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Orbit of Comet C/1854 L1 (Klinkerfues)

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ABSTRACT

Comet C/1854 L1 (Klinkerfues) is one of a large number of comets with parabolic orbits. Given that there are sufficient observations of the comet, 262 in right ascension and 260 in declination, it proves possible to calculate a better orbit. The calculations are based on a 12th order predictor–corrector method. The comet's orbit is highly elliptical, $e=0.99866$ and, from calculated mean errors, statistically different from a parabola. The comet will not return for at least 10,400 years and thus represents no immediate NEO threat.

1. Introduction

The reasons why 19th century cometary orbits should be studied have been explained previously, see Branham (2007) for example, and will not be repeated. It is sufficient to reiterate that most of the catalogued orbits, see Marsden and Williams (2003), are parabolic. A comet with a parabolic orbit *may* be, should a refined orbit turn out to be elliptical and depending on factors such as perihelion distance, a Near Earth Object (NEO). If a more refined orbit proves to be a hyperbola, the comet might potentially be of extra-solar origin. This should be addressed. Many, perhaps most, parabolic orbits were calculated by the method of Olbers (Dubyago, 1961, Chapter 8) as a computational convenience and used normal places. With modern computers normal places are an anachronism that degrade, if only slightly, the solution. Better can be done. And better orbits mean better statistics for studying the origin of comets.

Why study Comet C/1854 L1 (Klinkerfues) in particular? Over 260 observations in both coordinates are available, and the perihelion distance of 0.65 AU means that the object *might* be a potential NEO should the final orbit prove to be elliptical. If hyperbolic then one should investigate whether its origin might be extra-solar.

2. Preliminary data reduction and ephemerides

I conducted a literature search of the journals published in the 19th century that include comet observations and also annual reports of some of the major observatories. Observations of Comet C/1854 L1, henceforth simply Klinkerfues, were found in *The Astronomical Journal*, *Astronomische Nachrichten*, *Monthly Notices RAS*, *Annalen der Sternwarte Wien*, and the *Comptes rendus hebdomadaires des séances de l'Académie des sciences*. The first four journals are found on the ADS database (http://adswww.harvard.edu/), and the Bibliothèque nacional de France (http://gallica.bnf.fr) has made available on the internet nearly all of the volumes of the *Comptes rendus*. This is fortunate because the journal contains many observations of comets and minor planets. A series of observations by Hind (*Monthly Notices RAS*, 1854, vol. 14, p. 215) could not be used because of insufficient precision. Altogether there were a total of 522 observations, 262 in α and 260 in δ made between 6 June and 31 July 1854. The Vienna observers give a series of observations made closely in time and then average the series. I used the original series rather than the average. Table 1 summarizes the observations, and Fig. 1 graphs them.

Rectangular coordinates needed to calculate observed minus calculated positions, (O–C)'s, were initially generated, along with numerically integrated partial derivatives to correct the comet's orbit, from a 12th order predictor–corrector integrator.

3. Errors or missing information in the observations

Processing 19th century observations is a far from trivial task because the observations are published in different languages, English, French, German, Italian, and even Latin, do not conform to a standard format, and contain many errors. The reader may refer to a recent article of mine that discusses the matter in detail (Branham, 2011).

Table 2 exhibits the errors in the observations that could be corrected. Also given are the identifications for previously

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unidentified or misidentified reference stars although not all reference stars could be identified, most likely caused by an erroneous published position.

4. Treatment of the observations

Assigning weights to the observations becomes necessary because of the disparity in their quality. The first orbit was calculated, to minimize the effect of discordant observations, by use of the robust L_1 criterion (Branham, 1990, Chapter 6). Then the final orbit came from weighting the residuals with the Welsch function (Branham, 1990, p. 117). If **r** represents the vector of the post-fit residuals, scale an individual residual r_i by the median of the absolute values of the residuals, $r_i = r_i/median(|\mathbf{r}|)$. Then calculate weighting factors w_i by

$$w_i = \exp(-[r_i/2.985]^2), \quad (1)$$

the Welsch weighting function. Eq. (1) incorporates the advantages of being impersonal and recognizes that smaller residuals are more probable than larger ones and assigns them higher weight. In theory no residuals are eliminated, but in practice large residuals receive such low weight that they in effect represent no contribution to the solution. Fig. 2 shows a histogram of the weights. Fourteen of the weights, or 2.7%, are lower than the machine ε, 396, or 75.9%, are greater than 0.5, and 252, or 48.3%, greater than 0.9. The final mean error of unit weight becomes $\sigma(1) = 4.''68$, a value that falls within the range of the mean errors of other comets I have studied that use only 19th century observations; they vary from a low of $3.''25$ for comet C/1860 M1 (Great comet) (Branham, 2007) to a high of $12.''11$ for comet C/1845 L1 (Great June comet) (Branham, 2009).

The residuals before application of weighting factors and trimmed to eliminate 13 discordant residuals as determined by Pierces's criterion (Branham, 1990, pp. 70–80) were analyzed to calculate certain statistics. The distribution appears skewed with tails not too heavy: coefficient of skewness, 0.47, differs from the normal distribution's 0; the kurtosis of 4.18, 0 for a normal distribution, indicates a leptokurtic distribution, and the Q factor of 0.44 versus the normal's 2.58 demonstrates that the tails are lighter than those of the normal. We thus have a distribution that is more narrow peaked and lighter tailed than a normal distribution. Application of a runs test for randomness of the residuals (Wonnacott and Wonnacott, 1972, pp. 409–411) indicates 263 runs out of an expected 261, extremely random residuals: 86.1% of being random with a 2-sided probability distribution. Fig. 3 shows histograms of the original residuals, and Fig. 4 graphs the weighted residuals.

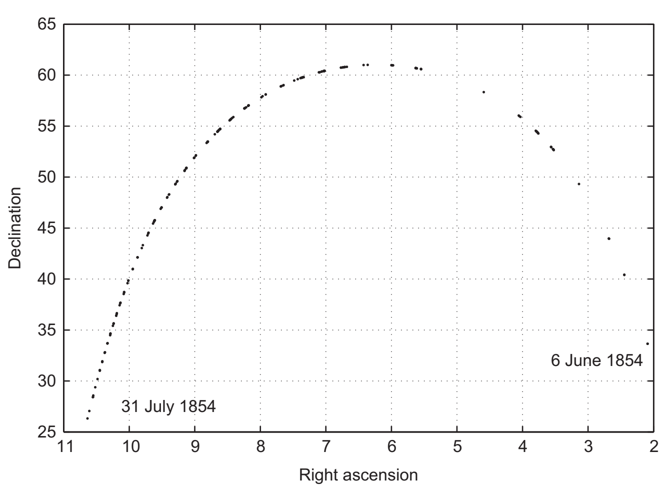

Fig. 1. The observations.

5. The solution

Table 3 shows the final solution for the rectangular coordinates, x_0, y_0, z_0, and velocities, $\dot{x}_0, \dot{y}_0, \dot{z}_0$, along with their mean errors for epoch JD 2398360.5 and the mean error of unit weight, $\sigma(1)$.

Table 1
Observations of Comet C/1854 L1 (Klinkerfues).

Observatory	Obsns. in α	Obsns. in δ	Reference[a]
Kremsmünster, Austria	9	9	AN, 1854, 38, 133
Vienna, Austria	43	43	Annalen Wien, 1858, 7, 95
Olmütz, Czech Republic	18	18	AN, 1854, 39, 103
Berlin, Germany	24	24	AN, 1854, 38, 349; 1855 40, 153
Bonn, Germany	8	8	AN, 1854, 38, 345; 39, 44; AJ, 1854, 4, 5
Göttingen, Germany	5	5	AN, 1854, 38, 353; AJ, 1854, 4, 5
Hamburg, Germany	4	4	AJ, 1854, 4, 46
Mannheim, Germany	8	8	AN, 1854, 39, 47
Paris, France	49	49	CR, 1854, 39, 158; AN, 1854, 38, 349
Florence, Italy	19	17	AN, 1854, 39, 45, 253
Padua, Italy	35	35	AN, 1854, 39, 119
Leiden, Netherlands	1	1	CR, 1854, 39, 1083
Cloverden, USA	17	17	AJ, 1854, 4, 14
Washington, D.C., USA	22	22	AJ, 1854, 4, 12
Total	262	260	

[a] AN: *Astron. Nachr.*; AJ: *Astron. J.*; CR: *Comptes rendux*.

Table 2
Errors/missing information in the observations of comet Klinkerfues.

Reference	Date	Error or missing data
AJ, 1854, vol. 4, p. 14	3 July $13^h43^m2.^s$	Star is Tycho 3798-02146-1
AN, 1854, vol. 39, pp. 47/48	5 July (both dates)	Star c cannot be identified
AN, 1857, vol. 39, pp. 103/104	7 July $10^h42^m10.^s7$	Time should be $10^h46^m10.^s7$
AN, 1857, vol. 39, pp. 105/106	…	Stars λ π cannot be identified
AN, 1857, vol. 39, pp. 105/106	…	Star μ is Tycho 2999-00506-1
AN, 1857, vol. 39, pp. 105/106	…	Star ν is Tycho 2999-00536-1
AN, 1857, vol. 39, pp. 105/106	…	Star σ is Tycho 2511-00649-1
CR, 1854, vol. 39, p. 159	…	Star b is Tycho 3717-00633-1
CR, 1854, vol. 39, p. 159	…	Star g is Tycho 4096-01960-1
CR, 1854, vol. 39, p. 159	…	Star i is Tycho 3794-00288-1
CR, 1854, vol. 39, p. 159	…	Star k is Tycho 3797-00965-1
CR, 1854, vol. 39, p. 159	…	Star l is Tycho 3797-00508-1
CR, 1854, vol. 39, p. 159	…	Star n is Tycho 3429-01008-1
CR, 1854, vol. 39, p. 159	…	Star o is Tycho 3433-00105-1

Table 4 shows the covariances and the correlations. The correlations are high, but the condition number of the matrix of the equations of condition, 1.11×10^4, remains relatively low. The linear system, therefore, seems well-conditioned and should result in a reliable solution.

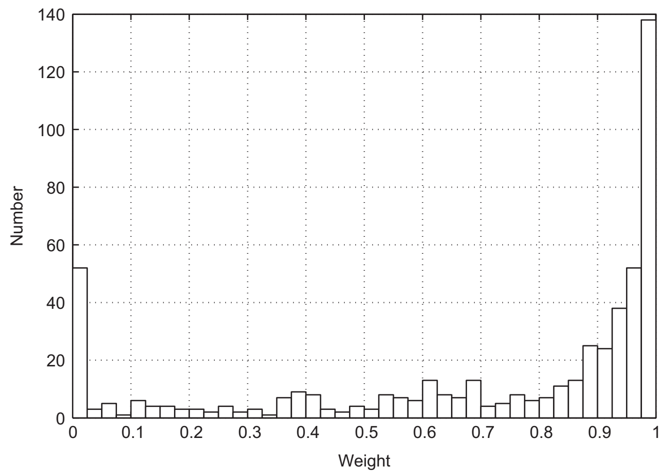

Fig. 2. Histogram of weights.

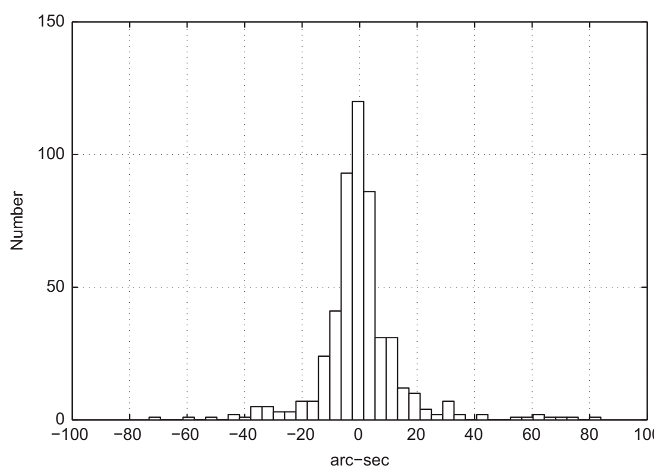

Fig. 3. Histogram of residuals before weighting.

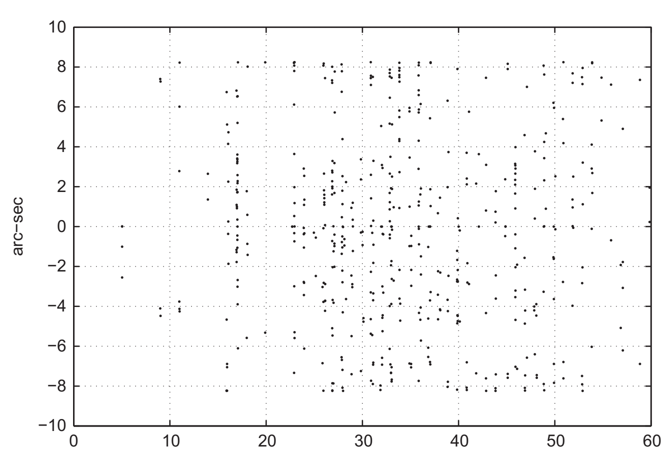

Fig. 4. Distribution of weighted residuals.

Table 5 gives the orbital elements corresponding with the rectangular coordinates of Table 3: the mean anomaly at epoch, M_0; the eccentricity, e; the semi-major axis, a; perihelion distance, q; the inclination, i; the node, Ω; and the argument of perihelion, ω. Rice's (1902) procedure, expressed in modern notation, calculates the mean errors for the elliptical elements and uses $\mathbf{C}$, the covariance matrix for the least squares solution for the rectangular coordinates and velocities. Identify the errors in a quantity such as the node Ω with the differential of the quantity, $d\Omega$. Let $\mathbf{V}$ be the vector of the partial derivatives $(\partial\Omega/\partial x_0 \ \ \partial\Omega/\partial y_0 \ \cdots \ \partial\Omega/\partial \dot{z}_0)$. Then the error can be found from

$$(d\Omega)^2 = \sigma^2(1)\mathbf{V}\cdot\mathbf{C}\cdot\mathbf{V}^T. \quad (2)$$

The partial derivatives in Eq. (5) are calculated from the well known expressions linking orbital elements, whether elliptical or hyperbolic, with their rectangular counterparts. The solution shows a highly elliptical orbit, and the mean errors indicate that the ellipse is statistically distinguishable from a parabola.

The comet's period P comes from the relation

$$P = 2\pi a^{1.5}/k, \quad (3)$$

where k is the Gaussian gravitational constant. It also seems evident that no close approach to the earth will take place. Integration of the orbit using the values for the rectangular coordinates and velocities from Table 3 shows that after 10,000 year the comet remains elliptic with closest approach of 0.507 AU on 27 August 10,894. Comet Klinkerfues, therefore, cannot be considered an NEO.

Given that the orbit is highly elliptic, could the comet possibly be of extra-solar origin, an initially hyperbolic orbit converted by planetary perturbations to elliptical? To check this possibility I integrated the orbit backwards, using barycentric coordinates, from epoch to JD -4052409.5 (11 October -15808), an interval of a over 17,600 years. The comet finds itself at 218 AU from the earth with a still elliptical orbit, barycentric eccentricity of 0.99857. It is possible that integrating backwards to an even more remote date might change this conclusion, but such an integration would be time consuming. The evidence from nearly 18,000 years, covering nearly two complete revolutions of the comet, favors the hypothesis that the orbit has always been elliptical.

Table 3
Solution for rectangular coordinates and velocities for epoch JD 2398360.5 and equinox J2000.

Unknown	Solution	Mean error
x_0	8.833512e−01	2.014014e−05
y_0	−2.436300e−01	1.232606e−05
z_0	4.121236e−02	1.698795e−05
$\dot{x}_0$	−1.770950e−02	7.691246e−07
$\dot{y}_0$	−9.857056e−03	4.150611e−07
$\dot{z}_0$	1.529086e−02	3.989189e−07
$\sigma(1)$	4.″61	

Table 4
Unscaled covariance (upper triangle) and correlation (lower triangle) matrices.

0.8129	−0.1087	0.5171	−0.0306	0.0139	−0.0142
−0.2184	0.3045	−0.1679	0.0066	−0.0071	0.0043
0.7541	−0.4000	0.5784	−0.0208	0.0114	−0.0127
−0.9868	0.3499	−0.7949	0.0012	−0.0006	0.0006
0.8299	−0.6929	0.8091	−0.8940	0.0003	−0.0003
−0.8827	0.4362	−0.9342	0.9216	−0.8771	0.0003

Table 5
Elliptic orbital elements and mean errors for epoch JD 2398360.5 and equinox J2000.

Unknown	Value	Mean error
M_0	$359.^\circ 99707$ $JD2398391.99353 \pm 0.^d 00063$ $(1854\ June 23.48708)$	$0.^\circ 00021$
a	482.63903	23.44969
e	0.99866	0.00006
q	0.64805	0.01526
Ω	$346.^\circ 87757$	$0.^\circ 06688$
i	$131.^\circ 69297$	$0.^\circ 05706$
ω	$344.^\circ 95639$	$0.^\circ 10509$
$P(year)$	10603.55	636.55

6. Conclusions

An orbit for Comet C/1854 L1 (Klinkerfues), based on available observations, 262 in α and 260 in δ, is given. The orbit is highly elliptical and statistically different from a parabola. The comet cannot be considered an NEO unless we wait for at least another 10,000 years. Nor is it likely that the comet has an extra-solar origin.

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