PD-like controller with impedance for delayed bilateral teleoperation of mobile robots

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Abstract—**This paper proposes a control scheme applied to the delayed bilateral teleoperation of mobile robots with force feedback in face of asymmetric and time-varying delays. The scheme is managed by a velocity PD-like control plus impedance and a force feedback based on damping and synchronization error. A fictitious force, depending on the robot motion and its environment, is used to avoid possible collisions. In addition, the stability of the system is analyzed from which simple conditions for the control parameters are established in order to assure stability. Finally, the performance of the delayed teleoperation system is shown through experiments where a human operator drives a mobile robot. .**

***Index Terms*—bilateral teleoperation, fictitious force, force feedback, mobile robot, PD-like controller, time delay.**

I. INTRODUCTION

ROBOT bilateral teleoperation allows us to carry out different tasks from a distant workstation [8]. In force-feedback robot teleoperation systems, a user drives a robot while simultaneously feels through a master device the closeness of objects in the remote environment. There are many applications for robot teleoperation, including telemedicine, exploration, entertainment, tele-services, tele-manufacturing, among other [17]. Additionally, the use of the Internet as a low-cost communication channel increases the applications of these systems. However, the presence of time-delays can induce instability or low performance [2,7,9] as well as a poor transparency [20].

There are many control schemes for standard teleoperation between two manipulators with time delay [2]. For example, [1] proposed to send the scattering signals to transform the transmission delays into a passive transmission line. In [4] and [5], wave transformations are used to keep the passivity of the two-port channel in front of time delay. These strategies inject the so called apparent damping. In [16], a simple PD-like scheme, that does not require scattering or wave variable transformations, yields a stable operation including the position coordination. From this, [6] and [18] proved asymptotic stability of PD-like schemes by using a sufficiently large damping injected into the master and slave for the case of constant delays and asymmetric time-varying delays, respectively. Recently, in [24] a reaction model of the human operator is included in the teleoperation system to decrease the necessary damping to achieve stability.

On the other hand, the state of the art for delayed mobile robot teleoperation generally will inherit the mentioned strategies; but, many times the mismatch between the models of the master and slave adds problems in the theoretical analysis, for example, if the master does not move, the mobile robot generally goes at a constant speed. The proposed strategies include compensation based on a human operator model [10], control based on virtual-mass plus wave variable[19], event-based control [3], signals fusion [21], control based on variable force-feedback gain [11], control based on virtual force [12], impedance control [13], control based on the r-passivity [14,15,16]. Recently, the concept of absolute transparency was proposed for bilateral teleoperation of wheeled robots in order to analyze such feature [22].

This paper proposes a control scheme for the delayed bilateral teleoperation of wheeled robots; the system includes a velocity PD-like control plus impedance at the remote site, and damping and a force feedback based on the synchronization error at the local site. This work considers the dynamics of the master and mobile robot as well as time-varying and asymmetric delays. Furthermore, the controller is evaluated from tests where a human operator drives via network a real mobile robot in order to follow a reference path avoiding collisions with the objects present in the environment. These experiments are made to verify the achieved theoretical analysis.

The paper is organized as follows: Section II presents some preliminary aspects such as the master and slave dynamic models and properties, assumptions and lemmas. In Section III a control scheme applied to bilateral teleoperation of unicycle-like wheeled robots is proposed. The stability analysis based on Lyapunov-Krasovskii functionals is carried out in Section IV. Section V shows experimental results, where a user drives a wheeled robot; and finally, in section VI, the conclusions of this work are given.

II. PRELIMINARY

This paper will analyze teleoperation systems in which a human operator drives a wheeled robot while he or she feels the environment near the robot through visual and force feedback. For example, the user could feel the closeness of an object respect to the mobile robot, which is remotely driven using velocity commands generated by the master position.

Notation: We use standard notations throughout the paper. If x is a scalar, $\mathbf{x}$ is a vector and $\mathbf{X}$ is a matrix, then, $|x|$ is the absolute value of x, $\mathbf{x}^T$ is the transpose of the vector, $\mathbf{X}^T$ is the transpose of the matrix, $|\mathbf{x}|$ is the Euclidean norm of $\mathbf{x}$, $|\mathbf{X}|$ is the induced norm of $\mathbf{X}$, $\mathbf{X} > 0$ ($\mathbf{X} < 0$) means that $\mathbf{X}$ is positive definite (negative definite), and $\lambda_{\min}(\mathbf{X})$ and $\lambda_{\max}(\mathbf{X})$ represent the minimum and maximum eigenvalue of matrix $\mathbf{X}$, respectively. In addition, $\|\mathbf{x}\|_1$ and $\|\mathbf{x}\|_2$ represent the L1-norm and L2-norm of $\mathbf{x}$, respectively.

The typical nonlinear dynamic model to represent the master or local device is used, that is,

$$\mathbf{M_m}(\mathbf{q_m})\ddot{\mathbf{q}}_\mathbf{m} + \mathbf{C_m}(\mathbf{q_m}, \dot{\mathbf{q}}_\mathbf{m})\dot{\mathbf{q}}_\mathbf{m} + \mathbf{g_m}(\mathbf{q_m}) = \boldsymbol{\tau}_\mathbf{m} + \mathbf{f_h} \tag{1}$$

where $\mathbf{q_m}(t) \in R^{n\times 1}$ is the joint position of the master, $\dot{\mathbf{q}}_\mathbf{m}(t)$ is the joint velocity, $\mathbf{M_m}(\mathbf{q_m}) \in R^{n\times n}$ is the inertia matrix, $\mathbf{C_m}(\mathbf{q_m}, \dot{\mathbf{q}}_\mathbf{m}) \in R^{n\times n}$ is the matrix representing centripetal and Coriolis torques, $\mathbf{g_m}(\mathbf{q_m}) \in R^{n\times 1}$ is the gravitational torque, $\mathbf{f_h} \in R^{n\times 1}$ is the torque caused by the human operator force and $\boldsymbol{\tau}_\mathbf{m} \in R^{n\times 1}$ is the control torque applied to the master.

For the case of wheeled robot teleoperation, the dynamic model of a unicycle-type mobile robot is considered [15]. It has two independently actuated rear wheels and is represented by,

$$\mathbf{D}\dot{\boldsymbol{\eta}} + \mathbf{Q}(\boldsymbol{\eta})\boldsymbol{\eta} = \boldsymbol{\tau}_\mathbf{s} + \mathbf{f_e} \tag{2}$$

where $\boldsymbol{\eta} = \begin{bmatrix} v \\ \omega \end{bmatrix}$ is the robot velocity vector with v and ω representing the linear and angular velocity of the mobile robot, $\mathbf{f_e} \in R^{n\times 1}$ is the force caused by the elements of the environment on the robot, $\mathbf{D} = \begin{bmatrix} m & 0 \\ 0 & i \end{bmatrix}$ is the inertia matrix and $\mathbf{Q} = \begin{bmatrix} 0 & -ma\omega \\ ma\omega & 0 \end{bmatrix}$ is the Coriolis matrix. m is the mass of the robot, i is the rotational inertia, and a is the distance between the mass center and the geometric center. In addition, $\boldsymbol{\tau}_\mathbf{s} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ involves a control force u_1 and a control torque u_2, with $u_1 = \frac{1}{r_w}(u_{left} + u_{right})$ and $u_2 = \frac{c}{r_w}(u_{right} - u_{left})$ where $r_w > 0$ is the radius of the wheels, $c > 0$ is the half-width of the cart, and u_{left} and u_{right} are the torques of the left and right rear wheels respectively. Furthermore, the communication channel adds a forward time delay h_1 (from the master to the slave) and a backward time delay h_2 (from the slave to the master). Generally, these delays are time-varying and different between them (asymmetric delays).

On the other hand, the following ordinary properties, assumptions and lemmas will be used in this paper [6,18]:

Property 1: The inertia matrices $\mathbf{M_m}(\mathbf{q_m})$ and $\mathbf{D}$ are symmetric positive definite. The matrix $\mathbf{D}$ is assumed constant.

Property 2: The matrix $\dot{\mathbf{M}}_\mathbf{m}(\mathbf{q_m}) - 2\mathbf{C_m}(\mathbf{q_m}, \dot{\mathbf{q}}_\mathbf{m})$ is skew-symmetric.

Property 3: There exists a $k_r > 0$ such that $\|\mathbf{C_m}(\mathbf{q_m}, \dot{\mathbf{q}}_\mathbf{m})\dot{\mathbf{q}}_\mathbf{m}\| \leq k_r |\dot{\mathbf{q}}_\mathbf{m}|$ for all t.

Assumption 1: The time delays $h_1(t)$ and $h_2(t)$ are bounded. Therefore, there exist positive scalars $\bar{h}_1$ and $\bar{h}_2$ such that $0 \leq h_1(t) \leq \bar{h}_1$ and $0 \leq h_2(t) \leq \bar{h}_2$ for all t.

Assumption 2: The human operator and the environment behave in a non-passive way and they are represented by the following models,

$$\mathbf{f_h} = -\alpha_h \dot{\mathbf{q}}_\mathbf{m} + \mathbf{f}_{a_h} \tag{3}$$

$$\mathbf{f_e} = -\alpha_e \boldsymbol{\eta} + \mathbf{f}_{a_e} \tag{4}$$

where α_h is the damping of the human operator model, and α_e is the environment's damping (passive components). On the other hand, $\mathbf{f}_{a_h}$ and $\mathbf{f}_{a_e}$ are the non-passive components of them which we assumed bounded, that is $|\mathbf{f}_{\mathbf{a}_h}| \leq \bar{f}_{a_h}$ and $|\mathbf{f}_{\mathbf{a_e}}| \leq \bar{f}_{a_e}$, with $\bar{f}_{a_h}$ and $\bar{f}_{a_e}$ positive constants.

Lemma 1 [18]**:** For real vector functions $\mathbf{a}(.)$ and $\mathbf{b}(.)$ and a time-varying scalar $h(t)$ with $0 \leq h(t) \leq \bar{h}$, the following inequality holds,

$$-2\mathbf{a}^T(t)\int_{t-h(t)}^{t}\mathbf{b}(\xi)d\xi - \int_{t-h(t)}^{t}\mathbf{b}^T(\xi)\mathbf{b}(\xi)d\xi \leq h(t)\mathbf{a}^T(t)\mathbf{a}(t) \leq \bar{h}(t)\mathbf{a}^T(t)\mathbf{a}(t) \tag{5}$$

In the next section, the control scheme will be introduced.

III. PD-LIKE CONTROLLER FOR TELEOPERATION

The PD-like controllers are simple structures that generally have a good performance in practice for common applications and are calibrated quickly. Lately, the performance of these schemes was evaluated for the position control in bilateral teleoperation systems of manipulator robots [6,18]. In these cases, if the damping of the master and slave are sufficiently big, then the stability is assured. If the damping increases, the system is better in terms of stability but the transparency is worst [18].

Here, the teleoperation system is used to control the velocity of a mobile robot, where the user permanently sends commands and perceives by means of force feedback the objects near the robot. The proposed control scheme, establish the control actions as follows,

$$\begin{cases} \boldsymbol{\tau}_{\mathbf{m}} = -k_m\left(k_g\mathbf{q}_{\mathbf{m}}(t) - \boldsymbol{\eta}(t-h_2)\right) - \alpha_m\dot{\mathbf{q}}_{\mathbf{m}} - k_p\mathbf{q}_{\mathbf{m}} + \mathbf{g}_{\mathbf{m}}(\mathbf{q}_{\mathbf{m}}) \\ \boldsymbol{\tau}_{\mathbf{s}} = k_s\left(k_g\mathbf{q}_{\mathbf{m}}(t-h_1) - \boldsymbol{\eta} - k_z\mathbf{f}_{\mathbf{v}}\right) - \sigma_s\mathbf{z} + \mathbf{Q}(\boldsymbol{\eta})\boldsymbol{\eta} \end{cases} \tag{6}$$

where the controller is formed by $\boldsymbol{\tau}_{\mathbf{m}}$ and $\boldsymbol{\tau}_{\mathbf{s}}$. The parameters k_s and σ_s are positive constant and they represent the proportional gain and acceleration-dependent damping added by the velocity controller, α_m and k_p are the damping and spring injected in the master, k_m represents a relative spring depending on the mismatch between the master reference and the mobile robot velocity, and k_z represents an impedance applied to a virtual force $\mathbf{f}_{\mathbf{v}}$ which is generated by the obstacles near the robot. Last force is assumed bounded by $|\mathbf{f}_{\mathbf{v}}| \le \bar{f}_v$ and will be described in Section V.

Besides, the parameter k_g linearly maps the master position to a velocity reference, and $\mathbf{z}$ is defined by,

$$\dot{\boldsymbol{\eta}} = \mathbf{z} + \gamma\dot{\mathbf{z}} \tag{7}$$

with $\gamma \to 0^+$. The signal $\mathbf{z}$ represents the mobile robot acceleration $\dot{\boldsymbol{\eta}}$ at an infinitesimal time instant before t, so it is assumed $\dot{\boldsymbol{\eta}} \approx \mathbf{z}$ considering $\dot{\mathbf{z}}$ without discontinuities.

Next, the stability of the delayed bilateral teleoperation system modeled by (1), (2), (3), (4), the communication channel and the PD-like controller (6) will be analyzed.

IV. Stability of the Delayed Closed-Loop System

The stability analysis is based on the theory of Lyapunov-Krasovskii [25]. It is important to remark that there is not an equilibrium point but a Krasovskii-like equilibrium solution that depends on the state in the time interval $[t-h_1-h_2, t]$. Now, we present the main result of this work as follows.

Theorem 1: Consider a delayed teleoperation system, where a human operator (3) using a master device (1) drives a remote mobile robot described by (2) and (7), interacting with an environment (4) and including the control law (6). For positive constant parameters k_m, k_s, k_g; considering Assumptions **1**, **2** and **3** and Properties **1**, **2** and **3**; if the control parameters α_m and σ_s are such that the following inequalities hold:

$$-\lambda_m = -\alpha_m + \bar{h}_1 + \frac{1}{4}\bar{h}_2 k_m{}^2 < 0$$

$$-\lambda_s = \frac{k_m}{k_s k_g}\left(-\sigma_s - |\mathbf{D}|\right) + \frac{1}{4}\bar{h}_1 \frac{k_m{}^2}{k_g{}^2} + \bar{h}_2 < 0$$

then the state defined by $\mathbf{x} := \left[\mathbf{q_m} \quad \dot{\mathbf{q}}_\mathbf{m} \quad k_g\mathbf{q_m} - \boldsymbol{\eta} \quad \mathbf{z} \quad \boldsymbol{\eta}\right] \in L_\infty$. In addition, the variables $\dot{\mathbf{q}}_\mathbf{m}$ and $\mathbf{z}$ are ultimately bounded with ultimate bound $\max\left\{\frac{\rho_m}{\lambda_m}, \frac{\rho_s}{\lambda_s}\right\}$, where $\rho_m = \bar{f}_{a_h}$ and $\rho_s = \frac{k_m}{k_s k_g}\left(\bar{f}_{a_e} + k_z \bar{f}_v\right)$.

Proof: First, a functional $\mathrm{V} = \mathrm{V}_1 + \mathrm{V}_2 + \mathrm{V}_3 + \mathrm{V}_4 + \mathrm{V}_5 + \mathrm{V}_6 > 0$ is proposed in order to analyze its evolution along the system's trajectories. The first five sub-functionals depend on the variables of $\mathbf{x}$ and are defined in the following manner:

$$\mathrm{V}_1 = \frac{1}{2}\dot{\mathbf{q}}_\mathbf{m}{}^\mathrm{T}\mathbf{M_m}(\mathbf{q_m})\dot{\mathbf{q}}_\mathbf{m} \tag{8}$$

$$\mathrm{V}_2 = \frac{1}{2}\frac{k_m}{k_g}\left(k_g\mathbf{q_m} - \boldsymbol{\eta}\right)^\mathrm{T}\left(k_g\mathbf{q_m} - \boldsymbol{\eta}\right) \tag{9}$$

$$\mathrm{V}_3 = \frac{1}{2}\alpha_e \frac{k_m}{k_s k_g}\boldsymbol{\eta}^{\mathbf{T}}\boldsymbol{\eta} \tag{10}$$

$$\mathrm{V}_4 = \frac{1}{2}\gamma\frac{k_m}{k_s k_g}\mathbf{z}^{\mathbf{T}}\mathbf{D}\mathbf{z} \tag{11}$$

$$\mathrm{V}_5 = \frac{1}{2}k_p \mathbf{q_m}^{\mathbf{T}}\mathbf{q_m} \tag{12}$$

The time derivative of V_1 (8) along the master dynamics (1), taking into account properties **1** and **2**, is the following one,

$$\begin{aligned}\dot{\mathrm{V}}_1 &= \frac{1}{2}\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\dot{\mathbf{M}}_{\mathbf{m}}\dot{\mathbf{q}}_{\mathbf{m}} + \dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\mathbf{M}_{\mathbf{m}}\ddot{\mathbf{q}}_{\mathbf{m}} \\ &= \frac{1}{2}\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\dot{\mathbf{M}}_{\mathbf{m}}\dot{\mathbf{q}}_{\mathbf{m}} + \\ &\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\mathbf{M}_{\mathbf{m}}\mathbf{M}_{\mathbf{m}}^{-1}\left(\boldsymbol{\tau}_{\mathbf{m}} + \mathbf{f}_{\mathbf{h}} - \mathbf{g}(\mathbf{q}_{\mathbf{m}}) - \mathbf{C}_{\mathbf{m}}\dot{\mathbf{q}}_{\mathbf{m}}\right) \\ &= \dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\left(\boldsymbol{\tau}_{\mathbf{m}} + \mathbf{f}_{\mathbf{h}} - \mathbf{g}(\mathbf{q}_{\mathbf{m}})\right)\end{aligned} \tag{13}$$

Now, if the control action $\boldsymbol{\tau}_{\mathbf{m}}$ of (6) is included in (13) considering also (3), it yields,

$$\begin{aligned}\dot{\mathrm{V}}_1 &= \dot{\mathbf{q}}_{\mathbf{m}}^{\mathrm{T}}\left(\boldsymbol{\tau}_{\mathbf{m}} - \mathbf{g}_{\mathbf{m}}(\mathbf{q}_{\mathbf{m}})\right) + \dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\mathbf{f}_{\mathbf{h}} \\ &= \dot{\mathbf{q}}_{\mathbf{m}}^{\mathrm{T}}\left(-k_m\left(k_g\mathbf{q}_{\mathbf{m}} - \boldsymbol{\eta}(t-h_2)\right) - \alpha_m\dot{\mathbf{q}}_{\mathbf{m}}\right) \\ &+ \dot{\mathbf{q}}_{\mathbf{m}}^{\mathrm{T}}\left(\mathbf{f}_{\mathbf{a_h}} - \alpha_h\dot{\mathbf{q}}_{\mathbf{m}} - k_p\mathbf{q}_{\mathbf{m}}\right) \\ &= -k_m\dot{\mathbf{q}}_{\mathbf{m}}^{\mathrm{T}}\left(\left(k_g\mathbf{q}_{\mathbf{m}} - \boldsymbol{\eta}(t-h_2) + \boldsymbol{\eta} - \boldsymbol{\eta}\right)\right) - (\alpha_m + \alpha_h)\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{m}} \\ &- k_h\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\mathbf{q}_{\mathbf{m}} + \dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\mathbf{f}_{\mathbf{a_h}} \\ &= -(\alpha_m + \alpha_h)\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{m}} - k_m\dot{\mathbf{q}}_{\mathbf{m}}^{\mathrm{T}}\left(k_g\mathbf{q}_{\mathbf{m}} - \boldsymbol{\eta}\right) \\ &- k_m\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\int_{t-h_2}^{t}\dot{\boldsymbol{\eta}}(\xi)d\xi - k_p\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\mathbf{q}_{\mathbf{m}} + \dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\mathbf{f}_{\mathbf{a_h}}\end{aligned} \tag{14}$$

Next, $\dot{\mathrm{V}}_2$ is obtained from (9) considering (7) $\mathbf{z} \approx \dot{\boldsymbol{\eta}}$ from (7), as follows,

$$\begin{aligned}\dot{\mathrm{V}}_2 &= \frac{k_m}{k_g}\left(k_g\mathbf{q}_{\mathbf{m}} - \boldsymbol{\eta}\right)^{\mathrm{T}}\left(k_g\dot{\mathbf{q}}_{\mathbf{m}} - \dot{\boldsymbol{\eta}}\right) \\ &\approx -\frac{k_m}{k_g}\left(k_g\mathbf{q}_{\mathbf{m}} - \boldsymbol{\eta}\right)^{\mathrm{T}}\mathbf{z} + k_m\left(k_g\mathbf{q}_{\mathbf{m}} - \boldsymbol{\eta}\right)^{\mathrm{T}}\dot{\mathbf{q}}_{\mathbf{m}}\end{aligned} \tag{15}$$

On the other hand, $\dot{\mathrm{V}}_3$ is computed from (10) taking into account $\mathbf{z} \approx \dot{\boldsymbol{\eta}}$ from (7), as follows,

$$\dot{\mathrm{V}}_3 = \alpha_e \frac{k_m}{k_s k_g} \boldsymbol{\eta}^{\mathbf{T}} \dot{\boldsymbol{\eta}} \approx \alpha_e \frac{k_m}{k_s k_g} \boldsymbol{\eta}^{\mathbf{T}} \mathbf{z} \tag{16}$$

Besides, $\dot{\mathrm{V}}_4$ along the mobile robot dynamics (2) can be written including (6) into the derivative of (11), in the following way,

$$\begin{aligned}
\dot{\mathrm{V}}_4 &= \gamma \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \mathbf{D} \dot{\mathbf{z}} = \gamma \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \mathbf{D} \left(\frac{\dot{\boldsymbol{\eta}}}{\gamma} - \frac{\mathbf{z}}{\gamma} \right) \\
&= \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \mathbf{D} \dot{\boldsymbol{\eta}} - \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \mathbf{D} \mathbf{z} \\
&= -\sigma_s \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \mathbf{z} + \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \left(\mathbf{f_e} + k_z \mathbf{f_v} \right) - \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \mathbf{D} \mathbf{z} \\
&\quad + \frac{k_m}{k_g} \mathbf{z}^{\mathbf{T}} \left(k_g \mathbf{q_m}(t - h_1) + \mathbf{q_m} - \mathbf{q_m} - \boldsymbol{\eta} \right) \\
&= -\sigma_s \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \mathbf{z} + \frac{k_m}{k_g} \mathbf{z}^{\mathbf{T}} \left(k_g \mathbf{q_m} - \boldsymbol{\eta} \right) - \alpha_e \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \boldsymbol{\eta} \\
&\quad + \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \left(\mathbf{f}_{\mathbf{a_e}} + k_z \mathbf{f_v} \right) - \frac{k_m}{k_s k_g} \mathbf{z}^{\mathbf{T}} \mathbf{D} \mathbf{z} - \frac{k_m}{k_g} \mathbf{z}^{\mathbf{T}} \int_{t-h_1}^{t} \dot{\mathbf{q}}_{\mathbf{m}}(\xi) d\xi
\end{aligned} \tag{17}$$

Furthermore, $\dot{\mathrm{V}}_5$ is obtained from (12) as follows,

$$\dot{\mathrm{V}}_5 = k_p \mathbf{q_m}^{\mathbf{T}} \dot{\mathbf{q}}_{\mathbf{m}} \tag{18}$$

It is possible to appreciate in (14) and (17) that there are terms with delayed variables that make the stability analysis difficult. To solve this, V_6 is proposed as follows:

$$\begin{aligned}
\mathrm{V}_6 &= \int_{-h_2}^{0} \int_{t+\theta}^{t} \mathbf{z}(\xi)^{\mathbf{T}} \mathbf{z}(\xi) d\xi d\theta \\
&\quad + \int_{-h_1}^{0} \int_{t+\theta}^{t} \dot{\mathbf{q}}_{\mathbf{m}}(\xi)^{\mathbf{T}} \dot{\mathbf{q}}_{\mathbf{m}}(\xi) d\xi d\theta
\end{aligned} \tag{19}$$

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From (19), and considering assumption **1**, $\dot{\mathrm{V}}_6$ is computed by,

$$\dot{V}_6 \leq \bar{h}_2 \mathbf{z}^{\mathbf{T}}\mathbf{z} - \int_{t-h_2}^{t} \mathbf{z}^{\mathbf{T}}(\xi)\mathbf{z}(\xi)d\xi + \bar{h}_1 \dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{m}} - \int_{t-h_1}^{t} \dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}(\xi)\dot{\mathbf{q}}_{\mathbf{m}}(\xi)d\xi \quad (20)$$

The terms with integrals in (20) can be linked with the third term of (14) and the sixth term of (17) by using Lemma **1** (5), and considering (7), it yields,

$$-\frac{k_m}{k_g}\mathbf{z}^{\mathbf{T}}\int_{t-h_1}^{t}\dot{\mathbf{q}}_{\mathbf{m}}(\xi)d\xi - \int_{t-h_1}^{t}\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}(\xi)\dot{\mathbf{q}}_{\mathbf{m}}(\xi)d\xi \leq \frac{1}{4}\bar{h}_1\frac{k_m{}^2}{k_g{}^2}\mathbf{z}^{\mathbf{T}}\mathbf{z} \quad (21)$$

$$\begin{aligned}&-\int_{t-h_2}^{t}\mathbf{z}^{\mathbf{T}}(\xi)\mathbf{z}(\xi)d\xi - k_m\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\int_{t-h_2}^{t}\dot{\boldsymbol{\eta}}(\xi)d\xi \\ &= -\int_{t-h_2}^{t}\mathbf{z}^{\mathbf{T}}(\xi)\mathbf{z}(\xi)d\xi - k_m\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\int_{t-h_2}^{t}\left(\mathbf{z}(\xi)+\gamma\,\dot{\mathbf{z}}(\xi)\right)d\xi \\ &\leq \frac{1}{4}\bar{h}_2 k_m{}^2\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{m}} - \gamma k_m\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\int_{t-h_2}^{t}\dot{\mathbf{z}}(\xi)d\xi\end{aligned} \quad (22)$$

In the last term of (22), the integral function is applied in a closed interval to $\dot{\mathbf{z}}(t)$ (assumed without discontinuities), so the function $\int_{t-h_2}^{t}\dot{\mathbf{z}}(\xi)d\xi$ has a maximum real value β that bounds such function and therefore holds $\left|\int_{t-h_2}^{t}\dot{\mathbf{z}}(\xi)d\xi\right| \leq h_2\beta$. From this, (22) can be re-written by,

$$\frac{1}{4}\bar{h}_2 k_m{}^2\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{m}} - \gamma k_m\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\int_{t-h_2}^{t}\dot{\mathbf{z}}(\xi)d\xi \leq \frac{1}{4}\bar{h}_2 k_m{}^2\dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{m}} + k_m\gamma\beta\bar{h}_2\left|\dot{\mathbf{q}}_{\mathbf{m}}\right| \quad (23)$$

That is, the terms with integrals were replaced by common quadratic terms. Finally, $\dot{V}$ can be built joining (14), (15), (16), (17), (18) and (20), considering $k_m\gamma\beta\bar{h}_2 \rightarrow 0$ and the relations (21) and (23) as follows,

$$
\begin{aligned}
\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 + \dot{V}_5 \\
&\le \dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}} \left(-\alpha_m \mathbf{I} + \bar{h}_1 \mathbf{I} + \frac{1}{4} \bar{h}_2 k_m^{\,2} \mathbf{I} \right) \dot{\mathbf{q}}_{\mathbf{m}} \\
&+ \mathbf{z}^{\mathbf{T}} \left(\frac{k_m}{k_s k_g} \left(-\sigma_s \mathbf{I} - \mathbf{D} \right) + \frac{1}{4} \bar{h}_1 \frac{k_m^{\,2}}{k_g^{\,2}} \mathbf{I} + \bar{h}_2 \mathbf{I} \right) \mathbf{z} \\
&+ \frac{k_m}{k_s k_g} \left(\bar{f}_{a_e} + k_z \bar{f}_v \right) |\mathbf{z}| + \bar{f}_{a_h} |\dot{\mathbf{q}}_{\mathbf{m}}| \\
&= -\lambda_m \dot{\mathbf{q}}_{\mathbf{m}}^{\mathbf{T}} \dot{\mathbf{q}}_{\mathbf{m}} - \lambda_s \mathbf{z}^{\mathbf{T}} \mathbf{z} + \rho_m |\dot{\mathbf{q}}_{\mathbf{m}}| + \rho_s |\mathbf{z}|
\end{aligned} \tag{24}
$$

Given positive constant parameters for k_m, k_s, and k_g, as well as bounded values for $\bar{h}_1$, $\bar{h}_2$, $\bar{f}_{a_e}$ and $\bar{f}_{a_h}$; the control parameters α_m and σ_s can be set to guarantee that the first two terms of (24) are negative definite and therefore the variables $\mathbf{q_m}, \dot{\mathbf{q}}_{\mathbf{m}}, k_g \mathbf{q_m} - \boldsymbol{\eta}, \mathbf{z}, \boldsymbol{\eta} \in L_\infty$. For this condition, it is possible to appreciate from (24) that the state variables $\dot{\mathbf{q}}_{\mathbf{m}}$ and $\mathbf{z}$ are ultimately bounded with ultimate bound $\max\left(\frac{\rho_m}{\lambda_m}, \frac{\rho_s}{\lambda_s} \right)$.The proof is completed.

Remark 1: If the active components of the human operator $\mathbf{f}_{\mathbf{a_h}}$, environment $\mathbf{f}_{\mathbf{a_e}}$ and fictitious force $\mathbf{f_v}$ are null ($\bar{f}_{a_h} = \bar{f}_{a_e} = \bar{f}_v = 0$), then $\rho_m = \rho_s = 0$ and therefore the system is stable. For this particular case, Barbalat's lemma can be used in (24); taking into account assumptions **1**, **2** and **3**, property **3** and the fact that $\mathbf{q_m}, \dot{\mathbf{q}}_{\mathbf{m}}, k_g \mathbf{q_m} - \boldsymbol{\eta}, \mathbf{z}, \boldsymbol{\eta} \in L_\infty$; it is possible to deduce that $\ddot{\mathbf{q}}_{\mathbf{m}}$ and $\dot{\mathbf{z}}$ are bounded and, consequently, $\ddot{V}$ is bounded too. Then, $\dot{\mathbf{q}}_{\mathbf{m}}$ and $\mathbf{z}$ will tend to zero as $t \to \infty$.

V. IMPEDANCE BASED ON FICTITIOUS FORCE

This section describes how the fictitious force $\mathbf{f_v}$ is computed in order to be used in $\boldsymbol{\tau}_{\mathbf{s}}$ (6). The virtual force $\mathbf{f_v}$ is calculated using robot motion prediction and the data provided by a 2D laser scanner. First, the path predicted for the mobile robot considering that and remain constant, is computed. Then, the 2D laser sensor gets n measurements of distance between the robot and the obstacles near it. Each measure is associated to one given direction, as it is shown in Fig. 1. From this, the points () closer to the positions on the robot's future path are obtained. The distance between and is named . Now, for each , the angle () measured from the center point of the circumference-type path of curvature radius defined by the robot velocity is obtained. Next, the distance along the path from the current position of the mobile robot to is calculated.

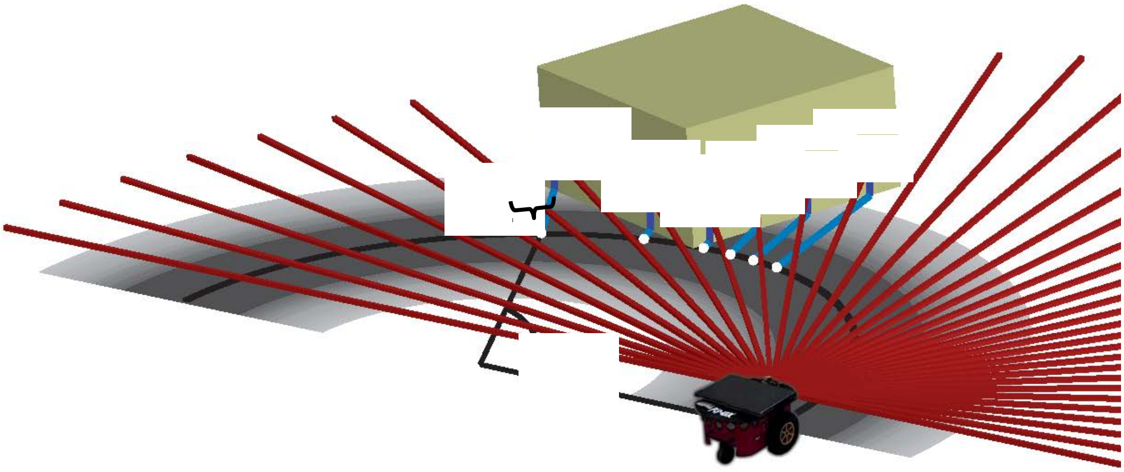

Fig. 1. Fictitious force based on 2D laser scanner and mobile robot motion.

Now, a weigh factor is defined depending on () as follows:

(25)

where is the robot width. If the robot will collide with the obstacle if it keeps its velocity vector. Otherwise is decreased as the points are further away from the path. Finally, the fictitious force $\mathbf{f}_\mathbf{v}$ vector $\mathbf{f}_\mathbf{v}$ is computed according to and of the following manner:

$$\mathbf{f}_\mathbf{v} = \frac{k}{n}\left[\sum_{i=1}^{n} p_i\left(s_{\max} - s_i\right) \quad 0\right] \tag{26}$$

where n is the quantity of measurements provided by the 2D laser scanner, $s_i = r(t)\frac{\pi}{2}$ and $k > 0$ is a gain to scale the force.

VI. Experimental results

In this section, the proposed control scheme is tested. In the experiments, a human operator drives a Pioneer 2DX mobile Robot employing a hand-controller with force feedback. The master device is a Novint Falcon running at 1 Khz. On the other hand, the mobile robot has incremental encoders in both DC motors and a Hokuyo 2D laser scanner to compute the fictitious force. In addition, a webcam onboard the robot is used to send visual feedback to the user. The layout of the experiment is shown in Fig. 2, where it is possible to appreciate the reference path (white line marked in the floor) established as goal for the delayed bilateral teleoperation system test.

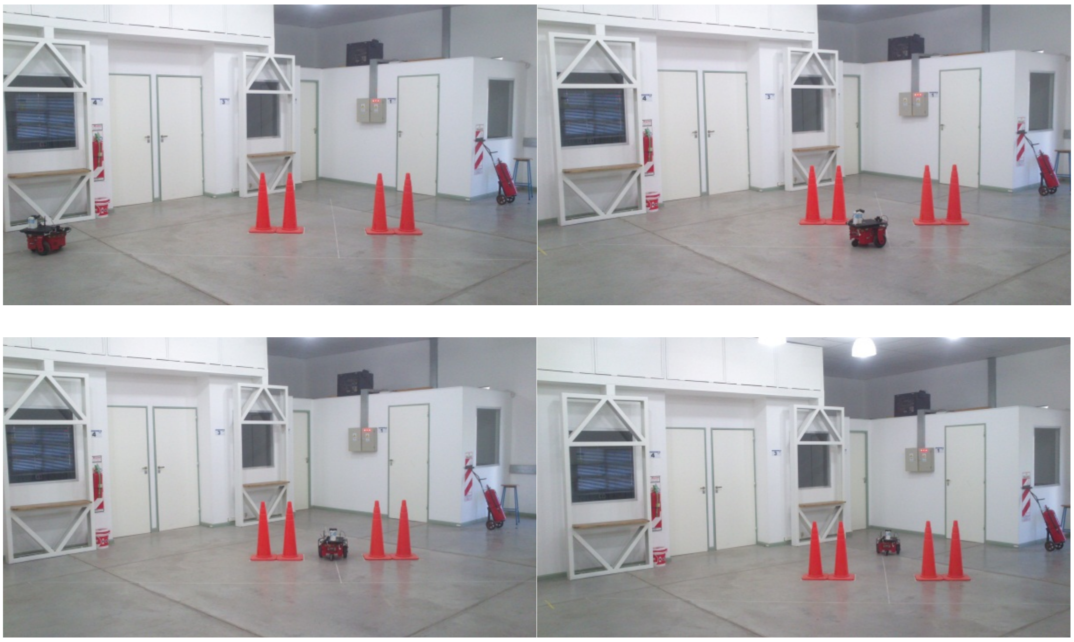

Fig. 2. Reference path for the teleoperated mobile robot.

The parameters $k_g, k_p, k_m, k_s, \alpha_m, \sigma_s$ in section III were taken as scalars, but in general they can be diagonal matrices called $K_g, K_p, K_m, K_s, \alpha_m, \sigma_s$ respectively. Without time delay, the following parameters are calibrated empirically:

$$\mathbf{K_g} = \begin{bmatrix} 14\ 1/s & 0 \\ 0 & 14\ 1/(m \cdot s) \end{bmatrix} \qquad \mathbf{K_m} = \begin{bmatrix} 15\ kg/s & 0 \\ 0 & 5\ kg \cdot m/s \end{bmatrix}$$

$$\mathbf{K_p} = \begin{bmatrix} 0.01\ kg/s^2 & 0 \\ 0 & 0.01\ kg/s^2 \end{bmatrix} \qquad \mathbf{K_s} = \begin{bmatrix} 34\ kg/s & 0 \\ 0 & 7\ kg \cdot m^2/s \end{bmatrix}$$

It is important to remark that without time delay the master damping and acceleration-dependent damping of the mobile robot are not necessary to assure stability. Besides, depending on the inertia matrix D of the robot, the values previously set and the maximum time delays, generally α_s is small in practice.

A) *Testing methodology*

The methodology used to carry out and evaluate the experiments is the following:

1) The equipment used is the following: 3D master device with force feedback, mobile robot, 2D laser sensor, vision camera, and two notebooks connected via network.

2) The goal of the test is for a user to drive a mobile robot in order to follow a reference path marked in the floor avoiding two static obstacles located in the workspace, and lastly placing the robot in the final position of the reference path.

3) The PD-like control scheme is evaluated for three different conditions. Each condition is tested 10 times. The conditions tested are the following:

A) Null both time delay and master damping

B) $h_1 = 0.75\ s + 0.25\sin(0.2\pi\ t)$ and $h_2 = 1$ (both in seconds) and null master damping.

C) $h_1 = 0.75\ s + 0.25\sin(0.2\pi\ t)$ and $h_2 = 1$ (both in seconds) and master damping $\mathbf{D} = \begin{bmatrix} 28.05\ kg & 0 \\ 0 & 0.175\ kg \cdot m^2 \end{bmatrix}$ set by Theorem 1, where taking $\boldsymbol{\alpha}_\mathbf{m} = \begin{bmatrix} 58\ kg/s & 0 \\ 0 & 8\ kg/s \end{bmatrix}$ (parameters of the used mobile robot) results in .

4) The time-to-complete the task and the time-including-force fictitious are computed for each testing and the average values of both metrics are calculated.

B) Results

Fig. 3 shows the paths followed by the mobile robot for the conditions A, B and C. Test B has more fluctuations that the others due to the presence of time delay and the lack of master damping. The absence of damping causes the user to execute fast change commands which in a delayed bilateral teleoperation loop generally produces motions with some oscillations making difficult the action of crossing the two obstacles placed in the workspace. Table I summarize the average time to complete the task and the average time to cross the two obstacles (non-null fictitious force). Such values show that the inclusion of damping into the master decreases the time used for C to complete the task and cross the two obstacles with respect to B. Of course, the better performance is obtained with A because there is no time delay. In Fig. 4, the evolution in time of the linear component of the state variables and fictitious force for condition C, are shown. It is important to signal that in the time interval [0, 20.1] seconds, the human operator drives the mobile robot speed $\boldsymbol{\eta}$ in synchronism with his commands $\mathbf{q_m}$. Later, during the time interval between 20.1 and 32.6 seconds, the mobile robot is crossing the two obstacles and therefore, the impedance based on fictitious force increases the error $k_g\mathbf{q_m} - \boldsymbol{\eta}$. Posteriorly, the user drives the mobile robot faster to the position goal and later, at 37.1 seconds, he or she decelerates it stopping the mobile robot.

Test	Average time to complete the task	Average time to cross the two obstacles
A	20.52 seconds	5.6 seconds
B	37.45 sec onds	15.1 seconds
C	34.03 seconds	11.9 seconds

Table I. Time metrics for the A, B and C conditions.

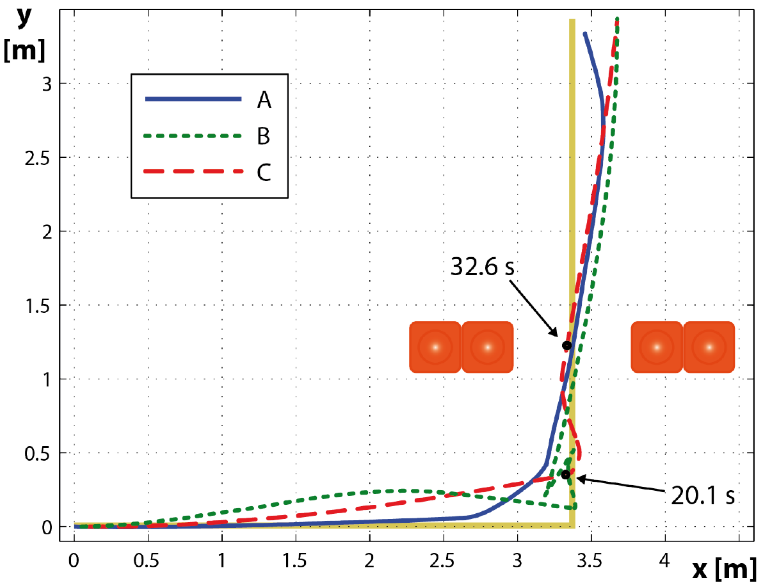

Fig. 3. Paths followed by the mobile robot for A, B and C tests.

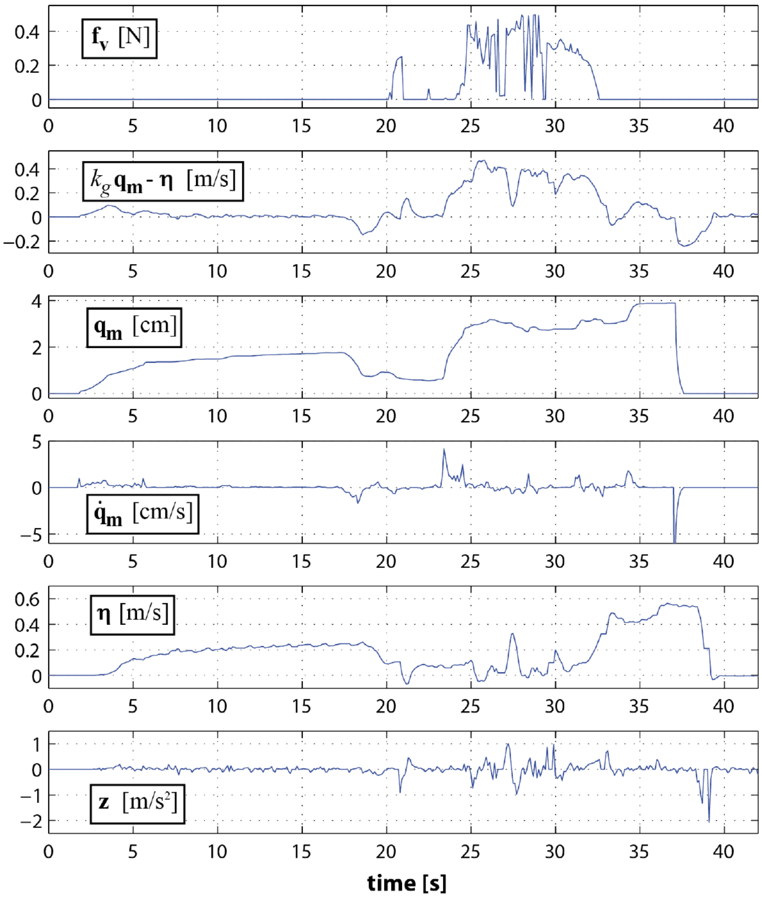

Fig. 4. Fictitious force and linear components of the state for C case.

VII. CONCLUSIONS

In this paper, a control scheme for a mobile robot bilateral teleoperation system has been proposed considering asymmetric and time-varying delays. The stability analysis carried out gives as a result the level of necessary damping to be applied into the master and mobile robot to assure the system stability, mainly depending on the forward and reverse time delays added by the communication channel. On the other hand, the impedance based on fictitious force helps to avoid collisions but it adds a perturbation in the synchronization error which can be felt by the human operator. This perception can be used to modify the velocity command in order to decrease such error between the master position and mobile robot velocity. Finally, experiments were made, whose results showed bounded errors of the state variables which are in agreement with the performed theoretical analysis.

REFERENCES

[1] R.J. Anderson, and Spong, M., "Bilateral control of Teleoperators with time delay". *IEEE Trans and Automatic Control,* 34(5):494-501 (1989).

[2] P.F. Hokayem and M.W. Spong. “Bilateral: an historical survey”. *Automatica* , 42: 2035-2057, (2006).
[3] I. Elhajj, Xi, N., Fung W., Liu Y., *et al*., *Supermedia-Enhanced Internet-Based Telerobotics.* Proceedings of the IEEE, Vol. 91, Nº3, pp. 396-421, March (2003).
[4] G. Niemeyer, and Slotine, J.J.E., “Stable Adaptive Teleoperation”, *IEEE Journal of Oceanic Engineering* , 16(1):152-162 (1991).
[5] G. Niemeyer, and Slotine, J., *Telemanipulation with time delays*. Int. Journal of Robotics Research, 23(9): 873-890, (2004).
[6] E. Nuno, R. Ortega, N. Barabanov, and L. Basanez, “A globally stable PD controller for bilateral teleoperators”, *IEEE Trans. on Robotics*, vol. 22, no. 3, pp. 753-758, (2008).
[7] J.P. Richard, *Time-delay systems: an overview of some recent advances and open problems*. Automatica 39, pp. 1667-1694 (2003).
[8] T.B. Sheridan, *Telerobotics, Automation, and Human Supervisory Control*. The MIT Press, Cambrige, MA (1992).
[9] T.B. Sheridan, *Space Teleoperation through Time Delay: Review and Prognosis*. IEEE Transactions on Robotics and Automation, Vol. 9, No 5, October (1993).
[10] E. Slawinski, V. Mut and J.F. Postigo, “Teleoperation of mobile robots with time-varying delay”, IEEE Trans. on Robotics 23(5):1071-1082, (2007).
[11] I. Farkhatdinov, J-H. Ryu, Jinung An, ”A Preliminary Experimental Study on Haptic Teleoperation of Mobile Robot with Variable Force Feedback Gain,” In Proc. of IEEE Haptics Symposium 2010, March 24-25, Waltham, Boston, US, (2010).
[12] N. Diolaiti and C. Melchiorri ”Haptic teleoperation of a mobile robot,” In Proc. of the 7th IFAC SYROCO, pp. 2798-2805, 2003.
[13] Janabi-Sharifi, F.; Hassanzadeh, I., "Experimental Analysis of Mobile-Robot Teleoperation via Shared Impedance Control," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on* , vol.41, no.2, pp.591,606, April 2011 doi: 10.1109/TSMCB.2010.2073702
[14] J. N. Lim, J. P. Ko, and J. M. Lee, “Internet-based Teleoperation of a Mobile Robot with Force-reflection”, CCA 2003, Proceedings of 2003 IEEE Conf. on Control Applications, Istanbul, Turkey, June 2003 vol.1 pp.680-685, (2003).
[15] D. Lee, O. Martinez-Palafox, M. W. Spong, ”Bilateral teleoperation of a wheeled mobile robot over delayed communication network,” In Proc. of IEEE ICRA 2006, pages 3298 – 3303, (2006).
[16] D. Lee and M. Spong, “Passive bilateral teleoperation with constant time delay” IEEE Trans. Robot., vol. 22, no. 2, pp. 269–281, Apr. 2006.
[17] M. Ferre, M. Buss, R. Aracil, C. Melchiorri, and C. Balaguer. *Advances in Telerobotics*. Springer, (2007).
[18] Chang-Chun Hua; Liu, X.P.; "Delay-Dependent Stability Criteria of Teleoperation Systems with Asymmetric Time-Varying Delays," *Robotics, IEEE Trans*, vol.26, no.5, pp.925-932, Oct. 2010.
[19] Xu Z., Ma L, Schilling K. “Passive bilateral teleoperation of a car-like mobile robot”. In: Proceedings of 17th mediterranean conference on control & automation; 2009. p. 790–6.
[20] D. A. Lawrence, “Stability and transparency in bilateral teleoperation,” IEEE Trans. Robot. Automat., vol. 9, pp. 624–637, Oct. 1993.
[21] E. Slawiñski, V. Mut, L. Salinas and S. García (2012). Teleoperation of a mobile robot with time-varying delay and force feedback. Robotica, 30, pp 67-77.
[22] E. Slawiñski, V. Mut, P. Fiorini and L. Salinas, "Quantitative Absolute Transparency for Bilateral Teleoperation of Mobile Robots," *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on* , vol.42, no.2, pp.430-442, March 2012.
[23] D. Lee; D. Xu; , "Feedback r-passivity of Lagrangian systems for mobile robot teleoperation," *Robotics and Automation (ICRA), 2011 IEEE International Conference on* , vol., no., pp.2118-2123, 9-13 May 2011.
[24] Slawiñski, E. and Mut, V., “PD-like controllers for delayed bilateral teleoperation of manipulators robots”, Int. J. Robust Nonlinear Control, Article first published online: 4 April 2014, DOI: 10.1002/rnc.3177.