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# Martianus Capella's calculation of the size of the moon

**Christián C. Carman**[1]

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**Abstract** The eighth book of Martianus Capella's famous De Nuptiis Philologiae et Mercurii deserves a prominent place in the history of astronomy because it is the oldest source that came down to us unambiguously postulating the heliocentrism of the inner planets. Just after the paragraph in which Capella asserts that Mercury and Venus revolve around the Sun, he describes a method for calculating the size of the Moon, as well as the proportion between the size of its orbit and the size of the Earth. It is possible to find some descriptions of the argument in general histories of astronomy or in books dedicated to Capella's work, but usually they do not try to make sense of the argument. Rather, they limit themselves to describe or paraphrase what Capella says. As far as I know, there is no single study of the argument itself. The explanation for this absence is simple: the calculation offers many difficulties in its interpretation, for it shows obvious inconsistencies in the steps of the argument and apparent arbitrariness in the selection of the data used. In this article, I offer an interpretation that tries to discover, behind Capella's confusing presentation, a well-sound argument for calculating the Moon's absolute size. Interestingly, we have no records of this argument in other sources, at least in the form described by Capella.

Martianus Capella's famous *De Nuptiis Philologiae et Mercurii* (Stahl and Johnson 1977; Willis 1983) was one of the most popular school-books in Western Europe for about thousand years, probably because it offered a well-proportioned and compre-

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✉ Christián C. Carman
ccarman@gmail.com

1 Centro de Estudios de Filosofía e Historia de la Ciencia (CEFHIC), Universidad Nacional de Quilmes (UNQ); CONICET, Consejo Nacional de Investigaciones Científicas y Técnicas, Buenos Aires, Argentina

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hensive introduction to the seven liberal arts that make up the *trivium* and *quadrivium* (Stahl 1971: 22). The eighth book is dedicated to astronomy. Its main source is probably Book VI (*De Astrologia*) of Varro's *Nine Books of the Disciplines*, now lost (Stahl 1971: 44, 50–53). The book is a general introduction to astronomy, by far simpler than, say, Geminus's *Introduction to Phenomena* (Evans and Berggren 2006) and full of misunderstandings and obscure passages. Nevertheless, it deserves a prominent place in the history of astronomy because it is the oldest source that came down to us unambiguously postulating the heliocentrism of the inner planets, i.e., Mercury and Venus (*De Nuptiis*, L. VIII, 856–857; Stahl and Johnson 1977: 332–333). This text was one of the few that Copernicus recognizes as an influence (*De Revolutionibus*, I, 10, Rosen 1992: 4–5). So, as Delambre (1817: 312) says, in writing this passage, Capella made more services to astronomy than many astronomers by far more skilled than him, and so we should forgive his verbiage, his blunders and his complicated language. Just after the paragraph in which he asserts that Mercury and Venus revolve around the Sun, Capella describes a method for calculating the size of the Moon, as well as the proportion between the size of its orbit and the size of the Earth. It is possible to find some descriptions of the argument in general histories of astronomy or in books dedicated to Capella's work, but usually they do not try to make sense of the argument. Rather, they limit themselves to describe or paraphrase what Capella says. As far as I know, there is no single study of the argument itself. The explanation for this absence is simple: The calculation offers many difficulties in its interpretation, for it shows obvious inconsistencies in the steps of the argument and apparent arbitrariness in the selection of the data used. In this paper, I offer an interpretation that tries to discover, behind Capella's confusing presentation, a sound argument for calculating the Moon's absolute size. Interestingly, we have no records of this argument in other sources, at least in the form described by Capella.

Let us start quoting *in extenso* the text in which Capella offers the argument (the translation is taken from Stahl and Johnson 1977: 333–335, with minor modifications):

> To ascertain the dimensions of all these orbits—an undertaking which astronomers consider a difficult one—a basic assumption must be drawn from geometry, one which the bridesmaid Geometry herself offers in the present work and which has been approved by Eratosthenes and Archimedes; namely, that there are 406,010 stadia in the Earth's circumference [*circuitu terrae*]. By irrefutable reckonings it is found that the Moon's orbit [*Lunae circulum*] is one hundred times greater than the Earth [*terra*]. This orbit is also found to be six hundred times as great as the Moon itself [*ipsa Luna*]. [859] During repeated eclipses of the Sun, by comparing the extent of the shadow which the Moon [*Luna*], lying directly beneath, casts upon the Earth, with the size of the Moon itself [*cum ipso Lunae corpore vera dimensione*], we obtain these two accurate dimensions. If this subject is not tedious, I shall explain how I obtained these measurements.
>
> It often happens that an eclipse of the Sun occurring at the latitude of Meroë darkens the entire orb, but at a nearby climate—that is, one passing through Rhodes—the obscuration is partial, and at the latitude of the mouth of the Borysthenes [Dnieper] there is no obstruction and the full orb shines forth. Since the

> correct distance in stadia at which the latitude of Rhodes is located is known, I have found that the breadth of the shadow which the Moon [*lunae*] casts is one-eighteenth part of the Earth [*terrae*]. Now since the body which casts the conical shadow is larger than the shadow itself, it has been ascertained from the latitudes of either side at which the Sun was partially obscured, that the Moon itself [*ipsam Lunam*] is three times as large as its shadow.
> Thus, it has been determined by the foregoing calculations that the Moon [*Lunam*] is one-sixth as large as the Earth [*orbe terrae*]. [860] That the Moon [Luna] is one six-hundredth of its orbit [*orbe suo*] is determined by the use of clepsydras … We conclude, then, that the lunar orbit [*Lunarem circulum*] is one hundred times as great as the Earth [*tellure*].

Capella's argument seems to follow this path. He wants to prove two things: (1) The lunar orbit is one hundred times greater than the Earth (it is not clear whether he is talking about the Earth's diameter or the Earth's circumference; he uses the ambiguous word *terra*); (2) the lunar orbit is six hundred times greater than the Moon. Again, at least from the words he uses it is not clear whether he refers to the diameter or to the circumference of the Moon. But, because he says that this is proved by the use of clepsydras, it is reasonable to interpret that he is claiming that the apparent size of the Moon is 1/600 of its orbit and, therefore, that he is speaking of the lunar diameter. He claims that these two conclusions can be reached by comparing the extent of the shadow, which the Moon, lying directly beneath, casts upon the Earth, with the size of the Moon itself, during repeated solar eclipses. He then develops the role that solar eclipses play in the argument. He asserts that a solar eclipse that is seen as total from Meroë is usually seen as partial from Rhodes, and that there is no eclipse at all at the latitude of the mouth of the Borysthenes. Because he knows the distance in stadia to Rhodes (and presumably also to the mouth of the Borysthenes), he obtains that the breadth of the shadow cast by the Moon is one-eighteenth of the Earth's circumference. He assumes that the Moon's shadow is conical and correctly asserts that the body which casts a conical shadow is larger than the shadow itself. He states that the Moon itself is three times as large as its shadow. Now, if the shadow is 1/18 of the Earth's circumference and the Moon is three times greater than its shadow, then the Moon is 1/6 of the Earth's circumference. Because he compared the size of the shadow with the size of the Moon, we have to assume that he is talking about the diameter of the Moon (it does not make sense to compare the diameter of the cone with the circumference of the Moon). Therefore, he concludes that the diameter of the Moon is 1/6 of the circumference of the Earth. Moreover, by the use of clepsydras we obtain that the Moon's diameter is one six-hundredth of its orbit. If, as we already proved, the Moon's diameter is 1/6 of the Earth's circumference, it follows that the lunar orbit is 100 times larger than the Earth's circumference.

In summary, he knows that the lunar shadow projected to the Earth is 1/18 of the Earth's circumference and that the lunar diameter is three times its shadow. Therefore, the lunar diameter is 1/6 of the Earth's circumference. He also knows that the lunar orbit is 600 greater than the lunar diameter and, therefore, that the lunar orbit is 100 times greater than the Earth's circumference.

This is how the argument is described in the most developed analysis published so far (Stahl 1971: 191–192). Stahl and Burge limit their analysis to a paraphrase of the text and to a criticism insofar as "in this would-be virtuoso demonstration, astronomy is unaware that she is in effect comparing the Moon's diameter with the Earth's circumference" (note 66). Delambre (1817: 312) also restricts his brief mention of Capella's argument to a description of the main steps, without any criticism or comment. Dreyer (1953: 181–182) says that Capella's estimates are wild and his description is shorter than those of the other two studies: just a paragraph. Neugebauer's treatment is even shorter (1975: 664). He says that "Martianus Capella concludes from alleged observations of solar eclipses (without telling us how) that" the diameter of the Moon is equal to 1/6 of the circumference of the Earth and then he deduces from this and from the apparent size of the Moon that the lunar orbit is equal to 100 Earth circumferences. Dicks's edition (Dicks 1925: 450–453) has only a few references to other classical authors, but without any explanation. And the same is true of James Willis's new edition (Willis 1983: 324–327), where we only find a few more contemporary references. Hans Zekl's German translation (Zekl 2005: 287–289, 345) offers just a few endnotes to this passage, but none of them is explanatory. Ilaria Ramelli devotes two long notes to this passage (2001: 976–79), where she quotes Remigius of Auxerre's comments on Capella's text but, again, there is no explanation of the argument.

Capella's description of the argument has several problems that avoid a rational reconstruction of the calculation and explains why it has been almost ignored by scholars. The last part of the argument, in which he finds the proportion between the lunar orbit and the Earth's circumference, is quite clear and does not present problems. But the calculation arriving at the comparison between the lunar diameter and the Earth's circumference is certainly obscure. Let me enumerate its main problems. First of all, it is not clear what he means by the 'shadow of the Moon'. See Fig. 1. $S$ is the center of the Sun, $L$ is the center of the Moon, and $C$ is the center of the Earth. $M$ is Meroë and $B$ the Mouth of Borysthenes. Because the eclipse is total from Meroë, the centers of both the Sun and the Moon are aligned with $M$. From $B$, instead, you can see the entire orb of the Sun, presumably, next to the Moon (i.e., it is the first latitude at which the eclipse is not visible). Therefore, when Capella identifies "the breadth of the shadow which the Moon casts upon the Earth" with the arc from Meroë to Borysthenes, he is talking about the penumbra (actually, half of the penumbra) and not the umbra, as is clear from the figure. But, when he later says that the conical shadow is smaller than the object that produces the shadow, he is presumably talking about the umbra, which certainly decreases as it moves away from the object. The penumbra, on the contrary, grows as it moves away from the object. So, in his argument Capella confuses the umbra with the penumbra: When he identifies the extreme of the shadow projection with Meroë and Borysthenes, he refers to the penumbra, but when he says that the shadow is smaller than the Moon, he refers to the umbra.

Moreover, even if it is true that the umbra would be smaller than the Moon when it touches the Earth, he does not explain why he assumes that it is exactly one-third of the lunar diameter. See Fig. 2. Again, $C$ is the center of the Earth and $L$, the center of the Moon. Suppose that the apparent sizes of the Moon and the Sun are the same seen from the center of the Earth (if their apparent sizes were the same seen from the

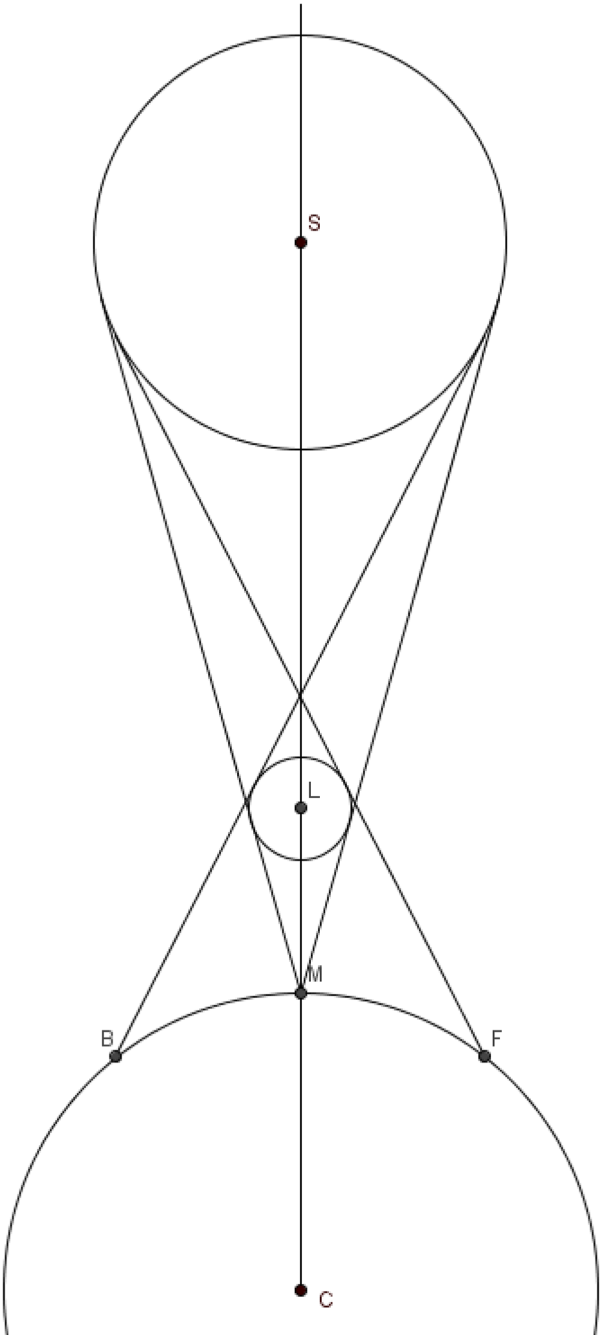


**Fig. 1** Umbra and Penumbra at a total solar eclipse. $C$ is the center of the Earth, $L$ is the center of the Moon, and $S$ is the center of the Sun. The umbra vertex touches the surface of the Earth and $M$ (Meroë). $B$ (Borysthenes) and $F$ are the limits of the penumbra

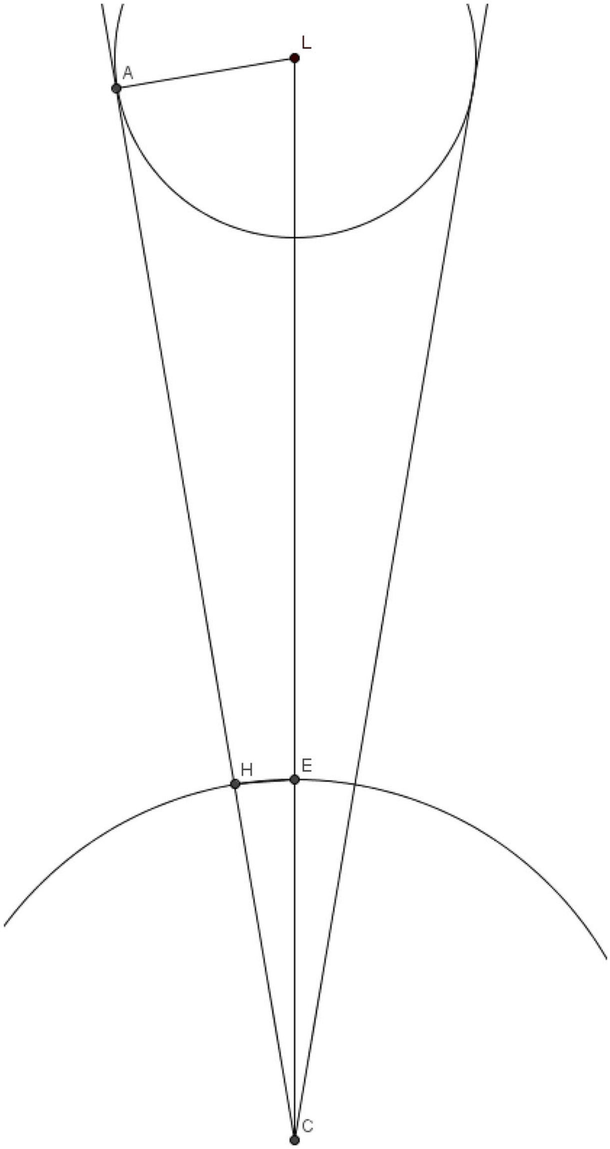


**Fig. 2** *C* is the center of the Earth and *L* the center of the Moon. *LA* is one lunar radius. *EH* is 1/3 of *LA*. The distance *CL* is three times the distance *CE*, i.e, three terrestrial radii

surface of the Earth, the umbra would be just a point at Earth's surface and it would not make any sense to say that the shadow is 1/3 of the Moon). Now, the shadow cone would be 1/3 of the diameter of the Moon at the surface of the Earth if the distance from $C$ to $L$ is 3 times the distance from $C$ to $M$, i.e., if the lunar distance is just 3 terrestrial radius. This is in plain contradiction to assuming that the lunar orbit is 100 times the Earth's circumference.

Finally, the comparison of the lunar diameter with the Earth's circumference is at least odd, as Stahl (1971: 191, note 6) already noted. He seems to assert that the lunar diameter is 1/6 of the Earth's circumference. But this is almost the same as to say that the lunar diameter is one half of the Earth diameter, i.e., that the Moon is half the Earth (noted by Neugebauer 1975: 664). To say that the lunar diameter is one-sixth of the Earth's circumference is certainly an odd way to express that the Moon is half the Earth!

With so many inconsistencies and oddities, we have two options: We can conclude that it is a really bad argument proposed for people who did not understand almost anything of astronomy and that it does not deserve the attention of historians of science, or we can suppose that it is a good argument but incorrectly explained, i.e., that Capella or his source wrongly understood a good argument that deserves to be explored. When reading Capella's description of the argument, one feels what teachers experience reading exams of bad students: There are some elements that certainly form part of the subject, but linked in an inconsistent way and mixed with bad explanations and wrong data added by the student. This feeling can be corroborated by analyzing other passages where Capella (or his source) wrongly explains good arguments. This is the case, for example, of his brief explanation of Eratosthenes's calculation of the Earth's circumference. Fortunately, we know Eratosthenes's method from other sources (mainly Cleomedes I.7; Todd 1990; 33–38; Bowen and Todd 2004: 78–84), so we can compare the correct argument with Capella's description. Capella introduces Eratosthenes's calculation in book VI, devoted to Geometry. He says:

> Eratosthenes, a most learned man, used a gnomon in calculating the Earth's circumference as 252,000 stadia. [597] There are bronze hemispherical bowls called scaphia, which mark the passage of the hours by means of a tall, upright stylus, located at the center of the bottom of the bowl. This stylus is called a gnomon. The length of its shadow, measured at the equinox by a determination of its distance from the center, when multiplied twenty-four times, gives the measure of a double circle. [598] Eratosthenes, upon being informed by official surveyors in the employ of king Ptolemy as to the number of stadia between Syene and Meroë, noted what portion of the Earth's surface that distance represented; and multiplying according to the proportionate amount, he straightway determined how many thousands of stadia there were in the Earth's circumference (Stahl and Johnson 1977: 223–224).

It is easy to note that the text mixes relevant data used by Eratosthenes and partially correct explanations with erroneous data and bad explanations. Capella certainly does not understand Eratosthenes's geometrical procedure, but this does not prevent him from trying to explain it. For example, the value reported for the Earth's circumference is the correct one[1]; it is also true that he used the shadow of a gnomon and that he then multiplies the distance in stadia between two cities in order to obtain the value

[1] For the differences between the 250,000 stadia reported by Cleomedes and the 252,000 stadia informed in many other sources see (Carman and Evans 2015). Because Capella knew that the value of Eratosthenes was 252,000 stadia, it remains a mystery why he attributes to Eratosthenes (and Archimedes) the value of 406,010 stadia that he will use in the calculation of the size of the Earth.

of the Earth's circumference in stadia. But these partially correct explanations and data are mixed with mistakes and bad explanations: He says that Eratosthenes used Syene and Meroë when we know that he used Syene and Alexandria; he talks about the shadow at the equinox, and we know that Eratosthenes used the shadow at the summer solstice[2]; and he says that the length of the shadow, if multiplied by 24, gives the measure of a double circle, which is not consistent with the 1/50 of the circle reported by Cleomedes. Moreover, all these errors and oddities are interwoven by an unintelligible reasoning. Therefore, it is a clear example of how Capella (or his source) could have misunderstood a good argument. Assuming that something similar happened with his description of the calculation of the size of the Moon, I will try to infer the characteristics of the argument that could be behind Capella's obscure description.

We know that a solar eclipse that is total at Meroë, while still visible at Alexandria, is not any longer visible from the Mouth of Borysthenes. The arc between the two cities corresponds to 1/18 of the Earth's circumference. From these data Capella inferred that the Moon is 1/6 of the Earth. So, how can we obtain this result from these data in a consistent way? That the arc from Meroë to Borysthenes is 1/18 of the Earth's circumference could be easily obtained. According to Strabo (*Geography* I.4.2; Jones 1960: 233), Eratosthenes asserted that the distance between Meroë and Borysthenes is 23.100 stadia (from Meroë to Alexandria: 10,000 stadia; from Alexandria to the Hellespont: 8100 stadia; and from the Hellespont to the Mouth of Borysthenes: 5000 stadia). If the Earth's circumference is, as stated by Capella, 406,010 stadia, then this arc represents 1/17.58 of the whole circumference that could reasonably be rounded to 1/18.

In Fig. 3, the center of the Earth is at $C$, Meroë is at $M$, and the Mouth of Borysthenes is at $B$. Angle $MCB$ is, therefore, 1/18 of the circumference, or 20°. Let us assume that the Moon is at the zenith of Meroë. The center of the Moon is $L$, at the zenith of $M$, the center of the Sun, not visible in the figure, is along line $ML$, because at Meorë the solar eclipse is total. But from $B$ you can see the entire Sun just next to the Moon. The distance of the Sun is assumed to be infinite; therefore, from $B$ the Sun will be along line $BE$, parallel to line $ML$. So, from Borysthenes the center of the Moon is at $L$ and the center of the Sun is in line $BE$ (but farther than $E$). If we assume that the Sun and the Moon have the same apparent radius, distance $EL$ is one lunar diameter, because $L$ represents the center of the Moon and $E$ is aligned with the center of the Sun. Line $BF$ is parallel to $EL$. Because $ELBF$ is a parallelogram, $BF$ is equal to $EL$. This means that $EF$ is also one lunar diameter. If we assume that $BF$ is equal to arc $BM$, then the distance between $B$ and $M$ is equal to one lunar diameter. Consequently, one lunar diameter is 1/18 of the Earth circumference. Because $BM$ is half the lunar

[2] It is interesting to note that Cleomedes, after describing Eratosthenes' method that used the summer solstice, asserts that others used the same method but with the winter solstice (I;7;111; Bowen & Todd: 84). Therefore, it is not unlikely that Capella's source used the equinox. Interestingly, the shadow cast by the gnomon at the equinox at Alexandria is around 30° (equal to its longitude) and 30° multiplied by 24 is equal to 720°, i.e., to a double circle. So, maybe Capella is referring to the size of the shadow of Alexandria, but he says that the calculation has been made using Syene and Meroë!

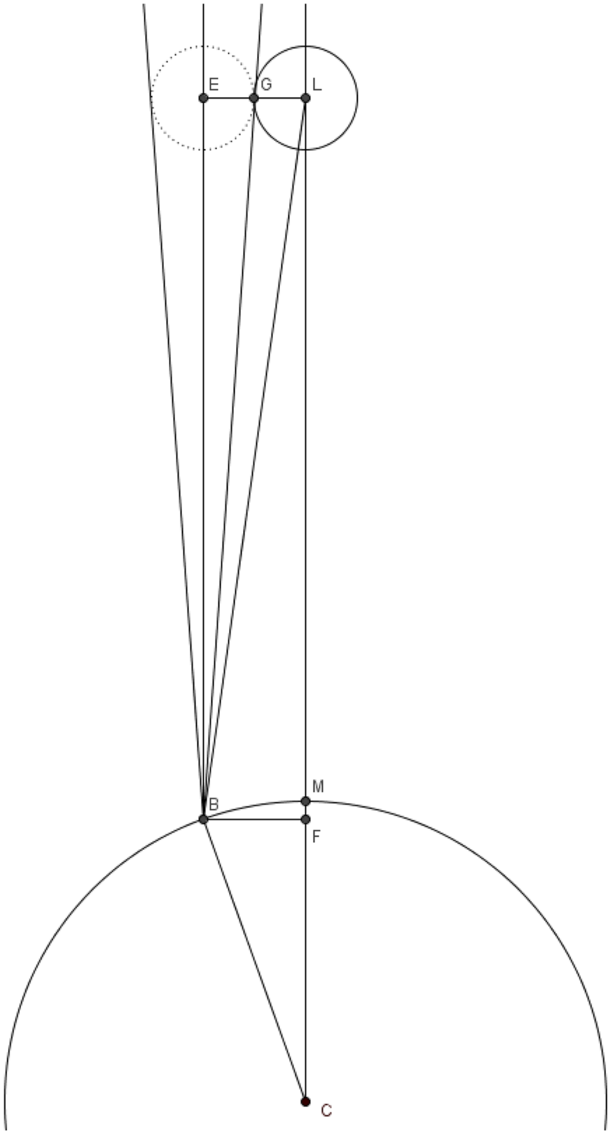


**Fig. 3** C is the center of the Earth and *L* is the center of the Moon; *M* is Meroë and *B* is Borysthenes. The Sun is at the zenith of Meroë, and therefore, the Sun is in line *CML*. Because the eclipse is total from Meroë, also the Moon is in this line, at L. From Borysthenes, the Sun is in line *BE*, parallel to *CL*, because it is assumed that the Sun has no parallax. Line *BF* is parallel and equal to line *LE*

penumbra (the line of penumbra is *GB*), the argument shows that the lunar penumbra projected to the Earth is equal to two Moons.[3] The circumference of the Moon will

[3] It is interesting to note that while the penumbra of the Moon's shadow projected over the Earth is equal to two Moons, the umbra of the Earth projected over the Moon also equals about two Moons (exactly 2 Moons for Aristarchus, 2,5 to Hipparchus and 2,6 to Ptolemy).

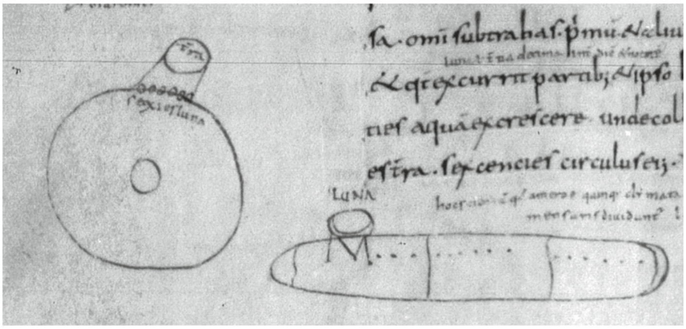


**Fig. 4** Two figures trying to represent Capella's argument in the Vossianus manuscript (*ms*. Voss. lat. F.48, f. 79, Leiden, Universiteitsbibliotheek, Special Collections)

be, obviously, 3 ($\pi$) times its diameter. Therefore, the circumference of the Moon is 1/6 the circumference of the Earth.

So, according to this reconstruction, the original argument concludes that the lunar circumference and not the lunar diameter is 1/6 of the Earth's circumference. Of course, also the diameters of the Moon and Earth are in the same proportion: The lunar diameter is 1/6 of the Earth's diameter. This has the advantage of comparing the same geometrical elements of both bodies. The reason that explains why the Moon is three times its shadow has nothing to do with the conical shape of the umbra, which does not play any role in the argument, but simply with $\pi$, taken as 3, because the shadow represents the lunar diameter.

The second part of the argument does not present any problem. But, of course, if related to this new interpretation of the results, the orbit of the Moon would be equal to 100 terrestrial diameters and not 100 terrestrial circumferences. By the use of clepsydras, we know that the Moon's diameter is one six-hundredth of its orbit. If the Moon's diameter is 1/6 of the Earth's diameter, it follows that the lunar orbit is 100 times larger than the Earth's diameter. Therefore, the lunar distance would be $33\frac{1}{3}$ and not 100 terrestrial radii.

While all the interpretations of Capella's description assume that the argument concludes that the Earth's circumference is six times the lunar diameter, our reconstruction proposes that this proportion must be applied to the lunar circumference. I am not proposing that Capella's description must be interpreted as attributing this proportion to the circumference and not to the diameter. I think that Capella's text is certainly ambiguous but probably what he wants to say is what is usually attributed to him, i.e., that the relationship is established with the lunar diameter. What I do propose is that the hidden argument that he or his source misunderstood established that the lunar circumference is 1/6 of the Earth's circumference and, therefore, that the lunar diameter is 1/6 of the Earth's diameter.

Some support for my interpretation can be found in two diagrams of a ninth-century scribe that tries to make sense of Capella's argument in Vossianus manuscript (ms. Voss. lat. F.48, f. 79, Leiden, Universiteitsbibliotheek, Special Collections). In the first diagram (Fig. 4, left), we can identify the Earth at the center of the lunar orbit. Along the lunar orbit, there are six contiguous Moons, which all together are compared with a second Earth that is now located outside the lunar orbit and close to the six Moons to facilitate the comparison with them. It is clear from the figure that six lunar diameters are equal to one Earth's diameter, exactly as our interpretation assumes. (Eastwood 2007: 262) comments this diagram saying: "in a confusing diagram mistakenly depicts the lunar diameter as one-sixth of the Earth's diameter (misrepresenting Martianus)". But there is another diagram (Fig. 4, right), not commented by Eastwood, which is also very interesting. In it we see what seems to be some kind of oval divided in three equal parts (plus small parts at the extremes) with eighteen dots (six by section). To the left of the diagram, there is an *M* that touches three of these points at the base; above the *M* there is a small Moon clearly identified by the Latin word *luna.* It seems to me that the oval represents the Earth's circumference stretched for convenience. The Earth's circumference is divided into eighteen sections, represented by the eighteen dots. I think that the *M* that joins the Moon and two sections of the Earth's circumference represents the umbra and penumbra.[4] The umbra's vertex touches the surface of the Earth in just one point, but the penumbra occupies two sections, i.e., 1/9 of the Earth's circumference, exactly as in our proposal (in which half of the penumbra is 1/18 of the Earth's circumference). Therefore, it seems that the scribe detected the equivocal use of shadow as umbra and penumbra in Capella's text and interpreted that what occupies 1/18 of the Earth's circumference is half of the penumbra. Moreover, the division in three (and the small parts at the extremes) probably indicates the relation between the circumference and the diameter of the Earth: The Earth's circumference is divided in 3 parts and a bit more, i.e., by $\pi > 3$. Each of the three sections, therefore, can also be interpreted as one Earth's diameter. As I already said, these three diameters are also divided in 6 sections each. Consequently, the figure shows at the same time that half of the lunar penumbra occupies 1/18 of the Earth's circumference and 1/6 of the Earth's diameter, exactly as I propose.

Let me analyze the argument in some detail. It has an interesting characteristic, not found in any other ancient argument for calculating the size of the Moon. All known calculations, those of Aristarchus (Heath 1913), Hipparchus (Swerdlow 1969; Toomer 1974) or Ptolemy (Almagest V, 12–16; Toomer 1988 : 247–257) one way or another use the apparent size of the Moon and its distance to the Earth for obtaining its absolute size. Capella's argument is the only one that is able to obtain the absolute size of the Moon using neither the lunar distance nor its apparent size. Then, adding the apparent size, he obtains the Moon's distance.

Our reconstruction of the method supposes a simplification that is perfectly justified: We assume that line *BF* is equal to arc *BM*. The difference between both can be calculated easily. Refer again to Fig. 3. In triangle *CBF*, side *BC* is one terrestrial radius and we know the three angles, because the angle in *F* is right and the angle in *C* is the difference in latitude ($\Delta$lat) between both cities. Therefore,

$$BF = r \cdot \sin(\Delta lat) \tag{1}$$

[4] I want to thank Anibal Szapiro, Diego Pelegrin and Gonzalo Recio for their invaluable help in interpreting the details of this diagram.

In this particular case, in which we assume that $r = 1$ and that $\Delta lat = 20°$ (1/18 of a circle), $BF = 0.3420tr$.

Now, arc *BM* is 1/18 of the circumference. If, again, we use as unit $r = 1$, we obtain:

$$\text{arc}BM = 2\pi/18 = 0.3490\,tr \tag{2}$$

The difference, therefore, is just 0.007 *tr*, which represents around 450 *st* according to the size of the circumference used by Capella (406,010 *st*) or just 280 *st* applying the value of Eratosthenes (252,000 *st*). So, the identification of line *BF* and arc *BM* is perfectly justified.

It is interesting to note that the argument, even if extremely simple, works perfectly well if we introduce correct input values. Take, for example, the solar eclipse of July 11, 1991, which reached its maximum immersion (total eclipse) at the zenith, as the argument asks (see Fig. 5). The eclipse was total at the zenith at longitude 105°W and latitude 22°N, at the same time and the same longitude, the eclipse ceased to be visible at latitude 14°S, and therefore $\Delta$lat is 36°, which implies a lunar diameter of 0.587 *tr* (if we calculate *BF*) or of 0.628 *tr* (if we calculate arc *BM*). The exact value of the lunar diameter is 0.545 *tr*,[5] very close to the values obtained with a difference of just 0.04 *tr* (*BF*) and 0.08 *tr* (arc *BM*). Capella's value for the diameter of the Moon is 0.33 *tr*, while the correct value, as we already mentioned, is 0.545 *tr*. If the calculation works well with accurate input data, the reason for the inaccuracy of Capella's result must be explained by the inaccuracy of the input data he used. From Eq. 1, we can infer that if we want *BF* = 0.545 *tr*, then $\Delta$lat must be 33.02°. If we want arc *BM* to be equal to the lunar diameter, then if *BM* = 0.545 *tr*, from Eq. 2 we can infer that $\Delta$lat must be 31.23°. Now, the latitude of Meroë is around 17° and that of the Mouth of Borysthenes (Dnieper) is around 46.5°. Therefore, the correct $\Delta$lat is 29.5°, which shows that what Capella says in the argument is actually true: A solar eclipse that is seen as total at the zenith in Meroë, is (almost) no longer visible in the Mouth of Borysthenes. As you can see in the graph of Fig. 6,[6] an eclipse that has magnitude 1 at Meroë starts to diminish its magnitude and arrives at 0 around Borysthenes.

So, the input error in Capella's argument is not in the cities used, but on the size of the circumference of the Earth, which gives 20° for $\Delta$lat instead of the correct 33°. Had he used the correct Eratosthenes' value, he would have found a much better value for the lunar diameter. Actually, with a circumference of 252,000 st. and a distance between the cities of 23,100 st., $\Delta$lat would be exactly 33°!

Consequently, it is reasonable to ask whether the original calculation has been based on observations of real solar eclipses. In order to analyze this option, we should look at eclipses that were total at Meroë, when the Sun was at the zenith. The most ancient eclipse registered in the *Almagest* is from −722 (Pedersen 1974-2010: 408) and Capella lived during the fifth century. So, if we look at all solar eclipses from −800 to 500, we will find that just 10 were total at Meroë. Just two of them had an altitude consistent with the argument: the first one on May 5, −769 with an altitude

[5] Value taken from Nasa. See: http://nssdc.gsfc.nasa.gov/planetary/factsheet/Moonfact.html.

[6] Thanks to Dennis Duke for suggesting and preparing this graph for me.

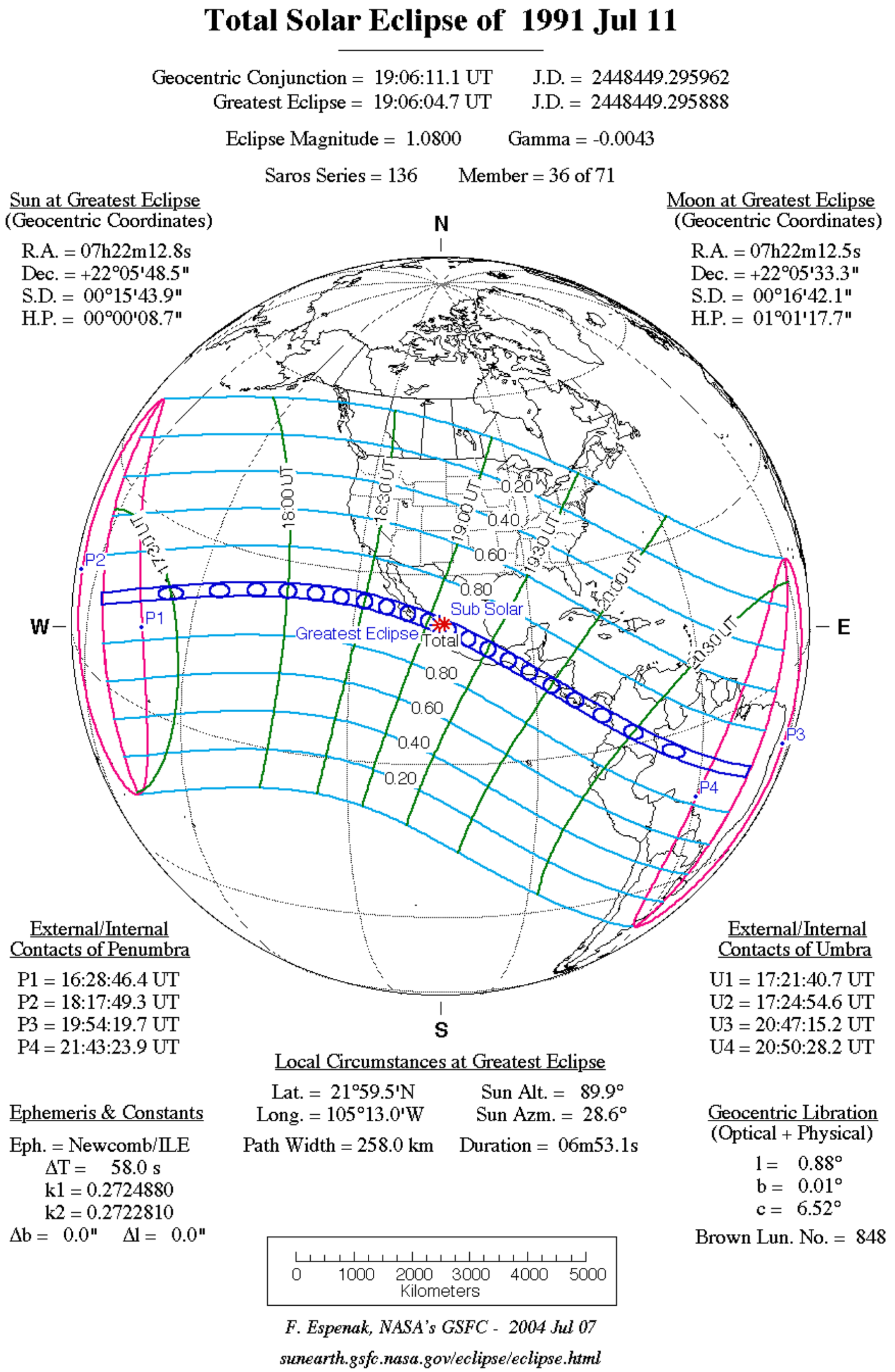


**Fig. 5** Details of the solar eclipse of July 11, 1991 taken from F. Espenak, NASA's GSFC

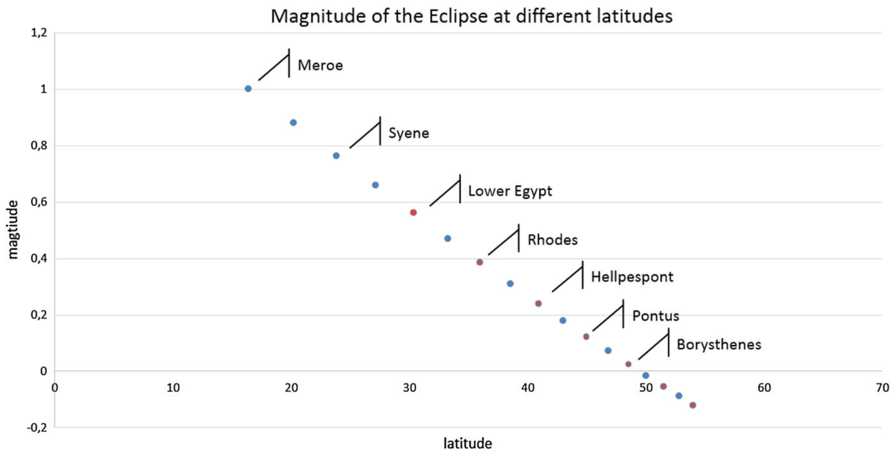


**Fig. 6** The change of the magnitude of a Solar eclipse that is total at Meroë in function of the latitude of the place

of 81° and the sixth one, 521 years later, on May 4, −248 with an altitude of 80°. So, there are two eclipses that could have been used in the calculation.

Nevertheless, I think that it is (very) unlikely that the argument has been built on real observations. The phenomena to be observed are obviously infrequent (just two in 1300 years!), but it has interesting advantages: For example, one does not need to record any value at the moment of the eclipse. It is enough to note that the eclipse was total close to the zenith at Meroë and that there was no eclipse at all at Borysthenes. But the point is that the argument requires (and actually this is the case) not just that the eclipse was not visible at Borysthenes, but that Borysthenes is the limit of the non-visibility, and this is by far much more difficult to observe. This is certainly something that could not be detected unless one is looking for it. Usually, solar eclipses that are not total are not observed by the naked eye. Therefore, it would be possible for an ancient astronomer to find a record of a total eclipse observed from Meroë, but unlikely to find any evidence that the limit of the visibility of that particular eclipse was around Borysthenes. So, in order to suppose that the argument has been based on real observations, we should postulate some kind of coordinate experiment in which the same day that someone at Meroë observed that the eclipse was total, another observer at Borysthenes observed that the eclipse was not visible (but trying to detect it), and a third observer a bit south of Borysthenes reported an eclipse with a magnitude close but not equal to zero. This is certainly implausible for year −248.

Solar eclipses from −800 to 500 seen as total from Meroë:[7]

| Calendar Date | Sun Altitude | Calendar Date | Sun Altitude |
|---|---|---|---|
| −769-May-05 | 81° | −248-May-04 | 80° |
| −701-Mar-05 | 54 | −125-Sep-19 | 46 |
| −636-Aug-29 | 02 | −23-Apr-07 | 15 |
| −563-Apr-07 | 40 | 5-Sep-22 | 04 |
| −371-Dec-06 | 10 | 52-Mar-19 | 44 |

If the argument was not based on observations, it was probably constructed post hoc. It would have been easy to build the argument based on Ptolemy's values. Probably the easiest way to do it is to take the absolute size of the Moon obtained in the *Almagest* and to reason backwards. In *Almagest* V,15 (H.425; Toomer 1998 : 255), Ptolemy obtains the value of 0;17,33 *tr* for the lunar radius. Therefore, the lunar diameter is twice as much: 0;35;6 *tr*. Multiplying this value by $360°/2\pi$, we obtain $\Delta\mathrm{lat} = 33.51°$. Then, we have to look for two cities of around the same longitude and 33.51° apart in latitude. There are not too many options, for this is approximately the difference in latitude of all the known world in antiquity. One would have to choose the cities that are at the northern and southern extremes of the habitable world, and these are Borysthenes and Meroë. Even for Ptolemy these two cities represent the extremes of the seven standard climates. Therefore, I think that the argument was based upon knowing the absolute size of the Moon and deducing the convenient input values backwards.

The calculation hidden in Capella's description supposes three simplifications that a more sophisticated version of the calculation could not assume. The first one is the already mentioned identification of line *BF* with arc *BM*. The second one is to ask one of the cities to be the limit at which the eclipse is no longer visible. The argument would also work if one has two cities: one in which the eclipse is total, and the other one in which the eclipse is partial, provided that one knows the magnitude of the eclipse at this second city. The third assumption is to ask that the eclipse takes place at the zenith. This allows us identifying arc *BM* (the distance between the cities) with line *BF* (refer again to Fig. 3). But if one knows how to calculate line *BF*, it is not necessary to ask the eclipse to be at zenith.

Take, for example, the calculation of the distance of the Moon that Ptolemy and Pappus attributed to Hipparchus (Swerdlow 1969; Toomer 1974). Hipparchus used an eclipse that was total at the Hellespont, but just 1/5 of the Sun was visible at Alexandria. This is apparently the only information that he used to obtain the lunar distance. According to Toomer (1974), the eclipse was that of March 14, -189, when the declination of the Sun was −3°, and the distance obtained was 71 *tr* (the lunar distance at perigee). With these values, it is easy to apply the sophisticated version of Capella's argument. See Fig. 7.

[7] This table has been calculated using Eclipse Predictions by Fred Espenak and Chris O'Byrne (NASA's GSFC), at http://eclipse.gsfc.nasa.gov/JSEX/JSEX-index.html. Data used for Meroë: lat: 16.9382°N, long 33.7488°E

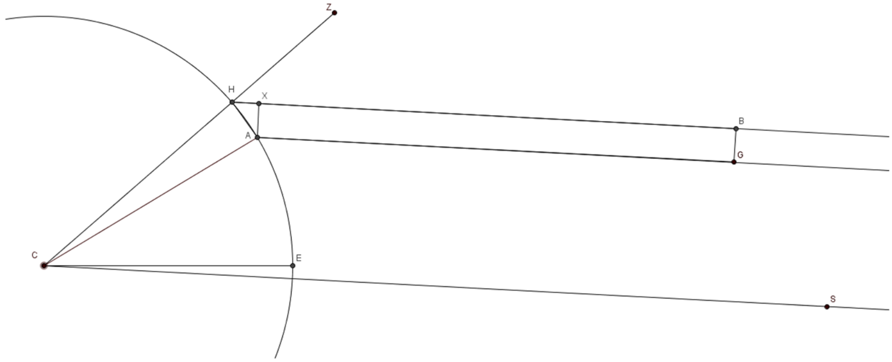


**Fig. 7** The sophisticated version of Capella's argument applied to the Hipparchian calculation of the lunar distance. $C$ is the center of the Earth, $H$ is the Hellespont, and $A$ is Alexandria. Line $CE$ is the equator. The Sun is in line $CE$. Because it is assumed that the Sun has no parallax, seen from $A$ the Sun is in line $AG$ and seen from H it is in line $HB$, all parallel to line $CS$. Because seen from $H$ the eclipse is total, the Moon is at $B$, in line $HB$. Therefore, seen from $A$ the Moon is at $B$ and the Sun at $G$. The distance between these two points is 1/5 of the lunar diameter. $XA$ is parallel and equal to $BG$. Therefore, the lunar diameter is equal to five times $XA$

The center of the Earth is $C$, $A$ is Alexandria, and $H$ is the Hellespont. Line $CE$ represents the equator. The Sun is at $S$. At the moment of the eclipse the declination was $-3°$, and therefore, the Sun is at line $CS$, and angle $ECS$ is $-3°$. Alexandria's latitude is 31° and that of the Hellespont, 41°; therefore, angle $HCA = \Delta\text{lat} = 10°$. Because the Sun is at an infinite distance, seen from $H$, the Sun is in line $HB$, parallel to $CS$. The eclipse is total from $H$, and consequently, the Moon is also in line $HB$. From Alexandria, 1/5 of the Sun was visible, therefore, from $A$ the Sun is in line $AG$, parallel to $CS$ and $HB$, but the Moon is at $B$. Assuming that the Sun and Moon have the same apparent size, the distance $BG$ is equal to 1/5 of the lunar diameter. $XA$ is parallel to $BG$, so $XAGB$ forms a parallelogram, and consequently $XA = BG$. Therefore, $XA$ is also 1/5 of the lunar diameter. In order to calculate the lunar diameter, we have to find $XA$ and multiply it by 5.

In triangle $CHA$, we know that sides $HC$ and $AC$ are 1 $tr$, and we also know angle $HCA = \Delta\text{lat} = 10°$. Therefore, we can calculate $HA$

$$HA = \frac{\sin HCA}{\sin CHA} = \frac{\sin(10)}{\sin(85)} = 0.1743\,tr$$

Now, in triangle $HXA$ we know that the angle at $X$ is right, and we can calculate $AHX$

$$AHX = 180 - ZHX - AHC$$

We know that angle $AHC = 85°$ and that angle $ZHX = \text{ZCS}$, i.e., to the latitude of $H$ plus the declination of the Sun, $41° + 3° = 44°$. Therefore,

$$AHX = 180° - 44° - 85° = 51°$$

Consequently,

$$XA = HA \cdot \sin AHX = 0.13546\,tr$$

And the lunar diameter is five times *XA*, i.e., 0.677 *tr*, the absolute size of the Moon from Hipparchus's report of visibilities of the eclipse. Now, if we want to calculate the lunar distance, we can use the apparent size of the Moon attributed by Ptolemy to Hipparchus, i.e., 360/650 (*Almagest* IV, 9; Toomer 1998: 205). Therefore, the lunar distance is:

$$HB = \frac{lunar\ diameter}{\tan\left(lunar\ apparent\ diameter\right)} = \frac{0.677}{0.0096} = 70.06\,tr$$

But this expresses the lunar distance from $H$, if we want it from the center of the Earth, we can add one terrestrial radius and obtain: 71.06 *tr*. As we already said, the value obtained by Hipparchus (according to Toomer 1974) is 71 *tr*.

We cannot be sure that Hipparchus followed this path. But the obvious similarity between the two maybe indicates that the argument that Capella tried to describe had been influenced by Hipparchus's method, incorporating the three simplifications already mentioned. Actually, If you want to explain Hipparchus' method to the broad public, it is a good idea to incorporate the three simplifications, because they render the reasoning by far simpler without losing the essence of the argument.[8] Maybe the fact that Capella's description mentions that the eclipse was partial at Alexandria, and that this datum does not play any role in his argument but plays a central role in Hipparchus', could be interpreted as a fossil of the Hipparchan origin of this argument.

In summary, I think that if we assume that what Capella describes when he offers an argument for calculating the absolute size of the Moon is not a bad argument, but a good argument badly explained, we can infer a nice method of calculation not previously attested, which asserts that the size of the penumbra of a total solar eclipse at the zenith is equal to two lunar diameters. This argument works perfectly if the input data are correct. Curiously, the cities in Capella's argument are adequate, but the problem is with the very odd value that he uses of the size of the Earth's circumference, which remains unexplained. Had he used the correct Eratosthenes's value, he would have found a very accurate value for the absolute size of the Moon.

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[8] Something similar could be said about Cleomedes' description of Eratosthenes's argument. It could also be understood as a simplification of what Eratosthenes actually could have done. But in this case it is a good description of the simplified argument. See (Carman and Evans 2015).

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