief base* $\mathcal{K}_A$ that stores a finite set of pairs (s, X), called *information objects* [8,9].

Definition 1 An *information object* is a tuple (s, X) where s is a sentence of $\mathcal{L}$, and $X \in \mathbb{A}$ represents the credibility that an agent assigns to the sentence s.

Given a belief base $\mathcal{K}_A$, $Exp(\mathcal{K}_A) = \{s \mid (s, X) \in \mathcal{K}_A\}$ is the set of sentences contained in the information objects of $\mathcal{K}_A$, whereas $Ag(\mathcal{K}_A) = \{X \mid (s, X) \in \mathcal{K}_A\}$ is the set of agent identifiers contained in $\mathcal{K}_A$. Note that $Exp(\mathcal{K}_A)$ is a finite set that corresponds to those beliefs that are explicitly represented in $\mathcal{K}_A$. The set of beliefs of A, denoted as $Bel(\mathcal{K}_A)$, will include all the sentences that can be inferred from $Exp(\mathcal{K}_A)$, i.e., $Bel(\mathcal{K}_A) = Cn(Exp(\mathcal{K}_A))$.

Example 1 Consider the set of agent identifiers $\{A_1, A_2, A_3, A_4, A_5\} \subseteq \mathbb{A}$ and that the agent identified with A_1 has the following belief base:

$$\mathcal{K}_{A_1} = \left\{ \begin{array}{ll} (\theta, A_1), & (\beta \to \alpha, A_3), \\ (\delta, A_2), & (\beta \to \theta, A_3), \\ (\gamma, A_3), & (\gamma \to \alpha, A_5), \\ (\gamma, A_5), & (\beta \to \gamma, A_4), \\ (\beta, A_5), & (\varphi, A_3), \\ (\beta, A_4), & (\varphi, A_4) \end{array} \right\}$$

That is, $\mathcal{K}_{A_1}$ represents that sentences $\gamma, \beta \to \alpha, \beta \to \theta$, and φ are as credible for A_1 as A_3 is; sentences $\beta, \beta \to \gamma$ and φ are as credible for A_1 as A_4 is; sentences γ, β and $\gamma \to \alpha$ are as credible for A_1 as A_5 is; the sentence δ is as credible for A_1 as A_2 is; and that the sentence θ is as credible as A_1 is for itself. Consider that the agent A_1 has the following credibility order among agents in $\mathbb{A}$: $A_1 <_{co}^{A_1} A_3$, $A_3 <_{co}^{A_1} A_5$, $A_3 <_{co}^{A_1} A_4$, $A_1 <_{co}^{A_1} A_4$, $A_1 <_{co}^{A_1} A_5$, and since the credibility order is a strict partial order, we also have that $A_4 \sim_{co}^{A_1} A_5$ and $A_1 \sim_{co}^{A_1} A_2$ hold. Figure 2 depicts the credibility order $<_{co}^{A_1}$ as a directed graph where nodes represent agents identifiers, and an arc from node n_1 to node n_2 represents that $n_1 <_{co}^{A_1} n_2$.

In Example 1, the set of sentences of the belief base of agent A_1 is $Exp(\mathcal{K}_{A_1}) = \{\theta, \delta, \gamma, \beta, \beta \to \alpha, \gamma \to \alpha, \beta \to \gamma, \varphi, \beta \to \theta\}$. A belief base could have several information objects with the same sentence but different agent identifier, e.g., $\{(\beta, A_4), (\beta, A_5)\} \subseteq \mathcal{K}_{A_1}$.

Observe that a belief can be obtained directly from an information object that explicitly contains it (e.g., β from (β, A_4)), but also a belief can be obtained as a result of an inference based on several information objects. For instance, $\beta \in Exp(\mathcal{K}_{A_1})$ and hence $\beta \in Bel(\mathcal{K}_{A_1})$, whereas $\alpha \notin Exp(\mathcal{K}_{A_1})$ but $\alpha \in Bel(\mathcal{K}_{A_1})$. Given a sentence s, there might exist several proofs for s, e.g., from $\mathcal{K}_{A_1}$, the sentence α can be inferred from $\{(\beta, A_5), (\beta \to \alpha, A_3)\}$, and also

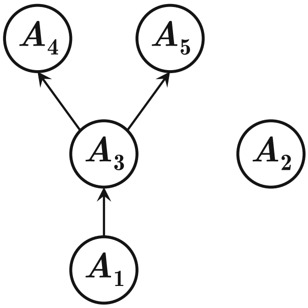


Fig. 2 Credibility partial order $<_{co}^{A_1}$

from $\{(\gamma, A_3), (\gamma \rightarrow \alpha, A_5)\}$. Since our aim was to forward a sentence s, it is necessary to consider all the possible forms of obtaining it accounting for the fact that different deductions may lead to attach different credibilities to s. Furthermore, the deduction sequences should only contain the beliefs relevant to obtain s, i.e., we must restrict ourselves to consider the minimal subsets of $\mathcal{K}$ entailing s. We will now introduce the notion of proof sets in our setting.

Definition 2 Let $\mathcal{K}_A$ be a belief base for an agent A and $s \in \mathcal{L}$. Then, $\mathcal{H}$ is a *minimal proof* of s from $\mathcal{K}_A$ if and only if (1) $\mathcal{H} \subseteq \mathcal{K}_A$; (2) $s \in Bel(\mathcal{H})$; and (3) if $\mathcal{H}' \subset \mathcal{H}$, then $s \notin Bel(\mathcal{H}')$. Given a sentence s, the function $\Pi(s, \mathcal{K}_A)$ returns the set of all the minimal proofs for s from $\mathcal{K}_A$.

Example 2 Consider Example 1. Then, $\Pi(\alpha, \mathcal{K}_{A_1}) = \{\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, \mathcal{H}_4, \mathcal{H}_5, \mathcal{H}_6\}$ represent six different proofs for α from $\mathcal{K}_{A_1}$ where:

$$
\begin{aligned}
\mathcal{H}_1 &= \{(\beta, A_5), (\beta \rightarrow \alpha, A_3)\} \\
\mathcal{H}_2 &= \{(\beta, A_4), (\beta \rightarrow \alpha, A_3)\} \\
\mathcal{H}_3 &= \{(\gamma, A_3), (\gamma \rightarrow \alpha, A_5)\} \\
\mathcal{H}_4 &= \{(\gamma, A_5), (\gamma \rightarrow \alpha, A_5)\} \\
\mathcal{H}_5 &= \{(\beta, A_5), (\beta \rightarrow \gamma, A_4), (\gamma \rightarrow \alpha, A_5)\} \\
\mathcal{H}_6 &= \{(\beta, A_4), (\beta \rightarrow \gamma, A_4), (\gamma \rightarrow \alpha, A_5)\}
\end{aligned}
$$

Note that $\Pi(\beta, \mathcal{K}_{A_1}) = \{\mathcal{G}_1, \mathcal{G}_2\}$ contains two different proofs for β from $\mathcal{K}_{A_1}$ where $\mathcal{G}_1 = \{(\beta, A_5)\}$ and $\mathcal{G}_2 = \{(\beta, A_4)\}$.

Remark 1 Given a belief base $\mathcal{K}_A$, $s \in Bel(\mathcal{K}_A)$ if and only if $\Pi(s, \mathcal{K}_A) \neq \emptyset$.

Observe that given a belief s of an agent A, i.e., $s \in Bel(\mathcal{K}_A)$, there are several possibilities: It may happen that for some $B \in \mathbb{A}$, there exists an information object (s, B) in $\mathcal{K}_A$; it might be the case that s is included in multiple information objects with different informants; or it may happen that there is no information object that contains the sentence s. In particular, if $s \in Bel(\mathcal{K}_A)$ but $s \notin Exp(\mathcal{K}_A)$, then there will be no information objects with the sentence s in $\mathcal{K}_A$, but there will be one or more proofs for s from $\mathcal{K}_A$. Finally, note that if $s \in Exp(\mathcal{K}_A)$, then there can be both explicit information objects with s and also proofs for s that use more than one object. The following example shows all these alternatives.

Example 3 Consider the base $\mathcal{K}_{A_1}$ of Example 1, in that we have:

(1) $\delta \in Exp(\mathcal{K}_{A_1})$, and there exists only one object in $\mathcal{K}_{A_1}$: (δ, A_2);
(2) $\varphi \in Exp(\mathcal{K}_{A_1})$, there are two objects in $\mathcal{K}_{A_1}$: (φ, A_3) and (φ, A_4), and the agents A_3 and A_4 are comparable;
(3) $\beta \in Exp(\mathcal{K}_{A_1})$, there exist two objects in $\mathcal{K}_{A_1}$: (β, A_5) and (β, A_4), but the agents A_5 and A_4 are incomparable;
(4) $\gamma \in Exp(\mathcal{K}_{A_1})$, there exist two objects (γ, A_3) and (γ, A_5), and also two other proofs for γ; and
(5) as shown in Example 2 although $\alpha \notin Exp(\mathcal{K}_{A_1})$, there are five proofs for α.

Our communication model is loosely based on [11]; thus, it follows the directives of speech act theory [12]. In particular, in our approach, a message from an agent S (sender) to an agent R (receiver) will be denoted (S, R, F) where $S, R \in \mathbb{A}$. The third element, F, called *forwarded object* will be a pair $[b, \mathbb{C}]$ that contains a sentence $b \in Bel(\mathcal{K}_S)$ and also other information $\mathbb{C}$ that corresponds to the credibility that S assigns to b. The structure of $\mathbb{C}$ will be explained and discussed in detail in next section (see Definition 3), but first we will introduce a motivating example that will be used throughout the rest of the paper.

Example 4 Let us consider a scenario where an agent called Vincent (represented as V) wants to travel to a mountain resort from a hotel in a nearby town where he is currently staying. Since it is winter, the road connecting that town to the resort could be closed because of snow and V has to decide whether to go to the resort. Vincent has the following beliefs: "if the road is not open, he will not go to the resort" $(\neg o \rightarrow \neg g, V)$, "if the road is open, he will go to the resort" $(o \rightarrow g, V)$, and that "it will snow today" (s, V). Vincent has also received information from other agents: The employee of the gas station (named G) told him that "the road is not open" $(\neg o, G)$, and the Tourist information office (T) informed him that "if there is snow the road will not be open." Figure 3 (right) describes the belief base of agent V. Also, agent Sam (S), who is a close friend of V, has been staying in the same hotel as V. As depicted in Fig. 3 (left), Sam believes that: "the road is open." He has also received information from other agents: The hotel receptionist (H) told him that "the road is open," the Tourist information office (T) told him that "if there is no snow, the road will be open," the weather report (W) informed him that "there is no snow", and the hotel receptionist (H) additionally told him that "there is no snow." Finally, Sam wants to send a message to Vincent with his belief about the status of the road. The scenario is fully described in Fig. 3,

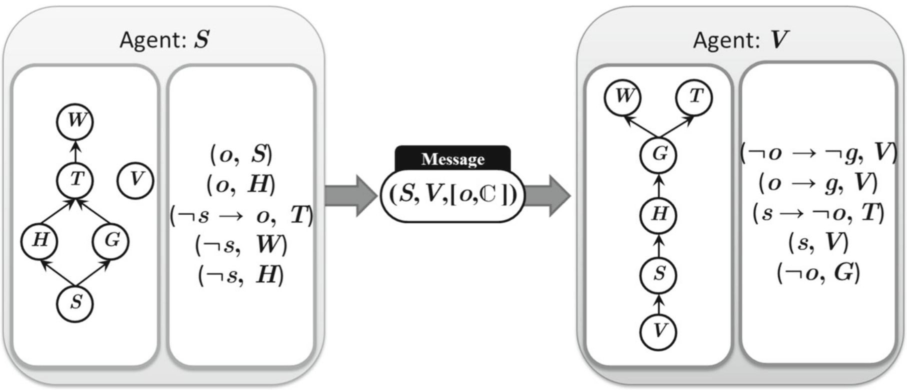


Fig. 3 Scenario of Example 4 where agent S sends a message to V

showing the belief bases corresponding to agents V and S, the two particular credibility orders each one establishes among their informants regarding the topic "Road availability for the Mountain Resort," and the message that S intends to send to V.

In the following section, we will advance on different criteria that an agent S can use for computing the credibility information $\mathbb{C}$ for a belief b that will be sent to R in a message $(S, R, [b, \mathbb{C}])$. For instance, consider the base $\mathcal{K}_{A_1}$ in Example 1 and suppose that agent A_1 wants to forward its belief δ to A_3. One alternative is to send the message $(A_1, A_3, [\delta, A_2])$, considering just the identifier A_2 included in the information object $(\delta, A_2) \in \mathcal{K}_{A_1}$; however, these simple criteria are only applicable to a sentence $b \in Exp(\mathcal{K}_{A_1})$. In particular, as it was shown in Example 2, there are six proofs for α from $\mathcal{K}_{A_1}$, but $\alpha \notin Exp(\mathcal{K}_{A_1})$. A different alternative is to send the message $(A_1, A_3, [\alpha, A_1])$, just considering the identifier A_1 of the sender; nevertheless, by doing that, a lot of information related to the beliefs that entail α is lost. This discussion suggests that a more elaborated criterion for selecting the more appropriate identifiers is needed.

3 Criteria for computing forwarded objects

In this section, four criteria for computing forwarded objects will be introduced. As described above, agents will communicate with each other exchanging their beliefs. In this framework, an agent can only communicate to its peers a sentence that it believes, and this sentence will be sent together with information that represents the credibility that the sender associates with the belief it is sharing. The structure of a forwarded object is presented next.

Definition 3 (*Forwarded object*) Given an agent $A \in \mathbb{A}$, a forwarded object from A is a pair $[s, \mathbb{C}]$ where $s \in Bel(\mathcal{K}_A)$ and $\mathbb{C}$, referred to as the credibility profile of s, is a set of sets of agent identifiers, i.e., $\mathbb{C} \subseteq 2^{\mathbb{A}}$.

The credibility profile plays an important role as a uniform structure for all the criteria we will present. We will introduce now two auxiliary functions, both will take a set of agent identifiers, and in the context of a given credibility order, the first will determine the subset of the least credible agent identifiers, and the second the subset of the most credible agent identifiers.

Definition 4 ($min_{<^A_{co}}$) Given the credibility order $<^A_{co}$ and a set of agent identifiers $\mathbb{I} \subseteq \mathbb{A}$, $min_{<^A_{co}}(\mathbb{I}) = \{X : X \in \mathbb{I}$ and for all $Y \in \mathbb{I}, Y \not<^A_{co} X\}$.

Definition 5 ($max_{<^A_{co}}$) Given the credibility order $<^A_{co}$ and a set of agent identifiers $\mathbb{I} \subseteq \mathbb{A}$, $max_{<^A_{co}}(\mathbb{I}) = \{X : X \in \mathbb{I}$ and for all $Y \in \mathbb{I}, X \not<^A_{co} Y\}$.

Observe that $min_{<^A_{co}}(\mathbb{I})$ (respectively, $max_{<^A_{co}}(\mathbb{I})$) will always return a subset of $\mathbb{I}$. Also, note that if $\mathbb{I}$ is an empty set, then both (min and max) will return an empty set. The returned set will be a singleton only if the least (respectively, most) credible identifier is comparable with all the other elements of $\mathbb{I}$. For instance, in Example 1, we have that $A_1 <^{A_1}_{co} A_3$, $A_3 <^{A_1}_{co} A_5$, $A_3 <^{A_1}_{co} A_4$, $A_1 <^{A_1}_{co} A_4$, $A_1 <^{A_1}_{co} A_5$; therefore, $min_{<^A_{co}}(\{A_1, A_3, A_4, A_5\}) = \{A_1\}$, $max_{<^A_{co}}(\{A_1, A_3, A_4, A_5\}) = \{A_4, A_5\}$, $min_{<^A_{co}}(\{A_4, A_5\}) = \{A_4, A_5\}$, and $max_{<^A_{co}}(\{A_1, A_3, A_5\}) = \{A_5\}$. Note that if we consider every agent in Fig. 2, then $min_{<^A_{co}}(\{A_1, A_2, A_3, A_4, A_5\}) = \{A_1, A_2\}$ and $max_{<^A_{co}}(\{A_1, A_2, A_3, A_4, A_5\}) = \{A_2, A_4, A_5\}$.

In the following subsections, two pairs of criteria for computing the credibility profile $\mathbb{C}$ in a forwarded object $[s, \mathbb{C}]$ will be proposed. The criteria will be defined in such a way that for any sentence that corresponds to an agent's belief, they will return a credibility profile; in some cases, this credibility profile can be empty. The first pair of criteria will use only the credibility information that is explicitly stored in the sender belief base. In contrast, the second pair will consider all the different proofs that can be obtained from the sender belief base for a particular belief. An analysis and comparison of the proposed criteria will follow these developments.

3.1 Forwarding criteria based on explicit information

The following two criteria will use only the credibility information that is explicitly stored in the sender belief base to compute an object to be forwarded. They mainly differ in that the first one uses the sender credibility order to compute the more credible informants, whereas the second criterion leaves this computation to be performed by the receiver with its own credibility order.

As explained in the previous section, a belief of an agent A can be explicitly included in one, in multiple information objects in $\mathcal{K}_A$, or in none. For instance, as shown in Example 3, the belief δ is explicitly stored in only one information object, the beliefs φ and β are explicitly stored in two information objects, whereas the belief α is not explicitly included in a information object.

In the first criterion, called *Explicit Sender Criterion* (or *ESC* for short), the sender uses its own credibility order to compute the set of agent identifiers that will be attached to the belief to be forwarded.

Explicit Sender Criterion (*ESC*): Let $\mathcal{K}_A$ be the belief base and $<^A_{co}$ the credibility order of $A \in \mathbb{A}$. Let $s \in Bel(\mathcal{K}_A)$ and $\mathbb{I} = \{X : (s, X) \in \mathcal{K}_A\}$. Then, the forwarded object for s is $[s, \mathbb{C}]$ where the credibility profile is $\mathbb{C} = \{\{Y\} : Y \in max_{<^A_{co}}(\mathbb{I})\}$.

Example 5 shows the application of *ESC* to some beliefs of Example 1. The second proposed criterion is called *Explicit Receiver Criterion* (or *ERC* for short), and it is introduced next. Using *ERC* the sender will gather all the information it has and the receiver can use its own credibility order to compute the credibility it will associate to the received belief.

Explicit Receiver Criterion (*ERC*): Let $\mathcal{K}_A$ be the belief base of $A \in \mathbb{A}$. Let $s \in Bel(\mathcal{K}_A)$ and $\mathbb{I} = \{X : (s, X) \in \mathcal{K}_A\}$. Then, the forwarded object for s is $[s, \mathbb{C}]$ where the credibility profile is $\mathbb{C} = \{\{Y\} : Y \in \mathbb{I}\}$.

Remark 2 Let $A \in \mathbb{A}$ and $s \in Bel(\mathcal{K}_A)$ be a belief to be forwarded. If $s \notin Exp(\mathcal{K}_A)$ and a forwarded object $[s, \mathbb{C}]$ is computed with *ESC* or *ERC*, then $\mathbb{C} = \emptyset$.

This observation shows that when the sender forwards a purely derived belief, i.e., a belief that is not explicitly contained in the sender's knowledge base (such as α in Example 3), whenever *ESC* or *ERC* are used, the second part of the forwarded object will be the empty set. These cases will be treated in a special way by the receiver and will be explained in detail in Sect. 4.

Example 5 Consider the belief base $\mathcal{K}_{A_1}$ of Example 1 and recall Example 3. The following table shows some forwarded objects that can be obtained with the explicit sender criterion (left) and the explicit receiver criterion (right). Observe that for the same belief base and the same sentence, the result of applying each criterion is different.

ESC	*ERC*
$[\delta, \{\{A_2\}\}]$	$[\delta, \{\{A_2\}\}]$
$[\varphi, \{\{A_4\}\}]$	$[\varphi, \{\{A_3\}, \{A_4\}\}]$
$[\beta, \{\{A_4\}, \{A_5\}\}]$	$[\beta, \{\{A_4\}, \{A_5\}\}]$
$[\gamma, \{A_5\}\}]$	$[\gamma, \{\{A_3\}, \{A_5\}\}]$
$[\alpha, \emptyset]$	$[\alpha, \emptyset]$

The main advantage of these two criteria is that they perform a simple computation. In the case of *ESC*, this computation is done at sender's side using the sender credibility order. Observe that when it is possible, *ESC* computes a singleton set (see left column at Example 5). However, if incomparable identifiers are found, the computed set may have more than one element (e.g., $[\beta, \{\{A_4\}, \{A_5\}\}]$). In the case that a set of more than one element is received, the receiver could also apply its own credibility order. In those scenarios where it is convenient to receive all the information that the sender has, and then to perform the computation using the receiver credibility order, the *ERC* can be used. In the following section, an approach to how the receiver can handle the incoming information is proposed. One disadvantage of *ESC* and *ERC* is that, in some cases, not all the information stored in the sender's belief base is used. In the following subsection, two criteria that consider all the information will be proposed. But first, we will show some properties of the two criteria introduced above.

We will show along the paper that for different circumstances, different criteria can be preferred. As a particular case, it is clear that when the sender has no way of comparing its peers, the decision of which of these two criteria to use is not relevant; the following proposition shows exactly that.

Proposition 1 *Let $A \in \mathbb{A}$, $<^A_{co}$ be the credibility order of agent A and $s \in Bel(\mathcal{K}_A)$. Let $[s, \mathbb{C}]$ be the forwarded object computed by A using ESC and let $[s, \mathbb{C}']$ be the forwarded object computed by A using ERC. If $<^A_{co} = \emptyset$, then $\mathbb{C} = \mathbb{C}'$.*

Proof See Appendix. □

This proposition remarks that if according to the sender agent A all agents are incomparable ($<^A_{co} = \emptyset$), then, for any belief s, the criteria *ESC* and *ERC* will build the same forwarded object.

A total order gives more information and hence gives the sender agent more accuracy for obtaining the credibility profile that includes the identifier that represents the credibility of the sentence to be sent. The following proposition characterizes why.

Proposition 2 *Let $A \in \mathbb{A}$, $s \in Exp(\mathcal{K}_A)$, and let $[s, \mathbb{C}]$ be a forwarded object computed by A using ESC. If $<^A_{co}$ is a total order, then $\mathbb{C}$ contains just one singleton.*

Proof See Appendix. □

That is to say, when the credibility order of the sender agent is a total order, then all the forwarded objects containing explicit sentences which are computed using *ESC* will only have one element which is a singleton.

3.2 Forwarding criteria based on all possible proofs

To compute an object to be forwarded, the two criteria that will be introduced here consider all the possible minimal proofs that can be obtained from the sender's belief base for a particular belief. The difference between them is that the first criterion will use the sender's credibility order for computing the information to be sent, whereas the second criterion leaves that computation to be performed in the receiver's side with its own credibility order.

To define the first criterion, we will adapt the concept of *plausibility function* proposed in [10] to suit the needs of our framework. This function will be used to obtain a set of agent identifiers which will represent the credibility of a given belief, and it will consider all the agent identifiers involved in every minimal proof of that belief. Since we take a cautious approach, the function first will obtain the set with the least credible agent identifiers from each proof, and then, if there exist more than one proof, the most credible identifiers of the resulting set. Therefore, to compute the plausibility of a sentence, we will use the function $min_{<^A_{co}}$ (Definition 4) and the function $max_{<^A_{co}}$ (Definition 5). Based on a belief base $\mathcal{K}_A$, we will define a function $pl_{<^A_{co}}(s, \mathcal{K}_A)$ such that given a sentence $s \in Bel(\mathcal{K}_A)$ returns a set of agent identifiers that represents the credibility of s with respect to $\mathcal{K}_A$ and $<^A_{co}$. Recall that $Ag(\mathcal{X})$ returns the set of agent identifiers that are contained in the information objects of a set $\mathcal{X}$ and that $\Pi(s, \mathcal{K}_A)$ is the set of all minimal proofs of s obtained from $\mathcal{K}_A$.

Definition 6 (*Plausibility of a belief*) Let $<^A_{co}$ be the credibility order and $\mathcal{K}_A$ be the belief base of an agent $A \in \mathbb{A}$. Let $s \in Bel(\mathcal{K}_A)$, and let $\mathrm{P} = \Pi(s, \mathcal{K}_A)$, then:

$$pl_{<^A_{co}}(s, \mathcal{K}_A) = max_{<^A_{co}}\left(\bigcup_{\mathcal{X} \in \mathrm{P}} min_{<^A_{co}}(Ag(\mathcal{X}))\right)$$

The function pl requires that $s \in Bel(\mathcal{K}_A)$, and therefore $\mathrm{P} \neq \emptyset$ (see Remark 1). Also observe that the function $max_{<^A_{co}}$ can return more than one agent identifier, hence pl could return a set of pairwise incomparable agents. For instance, in the belief base $\mathcal{K}_{A_1}$ introduced in Example 1, as it was shown in Example 2, there are six proofs for the sentence α, then:

$$pl_{<^{A_1}_{co}}(\alpha, \mathcal{K}_{A_1}) = max_{<^{A_1}_{co}}\{A_3, A_4, A_5\} = \{A_4, A_5\}$$

In the following criterion, called *Plausibility Sender Criterion* (or *PSC* for short), the sender will use the function pl for computing the set that will be attached to the belief to be forwarded.

Plausibility Sender Criterion (*PSC*): Let $\mathcal{K}_A$ be the belief base of $A \in \mathbb{A}$ and $<^A_{co}$ its associated credibility order. Let $s \in Bel(\mathcal{K}_A)$, the forwarded object for s is $[s, \mathbb{C}]$ where $\mathbb{C} = \{\{Y\} : Y \in pl_{<^A_{co}}(s, \mathcal{K}_A)\}$.

For example, in the context of Example 1, using *PSC* the following forwarded objects can be created: $[\alpha, \{\{A_4\}, \{A_5\}\}]$, $[\varphi, \{\{A_4\}\}]$, and $[\gamma, \{\{A_4\}, \{A_5\}\}]$. Now, in the following criterion, called *Plausibility Receiver Criterion* (or *PRC* for short), the sender will gather all the information it has and send it to the receiver, which will use its own credibility order to compute the credibility to be associated with the received belief.

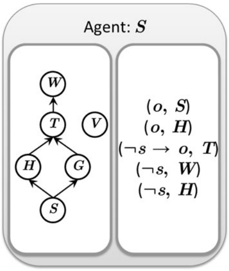
Plausibility Receiver Criterion (*PRC*): Let $\mathcal{K}_A$ be the belief base of $A \in \mathbb{A}$. Let $s \in Bel(\mathcal{K}_A)$, the forwarded object for s will be $[s, \mathbb{C}]$, where $\mathbb{C}$ is the credibility profile $\mathbb{C} = \{\mathbb{Y} : \mathbb{Y} = Ag(\mathcal{X})$ where $\mathcal{X} \in \Pi(s, \mathcal{K}_A)\}$.

For instance, in Example 2, there are six proofs of α, using *PRC* the forwarded object that is sent will be: $[\alpha, \{\{A_5, A_3\}, \{A_4, A_3\}, \{A_5\}, \{A_4, A_5\}\}]$.

Example 6 Consider the belief base $\mathcal{K}_{A_1}$ of Example 1, and recall Example 3. The following table shows the differences among the last two proposed criteria.

PSC	*PRC*
$[\delta, \{\{A_2\}\}]$	$[\delta, \{\{A_2\}\}]$
$[\varphi, \{\{A_4\}\}]$	$[\varphi, \{\{A_3\}, \{A_4\}\}]$
$[\beta, \{\{A_4\}, \{A_5\}\}]$	$[\beta, \{\{A_4\}, \{A_5\}\}]$
$[\gamma, \{\{A_5\}, \{A_4\}\}]$	$[\gamma, \{\{A_3\}, \{A_5\}, \{A_4\}, \{A_4, A_5\}\}]$
$[\alpha, \{\{A_5\}, \{A_4\}\}]$	$[\alpha, \{\{A_5, A_3\}, \{A_4, A_3\}, \{A_5\}, \{A_4, A_5\}\}]$

Example 7



Consider the scenario presented in Example 4, where agent S wants to send the message $(S, V, [o, \mathbb{C}])$ to V. Recalling the belief base and the credibility order of S (depicted again above), we will show how this agent can compute the credibility profile $\mathbb{C}$ for this message using the four proposed criteria. The ESC criterion computes the credibility profile as $\mathbb{C} = \{max_{<^S_{co}}(\{S, H\})\} = \{\{H\}\}$, while for the ERC is $\mathbb{C} = \{\{S\}, \{H\}\}$. The PSC and the PRC criteria consider the proofs for 'o' from $\mathcal{K}_S$; i.e., $\Pi(o, \mathcal{K}_S) = \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4\}$, where $\mathcal{O}_1 = \{(o, H)\}$, $\mathcal{O}_2 = \{(o, S)\}$, $\mathcal{O}_3 = \{(\neg s \rightarrow o, T), (\neg s, W)\}$, and $\mathcal{O}_4 = \{(\neg s \rightarrow o, T), (\neg s, H)\}$. In particular, the PSC criterion uses these proofs to compute the credibility profile as $\mathbb{C} = \{max_{<^S_{co}}(min_{<^S_{co}}(\{H\}) \cup min_{<^S_{co}}(\{S\}) \cup min_{<^S_{co}}(\{T, W\}) \cup min_{<^S_{co}}(\{T, H\}))\} = \{max_{<^S_{co}}(\{H\} \cup \{S\} \cup \{T\} \cup \{H\})\} = \{\{T\}\}$. Finally, the credibility profile applying PRC is $\mathbb{C} = \{\{H\}, \{S\}, \{T, W\}, \{T, H\}\}$.

As we discussed, in contrast to *ESC* and *ERC*, the last two criteria *PSC* and *PRC* consider all the possible proofs that a belief has from the sender belief base. Thus, the main advantage of these two criteria is that they perform a more informed computation. In the case of *PSC*, this computation is done at sender's side using the sender credibility order. At the end of Sect. 4.3, we will include formal and empirical results to show which are the best criteria that can be used in different circumstances. For instance, we will show that a sentence is more likely to be accepted by the receiver if the sender uses *PRC* rather than *ERC* or *ESC*. Observe that, when it is possible, *PSC* computes a singleton (see Example 6); however, if incomparable identifiers are found, the computed set will have more than one element. The receiver could also apply its own credibility order when a set of more than one element is received. In those scenarios where it is convenient for the receiver to perform the computation itself using its own credibility order, the *PRC* can be used. The obvious disadvantage of the two last criteria is that they need all the possible proofs to compute the credibility profile, requiring more computation than *ERC* and *ESC*.

Finally, in the case of *PRC* and *ERC*, the sender participates providing epistemic input; meanwhile, the receiver applies its credibility order to that input, whereas in the case of *PSC* and *ESC*, the sender provides epistemic input that is qualified by its credibility order.

As in Proposition 1, when the sender has no way of comparing its peers, then the choice of using *PSC* or *PRC* is not relevant; this is stated in the following proposition.

Proposition 3 *Let $A \in \mathbb{A}$, and let $<^A_{co}$ be the credibility order of agent A. Let $s \in Bel(\mathcal{K}_A)$ and let $[s, \mathbb{C}]$ be the forwarded object computed by A using PSC and $[s, \mathbb{C}']$ be the forwarded object computed by A using PRC. If $<^A_{co} = \emptyset$ and $s \notin (Bel(\mathcal{K}_A) \backslash Exp(\mathcal{K}_A))$, then $\mathbb{C} = \mathbb{C}'$.*

Proof See Appendix. □

Proposition 3 states that if according to the sender agent A all agents are incomparable ($<^A_{co} = \emptyset$) and the belief to be forwarded is not a derived belief, then the *PSC* and *PRC* criteria will compute the same forwarded object.

Remark 3 Let $A \in \mathbb{A}$ and $[s, \mathbb{C}]$ be a forwarded object computed by A using one of the criteria *PSC* or *PRC*. If $s \in Bel(\mathcal{K}_A)$, then $\mathbb{C} \neq \emptyset$ and $\mathbb{C}$ is unique.

Example 8 Consider the belief base $\mathcal{K}_{A_1}$ of Example 1, and suppose that A_1 has a credibility order of its peers such that $<^{A_1}_{co} = \emptyset$. The following tables show the differences among the four proposed criteria, when the sender has such a credibility order.

ESC = *ERC*	*PSC*	*PRC*
$[\delta, \{\{A_2\}\}]$	$[\delta, \{\{A_2\}\}]$	$[\delta, \{\{A_2\}\}]$
$[\varphi, \{\{A_3\}, \{A_4\}\}]$	$[\varphi, \{\{A_3\}, \{A_4\}\}]$	$[\varphi, \{\{A_3\}, \{A_4\}\}]$
$[\beta, \{\{A_4\}, \{A_5\}\}]$	$[\beta, \{\{A_4\}, \{A_5\}\}]$	$[\beta, \{\{A_4\}, \{A_5\}\}]$
$[\gamma, \{\{A_3\}, \{A_5\}\}]$	$[\gamma, \{\{A_3\}, \{A_4\}, \{A_5\}\}]$	$[\gamma, \{\{A_3\}, \{A_5\}, \{A_4\}, \{A_4, A_5\}\}]$
$[\alpha, \emptyset]$	$[\alpha, \{\{A_3\}, \{A_4\}, \{A_5\}\}]$	$[\alpha, \{\{A_5, A_3\}, \{A_4, A_3\}, \{A_5\}, \{A_4, A_5\}\}]$

Similar to the result of Proposition 2, since a total order gives more information, it also gives the sender agent more accuracy for obtaining the credibility profile.

Proposition 4 *Let $A \in \mathbb{A}$ and $[s, \mathbb{C}]$ be a forwarded object computed by A using PSC. If $<^A_{co}$ is a total order, then $\mathbb{C}$ contains just one singleton.*

Proof See Appendix. □

4 Processing and adopting incoming forwarded objects

In this section, we propose an approach that can be applied by an agent that receives a forwarded object from an agent that uses one of the forwarding criteria described above. First, we will propose how to process the received credibility information. Then, we will show how the credibility information can be used to decide if the incoming belief is accepted or rejected. Finally, if the decision is to accept the received belief, we will propose how to adopt it maintaining the consistency of the belief base.

4.1 Analyzing the incoming credibility information

Consider that an agent A sends to agent R the forwarded object $[s, \mathbb{C}]$. As stated in the last section, regardless of which of the four criteria is used, the sentence s will belong to $Bel(\mathcal{K}_A)$

and the credibility profile $\mathbb{C}$ (that can be empty) will have a uniform structure: a set of sets of agent identifiers. As shown in Example 6, the credibility profile $\mathbb{C}$ can be a singleton (e.g., $[\delta, \{\{A_2\}\}]$), it can have more than one singleton ($[\beta, \{\{A_4\}, \{A_5\}\}]$), or it can have several sets of identifiers ($[\alpha, \{\{A_5, A_3\}, \{A_4, A_3\}, \{A_5\}, \{A_4, A_5\}\}]$).

In those situations where R receives an object $[s, \mathbb{C}]$ and $\mathbb{C}$ has more than one identifier, R has to analyze the set $\mathbb{C}$ to decide which credibility profile will represent (for R) the credibility of s. For this analysis, we propose a cautious approach, analogous to the one explained in the last section for the computation of pl, now using the credibility order of the receiver agent R. The following function, called pr, will be used for this purpose. Given a set $\mathbb{C} \neq \emptyset$ from a received forwarded object $[s, \mathbb{C}]$, pr will return the set of preferred identifiers that will be computed selecting first the least credible identifiers with respect to $<^R_{co}$ of each set of $\mathbb{C}$, and then, the most credible of the selected ones.

Definition 7 (*Preferred identifiers*) Let $<^R_{co}$ be the credibility order of $R \in \mathbb{A}$ and $\mathbb{C} \neq \emptyset$ be a credibility profile, then the preferred identifiers of $\mathbb{C}$ with respect to $<^R_{co}$ are:

$$pr_{<^R_{co}}(\mathbb{C}) = max_{<^R_{co}}\left(\bigcup_{\mathbb{X}\in\mathbb{C}} min_{<^R_{co}}(\mathbb{X})\right)$$

Note that pr can be applied to any set $\mathbb{C} \neq \emptyset$ computed with any of the four criteria introduced in the previous section. Also observe that pr always returns a flattened set of agent identifiers, and when incomparable identifiers are found, the computed set may have more than one element.

Example 9 Consider a receiver agent $A_2 \in \mathbb{A}$ with the following credibility order (see Fig. 4): $A_4 <^{A_2}_{co} A_1$, $A_1 <^{A_2}_{co} A_3$, $A_2 <^{A_2}_{co} A_1$, $A_2 <^{A_2}_{co} A_6$, $A_4 <^{A_2}_{co} A_3$, $A_2 <^{A_2}_{co} A_3$. In the following table, the left column contains all the forwarded objects $[s, \mathbb{C}]$ shown in the right column of the table of Example 6 that were computed by the sender A_1 using *PRC*; the right column shows the result of applying $pr_{<^{A_2}_{co}}(\mathbb{C})$ to each set $\mathbb{C}$.

Forwarded objects $[s, \mathbb{C}]$	$pr_{<^R_{co}}(\mathbb{C})$
$[\delta, \{\{A_2\}\}]$	$\{A_2\}$
$[\varphi, \{\{A_3\}, \{A_4\}\}]$	$\{A_3\}$
$[\beta, \{\{A_4\}, \{A_5\}\}]$	$\{A_4, A_5\}$
$[\gamma, \{\{A_3\}, \{A_5\}, \{A_4\}, \{A_4, A_5\}\}]$	$\{A_3, A_5\}$
$[\alpha, \{\{A_5, A_3\}, \{A_4, A_3\}, \{A_5\}, \{A_4, A_5\}\}]$	$\{A_3, A_5\}$

Observe that a receiver agent R can receive a message $M = (A, R, [s, \mathbb{C}])$ where $\mathbb{C} = \emptyset$. In this case, R knows that A used a *ESC* or *ERC* criterion and $s \notin Exp(\mathcal{K}_A)$ (see Remark 2). As in other approaches found in the literature (see Sect. 5), in these cases, we consider that the sender of a forwarded object is, in a sense, responsible of the beliefs it retransmits. Hence, in our approach, the sender identifier will be considered as part of the credibility of the received sentence when the credibility profile is empty.

Definition 8 (*Incoming object*) Let $M = (A, R, [s, \mathbb{C}])$ be a message from a sender A and $<^R_{co}$ be the credibility order of the receiver R. An incoming object obtained from M is defined

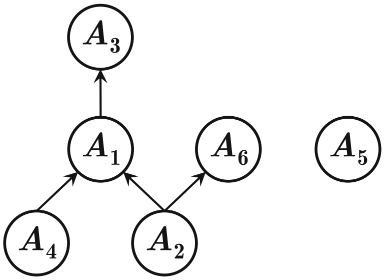


Fig. 4 Credibility order $<_{co}^{A_2}$

as $\langle s, \mathbb{C}' \rangle$, where s is the received sentence and

$$\mathbb{C}' = \begin{cases} \{A\} & \text{if } \mathbb{C} = \emptyset \\ pr_{<_{co}^{R}}(\mathbb{C}) & \text{otherwise} \end{cases}$$

For instance, if agent A_2 receives the message $M_1 = (A_1, A_2, [\varphi, \{\{A_3\}, \{A_4\}\}])$, then the incoming object for M_1 is $\langle \varphi, \{A_3\} \rangle$ (see Fig. 4). Furthermore, if agent A_2 receives the message $M_2 = (A_1, A_2, [\alpha, \{\{A_5, A_3\}, \{A_4, A_3\}, \{A_5\}, \{A_4, A_5\}\}])$, then the incoming object for M_2 will be $\langle \alpha, \{A_3, A_5\} \rangle$. Example 10 shows the incoming object obtained from the credibility profiles processed using *ERC*.

Example 10 Consider a receiver agent $A_2 \in \mathbb{A}$ with the credibility order proposed in Example 9 and shown in Fig. 4. The left column of the following table contains all the forwarded objects $[s, \mathbb{C}]$ shown in the right column of the table in Example 5 that were computed by the sender A_1 using *ERC*; the right column shows the computed incoming object.

Forwarded objects $[s, \mathbb{C}]$	$\langle s, \mathbb{C}' \rangle$
$[\delta, \{\{A_2\}\}]$	$\langle \delta, \{A_2\} \rangle$
$[\varphi, \{\{A_3\}, \{A_4\}\}]$	$\langle \varphi, \{A_3\} \rangle$
$[\beta, \{\{A_4\}, \{A_5\}\}]$	$\langle \beta, \{A_4, A_5\} \rangle$
$[\gamma, \{\{A_3\}, \{A_5\}\}]$	$\langle \gamma, \{A_3, A_5\} \rangle$
$[\alpha, \emptyset]$	$\langle \alpha, \{A_1\} \rangle$

Observe that when the receiver agent R maintains a total credibility order then for any incoming object $\langle s, \mathbb{C}' \rangle$, $\mathbb{C}'$ will always have just one element. Therefore, if the information is accepted, then the receiver agent will add only one information object.

4.2 Deciding acceptance or rejection of a received belief

At this point, we have described how from a received message $(A, R, [s, \mathbb{C}])$, an incoming object $\langle s, \mathbb{C}' \rangle$ can be computed. This computation of a sentence s is carried out taking into account both the information contained in the credibility profile $\mathbb{C}$ generated by the sender A and the credibility order $<_{co}^{R}$ of the receiver R. We will show now how, considering $\langle s, \mathbb{C}' \rangle$ and using the credibility information $\mathbb{C}'$ can be used by R to decide whether the sentence s is accepted or rejected as a new belief.

Considering the sentence s in a computed incoming object $\langle s, \mathbb{C}' \rangle$, it is possible to distinguish two cases. The agent R does not believe $\neg s$ ($\neg s \notin Bel(\mathcal{K}_R)$), in which case, since no contradiction arises, the decision is to accept s; or, the receiver believes $\neg s$ ($\neg s \in Bel(\mathcal{K}_R)$), and in this case, the acceptance will depend on the comparison of the credibility information about s contained in $\mathbb{C}'$ and the credibility information about $\neg s$ that can be computed with $pl_{<^R_{co}}(\neg s, \mathcal{K}_R)$. Again, in the next definition, we will follow a cautious approach.

Definition 9 (*Sufficiently credible*) Let $M = (A, R, [s, \mathbb{C}])$ be a received message from a sender A, $<^R_{co}$ be the credibility order of a receiver R, and $\langle s, \mathbb{C}' \rangle$ the computed incoming object for M. The sentence s is sufficiently credible with respect to $\mathcal{K}_R$, denoted $\langle s, \mathbb{C}' \rangle \rhd_R \mathcal{K}_R$, if either $\neg s \notin Bel(\mathcal{K}_R)$, or $\neg s \in Bel(\mathcal{K}_R)$ and for each $X \in pl_{<^R_{co}}(\neg s, \mathcal{K}_R)$ there exists some $I \in \mathbb{C}'$ such that $X <^R_{co} I$.

Note that the set $pl_{<^R_{co}}(\neg s, \mathcal{K}_R)$ represents the credibility that R attaches to $\neg s$ and can contain more than one element when the more credible identifiers are incomparable. Given an incoming object $\langle s, \mathbb{C}' \rangle$, the set $\mathbb{C}'$ represents for R the credibility of s and can also contain more than one element when the more credible identifiers are incomparable.

Example 11 Consider that the agent A_2 of Example 9 has the following belief base.

$$\mathcal{K}_{A_2} = \left\{ \begin{array}{ll} (\delta, A_2), & (\neg\alpha, A_2), \\ (\gamma, A_5), & (\varphi \rightarrow \neg\alpha, A_4), \\ (\gamma \rightarrow \neg\beta, A_3), & (\varphi, A_1), \\ (\neg\beta, A_4), & (\varphi, A_2), \\ (\gamma \rightarrow \delta, A_3), & \end{array} \right\}$$

If A_2 receives the message $(A_1, A_2, [\varphi, \{\{A_3\}, \{A_4\}\}])$, then, as it was mentioned above, the incoming object is $\langle \varphi, \{A_3\} \rangle$. Since $\neg\varphi \notin Bel(\mathcal{K}_{A_2})$, φ is sufficiently credible with respect to $\mathcal{K}_{A_2}$, i.e., $\langle \varphi, \{A_3\} \rangle \rhd_{A_2} \mathcal{K}_{A_2}$. If agent A_2 receives the message $(A_1, A_2, [\beta, \{\{A_4\}, \{A_5\}\}])$, the incoming object is $\langle \beta, \{A_4, A_5\} \rangle$. Since $\neg\beta \in Bel(\mathcal{K}_{A_2})$, $pl_{<^{A_2}_{co}}(\neg\beta, \mathcal{K}_{A_2}) = \{A_3, A_5\}$ and there is no agent identifier $I \in \{A_4, A_5\}$ such that $A_3 <^{A_2}_{co} I$, then β is not sufficiently credible with respect to $\mathcal{K}_{A_2}$. If A_2 receives $(A_1, A_2, [\alpha, \{\{A_5, A_3\}, \{A_4, A_3\}, \{A_5\}, \{A_4, A_5\}\}])$, the incoming object is $\langle \alpha, \{A_3, A_5\} \rangle$. Since $\neg\alpha \in Bel(\mathcal{K}_{A_2})$, $pl_{<^{A_2}_{co}}(\neg\alpha, \mathcal{K}_{A_2}) = \{A_2, A_4\}$ and, both A_2 and A_4 are less credible than A_3 ($A_2 <^{A_2}_{co} A_3$ and $A_4 <^{A_2}_{co} A_3$), then α is sufficiently credible with respect to $\mathcal{K}_{A_2}$, i.e., $\langle \alpha, \{A_3, A_5\} \rangle \rhd_{A_2} \mathcal{K}_{A_2}$. Note that the receiver A_2 does not know the agent A_5, but the input information is supported by A_3 which is one of the most credible agents according to A_2.

Figure 5 depicts the process of our proposed approach of sharing beliefs. There, we show how the sender agent A sends a message to forward a belief s to the receiver agent R. For this, first A computes the forwarded object $[s, \mathbb{C}]$ applying one of the four proposed forwarding criteria using its own belief base and its own credibility order; then A builds the message for s and sends it to R. When R receives this message, it applies pr function, using its own credibility order, to the credibility information $\mathbb{C}$ sent by A in the forwarded object. As a result, R computes the incoming object $\langle s, \mathbb{C}' \rangle$ for the received message, which will be used to determine whether s should be adopted by R's belief base.

4.3 Adopting a belief

As shown above, when an agent R receives a forwarded object from other agent, it can decide whether to accept the new belief using the associated credibility information and its

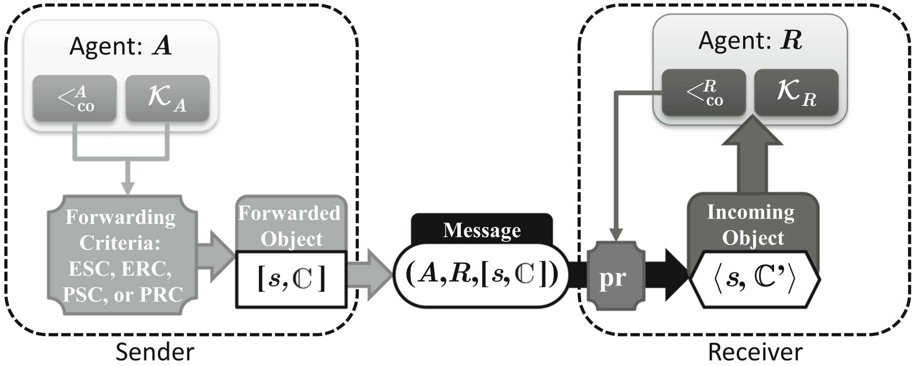


Fig. 5 Belief forwarding scheme

own order $<^R_{co}$. Next, we will introduce a proposal about how to proceed in the case that the decision taken is to accept a sentence s when $\neg s \in Bel(\mathcal{K}_R)$. In this particular case, to adopt the sentence s without generating a contradictory belief base, some beliefs will be removed, generating first a new and reduced belief base that does not entail $\neg s$. Then, one or more information objects with the sentence s will be added. For this removal process, we will propose a similar approach as the one presented in [10] that introduces a belief revision operator. In that work, the authors proposed to remove the least credible information objects using a mechanism, based on [13,14], to decide which tuples are erased from each minimal proof. In Sect. 5, we include a comparison of that approach with our current proposal. We refer the interested reader to [10] for a complete characterization and the properties of this kind of operator. The following function will be used in our conditional adoption operator and returns the least credible information objects of a given proof.

Definition 10 (*Least credible information objects*) Let $<^R_{co}$ be the credibility order of an agent R, and $\mathcal{H} \in \Pi(s, \mathcal{K}_R)$ a proof of a sentence $s \in Bel(\mathcal{K}_R)$, then $least_{<^R_{co}}(\mathcal{H}) = \{(e, X) : (e, X) \in \mathcal{H}$ and for all $(w, Y) \in \mathcal{H}, Y \not<^R_{co} X\}$.

Next, we introduce our conditional adoption operator. Recall that $\langle s, \mathbb{C}' \rangle$ is the incoming object computed by the receiver agent applying its credibility order over the forwarded object received in a message.

Definition 11 (*Conditional adoption operator*) Let $\mathcal{K}_R$ be a belief base and $<^R_{co}$ be the credibility order of the receiver R. Let $M = (A, R, [s, \mathbb{C}])$ be a message from a sender A, and $\langle s, \mathbb{C}' \rangle$ be the computed incoming object for M. Let $\mathrm{P} = \Pi(\neg s, \mathcal{K}_R)$ be all the proofs for $\neg s$ from $\mathcal{K}_R$ and $\mathcal{Z} = \{(s, A_i) : A_i \in \mathbb{C}'\}$. The operator "$\circledcirc^R$," called *conditional adoption operator*, is defined as follows:

$$\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}' \rangle = \begin{cases} \mathcal{K}_R \cup \mathcal{Z} & \text{if } \neg s \notin Bel(\mathcal{K}_R) \\ \left(\mathcal{K}_R \setminus \bigcup_{\mathcal{X} \in \mathrm{P}} least_{<^R_{co}}(\mathcal{X})\right) \cup \mathcal{Z} & \text{if } \neg s \in Bel(\mathcal{K}_R) \text{ and } \langle s, \mathbb{C}' \rangle \rhd_R \mathcal{K}_R \\ \mathcal{K}_R & \text{otherwise} \end{cases}$$

Example 12 Consider agent A_2 of Example 9. Suppose that A_2 receives the message $(A_1, A_2, [\varphi, \{\{A_3\}, \{A_4\}\}])$, then the incoming object for M is $\langle\varphi, \{A_3\}\rangle$. Thus, since $\neg\varphi \notin Bel(\mathcal{K}_{A_2})$, $\mathcal{K}_{A_2} \circledcirc^{A_2} \langle\varphi, \{A_3\}\rangle = \mathcal{K}_{A_2} \cup \{(\varphi, A_3)\}$.

Now, suppose A_2 receives the message $(A_1, A_2, [\beta, \{\{A_4\}, \{A_5\}\}])$, then, as was mentioned above, the incoming object is $\langle\beta, \{A_4, A_5\}\rangle$. Since $\neg\beta \in Bel(\mathcal{K}_{A_2})$, and according to Example 11, β is not sufficiently credible with respect to $\mathcal{K}_{A_2}$, then β is rejected by A_2 $(\mathcal{K}_{A_2} \circledcirc^{A_2} \langle\beta, \{A_4, A_5\}\rangle = \mathcal{K}_{A_2})$.

If A_2 receives $(A_1, A_2, [\alpha, \{\{A_5, A_3\}, \{A_4, A_3\}, \{A_5\}, \{A_4, A_5\}\}])$, the incoming object is $\langle\alpha, \{A_3, A_5\}\rangle$. Since $\neg\alpha \in Bel(\mathcal{K}_{A_2})$, and, as shown in Example 11, α is sufficiently credible with respect to $\mathcal{K}_{A_2}$ $(\langle\alpha, \{A_3, A_5\}\rangle \rhd_{A_2} \mathcal{K}_{A_2})$, α is accepted by A_2. In this case, to maintain the consistence, some information objects that entail $\neg\alpha$ need to be removed from $\mathcal{K}_{A_2}$ before adding the corresponding information objets that contain α. Then,

$$\Pi(\neg\alpha, \mathcal{K}_{A_2}) = \{\{(\neg\alpha, A_2)\}, \{(\varphi, A_1), (\varphi \rightarrow \neg\alpha, A_4)\}, \{(\varphi, A_2), (\varphi \rightarrow \neg\alpha, A_4)\}\},$$

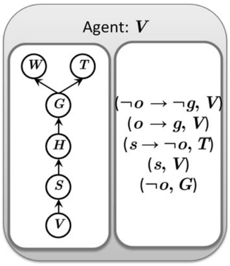
and

$$\bigcup_{\mathcal{X} \in \mathrm{P}} least_{<_{co}^{A_2}}(\mathcal{X}) = \{(\neg\alpha, A_2), (\varphi \rightarrow \neg\alpha, A_4), (\varphi, A_2)\},$$

hence,

$$\mathcal{K}_{A_2} \circledcirc^{A_2} \langle\alpha, \{A_3, A_5\}\rangle = (\mathcal{K}_{A_2} \backslash \{(\neg\alpha, A_2), (\varphi \rightarrow \neg\alpha, A_4), (\varphi, A_2)\}) \cup \{(\alpha, A_3)\} \cup \{(\alpha, A_5)\}.$$

Example 13



Consider the scenario described in Example 4, where agent S wanted to send the message $(S, V, [o, \mathbb{C}])$ to V. Using the belief base and the credibility order of V (as shown again above), we can analyze the effects of processing that message considering each credibility profile shown in Example 7. Note that $\neg o \in Bel(\mathcal{K}_V)$; thus, it is necessary to check whether the incoming objects are sufficiently credible for V. In the table below, we show for each credibility profile its associated incoming object, and if this incoming object is credible enough for V. Note that $pl_{<_{co}^{V}}(\neg o, \mathcal{K}_V) = \{G\}$.

Criterion	Credibility Profile	Incoming Object	Sufficiently Credible?
ESC	$\{\{H\}\}$	$\langle o, \{H\}\rangle$	No
ERC	$\{\{S\}, \{H\}\}$	$\langle o, \{H\}\rangle$	No
PSC	$\{\{T\}\}$	$\langle o, \{T\}\rangle$	Yes
PRC	$\{\{H\}, \{S\}, \{T, W\}, \{T, H\}\}$	$\langle o, \{T, W\}\rangle$	Yes

Observe that, when using the *ERC* or *ESC* criteria, the incoming objects are not sufficiently credible for V. Therefore, we have that $\mathcal{K}_V \circledcirc^V \langle o, \{H\}\rangle = \mathcal{K}_V$, i.e., in these cases, V will not adopt "o" as belief keeping his belief base as it was before processing the message. In contrast, the messages computed using the *PSC* and *PRC* criteria result in incoming objects that are sufficiently credible for V. In these cases, V will adopt "o" as a belief, adding/removing some information objects to/from his belief base. In particular, for the incoming object computed using *PSC*, it holds $\mathcal{K}_V \circledcirc^V \langle o, \{T\}\rangle = \{(\neg o \rightarrow \neg g, V), (o \rightarrow g, V), (s \rightarrow \neg o, T), (o, T)\}$; i.e., (o, T) was added, while (s, V) and $(\neg o, G)$ were removed. For the incoming object obtained using *PRC*, it holds $\mathcal{K}_V \circledcirc^V \langle o, \{\{H\}, \{S\}, \{T, W\}, \{T, H\}\}\rangle = \{(\neg o \rightarrow \neg g, V), (o \rightarrow g, V), (s \rightarrow \neg o, T), (o, T), (o, W)\}$; i.e., (o, T) and (o, W) were added, while (s, V) and $(\neg o, G)$ were removed.

Notice that, the credibility profile computed with *ERC* or *ESC* requires less information than when it is computed using *PSC* or *PRC*. Naturally, computing a credibility profile with more information may influence the acceptance decision of the receiver as shown in Example 13.

Next, we will give formal results that show which are the best criteria that can be used in different circumstances. We will show that a sentence is more likely to be accepted by the receiver if the sender uses *PRC* rather than *ERC* or *ESC*. The reason is that the sender with *PRC* uses more information that justify the sentence being forwarded. First, in Proposition 5, we show that if the sender agent uses *ESC* to forward a belief s and this sentence is accepted by the receiver, then s also would have been accepted if the sender used *ERC*. Observe that *PSC* does not appear in the following formal results; however, we have performed an empirical analysis in order to compare *PSC* with the other criteria for some particular interaction scenarios.

Proposition 5 *Let A and R be two agents in $\mathbb{A}$ and $s \in Exp(\mathcal{K}_A)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using ESC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using ERC, with $\langle s, \mathbb{C}'\rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}'\rangle$ the incoming object for M_2. If $s \in Bel(\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle)$, then $s \in Bel(\mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle)$.*

Proof See Appendix. □

Note that the inverse does not hold. For instance, suppose $\mathcal{K}_A = \{(s, A_1), (s, A_5)\}$ and $\mathcal{K}_R = \{(\neg s, A_6)\}$. Consider that A wants to forward s to R . Furthermore, suppose that $A_1 <^A_{co} A_5$ and $A_5 <^R_{co} A_6$, $A_6 <^R_{co} A_1$. So, if A uses *ERC*, then R accepts s, but if A uses *ESC* then R rejects s. The following proposition shows a result similar to Proposition 5 but considering the criteria *ERC* and *PRC*.

Proposition 6 *Let A and R be two agents in $\mathbb{A}$ and $s \in Exp(\mathcal{K}_A)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using ERC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using PRC, with $\langle s, \mathbb{C}'\rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}'\rangle$ the incoming object for M_2. If $s \in Bel(\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle)$, then $s \in Bel(\mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle)$.*

Proof See Appendix. □

In the same way as for Proposition 5, the inverse does not hold. For example, suppose now that $\mathcal{K}_A = \{(s, A_1), (p, A_2)(p \rightarrow s, A_3)\}$ and $\mathcal{K}_R = \{(\neg s, A_6)\}$. Consider that A wants to forward s to R. Furthermore, suppose that $A_1 <^A_{co} A_5$, and that $A_6 <^R_{co} A_3$ and $A_3 <^R_{co} A_2$. Then, if A uses *PRC* then R accepts s, but if A uses *ERC* then R rejects s.

Remark 4 Observe that, by Proposition 5 and Proposition 6, if agent A sends a sentence to agent R using *ESC* and this sentence is accepted by R, then if agent A uses *PRC* agent R accepts the sentence too.

The example below provides counterexamples that show that a formal relation similar to the results expressed in Propositions 5 and 6 cannot be established between *PSC* and the other three criteria.

Example 14 Let A and R be two agents in $\mathbb{A}$, and let $\mathcal{K}_A = \{(s, A_1), (p, A_2)(p \rightarrow s, A_3)\}$ and $\mathcal{K}_R = \{(\neg s, A_3)\}$ be their respective belief bases. Consider the scenario where agent A wants to send the message $M = (A, R, [s, \mathbb{C}])$ to agent R. Suppose that $A_1 <^A_{co} A_2$, $A_1 <^A_{co} A_3$, $A_2 <^A_{co} A_3$ and $A_3 <^R_{co} A_2$. Then, if A uses *PSC* then R will accept s, but if A uses *ESC*, *ERC*, or *PRC* then R will reject s. However, suppose that $A_3 <^R_{co} A_1$, in this case if A uses *PSC* then R will reject s, but if A uses *ESC*, *ERC*, or *PRC* then R will accept s.

Below, we will include an empirical analysis on how the forwarding criteria performs for some particular interaction scenarios. To do this, since the sender is responsible of deciding which forwarding criterion should be used, we will adopt its point of view. Recall that for any agent, the plausibility function is required to obtain the credibility of a given belief, that is, $pl_{<^A_{co}}(s, \mathcal{K}_A)$ contains those identifiers that A considers relevant to the credibility of s.

There are some particular interaction scenarios where an agent considers that the information it has is better than the information of its peers; for instance, the agent is an expert in some field, a teacher, *etc*. Intuitively, an expert would expect that, after an interaction, the information that it considers relevant for the credibility of the sentence to be forwarded should also be relevant for the receiver when considering the credibility of that sentence. Although our formalism does not provide concrete tools for establishing that some sender agent A has better information than some receiver R, we can assume that A believes that for some external reason. Then, following that intuition, in our formalism after an interaction, agent A should expect that, regardless of the credibility orders of both agents, the information contained in $pl_{<^A_{co}}(s, \mathcal{K}_A)$ will be considered as much as possible in $pl_{<^R_{co}}(s, \mathcal{K}_R)$. Therefore, to compare the four proposed criteria for this particular scenario, our empirical analysis will measure how similar are those plausibilities.

For our simulation, we have used a *similarity coefficient* between two sets: On one hand, the set $\mathbb{S}$ of agent identifiers representing the credibility (for the sender) of the sentence s to be forwarded, and on the other hand, the set $\mathbb{R}$ of agent identifiers representing the credibility (for the receiver) of the sentence s after adopting it in its belief base. That is, consider that A is the sender and R is the receiver, then we have compared the similarity of the sets $\mathbb{S} = pl_{<^A_{co}}(s, \mathcal{K}_A)$ and $\mathbb{R} = pl_{<^R_{co}}(s, \mathcal{K}'_R)$, where $\mathcal{K}'_R = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}' \rangle$. For computing the similarity between $\mathbb{S}$ and $\mathbb{R}$, we have used the *Jaccard similarity coefficient* [15], denoted $J(\mathbb{S}, \mathbb{R})$ and defined as the size of the intersection divided by the size of the union of the sets:

$$J(\mathbb{S}, \mathbb{R}) = \frac{|\mathbb{S} \cap \mathbb{R}|}{|\mathbb{S} \cup \mathbb{R}|}$$

Next, a brief explanation of the insights of our experimental analysis is included. Then, two charts with the obtained results will be shown. The simulation consists of a series of experiments each one called *test batch*. Briefly, a test batch is a set of 10,000 individual tests, each one representing the simulation of a particular scenario for exercising the construction of a message with the four proposed criteria and the processing of these messages by a receiver. For our simulation, we assume that both sender and receiver have 20 informants that can be involved in their credibility orders. We fixed this number to 20 because the results obtained were very similar when we increased or decreased the number of informants. The 10,000 tests were generated randomly, from a set of 20 agent identifiers, 100 pairs of credibility orders (one for the sender and one for the receiver); then, for each pair of orders, we randomly generate 100 pairs of derivations of a sentence s to be forwarded (one represents the sender's derivation of s and the other the receiver's derivation of s). For each forwarding criterion (*ESC*, *ERC*, *PRC*, *PSC*), the simulation computes the average of the Jaccard similarity $J(\mathbb{S}, \mathbb{R})$ of 10,000 tests. In all the tests, we assume that the forwarded sentence s is already present in the receiver belief base ($s \in Bel(\mathcal{K}_R)$) since this situation represents the worst-case scenario to compare the similarity. Since $s \in Bel(\mathcal{K}_R)$, we know in advance that the receiver will adopt the sentence. If new tuples are added to $\mathcal{K}_R$, then this change can have an impact over $pl_{<^R_{co}}(s, \mathcal{K}'_R)$. The value of $J(\mathbb{S}, \mathbb{R})$ will measure such impact.

Recall that in a test batch for each individual test, credibility orders and derivations for a forwarded sentence are generated randomly; then, for each forwarding criterion in the simulation, we will study how the variation in those elements affects $J(\mathbb{S}, \mathbb{R})$. Below, we show two charts describing the obtained results.

The chart shown in Fig. 6 pictures the variation in $J(\mathbb{S}, \mathbb{R})$ with respect to connectivity in the credibility orders of A and R, using separate curves for each of the four forwarding criteria (*ESC*, *PSC*, *ERC*, *PRC*). The horizontal axis represents the connectivity of the graph associated with the credibility order of each agent, expressed as the maximum number of outgoing arcs from each node in that graph. For instance, a value of 6 in the horizontal axis represents that the maximum number of agents that are more credible with respect to a particular agent identifier is 6. The vertical axis shows the average of $J(\mathbb{S}, \mathbb{R})$ for the 10,000 tests of the batch for the four criteria.

The connectivity of the graph representing the credibility order provides a strong indication of how many agent identifiers are incomparable with respect to that order and, as shown in previous sections, some criteria are more affected by such identifiers than others. The chart in Fig. 6 shows that as the connectivity increases, every criterion performs better; however, note that the growth of the *ESC* and *PSC* is greater than the growth of *ERC* and *PRC*, respectively. The reason that justifies this observation is that in the application of the *ESC* and *PSC* criteria, it is the sender that carries the burden of making the comparison; meanwhile, in the two other cases, it does not. Finally, observe that the *PSC*, where the sender makes the greater effort, outperforms the other criteria even in the scenarios where the credibility orders have low connectivity.

The chart in Fig. 7 shows the variation of $J(\mathbb{S}, \mathbb{R})$ (vertical axis) with respect to the maximum number of sentences that belong to the derivations of the sentence to be forwarded (horizontal axis). The value in the vertical axis is the average of $J(\mathbb{S}, \mathbb{R})$ for the 10,000 tests of the batch.

Both charts give strong indications that *PSC* is the best choice among the four criteria to obtain the greatest similarity. The similarity obtained with *PSC* is better with a more connected credibility order (Fig. 6), and *PSC* has also the best results when the size of the derivation is increased (Fig. 7). Although the sender has to make more efforts to obtain a derivation and its associated plausibility when a derivation is longer or the graph is more connected, the results of our simulation show that this effort is worthwhile.

Next, we include three propositions to show in which scenarios two criteria have the same outcome, and therefore, the simpler one can be used. In the particular case that the sender and the receiver have the same credibility order, the following proposition establishes that when using *ERC* or *ESC* for computing the forwarded objects, the receiver will end with the same belief base. Therefore, in those multi-agent systems where all agents share the same credibility partial order, it is indifferent for the sender to choose *ERC* or *ESC*.

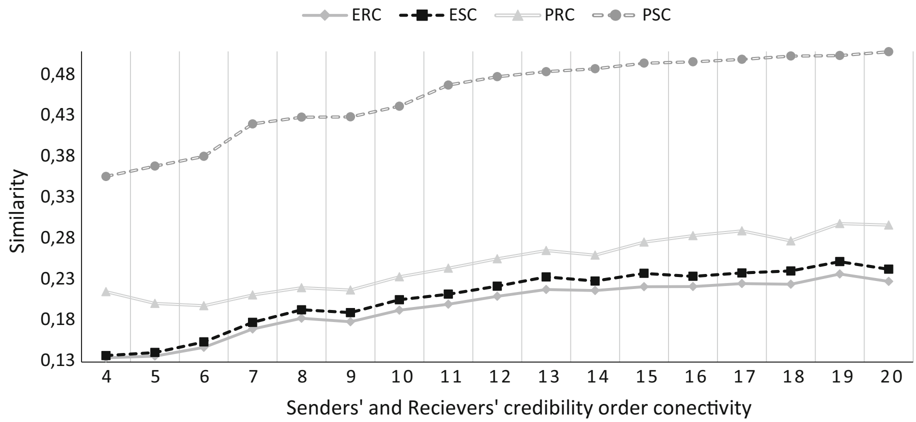


Fig. 6 Similarity analysis as the connectivity of credibility orders increases

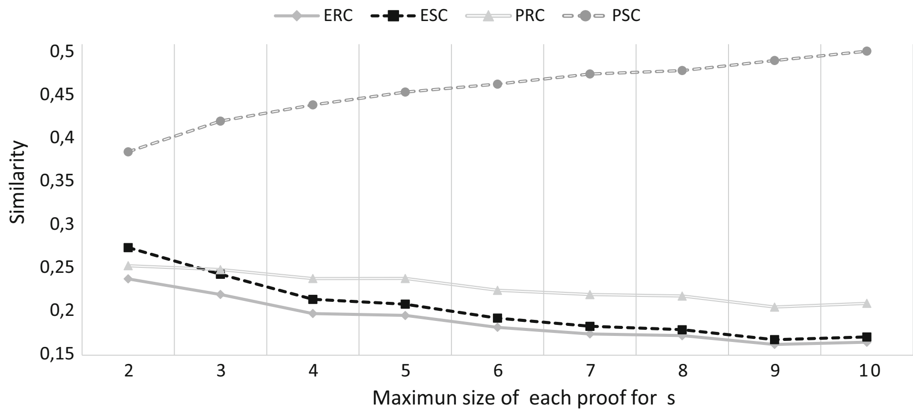


Fig. 7 Similarity analysis as the length of individual proofs for s increases

Proposition 7 *Let A and R be two agents in $\mathbb{A}$ and $s \in Bel(\mathcal{K}_A)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using ESC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using ERC, with $\langle s, \mathbb{C}' \rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}' \rangle$ the incoming object for M_2. If $<^A_{co} = <^R_{co}$ then $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}' \rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}' \rangle$.*

Proof See Appendix. □

If the sender and the receiver have the same credibility order, then an analogous situation for *PRC* and *PSC* occurs.

Proposition 8 *Let A and R be two agents in $\mathbb{A}$ and $s \in Bel(\mathcal{K}_R)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using PSC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using PRC, with $\langle s, \mathbb{C}' \rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}' \rangle$ the incoming object for M_2. If $<^A_{co} = <^R_{co}$, then $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}' \rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}' \rangle$.*

Proof See Appendix. □

Finally, the following proposition shows a particular case where using *ERC* and *PRC* for computing the forwarded objects yields the same belief base of the receiver.

Proposition 9 *Let A and R be two agents in $\mathbb{A}$ and $s \in Bel(\mathcal{K}_R)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using ERC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using PRC, with $\langle s, \mathbb{C}' \rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}' \rangle$ the incoming object for M_2. If $s \notin (Bel(\mathcal{K}_A) \setminus Exp(\mathcal{K}_A))$, then $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}' \rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}' \rangle$.*

Proof See Appendix. □

In the introduction, we have commented that our proposal is based on the principle that the credibility of forwarded information should not diminish. Next, we will introduce three propositions related to this principle. Proposition 10 shows that following our proposal, when an agent adopts a sentence, the credibility of this sentence will not decrease from its point of view.

Proposition 10 *Let $\mathcal{K}_D$ be the belief base of agent D, $s \in Bel(\mathcal{K}_D)$, $M = (S, D, [s, \mathbb{C}])$ a message computed using one of the criteria ESC, ERC, PSC, or PRC, $\langle s, \mathbb{C}' \rangle$ the incoming object for M, $\mathcal{K}'_D = \mathcal{K}_D \circledcirc^D \langle s, \mathbb{C}' \rangle$, and $s \in Bel(\mathcal{K}'_D)$. It holds that for each $A \in pl_{<^D_{co}}(s, \mathcal{K}_D)$ and each $A' \in pl_{<^D_{co}}(s, \mathcal{K}'_D)$, either $A = A'$, $A \sim^D_{co} A'$, or $A <^D_{co} A'$.*

Proof See Appendix. □

In the following proposition, we show that, when an agent adds a sentence s and this agent believed in $\neg s$, the plausibility of s will be better than the plausibility that this agent had for $\neg s$.

Proposition 11 *Let $\mathcal{K}_D$ be the belief base of agent D, $\neg s \in Bel(\mathcal{K}_D)$, $M = (S, D, [s, \mathbb{C}])$ a message computed using one of the criteria ESC, ERC, PSC, or PRC, $\langle s, \mathbb{C}' \rangle$ the incoming object for M, $\mathcal{K}'_D = \mathcal{K}_D \circledcirc^D \langle s, \mathbb{C}' \rangle$, and $s \in Bel(\mathcal{K}'_D)$. It holds that for each $A \in pl_{<^D_{co}}(\neg s, \mathcal{K}_D)$, there exists $A' \in pl_{<^D_{co}}(s, \mathcal{K}'_D)$ such that $A <^D_{co} A'$.*

Proof See Appendix. □

Next, we show that when an agent adopts a sentence, the information objects that it removes are less credible than those that it adopts.

Proposition 12 *Let $\mathcal{K}_D$ be the belief base of agent D, $(t, A) \in \mathcal{K}_D$, $M = (S, D, [s, \mathbb{C}])$ a message computed using one of the criteria ESC, ERC, PSC, or PRC, $\langle s, \mathbb{C}' \rangle$ the incoming object for M, $\mathcal{K}'_D = \mathcal{K}_D \circledcirc^D \langle s, \mathbb{C}' \rangle$, and $s \in Bel(\mathcal{K}'_D)$. If $(t, A) \notin \mathcal{K}'_D$, then there is an $A' \in \mathbb{C}'$ such that $A <^D_{co} A'$.*

Proof See Appendix. □

5 Discussion and related work

A common approach to the analysis of the reliability of information is obtained by integrating different sources that rely on the use of some form of a majority principle [5]. In those approaches that use majority, and oversimplifying the description of the decision mechanism they introduce, when two or more sources provide the same piece of information α, and a single agent gives $\neg\alpha$, then α will be preferred. It is clear that using majority in the process of deciding is a very useful and computationally efficient approach for many situations, but it might not be appropriate in some complex scenarios that require a qualitative analysis of the information; alternatively, in domains where there exists an order among informants, it is natural to prefer the information of the more credible one instead. As an example, consider the situation where an agent seeks information on a particular topic in an Internet children's health forum. Reading the forum, the agent finds out that four participants provide information α on the subject, but later the agent's pediatrician provides $\neg\alpha$. If the agent assigns a higher credibility to the pediatrician than the perceived credibility of the other four, then, clearly in this case, $\neg\alpha$ should prevail. Thus, our approach can be considered as complementary to those that use majority for taking decisions. This complementarity is important since siding with the majority is not always the right decision. Note also that sending everything to the receiver is a solution that can be unfeasible in many domains because of at least two reasons: the amount of information that would be necessary to send and the possible breach of confidentiality.

As was mentioned in Sect. 4, our conditional adoption operator was inspired in the one proposed in [10]. Both are non-prioritized, that is, an incoming sentence can be accepted or rejected, and use the credibility order for deciding the acceptance of the input. In contrast to our approach, the change operator proposed there was defined to change a credibility partial order and not a belief base as in ours. That is, the operator of our proposal consider minimal proofs and the one in theirs consider paths in the credibility order. Furthermore, the input there considers only one agent identifier and here the input can contain several agent identifiers. This last difference may lead to the addition of more than one information object to the receiver's belief base. In [10], the authors proposed the removal of the least credible information objects from each path using a mechanism based on [13,14]. Note that [10] uses an adapted notion of *safe element*, as proposed in [13] for contraction; however, in [13], an order among sentences is considered, and the contraction operator is defined over belief sets, whereas in [10] the contraction operator is defined over credibility bases.

Kernel contractions, which are based on a selection among the sentences that are relevant to derive the sentence to be retracted, were introduced in [14]. Note that kernel contractions are a generalization of safe contractions proposed in [13]; to perform a contraction, kernel contractions use incision functions which cut into the minimal subsets that entail the information to be given up. In [10] (and also in our current proposal), the definition of kernel was adapted to the new epistemic model allowing to define a comparison criterion among sentences (called plausibility) and hence contributing to the definition of incision functions.

The approach reported in [6,16] was one of the first works which pointed to the importance of considering information sources to deal with contradictory information in a multi-agent setting. Similarly to the work presented here, they argue that in this kind of scenario to solve a contradiction, it is useful to keep track of who informed a sentence, or, in general, about where did knowledge come from. To model this intuition, in their approach, an agent knowledge base contains assumption nodes, which are tuples of five elements: $\langle$*Node-identifier*, *Sentence*, *Origin-set*, *Source*, *Credibility*$\rangle$. In each assumption node, *Credibility* stores the computed plausibility for *Sentence* considering the reliability of the *Source* in the moment when the assumption node is asserted in the knowledge base, whereas *Origin-set* records the assumption nodes upon which *Sentence* depends (as derived by the theorem prover). In contrast to that representation, a tuple in our framework only stores two elements: a sentence and an agent identifier. An important difference with their proposal is that here the plausibility of a sentence is not explicitly stored, but is computed when is needed using the agent identifiers associated with all its possible derivations. This characteristic leads to another difference: When the credibility order of an agent changes, the plausibility of the agent's beliefs might automatically change accordingly. An additional advantage of computing on demand the plausibility of a belief considering all its proofs is that when an agent adopts a sentence s in its belief base, then the plausibility of every belief that uses s in its proof can automatically change (see Example 15 below).

Example 15 Consider the belief base $\mathcal{K}_{A_7} = \{ (\tau, A_7), (\tau \rightarrow \rho, A_9) \}$ and that the agent A_7 has the credibility order: $A_7 <_{co}^{A_7} A_8$, $A_8 <_{co}^{A_7} A_9$, $A_7 <_{co}^{A_7} A_9$; therefore, $pl_{<_{co}^{A_7}}(\rho, \mathcal{K}_{A_7}) = \{A_7\}$. If agent A_7 receives the message $(A_6, A_7, [\tau, \{\{A_8\}\}])$, then A_7 adopts (τ, A_8), and the plausibility of ρ changes to $\{A_8\}$.

Finally, another difference is that in [6], agents do not communicate the source of the assumption nodes: They present themselves as completely responsible for the information they are passing on. Therefore, the agent that receives information considers the sender as the source of all the assumption nodes it receives from it. In contrast, in our framework, we only follow this approach when the credibility profile sent by the sender is empty. As we have shown, this happens when the sender uses a explicit criterion (either *ERC* or *ESC*) and the sentence to be forwarded is not explicit; in these particular cases, the receiver establishes the sender as the responsible for the sentences.

In [17,18], a belief revision in a multi-agent environment is suggested. Similar to us, they introduce a non-prioritized operator considering a piece of information as a pair <source, information> in which the information comes from different sources. Differently to our approach, they propose that the agents maintain two knowledge bases. One of them stores all the pieces of information available to the agent (potentially inconsistent), and it cannot be used as a whole to support reasoning and decision processes. The other knowledge base is a maximally consistent subset of the previous one, and it is used to support reasoning and decision making. Also, in that work, the reliability assigned to each source is given in a range [0, 1]; therefore, the established order is a total order. They do not deal with the forwarding problem, they only suggest to use the source of information as part of the forwarded pair.

There is a connection between credibility orderings and possibility theory (at least in the case when there is a total ordering over agents). Possibility values, represented as real numbers in possibility theory, are used to characterize a preference ordering over items of information; for instance, see [19]. Clearly, introducing partially ordered labels to identify the trust level is a more general approach because it gives us the ability of representing the case where some elements are incomparable; in contrast, when real numbers are used, a total order is forced upon the labels.

In [20], Liau introduces a modal system that considers the influence of trust on the assimilation of acquired information to determine agent beliefs. Modal logic is used to characterize the relationship among beliefs, information acquisition, and trust. Like us, the credibility of informants is used to handle consistency in a multi-agent setting. However, like in [21,22] and in contrast to our approach, in [20] agents can deceive one another. To provide a suitable formalization for this kind of competitive setting, Liau proposes special axioms which disregard agents that contradict themselves. Since our approach is conceived for a collaborative setting, an agent that contradicts itself is not necessarily disregarded, and we assume that an agent with this behavior is rectifying itself. Unlike our proposal, the approach in [20] does not explicitly deal with the problem of forwarding information, and the notion of message among agents is not considered. However, that formalism allows to maintain the sources from where an agent acquires its information with the modal operator I; for instance, $I_{ij}\alpha$ means that the agent j has given agent i the information α. As modal operators can be nested, it is possible to track the original source of a piece of information. This allows to simulate some kind of forwarding similar to *ESC* and *ERC* in our approach. However, if an agent A wants to send a sentence α which it is not explicitly in its belief base, in contrast to us, the sender will always take responsibility for α, thus becoming the source of α for the receiver. In [23], a similar work to [20] is presented, which uses a modal approach to model the communication and assimilation of information between interacting agents. In contrast to [20], this approach considers the problem of forwarding information; however, it differs from our approach in that [23] aims to determine whether a source is believable or not in a multi-agent context that may be uncooperative. In this model, an agent can forward information that it does not believe, whereas in ours does not.

In [8], an approach was proposed that considers forwarding information; we have considered elements of that research as starting point. As in the present approach, in [8] agents have a finite belief base of information objects, and each agent has its own credibility order among its peers; nevertheless, there are several differences that we will describe below. Firstly, to model the credibility that an agent has about its peers, in [8] a total order among agent identifiers is used; this decision makes impossible to deal with incomparable agents. Secondly, in contrast to our proposal, that work does not consider ways for the receiver to deal with the received information; it only proposes alternatives for creating a forwarded object, and consequently, the criteria *ERC* and *PRC* were not considered. Four criteria for computing a forwarded object were introduced in [8]; nevertheless, since in that work a forwarded object is a pair $[sentence, A]$ where A is a single agent identifier, therefore the sender makes all the decisions contrasting with the present framework. As explained below, some of the criteria in [8] have some drawbacks that were addressed here. The first criterion presented in [8] is very simple and just assigns the sender identifier to every forwarded object. The second criterion proposed in [8] is not complete because it does not work with derived beliefs assigning the most credible agent identifier that is explicitly stored in the sender's belief base. The third criterion reported in [8] combines the first two, and the sender chooses the most credible identifier in the set those explicitly stored attached to the sentence and itself. Although their criterion is similar to *ESC*, the new criterion is more general because here we used a strict partial order; therefore, *ESC* can deal with incomparable agents. The fourth criterion proposed in [8], which is similar to *PSC*, computes the plausibility of the sentence to be forwarded; however, *PSC* works with a strict partial order. Notice that in our proposal, a forwarded object can contain more than one agent identifier. An advantage of this feature, in contrast to sending only one as proposed in [8], is that the receiver has the opportunity to store the forwarded sentence using the best identifier according to its own credibility order, or all of them if they are incomparable according to its order.

In [24,25], an argumentation formalism that can be used to reason using information about trust is proposed. This formalism is described as a set of graphs, and to determine agent's beliefs the authors propose a model which considers the trust in the information that is used for building arguments. Like ours, this approach is intended for a multi-agent setting, and informant agents can have different credibilities. In contrast to our work, where each agent has its own credibility order, they use a centralized notion of trust that is codified in a shared trust network. This global network holds information about how agents trust each other and can be used to obtain an agent-centric trust network that represents the viewpoint of a particular agent. Although from these graphs it is possible to determine a credibility order for each agent, these orderings are strongly dependent on the connections in the global network. In contrast, in our work each agent has its own credibility order which is completely independent of the credibility order of any other agent. Another significant difference is that they use numerical values to establish the trust relation among agents, leading to a total order on the set of agents, whereas our approach uses symbolic information to define the credibility order as a strict partial order. Similarly to us, each piece of information is linked to an agent and determines how credible this information is. However, in contrast to our proposal, their focus is not on information forwarding. They have no notion of message between agents, and all the agents share globally which is the connection of who provides each piece of information. Their approach assumes that the agent which provides a piece of information is its source. In our work, we do not make this assumption, and the receiver of a message can use both the credibility of the source and the credibility of the sender to determine whether the incoming information is accepted or rejected. The formalism in [24,25] also differs from ours because they use an argumentation inference mechanism, and therefore, they do not have to deal with the decision of accepting or rejecting a new incoming belief. That is, in their approach, agents can have a potentially contradictory belief base from where arguments are built, and then they use their trust measure to determine which arguments prevail. Nevertheless, similarly to us, their proposal aims to enrich the agent reasoning process by considering the credibility of the sources of the available information. In this sense, it could be interesting to study how to incorporate our forwarding mechanisms with their belief representation and reasoning approaches.

In [7,26], similarly to us, they use a symbolic approach to model credibility, using two global relations: the trust relation and the distrust relation; these relations together with the set of agents constitute a trust system. A pair (a, b) in the trust (distrust) relation determines that agent a trusts (distrusts) agent b. Unlike ours, this formalism is not concerned with the forwarding of information and adoption. Their formalism aims to determine whether an agent trusts another taking into account the potential conflicts that may appear when trust and distrust relations are analyzed together in the trust system. For doing this, they follow an argumentation approach, where arguments represent a position for an agent to either trust or distrust a peer. Additionally, when considering an advanced version of their system, each agent is also provided with a partial order of its peers, using this order to codify the efficacy in which this agent trusts it peers (aiming to model a grade of trustworthiness or reputation). Even if this can be seen as similar to the credibility ordering that agents have in our framework, the approaches use the ordering for different purposes. While they use these efficacy orders to provide strength to arguments in their argumentation formalism, in our approach the credibility order is used for information forwarding and adoption purposes.

In [27,28], a trust model used in document recommendation in the context of a multi-agent system is presented where the goal of such model is to maximize the utility and reuse of the recommended documents. For this, agents use several attributes (such as expertise, position, intuition, and previous experiences with an agent) to compute the trust they have of its peers; in this context, an agent can share information and evaluate information shared by its pairs. Similarly to us, in their framework, agents share information and use trust/credibility to asses information, but they do not deal with the problem of forwarding information. Another difference is that in their approach, it is the receiver who evaluates the relevance of a piece of knowledge, whereas in ours the sender also evaluates the knowledge using some criteria. Finally, some agents have different roles related to their trust mechanics, whereas in ours there is no such distinction.

6 Conclusion and future work

In the present proposal, agents can communicate with other agents in the system to share beliefs. Every agent has a belief base where each sentence is attached with an agent identifier representing the credibility of that sentence; besides that, each agent has its own strict partial order defined over the set of agents representing the credibility the agent has about its informants. This credibility order is used by a sender of a message to compute an object to be forwarded and by a receiver for deciding which information prevails upon reception of a forwarded belief.

We have introduced four different criteria that a sender agent S can use for computing the associated credibility information $\mathbb{C}$ for belief b to be sent to another agent R in a message $(S, R, [b, \mathbb{C}])$. The first two criteria, called *ESC* and *ERC*, use only the credibility information that is explicitly stored in the sender belief base; the main advantage of these two criteria is that the computation they perform is a simple one. In the case of *ESC*, this computation is done at sender's side using the sender credibility order. In those scenarios where it is convenient to receive all the information that the sender has, and then to perform the computation using the receiver credibility order, the *ERC* can be used. The other two proposed criteria, called *PSC* and *PRC*, consider all the possible proofs for a belief from the sender's belief base; thus, the main advantage of these two criteria is that they perform a more informed computation. In the case of *PSC*, this computation is done at sender's side using the sender credibility order. We have also introduced formal results that show which are the best criteria that can be used in different circumstances; for instance, we have shown that a sentence is more likely to be accepted by the receiver if the sender uses *PRC* rather than *ERC* or *ESC*.

We have also proposed an adoption approach that can be applied when an agent receives a forwarded object which was computed using any of the presented forwarding criteria. First, we have shown how the received message can be processed, then we have indicated how the credibility information can be used to decide whether the new incoming belief is accepted or rejected, and finally, we have formalized a way to adopt an incoming object in the receiver belief base. We have introduced some propositions for characterizing our proposal and for showing how it behaves in some particular cases. In particular, we have shown that following our approach when an agent adopts a belief that it was already entailed from the receiver belief base, the credibility of that belief does not decrease. We have also shown that upon the adoption of a new belief, the information objects that are eliminated are less credible than the new information.

An important decision we made, departing from the existing work already discussed, is to store in the knowledge base and forward to other agents, agent identifiers together with a sentence attaching additional information to the belief represented by that sentence. This decision was taken considering each agent has its own credibility order; thus, it is more suitable to send agent identifiers giving the receiver agent the possibility of evaluating the received belief based on its own credibility order. In that manner, the sending agent communicates that the information it forwards is as credible as is represented by the agent identifiers in the forwarded object. Upon reception, it becomes a choice of the receiver to assess how credible it considers each agent using its own credibility order. As importantly, another reason for this decision is that our framework can be applied in dynamic scenarios where the credibility orders among agents can change; hence, if the credibility order among agents changes, the plausibility of the beliefs also changes without having to modify the belief base of the agent.

In this paper, we have assumed that the agent's credibility order corresponds to a single topic. In [1], they offer the following example to show that trust is context-dependent: "*if we trust a doctor when she is recommending a medicine, it does not mean we have to trust her when she is suggesting a bottle of wine*." Our proposed model was designed to associate a single trust value per agent without taking into account the context. Note that a naïve solution for a multi-context approach could be to have for each agent a different credibility order for each topic and include the topic as part of the message. However, that does not cover what happens when a derivation considers several sentences of different topics, or how to deal with a sentence that can be considered in more than one topic. Hence, the consideration of a multi-topic approach deserves more attention, and we will leave it for future research. Nevertheless, we consider that the study of a single topic approach is valuable in and of itself, but also it can be used as part of the solution of a multi-topic approach.

As a future line of research, we will investigate how to combine and extend our proposal with the possibility of changing the credibility order of each agent as was proposed in [10]. Another interesting direction is to consider the way in which [20] models the information acquisition, which will allow us to keep track of every agent that was involved in the forwarding of a sentence.

Appendix

Proposition 1 *Let $A \in \mathbb{A}$, $<^A_{co}$ be the credibility order of agent A and $s \in Bel(\mathcal{K}_A)$. Let $[s, \mathbb{C}]$ be the forwarded object computed by A using ESC and let $[s, \mathbb{C}']$ be the forwarded object computed by A using ERC. If $<^A_{co} = \emptyset$, then $\mathbb{C} = \mathbb{C}'$.*

Proof Suppose that $s \in Bel(\mathcal{K}_A)$, $[s, \mathbb{C}]$ is computed by A using *ESC*, $[s, \mathbb{C}']$ is computed by A using *ERC* and $<^A_{co} = \emptyset$. Let $\mathbb{I} = \{X : (s, X) \in \mathcal{K}_A\}$ then:

- If $\mathbb{I} = \emptyset$, then by Definition 5 $max_{<^A_{co}}(\mathbb{I}) = \emptyset$ and $\mathbb{C} = \mathbb{C}' = \emptyset$ by *ESC* and *ERC*, respectively.
- If $\mathbb{I} = \{B\}$ is a singleton, then by Definition 5 $max_{<^A_{co}}(\mathbb{I}) = \{B\}$ and $\mathbb{C} = \mathbb{C}' = \{\{B\}\}$ by *ESC* and *ERC*, respectively.
- Otherwise, for all $D, E \in \mathbb{I}$, $D \not<^A_{co} E$ and $E \not<^A_{co} D$. Therefore, by Definition 5, $max_{<^A_{co}}(\mathbb{I}) = \mathbb{I}$. Thus, following *ESC*, $\mathbb{C} = \{\{X\} : X \in \mathbb{I}\}$. Therefore, $\mathbb{C}' = \mathbb{C}$ by *ERC*.

Proposition 2 *Let $A \in \mathbb{A}$, $s \in Exp(\mathcal{K}_A)$, and let $[s, \mathbb{C}]$ be a forwarded object computed by A using ESC. If $<^A_{co}$ is a total order, then $\mathbb{C}$ contains just one singleton.*

Proof Following *ESC* and $s \in Exp(\mathcal{K}_A)$, it holds that any element in $\mathbb{C}$ is a singleton. Then, if $[s, \mathbb{C}]$ was computed by A using the *ESC*, $\mathbb{C}$ was computed using the $max_{<^A_{co}}$ function over a set of agent identifiers $\mathbb{I}$. Thus, if $<^A_{co}$ is totally ordered, it holds that $max_{<^A_{co}}(\mathbb{I})$ returns a set with one element. □

Proposition 3 *Let $A \in \mathbb{A}$, and let $<^A_{co}$ be the credibility order of agent A. Let $s \in Bel(\mathcal{K}_A)$ and let $[s, \mathbb{C}]$ be the forwarded object computed by A using PSC and $[s, \mathbb{C}']$ be the forwarded object computed by A using PRC. If $<^A_{co} = \emptyset$ and $s \notin (Bel(\mathcal{K}_A) \backslash Exp(\mathcal{K}_A))$, then $\mathbb{C} = \mathbb{C}'$.*

Proof Suppose that $s \in Bel(\mathcal{K}_A)$ and $s \notin (Bel(\mathcal{K}_A) \backslash Exp(\mathcal{K}_A))$, $[s, \mathbb{C}]$ is computed by A using *PSC*, $[s, \mathbb{C}']$ is computed by A using *PRC* and $<^A_{co} = \emptyset$. Since $s \in Bel(\mathcal{K}_A)$ and $s \notin (Bel(\mathcal{K}_A) \backslash Exp(\mathcal{K}_A))$ each $\mathcal{P}_i \in \Pi(s, \mathcal{K}_A)$ is a set containing only one information object. Let $\mathbb{I} = \{X : X = Ag(\mathcal{P}_i)$ where $\mathcal{P}_i \in \Pi(s, \mathcal{K}_A)\}$, then for all $D, E \in \mathbb{I}$, $D \not<^A_{co} E$ and $E \not<^A_{co} D$. Therefore, by Definition 6, $pl_{<^A_{co}}(s, \mathcal{K}_A) = \mathbb{I}$. Thus, following *PSC*, $\mathbb{C} = \{\{X\} : X \in \mathbb{I}\}$. Then, since $\mathbb{C}' = \{\{X\} : X = Ag(\mathcal{P}_i)$ where $\mathcal{P}_i \in \Pi(s, \mathcal{K}_A)\}$ by *PRC*, $\mathbb{C} = \mathbb{C}'$. □

Proposition 4 *Let $A \in \mathbb{A}$ and $[s, \mathbb{C}]$ be a forwarded object computed by A using PSC. If $<^A_{co}$ is a total order, then $\mathbb{C}$ contains just one singleton.*

Proof Following *PSC* it holds that any element in $\mathbb{C}$ is a singleton. Then, if $[s, \mathbb{C}]$ was computed by A using the *PSC*, $\mathbb{C}$ was computed using the $max_{<^A_{co}}$ function over a set of agent identifiers $\mathbb{I}$. Thus, if $<^A_{co}$ is totally ordered, it holds that $max_{<^A_{co}}(\mathbb{I})$ returns a set with one element. □

Proposition 5 *Let A and R be two agents in $\mathbb{A}$ and $s \in Exp(\mathcal{K}_A)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using ESC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using ERC, with $\langle s, \mathbb{C}'\rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}'\rangle$ the incoming object for M_2. If $s \in Bel(\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle)$ then $s \in Bel(\mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle)$.*

Proof Let us suppose that $M_1 = (A, R, [s, \mathbb{C}])$ and $M_2 = (A, R, [s, \mathbb{D}])$ are messages computed by A using ESC and ERC, respectively. Also, let us assume that $s \in \mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle$. If $\neg s \notin Bel(\mathcal{K}_R)$, then $s \in \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$ by Definition 11. If $\neg s \in Bel(\mathcal{K}_R)$, then $\langle s, \mathbb{C}'\rangle \rhd_R \mathcal{K}_R$ by Definition 11. Thus, following Definition 9, there exists a set of agent identifiers $\mathbb{H} \subseteq \mathbb{C}'$ such that for each $Y \in pl_{<^R_{co}}(\neg s, \mathcal{K}_R)$, there exists some $X \in \mathbb{H}$ where $Y <^R_{co} X$. For all $Z \in \mathbb{H}$, since $s \in Exp(\mathcal{K}_A)$, by Definition 8, $\mathbb{C} \neq \emptyset$ and $Z \in pr_{<^R_{co}}(\mathbb{C})$. Then, following ESC and Definition 7, $\{Z\} \in \mathbb{C}$ and $(s, Z) \in \mathcal{K}_A$. Thus, by ERC, $\{Z\} \in \mathbb{D}$. If $Z \in pr_{<^R_{co}}(\mathbb{D})$, then $s \in \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$ due to $Z \in \mathbb{H}$. If $Z \notin pr_{<^R_{co}}(\mathbb{D})$, then there exists $W \in pr_{<^R_{co}}(\mathbb{D})$ such that $Z <^R_{co} W$. Then, $Y <^R_{co} W$ for each $Y \in pl_{<^R_{co}}(\neg s, \mathcal{K}_R)$ such that $Y <^R_{co} Z$. Then, $s \in \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$. □

Proposition 6 *Let A and R be two agents in $\mathbb{A}$ and $s \in Exp(\mathcal{K}_A)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using ERC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using PRC, with $\langle s, \mathbb{C}'\rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}'\rangle$ the incoming object for M_2. If $s \in Bel(\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle)$, then $s \in Bel(\mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle)$.*

Proof Let us suppose that $M_1 = (A, R, [s, \mathbb{C}])$ and $M_2 = (A, R, [s, \mathbb{D}])$ are messages computed by A using ERC and PRC, respectively. Also, let us assume that $s \in \mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle$. If $\neg s \notin Bel(\mathcal{K}_R)$, then $s \in \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$ by Definition 11. If $\neg s \in Bel(\mathcal{K}_R)$, then $\langle s, \mathbb{C}'\rangle \rhd_R \mathcal{K}_R$ by Definition 11. Thus, following Definition 9, there exists a set of agent identifiers $\mathbb{H} \subseteq \mathbb{C}'$ such that for each $Y \in pl_{<^R_{co}}(\neg s, \mathcal{K}_R)$, there exists some $X \in \mathbb{H}$ where $Y <^R_{co} X$. For all $Z \in \mathbb{H}$, since $s \in Exp(\mathcal{K}_A)$, by Definition 8, $\mathbb{C} \neq \emptyset$ and $Z \in pr_{<^R_{co}}(\mathbb{C})$. Then, following ERC and Definition 7, $\{Z\} \in \mathbb{C}$ and $(s, Z) \in \mathcal{K}_A$. Then, $\{(s, Z)\} \in \Pi(s, \mathcal{K}_A)$ by Definition 2. Thus, by PRC, $\{Z\} \in \mathbb{D}$. If $Z \in pr_{<^R_{co}}(\mathbb{D})$, then $s \in \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$ due to $Z \in \mathbb{H}$. If $Z \notin pr_{<^R_{co}}(\mathbb{D})$, then there exists $W \in pr_{<^R_{co}}(\mathbb{D})$ such that $Z <^R_{co} W$. Then, $Y <^R_{co} W$ for each $Y \in pl_{<^R_{co}}(\neg s, \mathcal{K}_R)$ such that $Y <^R_{co} Z$. Then, $s \in \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$. □

Proposition 7 *Let A and R be two agents in $\mathbb{A}$ and $s \in Bel(\mathcal{K}_A)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using ESC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using ERC, with $\langle s, \mathbb{C}'\rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}'\rangle$ the incoming object for M_2. If $<^A_{co} = <^R_{co}$, then $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$.*

Proof Let us suppose that $M_1 = (A, R, [s, \mathbb{C}])$ and $M_2 = (A, R, [s, \mathbb{D}])$ are messages computed by A using ESC and ERC, respectively. If $s \notin Exp(\mathcal{K}_A)$, then $\mathbb{C} = \mathbb{D} = \emptyset$ by ESC and ERC. Then, following Definition 8, the incoming object $\langle s, \mathbb{C}'\rangle$ for M_1 and the incoming object $\langle s, \mathbb{D}'\rangle$ for M_2 are such that $\mathbb{C}' = \mathbb{D}' = \{A\}$. Therefore, it holds that $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$ by Definition 11. If $s \in Exp(\mathcal{K}_A)$, then the elements of $\mathbb{D}$ are singletons by ERC. Following ESC and ERC it holds that $\mathbb{C} = \{\{X\} : X \in max_{<^A_{co}}(\mathbb{I})\}$ where $\mathbb{I} = \{Y : \{Y\} \in \mathbb{D}\}$. Then, since $<^A_{co} = <^R_{co}$, it holds $pr_{<^R_{co}}(\mathbb{C}) = pr_{<^R_{co}}(\mathbb{D})$. Thus, by Definition 8, the incoming object $\langle s, \mathbb{C}'\rangle$ for M_1 and the incoming object $\langle s, \mathbb{D}'\rangle$ for M_2 are such that $\mathbb{C}' = \mathbb{D}'$. Therefore, it holds that $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$ by Definition 11. □

Proposition 8 *Let A and R be two agents in $\mathbb{A}$ and $s \in Bel(\mathcal{K}_R)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using PSC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using PRC, with $\langle s, \mathbb{C}'\rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}'\rangle$ the incoming object for M_2. If $<^A_{co} = <^R_{co}$ then $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$.*

Proof Let us suppose that $M_1 = (A, R, [s, \mathbb{C}])$ and $M_2 = (A, R, [s, \mathbb{D}])$ are messages computed by A using *PSC* and *PRC*, respectively. Following *PSC*, $\mathbb{C} = \{\{Y\} : Y \in pl_{<^A_{co}}(s, \mathcal{K}_A)\}$. Thus, by Definition 6, it holds that

$$\mathbb{C} = \{\{Y\} : Y \in max_{<^A_{co}}(\bigcup_{\mathcal{X} \in \Pi(s, \mathcal{K}_A)} min_{<^A_{co}}(Ag(\mathcal{X})))\}.$$

Then, by *PRC*, $\mathbb{D} = \{\mathbb{Y} : \mathbb{Y} = Ag(\mathcal{X})$ where $\mathcal{X} \in \Pi(s, \mathcal{K}_A)\}$. Therefore, $\mathbb{C} = \{\{Y\} : Y \in max_{<^A_{co}}(\bigcup_{\mathbb{X} \in \mathbb{D}} min_{<^A_{co}}(\mathbb{X}))\}$. Following Definition 5, all agent identifiers contained in the singletons from $\mathbb{C}$ are incomparable according to A. Then, by Definition 7 and since $<^A_{co} = <^R_{co}$, $pr_{<^R_{co}}(\mathbb{C}) = \{Y : Y \in max_{<^R_{co}}(\bigcup_{\mathbb{X} \in \mathbb{C}} min_{<^R_{co}}(\mathbb{X}))\} = \{Y : \{Y\} \in \mathbb{C}\} = pr_{<^R_{co}}(\mathbb{D})$. Thus, by Definition 8, the incoming object $\langle s, \mathbb{C}'\rangle$ for M_1 and the incoming object $\langle s, \mathbb{D}'\rangle$ for M_2 are such that $\mathbb{C}' = \mathbb{D}'$. Hence, it holds that $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$. □

Proposition 9 *Let A and R be two agents in $\mathbb{A}$ and $s \in Bel(\mathcal{K}_R)$. Let $M_1 = (A, R, [s, \mathbb{C}])$ be a message computed by A using ERC and $M_2 = (A, R, [s, \mathbb{D}])$ be a message computed by A using PRC, with $\langle s, \mathbb{C}'\rangle$ the incoming object for M_1 and $\langle s, \mathbb{D}'\rangle$ the incoming object for M_2. If $s \notin (Bel(\mathcal{K}_A) \setminus Exp(\mathcal{K}_A))$, then $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$.*

Proof Let us suppose that $M_1 = (A, R, [s, \mathbb{C}])$ and $M_2 = (A, R, [s, \mathbb{D}])$ are messages computed by A using *ERC* and *PRC*, respectively. Lets also assume that $s \notin (Bel(\mathcal{K}_A) \setminus Exp(\mathcal{K}_A))$. Since $s \in Bel(\mathcal{K}_A)$ and $s \notin (Bel(\mathcal{K}_A) \setminus Exp(\mathcal{K}_A))$, each $\mathcal{P}_i \in \Pi(s, \mathcal{K}_A)$ is a set containing only one information object. Then, $\mathbb{D} = \{\mathbb{Y} : \mathbb{Y} = Ag(\mathcal{P}_i)\}$ by *PRC* where $\mathbb{Y}$ is a singleton. Let $\mathbb{I} = \{X : (s, X) \in \mathcal{K}_A\}$, then following *ERC*, $\mathbb{C} = \{\{Y\} : Y \in \mathbb{I}\}$. Thus, it holds that $\mathbb{C} = \mathbb{D}$. Therefore, we have that, by Definition 7, $pr_{<^R_{co}}(\mathbb{C}) = pr_{<^R_{co}}(\mathbb{D})$ holds. Thus, by Definition 8, the incoming object $\langle s, \mathbb{C}'\rangle$ for M_1 and the incoming object $\langle s, \mathbb{D}'\rangle$ for M_2 are such that $\mathbb{C}' = \mathbb{D}'$. Hence, it holds that $\mathcal{K}_R \circledcirc^R \langle s, \mathbb{C}'\rangle = \mathcal{K}_R \circledcirc^R \langle s, \mathbb{D}'\rangle$. □

Proposition 10 *Let $\mathcal{K}_D$ be the belief base of agent D, $s \in Bel(\mathcal{K}_D)$, $M = (S, D, [s, \mathbb{C}])$ a message computed using one of the criteria ESC, ERC, PSC, or PRC, $\langle s, \mathbb{C}'\rangle$ the incoming object for M, $\mathcal{K}'_D = \mathcal{K}_D \circledcirc^D \langle s, \mathbb{C}'\rangle$, and $s \in Bel(\mathcal{K}'_D)$. It holds that for each $A \in pl_{<^D_{co}}(s, \mathcal{K}_D)$ and each $A' \in pl_{<^D_{co}}(s, \mathcal{K}'_D)$, either $A = A'$, $A \sim^D_{co} A'$, or $A <^D_{co} A'$.*

Proof Let $\alpha \in Bel(\mathcal{K}_D)$ and $ISet = \{(\alpha, A_i) : A_i \in \mathbb{C}'\}$. If $\mathcal{K}'_D = \mathcal{K}_D \cup ISet$, then $\Pi(\alpha, \mathcal{K}_D) \subseteq \Pi(\alpha, \mathcal{K}'_D)$. Thus, by Definition 4,

$$\bigcup_{\mathcal{X} \in \mathrm{P}} min_{<^D_{co}}(Ag(\mathcal{X})) \subseteq \bigcup_{\mathcal{Y} \in \mathrm{P}'} min_{<^D_{co}}(Ag(\mathcal{Y}))$$

with $\mathrm{P} = \Pi(\alpha, \mathcal{K}_D)$ and $\mathrm{P}' = \Pi(\alpha, \mathcal{K}'_D)$. Then, by Definition 6, two cases arise:

- if $A' \in pl_{<^D_{co}}(\alpha, \mathcal{K}'_D)$ and $A' \in pl_{<^D_{co}}(\alpha, \mathcal{K}_D)$, then for every $A \neq A'$ with $A \in pl_{<^D_{co}}(\alpha, \mathcal{K}_D)$, we have that $A' \sim^D_{co} A$.
- if $A' \in pl_{<^D_{co}}(\alpha, \mathcal{K}'_D)$ and $A' \notin pl_{<^D_{co}}(\alpha, \mathcal{K}_D)$, then for every $A \neq A'$ with $A \in pl_{<^D_{co}}(\alpha, \mathcal{K}_D)$, it holds that $A' \not<^D_{co} A$ since both belong to $\bigcup_{\mathcal{Y} \in \mathrm{P}'} min_{<^D_{co}}(Ag(\mathcal{Y}))$ and $A' \in pl_{<^D_{co}}(\alpha, \mathcal{K}_D)$. Thus, either $A <^D_{co} A'$ or $A' \sim^D_{co} A$.

Proposition 11 *Let $\mathcal{K}_D$ be the belief base of agent D, $\neg s \in Bel(\mathcal{K}_D)$, $M = (S, D, [s, \mathbb{C}])$ a message computed using one of the criteria ESC, ERC, PSC, or PRC, $\langle s, \mathbb{C}'\rangle$ the incoming object for M, $\mathcal{K}'_D = \mathcal{K}_D \circledcirc^D \langle s, \mathbb{C}'\rangle$, and $s \in Bel(\mathcal{K}'_D)$. It holds that for each $A \in pl_{<^D_{co}}(\neg s, \mathcal{K}_D)$, there exists $A' \in pl_{<^D_{co}}(s, \mathcal{K}'_D)$ such that $A <^D_{co} A'$.*

Proof Since $\neg\alpha \in \mathcal{K}_D$ and $\alpha \in \mathcal{K}'_D$, then it holds that $\langle \alpha, \mathbb{C}' \rangle \rhd_D \mathcal{K}_D$ by Definition 11. Thus, following Definition 9, for each $A \in pl_{<^D_{co}}(\neg\alpha, \mathcal{K}_D)$, there is an $A' \in \mathbb{C}'$ such that it holds that $A <^D_{co} A'$. In addition, by Definition 11, it is holds that $\{(\alpha, I) : I \in \mathbb{C}'\} \subseteq \mathcal{K}'_D$. In particular, by Definitions 2 and 11, it holds that $\Pi(\alpha, \mathcal{K}'_D) = \{\{(\alpha, A')\} : A' \in \mathbb{C}'\}$. By Definition 8, it holds that every $A', A'' \in \mathbb{C}'$ are such that $A' \sim^D_{co} A''$. Thus, by Definition 6, $pl_{<^D_{co}}(\alpha, \mathcal{K}'_D) = \mathbb{C}'$ holds. Then, for each $A \in pl_{<^D_{co}}(\neg\alpha, \mathcal{K}_D)$, there is an $A' \in pl_{<^D_{co}}(\alpha, \mathcal{K}'_D)$ such that $A <^D_{co} A'$. □

Proposition 12 *Let $\mathcal{K}_D$ be the belief base of agent D, $(t, A) \in \mathcal{K}_D$, $M = (S, D, [s, \mathbb{C}])$ a message computed using one of the criteria ESC, ERC, PSC, or PRC, $\langle s, \mathbb{C}' \rangle$ the incoming object for M, $\mathcal{K}'_D = \mathcal{K}_D \circledcirc^D \langle s, \mathbb{C}' \rangle$, and $s \in Bel(\mathcal{K}'_D)$. If $(t, A) \notin \mathcal{K}'_D$, then there is an $A' \in \mathbb{C}'$ such that $A <^D_{co} A'$.*

Proof Since $(\gamma, A) \in \mathcal{K}_D$ and $(\gamma, A) \notin \mathcal{K}'_D$, by Definition 11, it holds that $\neg\alpha \in Bel(\mathcal{K}_D)$ and $\langle \alpha, \mathbb{C}' \rangle \rhd_D \mathcal{K}_D$. Following Definition 11, given that $(\gamma, A) \notin \mathcal{K}'_D$, it holds that $(\gamma, A) \in least_{<^D_{co}}(\mathcal{Z})$ for some $\mathcal{Z} \in \Pi(\neg\alpha, \mathcal{K}_D)$. Then, by Definition 4, it holds that $A \in min_{<^D_{co}}(\mathcal{Z})$. Then, by Definition 6, we have two cases:

- if $A \in pl_{<^D_{co}}(\neg\alpha, \mathcal{K}_D)$, then, since $\langle \alpha, \mathbb{C}' \rangle \rhd_D \mathcal{K}_D$, there is a $A' \in \mathbb{C}'$ such that $A <^D_{co} A'$,
- if $A \notin pl_{<^D_{co}}(\neg\alpha, \mathcal{K}_D)$, then there is an $A'' \in pl_{<^D_{co}}(\neg\alpha, \mathcal{K}_D)$ such that $A <^D_{co} A''$. Given that $\langle \alpha, \mathbb{C}' \rangle \rhd_D \mathcal{K}_D$, there is a $A' \in C'$ such that $A'' <^D_{co} A'$. Then, it holds that $A <^D_{co} A'$.

□

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Compliance with ethical standards

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