Physica A 467 (2017) 307–314

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Study of the critical behavior of the driven lattice gas model with limited nonequilibrium dynamics

Gustavo P. Saracco *, M. Leticia Rubio Puzzo, Marisa A. Bab

Instituto de Investigaciones Fisicoquímicas Teóricas y Aplicadas (INIFTA), UNLP, CONICET, Facultad de Ciencias Exactas, c.c. 16, Suc. 4, (1900) La Plata, Argentina

HIGHLIGHTS

- A nonequilibrium model is defined, where the dynamics is regulated by a parameter p.
- Short-time dynamics allows to determine the critical phase transitions as a function of p.
- The critical temperature follows a power law behavior, from a Ising-like to DLG behavior is observed.
- The obtained exponent is consistent with a that estimated analytically of the scalar-field model.
- The critical exponents were calculated and their values range from those of the Ising model's up to the DLG model.

ARTICLE INFO

Article history:
Received 7 April 2016
Received in revised form 11 August 2016
Available online 14 October 2016

Keywords:
Monte Carlo simulations
Nonequilibrium systems
Phase transitions
Driven diffusive systems

ABSTRACT

In this paper the nonequilibrium critical behavior is investigated using a variant of the well-known two-dimensional driven lattice gas (DLG) model, called modified driven lattice gas (MDLG). In this model, the application of the external field is regulated by a parameter $p\epsilon[0, 1]$ in such a way that if $p = 0$, the field is not applied, and it becomes the Ising model, while if $p = 1$, the DLG model is recovered.

The behavior of the model is investigated for several values of p by studying the dynamic evolution of the system within the short-time regime in the neighborhood of a phase transition. It is found that the system experiences second-order phase transitions in all the interval of p for the density of particles $\rho = 0.5$. The determined critical temperatures $T_c(p)$ are greater than the critical temperature of the Ising model T_c^I, and increase with p up to the critical temperature of the DLG model in the limit of infinite driving fields. The dependence of $T_c(p)$ on p is compatible with a power-law behavior whose exponent is $\psi = 0.27(3)$.

Furthermore, the complete set of the critical and the anisotropic exponents is estimated. For the smallest value of p, the dynamics and β exponents are close to that calculated for the Ising model, and the anisotropic exponent Δ is near zero. As p is increased, the exponents and Δ change, meaning that the anisotropy effects increase. For the largest value investigated, the set of exponents approaches to that reported by the most recent theoretical framework developed for the DLG model.

1. Introduction

In the last decades, the study of nonequilibrium phenomena has attracted the attention of many branches of science. However, from the point of view of physics, there is not a theoretical framework yet to investigate them systematically.

* Corresponding author.
E-mail address: gsaracco@inifta.unlp.edu.ar (G.P. Saracco).

http://dx.doi.org/10.1016/j.physa.2016.10.018
0378-4371/

In spite of that, several approaches have been proposed, such as the search of exact solutions, mean-field theories, renormalization group techniques, molecular dynamics, and Monte Carlo simulations, among others [1,2]. One of the simplest models that captures many essential features of nonequilibrium behavior is the driven lattice gas (DLG) model, proposed by Katz, Lebowitz and Spohn [3]. It consists of an interacting particle system driven by an external field – applied along one preferential axis of the lattice – that makes the system reach nonequilibrium steady states (NESS) in the long-time limit. In particular, the two-dimensional DLG model exhibits remarkable properties that contrast with those of its equilibrium counterpart, the conservative Ising model, such as its non-Hamiltonian nature, the violation of the fluctuation–dissipation theorem, the occurrence of anisotropic critical behavior [4], the existence of a unique relevant length scale in the anisotropic pattern formation at low temperatures and the consequent self-similarity in the system at different evolution times [5].

At high temperatures, the DLG exhibits a gas-like disordered state, but at low temperatures an anisotropic NESS emerges. This ordered phase is characterized by stripes of high and low particle density, aligned with the field direction. If the particle density is $\rho_0 = 1/2$, the order–disorder phase transition is of second order [4]. Extensive Monte Carlo simulations have shown that the critical temperature depends on the external field E, and it saturates for $E \to \infty$ at a value $T_c^{DLG} \simeq 1.41 T_c^I = 3.20$ ($T_c^I = 2.269\ldots$ is the critical temperature of the two-dimensional Ising model, in units of J/k_B, where J and k_B are the coupling and Boltzmann constants, respectively) [4]. The field generates a strong anisotropy, so a nonzero anisotropic exponent Δ must be defined in such a way that $\xi_\parallel \sim \xi_\perp^{1+\Delta}$, where $\xi_{\parallel(\perp)}$ are spatial correlation lengths in the longitudinal (transverse) direction to the field. Moreover, a double set of critical exponents is needed to take into account both the dynamics and the steady-state behavior of the correlation lengths at the critical point in each direction. These exponents are related by Δ, i.e., $\nu_\parallel = \nu_\perp(1+\Delta)$, $z_\parallel = z_\perp/(1+\Delta)$ [4], where $\nu_i, z_i, i = \parallel, \perp$ are the steady-state and dynamic correlation length exponents, respectively. On the other hand, if $\rho_0 < 1/2$, the transition is of first order with particle–hole invariance [4].

The critical behavior of the DLG model, in the limit $E \to \infty$, has become the subject of a long-standing debate. The first theoretical approach was carried out by Janssen et al. [6]. They considered that the anisotropic behavior in the Langevin equation was mainly originated by the particle current. Based on this, the set of critical exponents was calculated, and a new universality class was found [6–8]. Their results were confirmed by several numerical simulations [9]. This subject was later revisited from a different theoretical approach by Garrido et al. [10]. In this case, the Langevin equation was obtained from a coarse-grain process of the master equation and considering the external field as the main source of the anisotropic behavior. A different set of values for the critical exponents was obtained [10–12]; it corresponded to the previously found universality class of the random DLG model (RDLG), where the field direction changes randomly on each Monte Carlo trial [13]. These results were supported numerically by the simulations of Achahbar et al. [14] and Albano and Saracco [15,16]. In this last case, the critical behavior was analyzed by studying the critical short-time dynamics (STD) of the order parameter.

In all of the briefly commented works above, the field magnitude was fixed at a large value, which for practical purposes is equivalent to studying the model in the $E \to \infty$ limit. This means that the jumps of the particles to an empty nearest-neighbor site in the direction of E are always accepted, while jumps against it are forbidden. In addition, the dynamics imposed on this model makes it difficult to study it in the limit of small drives, i.e., in the $E \to 0$ limit. In order to describe the behavior in this regime, new models have been proposed. As an example, the nonconservative Ising model where an external driving field mimicking a shear profile is applied [17]. Here, this field has a magnitude $\dot{\gamma}$, and is decoupled from the Ising dynamics, making the study at any value of $\dot{\gamma}$ easier. It has been found that this model exhibits second-order phase transitions between stripe-ordered configurations and paramagnetic disordered ones. Furthermore, the critical temperature depends on $\dot{\gamma}$, in such a way that it decreases to T_c^I in the limit of $\dot{\gamma} \to 0$, while it saturates at large values. The set of critical exponents for each $\dot{\gamma}$ was estimated for the first time, and was later confirmed by the work using a similar model with friction by Hutch et al. [18].

In this work, a new model based on the DLG is proposed, and its critical behavior is studied by means of Monte Carlo simulations by using the STD technique [19]. In this case, the magnitude of the external field is fixed at a large value, but its application is controlled by a parameter $p \epsilon [0, 1]$, with the following conditions: (a) if $p = 0$ the drive is not applied, so the model becomes the Ising model, and (b) if $p = 1$ the standard DLG model is recovered. In this way the influence of the external field grows as p increases.

The paper is organized as follows: in Section 2 the proposed model is described; in Section 3 a brief description of STD is given. The obtained results are presented and discussed in Section 4, and finally, in Section 5 conclusions are stated.

2. The model

The DLG model [3] is defined on the square lattice of size $L_x \times L_y$ with periodic boundary conditions along both directions, and contained in a thermal bath at a temperature T. The driving field, E, is applied along the L_x-direction. Each lattice site can be empty or occupied by a particle. If the coordinates of the site are (i, j), then the label (or occupation number) of that site is $\eta_{ij} = \{0, 1\}$, where 0 (1) means that the site is empty (occupied). The set of all occupation numbers specifies a particular configuration of the lattice. The particles interact among themselves through a nearest-neighbor attraction with positive coupling constant ($J > 0$). So, in the absence of any field, the Hamiltonian is given by

$$H = -4J \sum_{\langle ij;i'j' \rangle} \eta_{ij}\eta_{i'j'}, \tag{1}$$

where $\langle \cdot \rangle$ means that the summation is made over nearest-neighbor sites only. The attempt of a particle to jump to an empty nearest-neighbor site, W_{jump}, is given by the Metropolis rates [20] modified by the presence of the driving field E, i.e.,

$$W_{jump} = \min[1,\ \mathrm{e}^{-[\Delta H - \epsilon E]/k_B T}], \tag{2}$$

where ΔH is the energy change after the particle–hole exchange, and $\epsilon = (-1,\ 0,\ 1)$ assumes these values when the direction of the jump is against, orthogonal or along the driving field E, respectively. The field is measured in units of J and temperatures are given in units of J/k_B. As was mentioned above, the critical temperature for $E = \infty$ was estimated to be $T_c \simeq 3.20$ [4]. The dynamics imposed does not allow elimination of particles, so their density ρ_0 is a conserved quantity. Also, in the absence of a driving field, the DLG model reduces to the Ising model with conserving (i.e. Kawasaki) dynamics.

In the model under study, called modified DLG model (hereafter MDLG), the dynamics equation (2) is defined in the following way:

(a) In the perpendicular to the field direction, the Kawasaki dynamics remains unchanged, i.e.,

$$W_{jump} = \min[1,\ \mathrm{e}^{-\Delta H/k_B T}], \tag{3}$$

(b) For jumps in the longitudinal direction, the field acts with a probability p:

$$W_{jump} = \begin{cases} \min[1,\ \mathrm{e}^{-[\Delta H - \epsilon E]/k_B T}] & \text{if } x \le p \\ \min[1,\ \mathrm{e}^{-\Delta H/k_B T}] & \text{if } x > p \end{cases} \tag{4}$$

where $\epsilon = (-1,\ 1)$ assumes these values if the direction of the jump is against or along the driving field E, respectively, and x is a random number taken from a uniform distribution between 0 and 1. Also, the external drive magnitude is large enough to be considered in the $E \to \infty$ limit.

In this way, the application of the external drive is regulated by the parameter p, so if $p = 1$, the external drive is always applied, so the model becomes the DLG. However, if $p = 0$, the model is a lattice gas with Kawasaki dynamics, which is equivalent to the (conserved) Ising model (model B in the classification of Hohenberg and Halperin [21]). Notice that jumps against the field direction are performed only if $x > p$ with a high acceptance rate, in view of the temperature range studied in this work (see below). This fact differs from the transition rates of the DLG model where the drive is always present in the dynamics.

3. Short-time dynamics

In order to investigate the phase transitions of the MDLG model, the STD technique [22,19] was employed. This is an effective tool to determine both the order of a phase transition and its transition temperature in the case of continuous transitions, or the transition zone in the case of a first-order transitions. The STD method is based on the fact that at the early stages of the time evolution of a system towards its nonequilibrium steady-state or equilibrium critical behavior, it is possible to measure dynamic scaling laws of the observables such as the order parameter and its moments. This is valid in the time range $t_{mic} < t < t_{mac}$, where the spatial correlation length ξ grows from one lattice spacing at $t = t_{mic}$ up to being of the order of the lattice size L at $t = t_{mac}$. Furthermore, the scale t_{mac} is very small compared with the equilibration or steady-state time, so the STD is free of both critical slowing down and – in the case of short range models like this – finite-size effects. It is also important to mention that the critical value of the control parameter corresponds to that in the thermodynamic limit.

This technique was already applied to study phase transitions in equilibrium and nonequilibrium models (see, for example, the review [19] and references therein). With respect to the DLG model, the STD was applied to determine the regions of the first-order transitions ($\rho_0 \neq 0.5$) [23], and both the critical point and anisotropic exponents of the second-order phase transition at $\rho_0 = 0.5$ [15]. The above-mentioned scaling laws can be observed by employing two different initial configurations, namely, (1) fully disordered configurations (FDC), which means that the system is initially placed in a thermal bath at $T \to \infty$ and on all particles are randomly placed (the appearance is similar to that observed in Fig. 1 (b)); (2) ground state configuration (GSC) where the thermal bath is set at $T = 0$, and all particles form one band at the middle of the lattice, directed along the external field axis.

According to the STD, if the evolution of the observables when the system is started from both initial configurations exhibits power-law behaviors at the same temperature T^*, the system undergoes a second-order phase transition, and T^* is identified as the critical point T_c [22,19]. In this case, the obtained exponents are combinations of the equilibrium or steady-state and dynamic critical exponents of the transition [22,19]. It is important to remark that if the system is not tuned at the critical point, the power-law behavior is modulated by universal functions that depend on the distance to the transition point $|(T - T_c)/T_c|$. These deviations are important since they can set the error bars in the determination of the transition points.

In the DLG model, the observable employed in order to record the dynamic behavior is the order parameter defined by Albano et al. [15,19], which takes into account both the stripe ordering and the gas-like configuration by measuring the

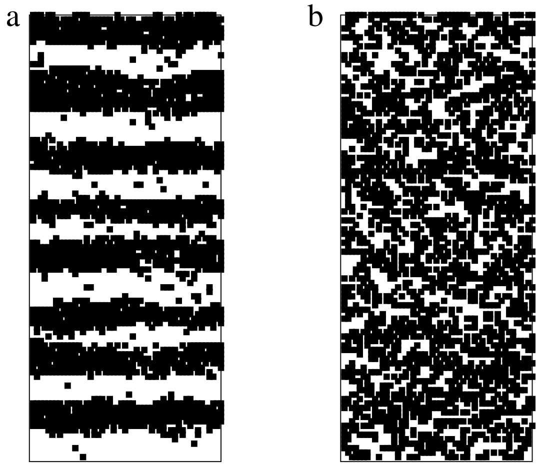

Fig. 1. Snapshot of the NESS configurations of the MDLG model with $p = 0.5$ at: (a) $T = 2.0$ and (b) $T = 4.0$, where an ordered configuration consisting of multiple stripes and a disordered configuration can be observed, respectively. The lattice size is $L_x \times L_y = 50 \times 100$.

density fluctuations along the field axis, i.e.,

$$OP = (RL_y)^{-1} \sum_{j=1}^{L_y} | P(j) - \rho_0 |, \tag{5}$$

where $L_{y(x)}$ is the lattice size in the perpendicular (parallel) direction to the applied field, $P(j) = (L_x)^{-1} \sum_{i=1}^{L_x} \eta_{ij}$ is the perpendicular density profile, and $R = 2\rho_0(1 - \rho_0)$ is a normalization constant. Since the MDLG model shows the same phase appearance, it is reasonable to employ the same observable.

In what follows a brief review of the main features of the STD method for nonequilibrium systems is given. For a detailed description see the review [19] and references therein. By assuming an anisotropic scaling, the ansatze proposed for the STD dynamic behavior OP when the system is initiated from FDC is the following [15,19]:

$$OP(t, \tau, L_x) = b^{\beta/\nu_\parallel} OP^*(b^{1/z_\parallel} t, b^{1/\nu_\parallel} \tau, b^{-1} L_x) \tag{6}$$

where $\tau = (T - T_c)/T_c$, b is a rescaling factor, β is the critical exponent of the order parameter, $\nu_{parallel}$ and $z_{parallel}$ are the NESS and dynamic correlation length exponents in the external drive direction, respectively. Furthermore, OP^* is a scaling function. By setting $b \sim t^{1/z_{parallel}}$ the final expression of OP is obtained:

$$OP(t, \tau, L_x) \propto L_x^{-1/2} t^{c_1} OP^{**}(t^{1/\nu_\parallel z_\parallel} \tau) \quad L_x, L_y \to \infty \tag{7}$$

where $c_1 = (1-2\beta/\nu_\parallel)/2z_\parallel$, and OP^{**} is a scaling function that depends on τ. Here, $L_x^{-1/2}$ must be added since it is originated from the construction of the FDC, according to the central limit theorem.

The logarithmic derivative of OP with respect to τ, evaluated at the critical point, behaves as

$$\partial_\tau \ln OP(\tau, t)|_{\tau=0} \propto t^{c_2}, \tag{8}$$

where $c_2 = 1/\nu_\parallel z_\parallel$.

On the other hand, if the system is initiated from the GSC [15,16] one has the following ansatze:

$$OP(t, \tau) = b^{\beta/\nu_\perp} OP^{***}(b^{1/z_\parallel} t, b^{1/\nu_\perp} \tau), \tag{9}$$

where $\nu_\perp$ is the critical exponent of the NESS correlation length in the transverse direction, and OP^{***} is a scaling function. Again, by setting $b \sim t^{1/z_{parallel}}$ the expression for the time evolution of OP is obtained:

$$OP(t, \tau) \propto t^{-c_3} OP^{****}(1, t^{1/\nu_\perp z_\parallel} \tau), \tag{10}$$

where $c_3 = \beta/\nu_\perp z_\parallel$ and OP^{***} is another scaling function. Here, the derivative with respect to τ evaluated at $\tau = 0$ follows the scaling law

$$\partial_\tau OP(t)|_{\tau=0} \propto t^{c_4}, \tag{11}$$

where $c_4 = (1 - \beta)/\nu_\perp z_\parallel$.

In this way, it is possible to estimate the critical exponents of the second-order phase transition by combining the STD exponents defined above. Notice that for the DLG model the spatial correlations develop first in the longitudinal direction [16], so this fact allows us to determine the anisotropic exponent Δ, defined in the introduction.

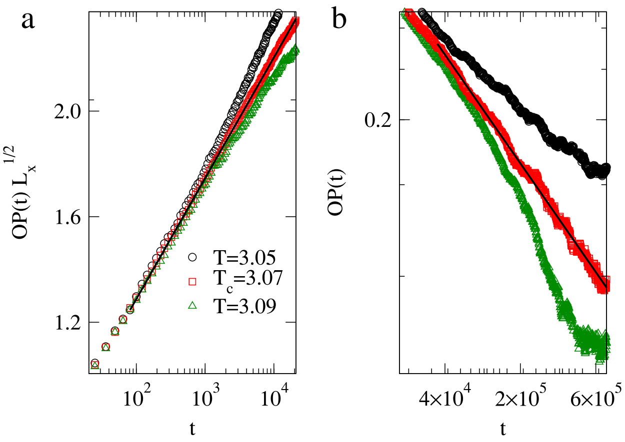

Fig. 2. (Color online) Log–log plots of the time evolution of OP (Eq. (5)), corresponding to $p = 0.5$ and initial configurations: (a) FDC, (b) GSC. As can be observed in both cases, the best power-law behavior occurs at $T = 3.07$ (red squares), while deviations from this behavior are observed at $T = 3.05$ (black circles), and $T = 3.09$ (green triangles), respectively. The solid lines are the fits of Eqs. (7) and (10), respectively. The model was simulated in a lattice with $L_x \times L_y = 240 \times 120$ ($L_x \times L_y = 960 \times 120$) for the FDC (GSC) initial conditions.

4. Results and discussion

The MDLG model was studied by using Monte Carlo simulations with lattice sizes in the range $50 \le L_x \le 960$ and $32 \le L_y \le 480$, according to the requirements of each simulation (see below). Periodic boundary conditions were implemented in both directions. The external field was applied in the horizontal (i.e., x) direction, with magnitude $E = 50J$ for all simulations, which is equivalent to $E = \infty$ for practical purposes. Furthermore, the density was fixed at $\rho_0 = 0.5$ for all simulations.

Fig. 1 shows snapshots of the MDLG model in the NESS states at low and high temperatures for $p = 0.5$. Both panels exhibit configurations that are similar to those observed in the DLG model, i.e. an ordered phase at low $T's$, characterized by stripes running along the field axis, and a disordered, gas-like phase at high $T's$. The same behavior was observed for all the studied values of p listed in Table 1. These results suggest the existence of a line of phase transitions at finite nonzero temperatures.

Fig. 2 exhibits the dynamic evolution of OP for the case $p = 0.5$ in a neighborhood of the phase transition point. In panel (a) the system was initiated from FDC in a lattice of size $L_x \times L_y = 240 \times 120$. According to the results, the scaling behavior of Eq. (7) emerges after a few MCS, about $t_{mic} \simeq 100$ MCS, and extends for more than two decades before it reaches a saturation value due to finite-size effects (not shown). The best power-law behavior – obtained by a least squares fit – occurs at $T_c = 3.07$, while the corresponding upward and downward bendings caused by universal functions for $T < T_c$ and $T > T_c$, respectively, are also displayed. These temperatures were used to determine the error bars. On the other hand, panel (b) shows the dynamic evolution of the model when the system was initiated from the GSC in a lattice of sizes $L_x \times L_y = 960 \times 120$. In this case, t_{mic} is larger than in the previous case by two orders of magnitude. As a consequence, larger lattices have to be used in order to obtain a measurable power-law evolution range. This means that t_{max} – which is not known a priori – has to be extended as much as possible without increasing the lattice size too much for a feasible simulation.

As in the case of FDC, the best power law was obtained at $T_c = 3.07$, and deviations were found at the same temperatures indicated in the panel (a). This coincidence indicates that $T_c = 3.07$ is in fact a critical point where the system undergoes a second-order phase transition. The described procedure was applied for the other studied values of p, in all cases obtaining the same result, with critical temperatures that depend on p (see Table 1).

It is important to remark that the critical dynamics was investigated by simulating the system at several lattice sizes with different aspect ratios, i.e., the relation between L_x and L_y is different for each case. As an example, the main plot of Fig. 3 shows the evolution of OP from FDC for $p = 0.1$ at the critical temperature $T_c = 2.81$ in different lattice sizes, as indicated in the legend. As can be observed, the power-law behavior of the evolution at the early stages is practically the same for all sizes in the time range [20, 5000] MCS. In this way, the behavior is neither affected by finite-size nor by lattice shape effects, or aspect ratio [15]. The inset shows the scaling described in Eq. (7). Furthermore, it may be noticed that the dynamic evolution from the GSC is also free of the above-mentioned effects, since the power-law behavior was obtained at the same temperature as that found for the dynamic evolution from the FDC. The independence of finite-size and shape effects was verified for all studied p's values. In this way, according to the STD results, it can safely be affirmed that the determined critical temperatures correspond to those of the thermodynamic limit.

The dependence of $T_c(p)$ on p can be better understood by means of the plot $T_c(p)/T_c^I - 1$ versus p, which is displayed in Fig. 4. The inset shows that $T_c(p)$ grows with p for all values of p. Near $p = 0$ and $p = 1$, it is similar to T_c^I and T_c^{DLG}, as expected. The fact that $T_c(p) \ge T_c^I$ can be related to the presence of the external field E, which induces ordered configurations

Table 1
Critical temperatures and STD exponents c_i, $i = 1 - 4$ for each p investigated. The initial condition is also indicated.

p	$T_c(p)$	c_1 (FDC)	c_2(FDC)	c_3(GSC)	c_4(GSC)
10^{-3}	2.43(3)	0.0733(1)	0.55(2)	0.0561(5)	0.43(1)
10^{-2}	2.50(2)	0.0867(2)	0.503(5)	0.2476(6)	0.474(1)
10^{-1}	2.81(2)	0.0944(3)	0.47(2)	0.372(1)	0.495(5)
0.25	2.94(2)	0.1141(1)	0.49(3)	0.3040(5)	0.49(1)
0.50	3.07(1)	0.1057(3)	0.43(1)	0.3046(6)	0.39(1)
0.75	3.15(1)	0.1187(4)	0.44(6)	0.3143(8)	0.56(1)
0.90	3.18(1)	0.1144(5)	0.38(3)	0.2540(5)	0.47(1)

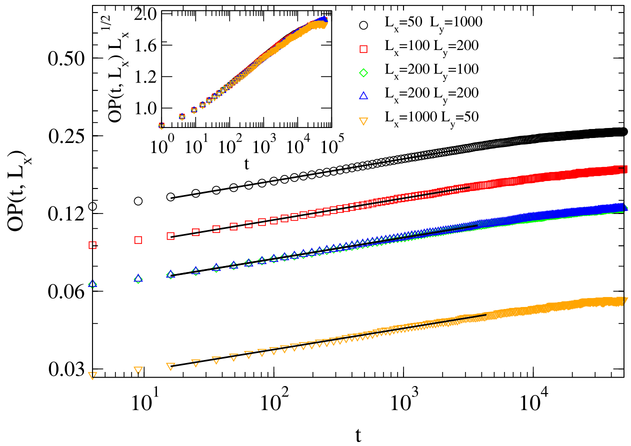

Fig. 3. Log–log plots of the time evolutions of OP at $T = 2.81$ for $p = 0.1$ and lattices with different aspect ratios, as indicated in the legend. The straight lines are fits of the data with Eq. (7). The inset shows the scaling $OP(t, L_x)L_x^{1/2}$ according to Eq. (7).

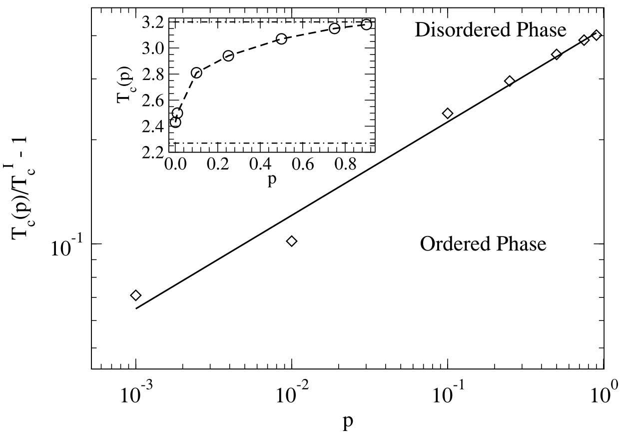

Fig. 4. Inset: Critical temperatures $T_c(p)$ of the MDLG model versus p, on a linear–linear scale. The dot-dashed lines show the critical temperature of the Ising and DLG models, i.e., $T_c(p = 0) \simeq 2.269$ and $T_c(p = 1) = 3.20$ respectively. Main plot: Log–log plot of $T_c(p)/T_c^I - 1$ versus p. The solid line shows a power-law fit.

at $T \geq T_c^I$. The main plot of the figure shows that $T_c(p)/T_c^I - 1$ is consistent with the power law $T_c(p)/T_c^I - 1 \sim p^{\psi}$ with $\psi = 0.27(1)$. The kind of behavior obtained was already observed in other models that allow the study of the critical behavior in the limit of low external fields, such as the Ising model with an external shear field [17]. Even more, it was also predicted theoretically in the critical transition of the scalar field model based on a convection–diffusion equation by using the $\varphi^3 \rightarrow \langle\varphi^2\rangle\varphi$ approximation in Ref. [24]. In this case, the exponent ψ is in good agreement with that obtained in this work for the conserved case, i.e. $\psi = 0.25$.

In order to estimate the STD exponents c_i, the attention must be directed to the behavior of OP and its derivatives at $T = T_c$, described by Eqs. (7)–(11). A representation of the fits can be observed in Figs. 2 and 3. Table 1 lists the obtained critical temperatures and the STD exponents.

By combining the exponents c_i, the critical exponents were calculated. Table 2 displays the obtained values as well as the estimation of the anisotropic exponent Δ and the dynamic correlation length critical exponent $z_\perp$.

Table 2
Critical exponents obtained by combining the exponents listed in Table 1 as a function of p. For the sake of comparison, the obtained values for the DLG model from Garrido et al. theoretical approach [10,11] and Albano et al. STD simulations [15], together with those of the Ising model are also included.

p	$T_c(p)$	β	$z_\parallel$	$\nu_\parallel$	$\nu_\perp$	Δ	$z_\perp$
10^{-3}	2.43(3)	0.115(3)	3.7(1)	0.497(8)	0.56(3)	−0.11(5)	3.2(2)
10^{-2}	2.50(2)	0.343(1)	1.92(3)	1.031(5)	0.72(1)	0.43(3)	2.77(4)
10^{-1}	2.81(2)	0.429(3)	1.69(9)	1.26(1)	0.68(4)	0.84(8)	3.1(3)
0.25	2.94(1)	0.383(4)	1.7(1)	1.23(2)	0.76(6)	0.6(1)	2.7(3)
0.50	3.07(1)	0.439(6)	1.69(6)	1.37(1)	0.84(3)	0.62(5)	2.7(1)
0.75	3.15(1)	0.359(4)	1.8(3)	1.26(6)	0.63(9)	1.0(3)	3.6(8)
0.90	3.18(1)	0.351(5)	2.0(2)	1.30(3)	0.68(6)	0.9(2)	3.8(5)
Ising ($d = 2$)	2.269	1/8	3.75	1	1	0	3.75
DLG (Garrido et al.)	3.20	0.315	2	1.247	0.626	0.992	3.984
DLG (Albano et al.)	3.20(1)	0.33(3)	2.02(4)	1.22(4)	0.64(4)	0.91(1)	3.85(8)

An overview of Table 2 allows us to do some observations. On the one hand, all the obtained values are a first estimate of the critical exponents, in view of both the variation of the values and its corresponding error bars (calculated by error propagation from the STD exponents). In spite of this, the exponents present a clear convergence when p increases to the values estimated theoretically [10,11] and numerically by using STD [15] for the DLG model in the limit $E \to \infty$. This is expected since the MDLG becomes the DLG model in $p = 1$.

For $p = 10^{-3}$, β and $z_\parallel$ approach to the values calculated for the Ising model, but the rest of the exponents do not follow this tendency. In particular, the exponents $\nu_\parallel \lesssim \nu_\perp$ – opposite to the other cases where $\nu_\parallel > \nu_\perp$ –, which leads to find an anisotropy exponent $\Delta < 0$ but near to zero. It is important to remark that the singular behavior of the ν exponents was already observed in the critical evolution of the Ising model with an external shear field in the limit of low shear magnitude [17]. Another fact to be mentioned is that both correlation exponents differ from that corresponding to the Ising model by a factor close to 2 (see Table 2). Unfortunately, there is no theoretical background, which allows us to give a convincing explanation of this difference. Nevertheless, the previous discussion suggests that the behavior of the correlations in the low-external drive regime as well as the T_c would be characteristic of nonequilibrium models. The negative value of Δ seems to be anomalous but the difference between the exponents $\nu_\perp$ and $\nu_\parallel$ is of about 10% and consequently Δ is near zero. On the one hand, this means that the anisotropic effects induced by the external field are heavily reduced, and the MDLG model behaves more like the Ising model than the DLG model. On the other hand, it cannot be ruled out that the negative value is a consequence of the uncertainty in the determination of the T_c, where a tiny variation – even within the error bars – leads to changes in the STD exponents, which affect the estimation of the critical exponents and consequently of Δ. In order to clarify this subject, extensive simulations are being developed whose results will be exposed in a forthcoming article.

5. Conclusions

In this work, a variant of the DLG model named MDLG was studied. In the MDLG model, the application of the external drive is regulated by introducing a parameter $p\epsilon[0, 1]$ in the dynamics of the DLG model. In this way, if $p = 0$ the model becomes the conservative Ising model, while if $p = 1$ the DLG model is recovered. So, in the interval $0 < p < 1$ the drive acts more frequently as p is higher, leading to an increasing anisotropy effect.

The critical behavior of the MDLG model was studied by means of short-time dynamics technique, and a second-order phase transition line was determined as a function of the parameter p. The critical temperatures $T_c(p)$ increase with p from T_c^I up to T_c^{DLG} according to a power law $T_c(p)/T_c^I - 1 \sim p^\psi$, with $\psi = 0.27(1)$. A similar behavior was already obtained theoretically and verified numerically in other nonequilibrium models [24,17], such as the scalar-field model [24],

The complete set of the critical exponents, as well as the anisotropic exponent was also obtained as a function of p. At the smallest value $p = 10^{-3}$ the set is similar to those calculated by Ising model except for the $\nu's$. As p is increased, the exponents change their values due to the increasing anisotropy, which is reflected by the increase of Δ. Finally, for the largest value of $p = 0.9$, the exponents approach those obtained from the theoretical framework proposed by Garrido et al. [10–12], and from Monte Carlo simulation using STD by Albano et al. [15].

Acknowledgments

This work was supported by projects No. PIP 0143 (CONICET) and X11-637 (UNLP). We also thank UnCaFiQT (SNCAD) for computational resources.

References

[1] J. Cardy, Scaling and Renormalization in Statistical Physics, Cambridge University Press, Cambridge, 1996.
[2] Z. Rácz, Nonequilibrium Phase Transitions, in: Lecture Notes, Les Houches, 2002.
[3] S. Katz, J.L. Lebowitz, H. Spohn, Phase transitions in stationary nonequilibrium states of model lattice systems, Phys. Rev. B 28 (3) (1983) 1655–1658.

[4] B. Schmittmann, R.K.P. Zia, Statistical Mechanics of Driven Diffusive Systems, Academic, London, 1995.
[5] P.I. Hurtado, J. Marro, P.L. Garrido, E.V. Albano, Kinetics of phase separation in the driven lattice gas: Self-similar pattern growth under anisotropic nonequilibrium conditions, Phys. Rev. B 67 (1) (2003) 014206.
[6] H.K. Janssen, B. Schmittmann, Field theory of critical behaviour in driven diffusive systems, Z. Phys. B 64 (4) (1986) 503–514.
[7] K.-t. Leung, J.L. Cardy, Field theory of critical behavior in a driven diffusive system, J. Stat. Phys. 44 (3–4) (1986) 567–588.
[8] K.-t. Leung, J.L. Cardy, Field theory of critical behavior in a driven diffusive system, J. Stat. Phys. 45 (5–6) (1986) 1087. (erratum).
[9] K.-t. Leung, Finite-size scaling of driven diffusive systems: Theory and Monte Carlo studies, Phys. Rev. Lett. 66 (4) (1991) 453–456.
[10] P.L. Garrido, F. de los Santos, M.A. Muñoz, Langevin equation for driven diffusive systems, Phys. Rev. E 57 (1) (1998) 752–755.
[11] P.L. Garrido, M.A. Muñoz, F. de los Santos, Universality classes of driven lattice gases, Phys. Rev. E 61 (5) (2000) R4683–R4686.
[12] F. de los Santos, P.L. Garrido, M.A. Muñoz, Entropic contributions in Langevin equations for anisotropic driven systems, Physica A 296 (2001) 364–374.
[13] B. Schmittmann, R.K.P. Zia, Critical properties of a randomly driven diffusive system, Phys. Rev. Lett. 66 (3) (1991) 357–360.
[14] A. Achahbar, P.L. Garrido, J. Marro, M.A. Muñoz, Is the particle current a relevant feature in driven lattice gases? Phys. Rev. Lett. 87 (19) (2001) 195702.
[15] E.V. Albano, G. Saracco, Dynamic behavior of anisotropic nonequilibrium driving lattice gases, Phys. Rev. Lett. 88 (14) (2002) 145701.
[16] E.V. Albano, G. Saracco, Albano and Saracco reply, Phys. Rev. Lett. 92 (2) (2004) 029602.
[17] G.P. Saracco, G. Gonnella, Monte carlo study of the phase transition in the critical behavior of the Ising model with shear, Phys. Rev. E 80 (2009) 051126.
[18] S. Angst, A. Hucht, D.E. Wolf, Strongly anisotropic nonequilibrium phase transition in ising models with friction, Phys. Rev. E 85 (2012) 051120.
[19] E.V. Albano, M.A. Bab, G. Baglietto, R.A. Borzi, T.S. Grigera, E.S. Loscar, D.E. Rodriguez, M.L. Rubio Puzzo, G.P. Saracco, Study of phase transitions from non-equilibrium behaviour, Rep. Progr. Phys. 74 (2011) 026501.
[20] N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller, E. Teller, Equation of state calculations by fast computing machines, J. Chem. Phys. 21 (6) (1953) 1087–1092.
[21] P.C. Hohenberg, B.I. Halperin, Theory of dynamic critical phenomena, Rev. Modern Phys. 49 (1977) 435–479.
[22] B. Zheng, Monte Carlo simulations of short-time critical dynamics, Internat. J. Modern Phys. B 12 (1998) 1419–1484.
[23] G.P. Saracco, E.V. Albano, Critical and dynamical behavior of a driven diffusive system, J. Chem. Phys. 118 (9) (2003) 4157.
[24] G. Gonnella, M. Pellicoro, J. Phys. A 33 (2000) 7043.