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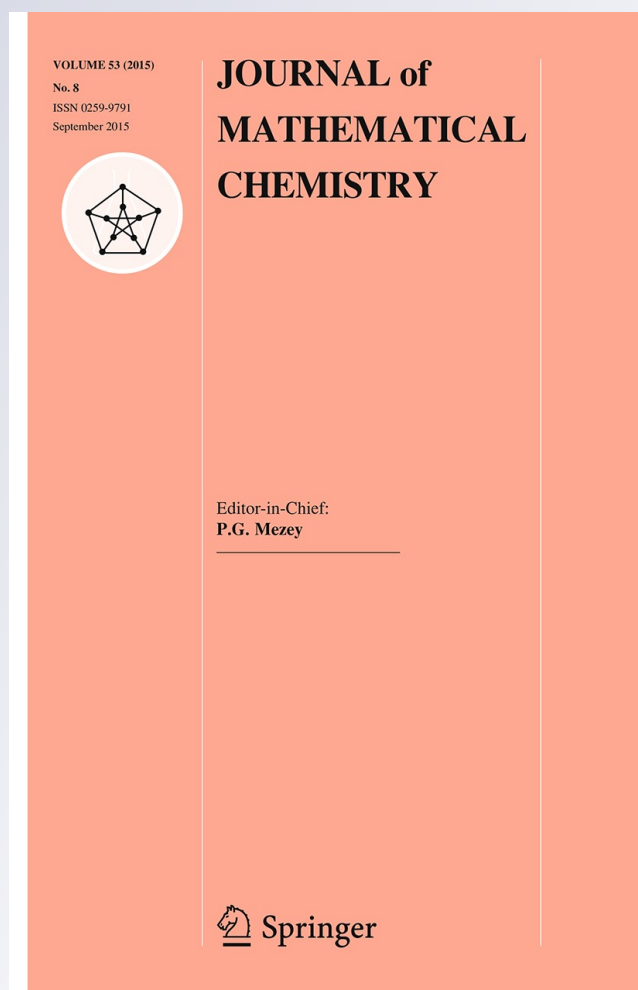
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About the crossing of the energy levels of a parameter-dependent quantum-mechanical Hamiltonian

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Abstract The purpose of this letter is to correct the lack of generality of one of the statements put forward in a recent article about the noncrossing rule in quantum mechanics. We illustrate the point by means of an exactly solvable model.

Keywords Avoided crossings · Noncrossing rule · Parameter-dependent operators

In a recent paper [1] we analyzed the avoided crossings between pairs of eigenvalues of parameter-dependent Hamiltonian operators and considered the well known fact that eigenvalues of the same symmetry do not cross. Typically two such eigenvalues $E_m(\lambda)$ and $E_n(\lambda)$ approach each other when $\lambda < \lambda_0$ and move apart when $\lambda > \lambda_0$ in such way that they appear to repel each other. We argued that under such conditions there is either $\lambda = \lambda_m$ such that $E'_m(\lambda_m) = 0$ or there is $\lambda = \lambda_n$ such that $E'_n(\lambda_n) = 0$ or both. Here we want to point out that this statement may only be valid when the slopes $E'_m(\lambda)$ and $E'_n(\lambda)$ have opposite signs for $\lambda < \lambda_0$. In general, we speak of an avoided crossing when $[E_m(\lambda) - E_n(\lambda)]^2$ exhibits a minimum nonzero value at some point $\lambda = \lambda_0$. The correct condition should therefore be

$$E'_m(\lambda_0) = E'_n(\lambda_0). \quad (1)$$

As a trivial illustrative example, consider the Hamiltonian matrix

$$\mathbf{H} = \begin{pmatrix} 1 + \lambda & 0.1 \\ 0.1 & -1 + 3\lambda \end{pmatrix}. \quad (2)$$

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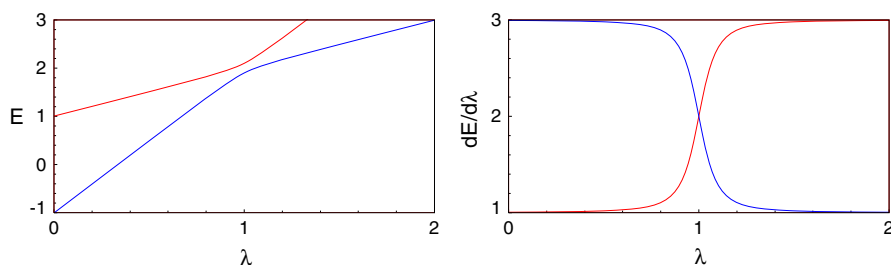


Fig. 1 Eigenvalues $E(\lambda)$ of the Hamiltonian matrix (2) (*left panel*) and their derivatives (*right panel*)

Its eigenvalues exhibit an avoided crossing at $\lambda_0 = 1$ as shown in Fig. 1 and their slopes never vanish. The right panel clearly shows that the slopes are always positive and become equal at $\lambda = 1$ in agreement with Eq. (1).

A nontrivial example with most interesting avoided crossings that illustrate what has just been said was recently discussed elsewhere [2].

References

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