AAPT PHYSICS EDUCATION
AMERICAN JOURNAL of PHYSICS

An experiment to measure the instantaneous distance to the Moon

Leonardo J. Pellizza, Mariano G. Mayochi, and Ligia Ciocci Brazzano

Citation: American Journal of Physics **82**, 311 (2014); doi: 10.1119/1.4856566
View online: http://dx.doi.org/10.1119/1.4856566
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/82/4?ver=pdfcov
Published by the American Association of Physics Teachers

An experiment to measure the instantaneous distance to the Moon

Leonardo J. Pellizza[a)]
Instituto de Astronomía y Física del Espacio (CONICET/UBA), C.C. 67, Suc. 28, C1428ZAA, Buenos Aires, Argentina and Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina

Mariano G. Mayochi
Scuola Italiana Cristoforo Colombo, Ramsay 2251, Buenos Aires, Argentina, and Dpto. de Física "J.J. Giambiagi" FCEyN, Universidad de Buenos Aires, Int. Güiraldes 2620, Buenos Aires, Argentina

Ligia Ciocci Brazzano
Facultad de Ingeniería, Universidad de Buenos Aires, Av. Paseo Colón 850, Buenos Aires, Argentina and Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina

(Received 4 February 2013; accepted 10 December 2013)

We propose an experimental technique for determining the distance to the Moon. Our technique is based on measuring the change in angular size of the lunar disk due to the variation of the observer-Moon distance, as caused by the rotation of the Earth over several hours. Using this method we obtained a value of 3.46×10^5 km with a precision of 7%. Additionally, our technique allows for the determination of the Moon radius (1678 km $\pm$ 7%), and the instantaneous radial velocity with respect to the Earth (26.4 m/s $\pm$ 26%). A unique advantage of this method is that it is performed from a single location with a single observer, unlike the traditional parallax-based measurements that require at least two observers with a large separation distance.

[http://dx.doi.org/10.1119/1.4856566]

I. INTRODUCTION

The Moon, which is our only natural satellite, is located at a mean distance of 3.84×10^5 km—about 60 Earth radii—from the Earth.[1] It is the cause of many natural phenomena, the most common of which are solar eclipses and ocean tides. Knowledge of the distance to the Moon is important for several reasons, and it can be used to test theories of gravitation, study the motion of the Moon, and investigate geophysical phenomena such as continental drift.[2,3] As with any astronomical object, the measurement of the distance to the Moon is a difficult task. The first precise measurements were made in the 18th century, using the parallax method, which was refined in the following centuries to attain a precision of a few kilometers.[4] The primary drawback of parallax-based approaches is that it requires time-coordinated observations from two different Earth-bound locations, separated by a maximal distance. The next major measurement advance was attained by the use of radar pulses, whose round-trip travel time was measured to determine the distance to the Moon with an uncertainty of about 1 km.[5] At present, lunar laser-ranging techniques determine the distance to within a few centimeters, thanks to the retroreflectors left on the Moon surface by Apollo astronauts.[2,6]

The measurement of the distance to the Moon offers an interesting opportunity to train undergraduate students in observational techniques and give them some insight into the particular problem of determining distances in astronomy. As with any observation in which the experimental conditions cannot be completely controlled, it requires both careful planning and the flexibility to adapt the plan to changing external conditions. At the same time, because the Moon is a bright object that is observable almost everyday (from most places in the world), the experiment is easy to fit into most course schedules. Past work required the use of lunar and solar eclipses to perform low-precision measurements of the distance to the Moon.[7–10] Lough[11] determined this distance by measuring the horizontal coordinates of the Moon at two points on the Earth's surface using theodolites, while Cenadelli[12] set up an international campaign to image the Moon from Italy and South Africa and derive its parallax. Their results agree with those of more precise methods to within a few percent. The experiments of Lough[11] and Cenadelli[12] have the advantage of a simple theoretical framework, but their drawback is that a large effort must be exerted to coordinate observations at two locations on Earth.

In this paper, we propose an experiment to obtain a relatively high-precision measurement of the instantaneous distance to the Moon, using only a small telescope and a digital camera. Our experiment has the advantage that it can be performed from a single location on the Earth and needs only a single set of measurements taken over several hours. Due to the rotation of the Earth, the Moon first approaches and then recedes from a particular location on Earth and this results in a change in its angular size. The experiment is based on the determination of this diurnal change. Roughly speaking, the fractional amount of this change depends on the ratio of the Earth's radius to the Earth-Moon distance. As this change is on the order of a few percent, very careful planning, setup, and execution of the experiment is critical for obtaining the desired result. Hence, the experiment poses an interesting challenge for students.

The organization of this paper is as follows: Sec. II gives the theoretical framework, Sec. III shows our experimental setup and procedure, and Sec. IV discusses our data analysis. Finally, Sec. V presents our conclusions.

II. THEORY

Let us consider the Earth-Moon system from a reference frame located at the center of the Earth, as shown in Fig. 1. The position of the Moon at time t is $\vec{r}$, while that of an observer at the surface of the Earth is $\vec{r}_{\rm obs}$. The distance between the centers of the Earth and the Moon is then $r = |\vec{r}|$, while the radius of the Earth (assumed to be

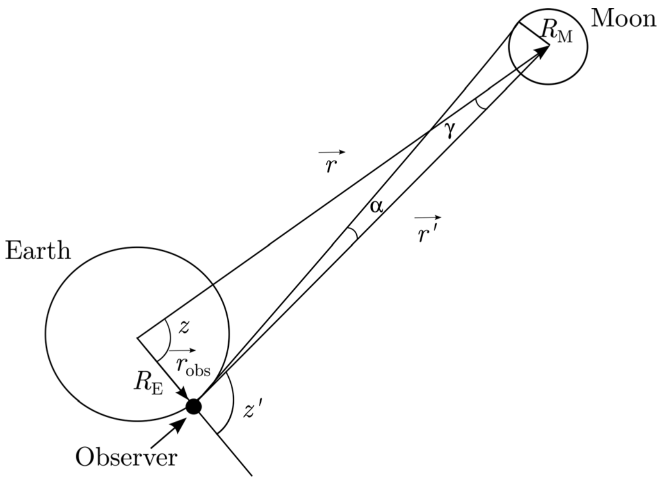

Fig. 1. Schematic of the geometry used to determine the Earth-Moon distance.

spherical) is $R_E = |\vec{r}_{obs}|$. The distance between the observer and the Moon is $r' = |\vec{r} - \vec{r}_{obs}|$, hence

$$r'^2 = r^2 + R_E^2 - 2r R_E \cos z, \tag{1}$$

where z is the geocentric zenith angle of the Moon. As z is not an observable, we aim to replace it by some function of the topocentric zenith angle of the Moon z'. Trigonometric identities lead us to

$$r' = r\sqrt{1 + x^2 - 2x(\cos z' \cos\gamma + \sin z' \sin\gamma)}, \tag{2}$$

where $\gamma = z' - z$, and $x = R_E/r$. As $x \ll 1$, we approximate Eq. (2) to first order in x in two steps. First, recalling that $\sqrt{1+\epsilon} \approx 1 + \epsilon/2$ and neglecting terms of order x^2 or higher, we arrive at

$$r' \approx r[1 - x(\cos z' \cos\gamma + \sin z' \sin\gamma)]. \tag{3}$$

Second, we take into account that γ is on the order of x (see Fig. 1); this implies that $\sin\gamma$ is at most of that same order, making the second term in parentheses in Eq. (3) of order x^2. Moreover, we can put $\cos\gamma \approx 1$, disregarding terms of order x^2 or greater. Hence, to first order in R_E/r, we have

$$r' \approx r\left[1 - \frac{R_E}{r}\cos z'\right]. \tag{4}$$

Because of the orbital motion of the Moon, the distance r (and hence r') depends on time with a time scale of a month; in addition, the distance r' varies due to the rotation of the Earth with a time scale of a day. As we aim to measure the effect of the Earth's rotation our experiment will last only a few hours; hence, we can approximate the variation of r by a linear function

$$r'(t) \approx r_0 + v_r(t - t_0) - R_E \cos z'(t), \tag{5}$$

where r_0 is the Earth-Moon distance at (an arbitrary) time t_0, and v_r the radial velocity of the Moon at t_0.

At any time t, the angular radius α of the visible disk of the Moon measured by the observer is

$$\alpha \simeq \sin\alpha = \frac{R_M}{r'}, \tag{6}$$

where R_M is the radius of the Moon (the linear approximation of the sine function is justified by the fact that $\alpha \approx 0.5°$). Thus Eq. (5) can be expressed in terms of the observables α and z' as

$$\alpha^{-1} \approx \frac{r_0}{R_M} + \frac{v_r}{R_M}(t - t_0) - \frac{R_E}{R_M}\cos z'(t), \tag{7}$$

which gives the time variation of the angular radius of the Moon measured by the observer. This variation has a component produced by the orbital motion of the Moon (the second term) and a component that depends on the Earth's rotation (the third term). The former has already been approximated as linear for a time scale of a few hours, but the latter is nonlinear on this time scale. The idea behind our experiment is to measure the dependence of α on z', from which we deduce r_0. This would be straightforward if we could drop the second term in Eq. (7), which we can if the experiment is performed when the Moon is near its perigee or apogee (when $v_r = 0$). In the general case, however, $v_r \neq 0$ and we must resort to a slightly different method.

Although the orbital motion term in Eq. (7) cannot be neglected, it can be estimated in the following way. Assume we measure the angular size of the Moon at a pair of times with the same zenith angle z', the first (t_a) when the Moon is ascending in the sky and the last (t_d) when it is descending. As the zenith angle is the same at both times, the term describing the Earth's rotation in Eq. (7) has the same value and, within our approximations, we get

$$\alpha^{-1}(t_d) - \alpha^{-1}(t_a) \approx \frac{v_r}{R_M}(t_d - t_a). \tag{8}$$

Hence, two observations with the same zenith angle will allow us to estimate the value of v_r/R_M. Given this value, the individual values of α can be corrected for the Moon's orbital motion to obtain the angular size β that the Moon would have if it were static at the distance r_0:

$$\beta^{-1}(t) = \alpha^{-1}(t) - \frac{v_r}{R_M}(t - t_0). \tag{9}$$

Substituting into Eq. (7), we find that

$$\beta^{-1} \approx \frac{r_0}{R_M} - \frac{R_E}{R_M}\cos z'. \tag{10}$$

We can now use our observations of α and z' to derive the dependence of β^{-1} on $\cos z'$. A linear fit to this relation would allow us to obtain the value of r_0/R_E as the ratio between the intercept and the slope. Two important characteristics of this method must be noted. First, it is not necessary to measure the angular radius of the Moon. In fact, the angular size of any feature in the Moon's surface can be used instead, as long as it does not change its projected physical size during the experiment. Second, as the method relies in the ratio of the first and second terms in the right-hand-side of Eq. (10), the measured angles can be replaced by any other proportional observables, such as the linear size of the image of the Moon on the focal plane of a telescope. The use of actual angles however, allows for the determination of the radius of the Moon and its instantaneous radial velocity as a by-product.

To assess the experimental requirements, it is interesting to compute the order of magnitude of the effect we want to measure. The values of three observables must be determined, the time, the zenith angle, and the angular size of the Moon. Times are easy to measure, and standard stopwatches have precisions well below a second for short periods of times, hence the measurement of times poses no problem in this experiment. A similar argument applies to the zenith angle of the Moon, as modern instruments allow us to measure it with a precision of less than a degree, which is good enough for our purposes. The measurement of the angular size of the Moon is then the major source of uncertainties in the present experiment. Not only it is a small angle, about 0.5°, but what we want to measure is its variation, which is much smaller. The mean distance between the Moon and the observer is $\sim 3.84 \times 10^5$ km, and we want to measure its diurnal variation due to the Earth's rotation, which amounts, at most, to the Earth's radius of 6.37×10^3 km. Hence, the maximum variation of the Moon's angular size we expect is just 1.5% of this size, or about 0.5 arc min. As we want to track this variation from near the horizon up to the Moon's maximum altitude, we need a resolution at least one order of magnitude better, or about a few arcseconds. Moreover, differential refraction in the turbulent atmosphere of the Earth produces random changes in the apparent position of celestial objects (the so-called "seeing"), which in the case of the Moon, translates into a random distortion of its disk. This effect amounts also to a few arcseconds, so it must be taken into account to obtain meaningful results. These are the real challenges of the present experiment and the reason behind the choice of our experimental setup.

III. EXPERIMENTAL SETUP

The goal of the present experiment is to accurately measure the Moon's angular size as a function of time and zenith angle, from its rise to its set. The best choice is to perform the experiment at full Moon phase, because the Moon is visible all night long, and its entire disk is illuminated, ensuring an easy measurement of its radius. The experiment was performed in Buenos Aires, Argentina, on April 6, 2012. The Moon rose that day at $18^\mathrm{h}\,26^\mathrm{m}$ local time ($21^\mathrm{h}\,26^\mathrm{m}$ UT), and set the next day at $8^\mathrm{h}\,04^\mathrm{m}$ ($11^\mathrm{h}\,04^\mathrm{m}$ UT), reaching a maximum altitude of approximately 65°.

The critical constraint of this experiment is to obtain a precision on the order of 10^{-3} in the angular size. This constraint limited our options, given that we wanted to use a small telescope and a high-resolution digital camera to image the Moon and measure its size. We decided to perform the experiment at the Galileo Galilei Observatory of the Cristoforo Colombo Italian School of Buenos Aires. The observatory is equipped with a 203-mm aperture, *f*/6.3-focal-ratio Meade LX200 Schmidt-Cassegrain telescope, and a 2048×3072-pixel Canon EOS Digital Rebel XT reflex camera was used to image the Moon. The image of the Moon in the focal plane of the telescope is about 11 mm in size, which corresponds to about 1700 pixels of the camera chip and allows us to almost exploit the full resolution of the camera. This will result a nominal precision on the order of 0.001 in the measurement of the Moon's angular radius, if a 1 pixel uncertainty is assumed.

The telescope has a motor-driven alt-azimuth mount, and an on-board computer that performs the tracking of celestial objects in both axes as they move across the sky. The mount has a setting circle fixed to its horizontal axis, which allows a direct measurement of the altitude (the complement of the zenith angle) of the celestial object at which the telescope is pointing, up to one-eighth of a degree, a precision high enough for our purposes. An indirect measurement of the altitude is given by the computer, determined using the geographic coordinates of the observatory, the time, and the coordinates of the object being observed. However, we preferred the direct measurement as its precision is high enough for our purposes. A crucial step to obtain a reliable measurement from the setting circles is a careful leveling of the telescope mount, which was accomplished using the bubble level integrated into the unit.

Given the method described in Sec. II, we used the telescope to image the Moon at a set of different zenith angles ranging from 16 to 65°, in 5° increments. For each zenith angle, two sets of images were obtained, one while the Moon was ascending in the sky and the other, while it was descending. Each set comprises a sequence of four images, taken one immediately after another to avoid zenith angle variations. Taking more than one image allows us to repeat the measurements and average the results to decrease the effect of image distortions due to atmospheric seeing, thus improving the precision of the experiment. Although more than four images per set would have been desirable, this limit was imposed both by the motion of the Moon and by the size of the storage memory of the camera. The mean time that the images were taken was recorded using a standard stopwatch synchronized to the local time. Before the experiment, the camera was focused on the Moon (a straightforward task for a reflex camera and such a bright object), and a set of test images were taken to determine the best exposure time. A value of 1/3200-sec exposure at ISO-800 was chosen because it provided a good contrast of the images and avoided overexposing the Moon.

The raw outcome of the experiment was therefore 22 sets of four images of the full Moon at 11 different zenith angles, and the mean times when the images were taken. We also took an image of a known stellar field with exactly the same telescope plus camera configuration so that we could determine the angular scale of the images and hence be able to transform linear sizes on the images into angular sizes in the sky.

IV. DATA ANALYSIS AND RESULTS

Many features of the lunar landscape, such as craters and mountain chains, are clearly visible in our images. As pointed out in Sec. II, the angular size corresponding to any fixed length on the surface of the Moon can be used to measure its distance. The obvious choice is the Moon radius, but the distance between two fixed points on the surface, such as the centers of a pair of craters, can be used as well. Each choice has its advantages and disadvantages, hence we decided to use both and then compare the results.

A. Measurement of Moon radius

The measurement of the Moon radius is somewhat tricky because the limb is not precisely defined in the image. A radial intensity profile of the Moon disk (Fig. 2) shows a smooth decline of the intensity at the limb, instead of a sharp cutoff. This is due to different factors, among them the angular dependence of the scattering of solar light by the Moon surface, and atmospheric refraction and extinction. Given these factors, a physically motivated working hypothesis for

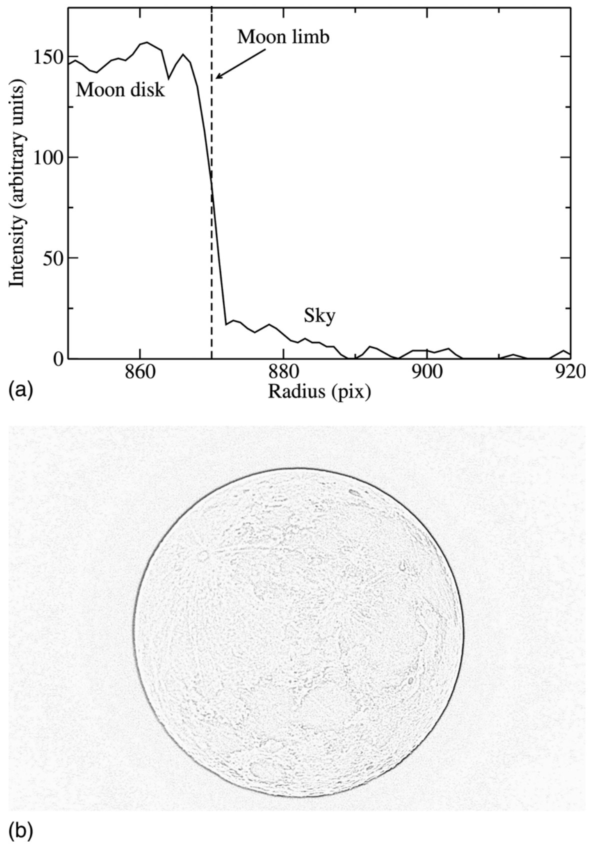

Fig. 2. Top: Radial intensity profile of an image of the Moon, showing the smooth decline in intensity that marks the position of the Moon limb. Bottom: The same image, processed using an edge-detection filter. The black ring around the Moon disk shows the position of the maximum slope of the intensity profile. The high contrast of this ring makes it easy to determine the position of the limb.

determining the location of the limb is to use the radius at which the intensity profile attains its maximum slope, as shown in Fig. 2. Despite this conceptual problem, the detection of the position of the limb in the image, defined in this way, is made easy by digital filtering techniques. Edge-detection filters that, when convolved with the image, give the positions where large intensity changes occur, have been known for many years. Figure 2 shows an example of a filtered image, computed by applying a standard edge-detection filter to the original image, using the GNU Image Manipulation Program (GIMP).[13] The abrupt changes from black to white in the image show the position of the Moon limb, which can be determined by eye with a precision of less that 2 pixels due to this extreme contrast. Using the Smithsonian Astrophysical Observatory DS9 software, which allows us to draw circles on any image and measure their centers and radii, we determined by eye the circle passing as close as possible to the limb position and took its radius as the Moon radius.

The advantages of this technique are twofold. First, it is straightforward to understand and apply. And second, compared to any method that relies on the measurement of a single radius, this technique reduces the experimental error—fitting a circle to the limb is equivalent to measuring and averaging several thousand radii at once, and therefore it is expected to minimize statistical uncertainties associated with the measurement of a single radius. The two main sources of random errors that this technique minimizes are the presence of mountains and valleys on the limb, and the atmospheric seeing that randomly distorts the limb shape. These errors would produce uncertainties on the order of 3–5 arc sec in the observed Moon radius, large enough to interfere with the phenomenon we want to measure. The circle-fitting procedure decrease these uncertainties to about 1 arc sec (see below).

Two sources of systematic errors are also present in our measurements. The first one is due to the difference between the equatorial and polar radius of the Moon, which amounts to $\sim$2.6 arc sec. The second source is atmospheric refraction, which, at low altitudes decreases the vertical radius of the Moon disk. Using normal refraction formulae,[14] we estimate that the difference between the vertical and horizontal radii is below 1 arc sec at altitudes greater than 30°, and reaches 3 arc sec at 16° (the lowest Moon altitude of our experiment). We note here that any of the two sources of systematic errors could be overcome by fitting ellipses, instead of circles, to the limb. However, this procedure cannot be used to avoid both sources together, and it would make the method more complex without providing a significative improvement in the results.

To estimate the total uncertainty (random plus systematic) of our measurements, we began with the best-fit circle and superimposed additional circles with the same center but with larger and smaller radii in steps of 0.25 pixels. We then took the smallest and largest radii that clearly leave the limb inside or outside the circle, and estimate the error of the Moon radius as half their difference. The value obtained in this way was 2 pixels. The average of the radii obtained from the four images in each set has then a mean uncertainty of 1 pixel. It is important to note that there is no advantage to using a more powerful fitting technique; the one described here already gives acceptable precision and it is simple enough to be easily understood by students.

B. Measurement of crater positions

The second method is based on the measurement of the positions of two craters in the image. We chose two craters whose rims are well defined, nearly symmetric (i.e., nearly circular or elliptical), and as far as possible from each other. For each crater, a fit of a circle or ellipse was made by eye, drawing the corresponding curve over the rim using the DS9 software. The center of the two circles was taken as the positions of the craters, and used to compute their distance. Once again, with this method, the position of the crater is defined through several points, hence being less sensitive to random distortions of the image or variations in the intensity of any individual point, thus increasing the precision of the measurement. However, it is still affected by the systematic error due to atmospheric refraction, which in this case is difficult to estimate. Therefore, given our previous results we assigned a conservative error of 2 pixels to the values of the distances between craters. It is important to note that this method could be affected also by libration of the Moon that changes the projection of any line in the Moon surface.

C. Results

Either using the craters or the Moon radius, what we are really measuring is the linear size of an image in the focal plane of the telescope, in pixels. Although as stated in Sec. II, this is enough for the purpose of determining the Moon distance, the true measure of the Moon angular radius would give us the Moon physical radius and radial velocity as by-products. From elementary optics,[15] the image size in pixels can be translated into an angular size if we know the pixel size and the focal length of the telescope. However, the Meade LX200 telescope has a non-standard design that uses a displacement of its primary mirror to focus the image; hence, the effective focal length is different from its nominal value and depends on the distance from the camera to the mirror system. Therefore, we used an empirical method to measure the angular scale of the images. We took an image of a known, moderately crowded stellar field, and fitted the linear distances (in pixels) of several pairs of stars in the image to their known angular distances, determined from their celestial coordinates taken from the Smithsonian Astrophysical Observatory Star Catalogue.[16] As expected, the relation is consistent with a straight line. The slope of the best-fit line gives the translation factor from image sizes in pixels to angular sizes in the sky, which for our setup is 1.1661 ± 0.0004 arc sec/pixel.

The upper panel of Fig. 3 shows the relation between the inverse of the Moon's angular radius and the zenith angle, while the lower panel displays the same relation for the inverse of the distance between two selected craters. In both cases, we see that the observed size increases as the zenith angle decreases, as expected from the effect of the rotation of the Earth on the observer-Moon distance. Moreover, in both cases the α-z relation presents two branches, one for the ascending and the other for the descending Moon. As explained, this is an effect of the Moon's radial velocity, which was not null at the date of our observations. Note also that the entire variation of the size amounts to only about 1%, as our previous analysis predicted.

As explained in Sec. II, the effect of the Moon's radial velocity must be removed to be able to determine its distance. Figure 4 shows the difference between the inverse of the descending and the ascending angular radii of the Moon at the same zenith angle [Eq. (8)] as a function of the time elapsed between both observations. Although the errors are large, a linear trend between these magnitudes is evident, supporting the approximation we made in Eq. (5). A linear fit of the data, constrained to have a null intercept, gives the ratio of the velocity of the Moon to its radius, $v_R/R_M = (1.57 \pm 0.41) \times 10^{-5}\ \mathrm{s}^{-1}$. It is important to note that the relative magnitude of the error is large in part because the Moon happened to be near perigee on the date of the observation, hence the radial velocity was small. However, the value of the goodness of fit, $\chi^2 = 1$ for 10 degrees of freedom, suggests that the measurement errors might have been overestimated.

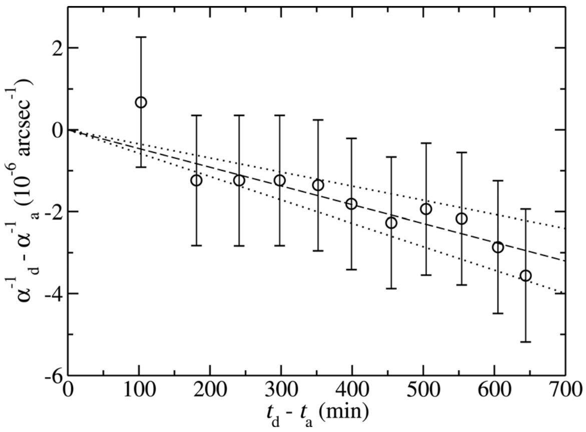

Fig. 4. Difference in the inverse of the angular radius of the Moon between the descending and ascending passages by a certain zenith angle, as a function of the time elapsed between these passages. The dashed line is the best-fit linear relation with null intercept, while the dotted lines correspond to the 99.7% (3σ) confidence region.

The value of v_R/R_M and its uncertainty is then used to correct the individual angular radius measurements, as given by Eq. (9). To do this, we take the time of our first measurement as t_0, hence the resulting Moon distance and radial velocity correspond to this time. Figure 5 shows the corrected angular sizes β^{-1} as a function of the zenith angle of the Moon, both for the Moon radius and for the distance between craters. The two branches seen in Fig. 3 have merged in both cases, indicating that the correction for the orbital motion of the

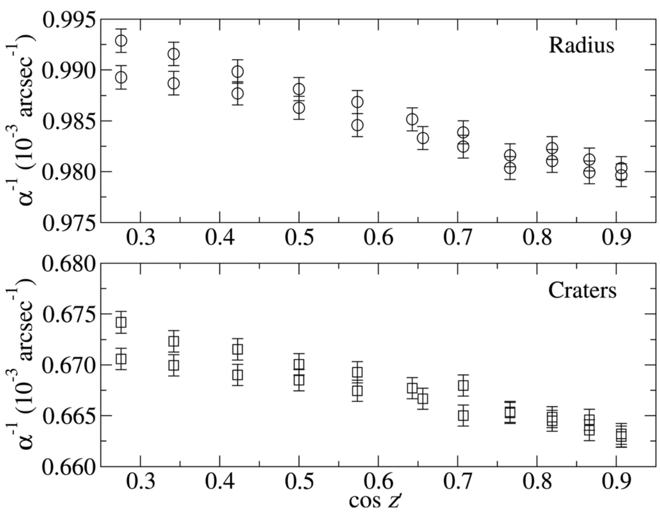

Fig. 3. Inverse of the angular size of the measured feature as a function of the zenith angle of the Moon (upper panel: Moon radius; lower panel: distance between craters). In both cases, the feature size increases as the Moon approaches the zenith, as expected for the effect of the rotation of the Earth. The two branches seen in each plot are due to the effect of the Moon's orbital motion.

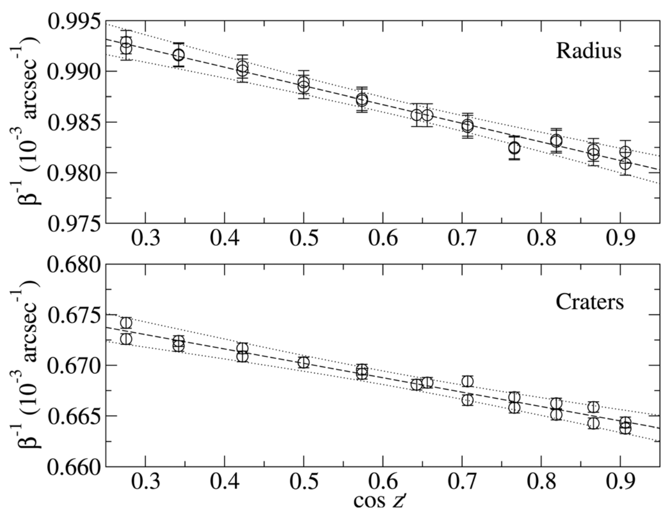

Fig. 5. Inverse of the angular size of the measured feature as a function of the zenith angle of the Moon, corrected for the orbital motion effect (upper panel: Moon radius; lower panel: distance between craters). The dashed line is the best linear fit to the data, while the dotted lines represent the 99.7% (3σ) confidence region.

Moon has been properly applied. The remaining linear trend is very tight and corresponds to the effect of the Earth's rotation. A linear fit to the data should then give the distance to the Moon in terms of the radius of the Earth as the negative of the ratio of the intercept to the slope. From the Moon angular radius data, we obtain $r_0/R_E = 54.2 \pm 3.6$ or, using $R_E = 6371$ km, $r_0 = (3.46 \pm 0.23) \times 10^5$ km. The crater data result in a lower value of $r_0/R_E = 47.6 \pm 3.6$, resulting in $r_0 = (3.04 \pm 0.23) \times 10^5$ km. The meaning of these values will be discussed in the Sec. V. As the value obtained from the angular radius measurements is closer to the mean Earth-Moon distance, we used this one to compute the Moon radius and radial velocity from the fitted parameters, yielding $R_M = (1.68 \pm 0.11) \times 10^3$ km and $v_r = (-26.4 \pm 6.9)$ m/s.

The results obtained from this experiment have a precision of within a few percent, which is fairly good for a training experiment. The Moon distance measured through its angular radius and the Moon physical radius have a 7% precision, while the Moon distance measured using the craters has an 8% precision. Only the radial velocity value has a lower precision of 26%. We attribute this lower precision in part to the fact that the Moon was near perigee at the date of observation—having an almost null radial velocity thus requires extremely precise observations to be measured—and in part to the measurement method that relies on the difference of two quantities of similar values, which is prone to cancellation effects with the corresponding loss of significance.

The accuracy of our results is fairly good. The United States Naval Observatory astronomical data service,[17] based on high-accuracy ephemeris, gives a value of 3.59×10^5 km for the distance to the Moon on April 6, 2012 at 22:50 UT. Hence, our result from the Moon angular radius is consistent with the ephemeris value to within 0.6σ, or 4%. On the other hand, the result obtained using the craters, $r_0 = (3.04 \pm 0.11) \times 10^5$ km, has an accuracy of only 15%, which is poor compared to the former other measurement. This relative inaccuracy could be attributed to sources of systematic error not taken into account and which are not present in the angular radius measurement. The main candidate is the Moon libration, which changes the position of the craters in the disk, and hence the projected size of the line that joins them. On the other hand, our measurement agrees with the accepted value of the Moon radius (1737 km)[1] to within 0.5σ, or 3%.

V. CONCLUSIONS

We present here a simple experiment to measure the instantaneous Earth-Moon distance from a single place on the surface of the Earth. The experiment, lasting for several hours, uses a small telescope and a high-resolution digital camera to track the variations in the Moon's angular radius during one night of observations. The raw output of the experiment is a set of digital images of the Moon, together with the mean time and zenith angle at which the images were taken, plus a single image of a stellar field to determine the angular scale of the images in the sky. The data analysis is simple, requiring only a personal computer with basic image processing and data fitting software to determine the instantaneous distance to the Moon, its physical radius, and its instantaneous radial velocity. Hence, we believe that this experiment allows students to gain first-hand insight on the problem of astronomical distance determination, particularly on these important astrophysical quantities.

Two crucial features of the present experiment must be kept in mind to succeed in obtaining useful data. First, the precision of each measurement must be high (about 0.1%) to detect the effect of the Earth's rotation on the distance between the Moon and the observer. This implies not only a careful instrumental setup, but also a thorough data analysis, investigating and quantifying the sources of uncertainties and taking the necessary steps to minimize their effects. Second, disentangling the effect of the Moon's orbital motion from that of the Earth's rotation poses a constraint on the experiment, as the observations must be made in pairs at the same zenith angle of the Moon. This, together with the usually varying weather conditions over such a long period of time, requires a careful planning of the entire experiment, producing a schedule strict enough to meet the constraint, but flexible enough to be adapted to the possibly changing weather. The level of precision and accuracy obtained by us (7% and 4%, respectively) is a direct consequence of careful analysis and planning of our experiment. Because of these features, we believe that the present experiment poses an interesting challenge to students.

ACKNOWLEDGMENTS

The authors wish to acknowledge Lic. J. Bertúa and the authorities of the Scuola Italiana Cristoforo Colombo of Buenos Aires, Argentina, for the strong support given to this project, and also to the anonymous referees for their detailed and helpful comments.

[a]Electronic mail: pellizza@iafe.uba.ar

[1]NASA Moon fact sheet, <http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html>.
[2]A. L. Hammond, "Laser Ranging: Measuring the Moon's Distance," Science **170**, 1289–1290 (1970).
[3]A. H. Cook and J. Kovalevsky, "Theories of lunar libration," Philos. Trans. R. Soc. London, Ser. A **284**, 573–585 (1977).
[4]J. A. O'Keefe and J. P. Anderson, "The Earth's equatorial radius and the distance of the Moon," Astron. J. **57**, 108–121 (1952).
[5]B. S. Yaplee *et al.*, "The Mean Distance to the Moon as Determined by Radar," in *Proceedings of the IAU Symposium, No. 21*, edited by J. Kovalevsky (Gauthier-Villars, Paris, 1965), pp. 81–93.
[6]P. L. Bender, "Laser Measurements of the lunar distance," Rev. Geophys. Space Phys. **13**, 271–272, doi:10.1029/RG013i003p00271 (1975).
[7]D. H. DeVorkin, "Determining the distance to the moon," Phys. Teach. **10**, 40–43 (1972).
[8]E. Roger Cowley, "A classroom exercise to determine the Earth, Moon distance," Am. J. Phys. **57**, 351–352 (1989).
[9]D. H. Bruning, "Determining Earth-Moon distance," Am. J. Phys. **59**, 850–850 (1991).
[10]M. Sawicki, "Using the solar eclipse to estimate earth's distance from the moon," Phys. Teach. **34**, 232–233 (1996).
[11]T. Lough, A. Kellie, and K. Hee, "Determining the instantaneous distance to the moon by direct triangulation," Eur. J. Phys. **27**, 1121–1132 (2006).
[12]D. Cenadelli *et al.*, "An international parallax campaign to measure distance to the Moon and Mars," Eur. J. Phys. **30**, 35–46 (2009).
[13]"GIMP" stands for "Gnu Image Manipulation Program," and can be found as a free download at <http://www.gimp.org>.
[14]J. Meeus, *Astronomical Algorithms* (Willmann-Bell, 1991).
[15]E. Hecht, *Optics*, 2nd. ed. (Addison-Wesley, Reading, MA, 1987).
[16]SAO Star catalog J2000, available online at <http://vizier.cfa.harvard.edu/viz-bin/VizieR?-source=I/131A>
[17]United States Naval Observatory data services, available online at <http://aa.usno.navy.mil/data/>.