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COMPLEX NETWORK REPRESENTATION OF MULTIAGENT SYSTEMS WITH COOPERATIVE AND COMPETITIVE INTERACTIONS

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ABSTRACT. The dynamic behavior of Multi-Agent Systems (MAS) is analyzed in the context of a modified Lotka-Volterra model. The interaction strength is determined by the difference of agent sizes: as the difference increases, the interaction is weaker. Competitive and cooperative scenarios are analyzed, showing clusters of agents in the stationary state. However, meantime in the competitive scenario the agent sizes are constrained to be non greater than the capacity value ($\beta = 1$), in the cooperative scenario, they are allowed to exceed such capacity making clear the advantages of cooperation. The complex network representation is introduced in order to enhance the role of agent sizes and their one-on-one interactions in the dynamic behavior of the system.

1. Introduction

The field of complex networks and Multi-Agent Systems (MAS) is today one of the most active areas in statistical physics. The reason for such a success is mostly due to the simplicity and broad significance of the approach that, through graph theory, allows researchers to tackle a variety of different fields of complex systems within a common framework. Many years of research after seminal works (see Albert and Barabási 2002; Amaral and Ottino 2004; Knyazeva and Kurdyumov 1994; Kocarev and Vattay 2005; Liljeros, Edling, and Amaral 2003; Sitharama Iyengar 1997; S. Strogatz 2004; S. H. Strogatz and Mirollo 1988; Watts and S. H. Strogatz 1998; Zhang and Tan 2005) initiated the modern study of networks and the interest in the field is, in fact, still growing as indicated by the increasing number of publications in this area.

In this work we treat both, competitive and cooperative scenarios, which are based on a set of generalized predator-prey Lotka-Volterra differential equations (see Lotka 1925; Maurer and Huberman 2003; Volterra 1926). This means that we are dealing with n interacting agents, all of them needing some common resources.

Particularly, here we describe the behavior of MAS analyzed by Caram *et al.* (2010), through complex networks. We introduce a graph representation, namely the complex network representation, where each node represents an agent and agent iterations are represented by links. The agent size is represented by the node's diameter, and the strength of interaction is represented by the thickness of the associated link.

Our model consider an interaction function that allows agents to interact in a selective way, it means that the interaction will be strong between those agents with similar or equal sizes. Meanwhile we will have weak interaction between agents with very different sizes (see Caram *et al.* 2010).

We have observed different behaviors like oscillations or chaotic evolution, but the more interesting case was the emergence of clusters of agents or the stratification of the system which is studied in this paper. Also we found different behaviors for a given set of parameters and different set of initial conditions.

In previous works (see, for example, Adamic and Huberman 2000; Maurer and Huberman 2003; Yanhui and Siming 2007) it was demonstrated the appearance of just two states of global behavior: on one side, one agent gets all the resources meaning that "the winner take all" while the rest of sites have nothing and, on the other side, each agent gets some level or portion of the common resources. With the modification described here, we have found that besides those behaviors, also there are clusters where many agents share different levels of common resources.

The main objective of the present work is to enhance, using complex networks representation, how the dynamics of the agents' size is, as well as how they are related, in both, the competitive and the cooperative scenarios. For arriving to this purpose, the time evolution of the agents size is simulated and the emerging complex network configuration is visualized.

2. The Model

We consider n agents with access to some common resource. If the agent i is able to get some portion of the common resource, its size increases, while if not, loosing a portion of the resource implies a reduction in its size. In our model, essentially based on the well known prey-predator model, the interaction parameter is not a constant (see Caram *et al.* 2010). Instead, it depends on the difference between agent sizes. This fact introduces a feedback phenomenon which induces a high complex dynamics. Each agent size s_i is represented by the fraction of common resource that the agent is able to get during system evolution.

Mathematically the model is the following:

$$\frac{ds_i}{dt} = \alpha_i s_i (\beta_i - s_i) - \sum_{i \neq j} \gamma_{ij}(s_i, s_j) s_i s_j \qquad \text{for } i = 1, \ldots, n \tag{1}$$

where s_i is the size of agent i, α_i is the agent's grow rate, β_i is the agent's maxima capacity without interaction and $\gamma_{ij}(s_i, s_j)$ is the interaction between the agent s_i and the agent s_j.

If $\gamma_{ij} = 0$ the system is uncoupled, and it is the basic Lotka-Volterra or Kolmogorov prey-predator model (see Lotka 1925; Volterra 1926). In this case each agent will exponentially grow according with his particular rate α_i, up to reach his maximum capacity β_i or the maximum size, as it happens in the population dynamics model of Verhulst (see Verhulst 1845, 1847). Once each agent has reached its maximum, the system is in the stationary state. By introducing a variable interaction given by $\gamma_{ij}(s_i, s_j)$, the dynamics is changed (see Caram *et al.* 2010). In fact, the interaction could be stronger or weaker depending on the

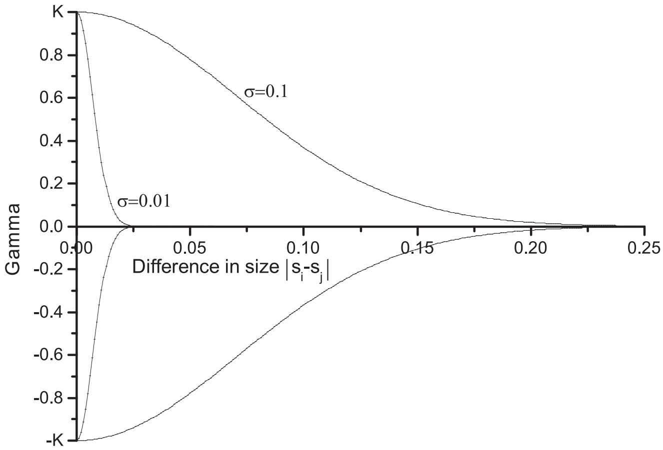

FIGURE 1. $\gamma_{ij}(s_i, s_j)$ interaction function depending on difference in sizes. K value defines the nature of competition or cooperation of pair model. If $K > 0$, we present a competitive scenario, while if $K < 0$ a cooperative scenario appears.

sizes of interacting agents. We use the following interaction function:

$$\gamma_{ij}(s_i, s_j) = K \exp\left[-\left(\frac{s_i - s_j}{\sigma}\right)^2\right], \tag{2}$$

where K and σ are fixed parameters. It means that, when a big agent is interacting with a small one, the intensity of interaction is weak or almost null. Instead, when two agents of the same or similar size are interacting, the intensity of interaction becomes strong. There is a relationship to a well known normal distribution, in which the variable is $\Delta_{ij} = (s_i - s_j)$. It is easy to see that the interaction $\gamma_{ij}(s_i, s_j)$ just varies between 0 and K. Parameter σ, is a positive parameter that controls the interaction levels. The K parameter determines the type of scenario: $K > 0$ competitive, or $K < 0$ cooperative as well as it defines the amplitude of the interaction.

In Fig. 1, the interaction function $\gamma(s_i, s_j)$ is shown as a function of Δ_{ij} for different values of σ. It is noted that, for $\sigma \to 0$, $\gamma(s_i, s_j)$ is more like a Dirac distribution.

We consider that all agents have the same dynamical properties, so $\alpha_i = 1$ and $\beta_i = 1$, in which case the model is given by:

$$\frac{ds_i}{dt} = s_i(1 - s_i) - \sum_{i \neq j} K \exp^{-\left(\frac{s_i - s_j}{\sigma}\right)^2} s_i s_j \qquad \text{for } i = 1, \ldots, n \tag{3}$$

Further details about fixed points analysis and stability points of such, can be found in Caram *et al.* (2010) for the competitive scenario and $n = 5$. In the present contribution we describe the previous results, in terms of complex networks representation and include the cooperative scenario as well ($K < 0$).

We restrict ourselves to examine the case of $n = 5$ agents. This restriction do not affect the possibility to apply the model to more agents, just demand more simulation time with different parameters values.

It can be seen that, for $n = 5$ agents, considering the degeneracy of several solutions, seven possible cases of fixed points appear representing different combinations in the number of agents that are involved in each cluster. These cases are summarized by Caram *et al.* (2010), for the competitive scenario, where the emerged fixed points are stable or not according with the σ value and the initial conditions. The type of stability has been determined by numerically evaluating the Jacobian matrix of the system at the fixed point, using a Newton-Raphson type algorithm (see Caram *et al.* 2010). In the following sections, we give different examples where the model parameters, σ, K, are kept fixed, but the set of initial conditions is different, just for stable fixed points. The size of the agent is given by s_i meantime the links' thickness is given by the corresponding $\gamma(s_i, s_j)$. Complex networks allow us to display the interaction between agents.

3. Competitive scenario

In order to represent a competitive scenario, K must be positive, this produces a negative feedback. So, according to the analysis made by Caram *et al.* (2010), the possible range is $0 < K < 1$. Three different network configurations emerge during the time evolution of the system which are shown in Fig. 2. In this case the time evolution corresponds to a set of initial conditions which were chosen very close to each other (Gaussian distributed with mean $\mu = 0.1$ and standard deviation equals to 0.05), $\sigma = 0.1$ and $K = 1$. With this σ value just only two fixed points are stable. This case is one of the two possible time evolutions according to the analysis made by Caram *et al.* (2010). It is clear that, in the stationary state two clusters of agents with strong interactions within the cluster appear. It is easy to see that two subnetworks well differentiated composed by three and two agents, respectively. One subnetwork has two agents with large sizes (agents 4 and 5), and the other has three agents with the same small size (agents 1, 2 and 3).

In Fig. 3 a different competitive scenario is shown. Parameters are $\sigma = 0.1$, $K = 1$ and random initial conditions are considered (uniformly distributed in $[0, 1]$). Here two different configurations emerge during the time evolution of the network, for $n = 5$ agents. This case corresponds to the other possible time evolution according to the analysis made by Caram *et al.* (2010). In the stationary state three clusters of agents with strong interactions within clusters emerge. Just only one agent (with the largest size) $s_1 \simeq 1$, dominates the configuration, and two subnetworks are well differentiated and composed by two agents each one, respectively. One subnetwork has two agents with medium sizes (agents 2 and 3), and the other has two agents with the same lower size (agents 4 and 5).

As it was discussed by Caram *et al.* (2010), Figs. 2 and 3 correspond to three levels of sizes, 1-1-3, (two agents with different sizes and three agents with the same size) and to four levels of sizes, 1-1-1-2, (three agents with different sizes and two agent with the same size), respectively. The difference between both time evolutions are due to the different set of used initial conditions, showing the sensitivity to them, in the competitive scenarios.

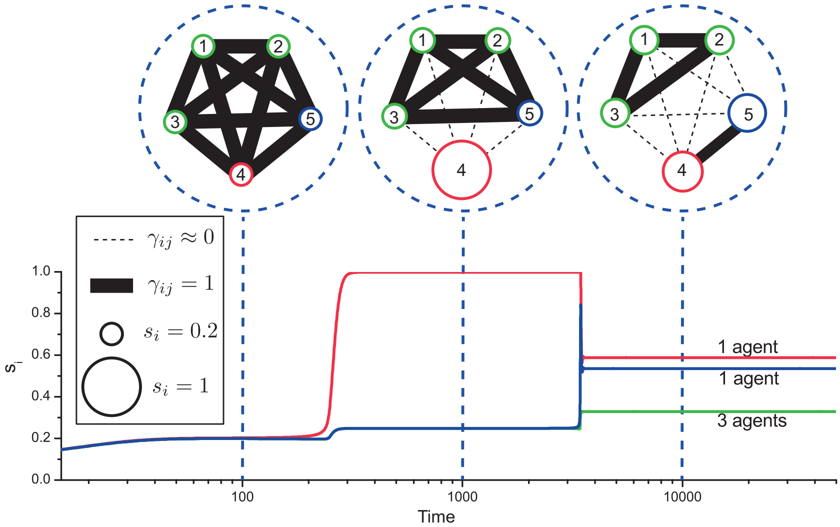

FIGURE 2. Top: network configurations. Bottom: s_i vs. time simulation results for $\sigma = 0.1$, $K = 1$ (competitive scenario). Initial conditions are taken for agent sizes quite close to each other. Three different network configuration emerge.

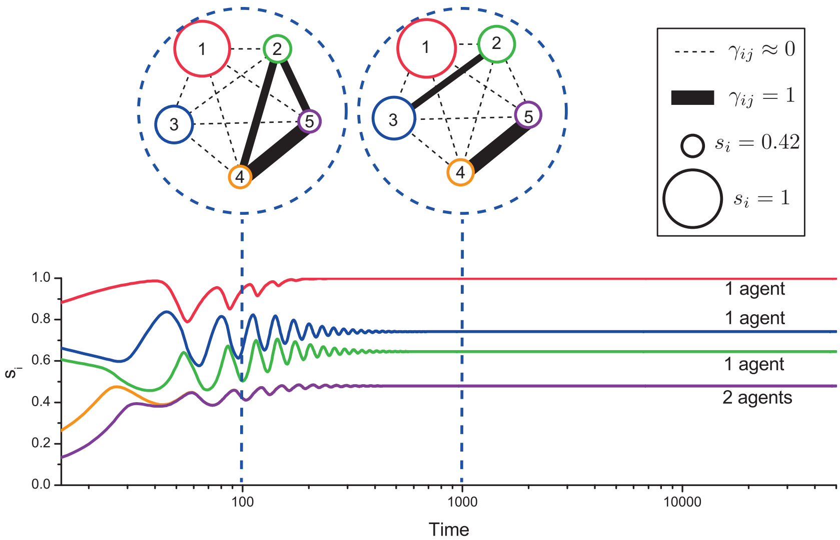

FIGURE 3. Top: network configurations. Bottom: s_i vs. time simulation results for $\sigma = 0.1$ and $K = 1$ (competitive scenario) and random initial conditions. Two different network configurations emerge.

4. Cooperative scenario

In order to represent a cooperative scenario, K must be negative, which creates a positive feedback. Instead of stabilizing the system, it leads to very unstable and complex behaviors and the possible values of K must be chosen very carefully in order to have a stable system behavior. When the cooperation between agents i and j is maximum, and equal to K, $s_i = s_j$ and, as the difference of sizes decreases, as absolute value, the cooperation decreases too, making only possible values into the range $-K < \gamma_{ij}(s_i, s_j) < 0$, in order to have a stable system behavior. Following a similar analysis as it was done by Caram *et al.* (2010), it is demonstrated that, the K values range is $-\frac{1}{n-1} < K < 0$, for getting a cooperative scenario with stable solutions, so for five agents case ($n = 5$), it results in $-\frac{1}{4} < K < 0$. In this case, four stable fixed points exits: four levels of sizes, 2-1-1-1, two agents with the same size and three agents with different sizes, two levels of sizes, 3-2, three agents with the same size and two agents with other lower same size, two levels of sizes, 4-1, four agents with the same size and one agent with other lower size, and one level of sizes, 5-0, five agents with the same size, respectively. Following a similar analysis as it was done by Caram *et al.* (2010), is possible to see that, the fixed point 2-1-1-1, is the strongest attractor compared to the rest of fixed points. In the following we show simulations in which each one of fixed points are reached in the stationary state. Initial conditions are chosen in order to assure the convergence to the desired fixed points.

Three different configurations emerge during the time evolution of the network, for $n = 5$ agents, which are shown in Fig. 4. In this case, the time evolution corresponds to $\sigma = 0.01$ and $K = -0.0625$ for a set of random initial conditions (uniformly distributed in $[0, 1]$). Following a similar analysis as it was done by Caram *et al.* (2010), and for these σ and K values, it is demonstrated that there are four possible stable fixed points. So this case is one of the four possible stationary states. We can see that all agents appear with almost the same sizes. One can assume that the interaction should be strong, but this is not true. It happens that for $\sigma = 0.01$, the random initial conditions lead to a stationary state with an average $\Delta s \geq 0.02$ and interaction $\gamma_{ij}(s_i, s_j)$ equals to almost zero (see Fig. 1). However all agents have similar sizes but very different strengths of interaction, for example agent 5 is disconnected of the rest. When agent sizes are exactly the same, the interaction is really strong, as it happens between agents 1 and 2. On the other side, agents 3 and 4 are well connected and the interactions between agents 1 and 4, and agents 2 and 4 are weak. Also, a medium interaction is observed between agents 1 and 3, and agents 2 and 3.

A different cooperative scenario is obtained for $\sigma = 0.01$ and $K = -0.0625$ for a set of initial conditions $s_1 = s_2 = s_3 = 1.14286$ and $s_4 = s_5 = 1.066$ very close to the fixed point at $s_1 = s_2 = s_3 = 1.14286$ and $s_4 = s_5 = 1.06667$. We observe that the system evolves quickly to the fixed point (see Fig. 5). This case is one of the four possible stationary states, three agent sizes in $s_1 = s_2 = s_3 = 1.14286$, and two agents in $s_4 = s_5 = 1.06667$, appear in stationary state. The resulting system configuration is easily described by a complex network representation. This representation shows two subnetworks with strong interactions within the clusters, composed by three and two agents, respectively. The difference of sizes between two subnetworks is $\Delta s = 0.07619$, and for this reason the subnetworks are disconnected.

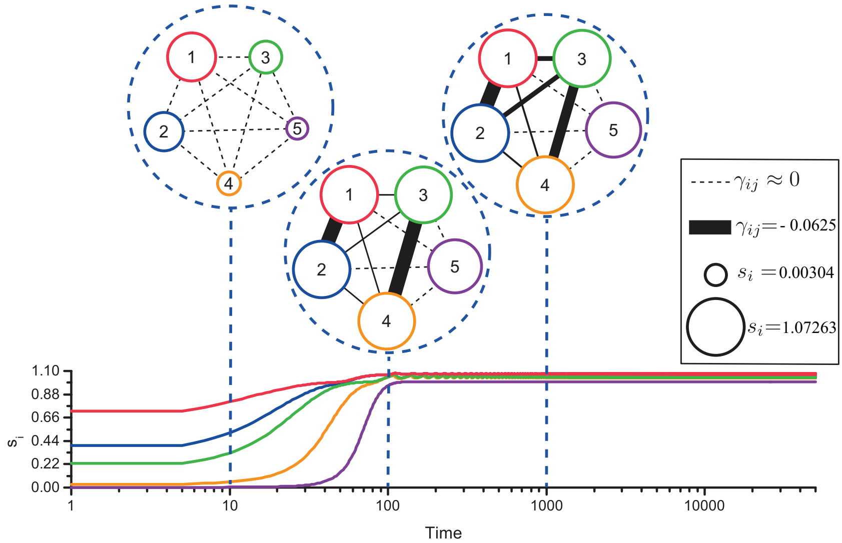

FIGURE 4. Top: network configurations. Bottom: s_i vs. time simulation results for $\sigma = 0.01$, $K = -0.0625$ (cooperative scenario) and random initial conditions. Three different network configurations emerge. From the given initial conditions, a maximum $\Delta s = 0.0547$ values appear for the stationary state.

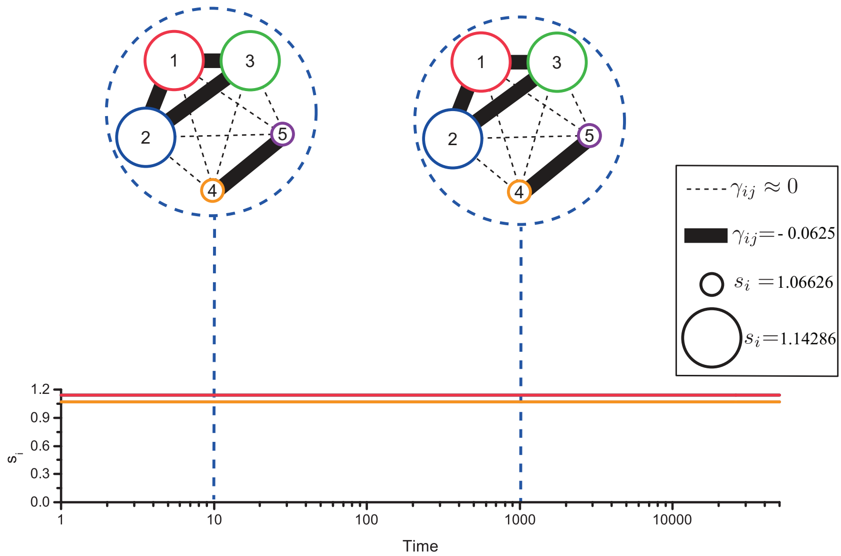

FIGURE 5. Top: network configurations. Bottom: s_i vs. time simulation results for $\sigma = 0.01$, $K = -0.0625$ (cooperative scenario) and initial conditions such that agent sizes are close to the stable fixed point. Two different network configurations emerge with $\Delta s = 0.07619$.

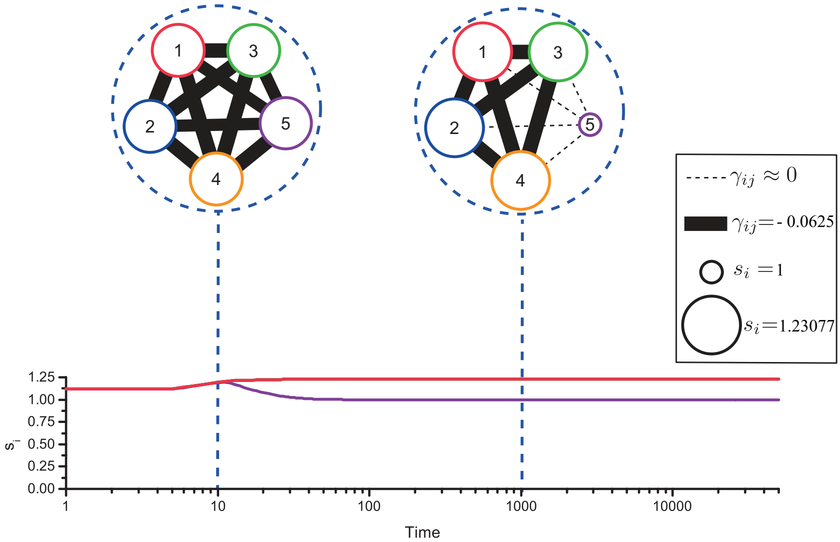

FIGURE 6. Top: network configurations. Bottom: s_i vs. time simulation results for $\sigma = 0.01$, $K = -0.0625$ (cooperative scenario) and a set of initial conditions such that agent sizes close to the stable fixed point. Network representations for $\Delta s = 0$ and $\Delta s = 0.23077$ (maximum) are shown.

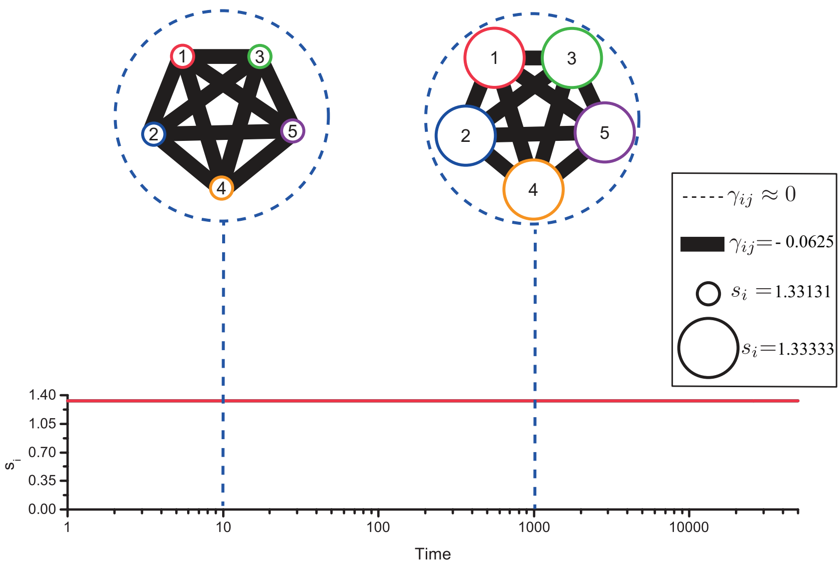

FIGURE 7. Top: network configurations. Bottom: s_i vs. time simulation results for $\sigma = 0.01$, $K = -0.0625$ (cooperative scenario) and a set of initial conditions such that agent sizes are chosen to be close to to the fixed point.

In Fig. 6, a different cooperative scenario is obtained for $\sigma = 0.01$, $K = -0.0625$ as before, and also the set of initial conditions were chosen very close to the fixed point at $s_1 = s_2 = s_3 = s_4 = 1.23077$ and $s_5 = 1$ (Gaussian distribution with mean $\mu = 1.0$ and standard deviation of 0.0005), but now $\Delta s = 0.23077$. This case is one of the four possible time evolution and in the stationary state two network configurations emerge. Now the system evolves quickly to four agent sizes with $s_1 = s_2 = s_3 = s_4 = 1.23077$, and one agent with $s_5 = 1$. Two clusters of agent sizes with strong interaction emerge ($t \approx 30$). A subnetwork well differentiated composed by four agents with exactly the same size, and agent 5 disconnected.

In Fig. 7, keeping $\sigma = 0.01$, $K = -0.0625$ and the set of initial conditions ($s_1 = s_2 = s_3 = s_4 = s_5 = 1.33$) near to the stable fixed point $s_1 = s_2 = s_3 = s_4 = s_5 = 1.33333$. This case is the another of the four possible time evolution. We observe that the sizes of agents are the same within each network configuration (see Fig. 7 at $t = 10$ and $t = 1000$) but, the sizes slightly change over time in order to evolve to the fixed point. Now, in the stationary state, five agents with sizes in $s_1 = s_2 = s_3 = s_4 = s_5 = 1.33333$ are found. However, the network representation allows one to enhance the role of $\gamma(s_i, s_j)$ which otherwise will be difficult to visualize.

The network representations described above (see Figs. 4, 5, 6, and 7) show, in the stationary state, four levels of size, 2-1-1-1, two agents with the same size and three agents with different sizes, two levels of sizes, 3-2, three agents with the same size and two agents with other lower same size, two levels of sizes, 4-1, four agents with the same size and one agent with other lower size, and one level of sizes, 5-0, five agents with the same size, respectively.

5. Conclusions

In this work we have described different behaviors of a particular multi-agent system, including both cooperative and competitive scenarios. The complex networks representation is introduced which allows us to give a more complete description of the model. While competitive scenario leads to stratification of agents with sizes lower or at least equal to β (in this paper $\beta = 1$), in the cooperative scenario clusters of agents emerge with individual capacities higher that $\beta = 1$, which makes clear the cooperative effect. The complex network representation allow us to appreciate how the difference value Δs impact on the interaction level $\gamma(s_i, s_j)$ for each pair of agents. The difference between time evolutions are due to the different set of initial conditions. The complex networks representation reflects how the initial conditions modified both, the number of interacting agents, and the links among them.

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