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A labeled argumentation framework

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A B S T R A C T

To increase the expressivity of an argumentation formalism, we propose adding meta-level information to the arguments in the form of labels representing quantifiable data such as reliability degree, strength measure, skill, time availability, or any other feature about arguments. The extra information attached to an argument is then used in the acceptability determination process.

We present a *Labeled Argumentation Framework* (LAF), combining the knowledge representation capabilities provided by the *Argument Interchange Format* (AIF) with an *Algebra of Argumentation Labels*, enabling us to handle the labels associated with the arguments. These labels are propagated through an argumentative graph according to the relations of support, conflict, and aggregation between arguments. Through this process we obtain final labels attached to the arguments that are useful to determine their acceptability.

1. Introduction

The study and implementation of systems that exhibit autonomous intelligence guiding their behavior has been a long term concern of Artificial Intelligence researchers. Argumentation, as an area of Knowledge Representation and Reasoning, specializes in modeling the process of human reasoning to determine which conclusions are acceptable in a context of disagreement. Broadly speaking, argumentation theories deal with the interactions between arguments that are in favor of or against a specific conclusion, such as support or attack, with the final goal of determining when a conclusion is acceptable (see [4,29] for a general account); these theories are extensively used in diverse domains such as legal reasoning, dialogue

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and persuasion, recommender systems, intelligent web search, autonomous agents and multi-agent systems, and many others [27,24,9,29,21].

In certain applications of argumentation, it is necessary to provide further details of the arguments that represent real-world features aiming to obtain more refined results. Moreover, the properties related to the intrinsic logical soundness of an argument are not always the only ones that matter in determining its acceptability; other qualities can be weighted in the decision process of acceptability. For instance, each argument may have associated different characteristics such as its strength [3,7], weight [13], or reliability varying on time [6]. In the domain of agents and multi-agent systems, it is important to associate each argument with the reliability degree of their source and an accuracy measurement associated with the information on which the arguments are based, increasing the information that is used to determine their acceptability.

Based on this intuition, the argumentation process is defined in two steps: the determination of the argument valuations, and the selection of the set of most acceptable arguments. The former can be obtained independently of the interactions with other arguments as [3,14], or those that are dependent on the relations (support and attack) that the argument has with other arguments [17,7]. For the latter, it is possible to analyze this in two ways: the individual acceptability, where the acceptability of an argument depends on its attributes [3,22], and the collective acceptability, where a set of arguments satisfies certain properties [12,8]. Recently, a combination of both points of view was considered, increasing the capability of representing real world applications, and providing more information about argument acceptability [7,13].

We will address here a combination of these proposals, generalizing them and providing a flexible structure which allows different instantiations of its elements to create models tailored for particular goals. Our formalization, called *Labeled Argumentation Framework* (LAF), combines the capabilities of knowledge modeling of the *Argument Interchange Format* (AIF) [10] with an *Algebra of Argumentation Labels* which allows to manipulate and propagate labels through a series of operations defined for that purpose. These labels will be combined and propagated through an argumentation graph according to the manner in which the interactions between arguments are defined: support, conflict, and aggregation. Each one of these has received extensive attention, in particular, aggregation has been studied in the form of argument accrual [28,33,25]; for each of these interactions, an associated operation in the algebra is introduced. Once completed the propagation process which produces the *definitive* argumentation labels associated with the arguments, we continue to establish the arguments' acceptability using the information on the labels to offer, for example, the reliability degree of an acceptability status or an explanation. In addition, using the information provided by the labels we will define an *acceptability threshold* to determine whether an argument satisfies certain conditions to be accepted in a particular query and also to specify when an argument is better to another.

This paper is structured as follows: in Section 2, we introduce a particular abstract algebra for handling the labels associated with the arguments that we call *Algebra of Argumentation Labels*; the core contribution of the paper is presented in Section 3 as the formalism characterizing *Labeled Argumentation Frameworks* (LAF) together with an example of application in the domain of agents and multi-agents systems; finally, in Section 4 we discuss related work, and in Section 5 we conclude and propose future work.

2. An initial example

The aim of our work is to increase the representation capability of argumentation systems through the use of labels that represent real world features of the arguments, bringing the possibility of operating over these labels to compute the corresponding labels of derived arguments. This information will be used to refine the assessment process providing additional details attached to the conclusions offered by the system. Additionally, we will show how our formalism is suitable to be used in different artificial intelligence applications, including but not limited to decision support systems, recommender systems, autonomous agents and multi-agents systems. Next, we will motivate the usefulness of our formalization in the context of autonomous agents and multi-agent systems as an example where the formalism could help achieve an enhanced behavior.

Consider the following scenario where an agent must decide whether it is riskier to invest in a real estate property or to invest in gold bullion. To reach a decision the agent ponders these arguments:

- $\mathbb{A}$ *I should invest my savings in land because it doesn't depreciate quickly, and this leads to financial safety.*
- $\mathbb{B}$ *Buying land is a good way to invest as long as it is a in quiet and safe area.*
- $\mathbb{C}$ *It is not wise to invest in land as involves a significant maintenance expenditure.*
- $\mathbb{D}$ *It is better to invest in gold bullion because it does not deteriorate, and it does not require maintenance.*
- $\mathbb{E}$ *I should not invest in gold bullion, since it is expensive and the results are unpredictable.*
- $\mathbb{F}$ *Investing in gold bullion is expensive as you have to store it in safekeeping.*
- $\mathbb{G}$ *Buying foreign currency is an investment of unpredictable results because it depends on the global economy.*

As it is possible to appreciate in this example, the knowledge used to make a decision can be naturally structured in arguments, and to reach a decision it will be necessary to consider how these arguments affect each other. This interaction include the support between arguments (*e.g.*, $\mathbb{E}$ and $\mathbb{F}$), their aggregation in the form of different arguments for the same conclusion (*e.g.*, $\mathbb{A}$ and $\mathbb{B}$), and the conflicts represented as contradictory conclusions (*e.g.*, $\mathbb{A}$ and $\mathbb{C}$).

In some applications of argumentation, like those in which the objective is to find all the arguments for and against a specific conclusion, it is often natural to consider all those arguments sharing the same claim as a single argument structure instead of assessing them individually. This is known as *aggregation* or *accrual*, and it is based on the intuition that having more reasons for a given conclusion makes it more credible [28,33,25]. In the example, the arguments supporting investing in land $\mathbb{A}$ and $\mathbb{B}$ provide reasons which should be considered together, against the reason given by $\mathbb{C}$ backing the contrary position.

The pros and cons to proceed with a particular decision (*i.e.*, *invest in property* or *in gold bullion*), have associated certain features that influence the final decision; that is to say, the agent uses this information to determine which of these options is the most convenient. Thus, the agent analyzes each case to obtain information on the advantages and disadvantages of each option determining which of them is the most appropriate. In our particular example, it would be interesting to assign to the arguments a reliability degree (the reliability that the agent gives to the information source) and an accuracy measurement (which represents the accuracy level for the information).

Usually, in argumentation systems the conflict between two arguments is resolved producing defeat in the following way: if two arguments $\mathbb{X}$ and $\mathbb{Y}$ are in conflict, and $\mathbb{X}$ is "better" (in some agreed sense) than $\mathbb{Y}$, then $\mathbb{Y}$ becomes defeated by $\mathbb{X}$; but, conflict is a symmetrical relation, *i.e.*, $\mathbb{X}$ and $\mathbb{Y}$ are in effect attacking each other. Nevertheless, in most argumentation formalisms the defeating argument $\mathbb{X}$ is regarded as unaffected by the attack of the defeated argument $\mathbb{Y}$. We propose a different way to treat disputes between arguments allowing the strength of $\mathbb{X}$ to become diminished by $\mathbb{Y}$'s attack, *i.e.*, when there is no counterargument, the strength of an argument that supports a decision should not be the same as when a counter-argument exist. Thus, it is interesting to model the diminishing effect over an undefeated argument when counterarguments exist; *i.e.*, the reliability and certainty that an argument provides to support a decision of the agent should not be the same when it is free of counterarguments as when it is controversial.

Next, we will introduce the elements necessary to develop our formalism, including the possibility of representing meta-information associated with arguments using labels, and providing the capability of defining acceptability by combining and propagating these labels according to the interactions of support, aggregation, and conflict. Finally, we will instantiate the proposed formalization to model the example presented in this section.

3. Algebra of argumentation labels

In this section, we will introduce the use of labels as a tool for aiding in the assessment of arguments. To be useful, these labels must represent information about the arguments and how the arguments interact. A natural way of representing this information is to use a scale that measures a particular feature of the argument, such as the trust the argument has, the temporal availability of the argument, *et cetera*. We will consider a scale, that will be further characterized below, where the valuations range between two distinguished elements: $\bot$ and $\top$, where $\bot$ represents the least possible degree in which an argument may possess a certain attribute, and $\top$ the maximum.

Proposed by L.A. Zadeh [36] in 1965, *Fuzzy Logic* provides a ready made set of tools for dealing with values in the real interval $[0, 1]$ as if it were a set of truth values of propositions. We borrow some of those tools as explained below, and add a new operation. The effectiveness of these methods has been proven by many applications, see for example [37].

We define an *algebra of argumentation labels* as an abstract algebraic structure, and over its carrier the set the necessary operations related to the argument manipulation will be performed. The effect of aggregation, support, and conflict of arguments will be reflected on their labels, and in that manner, the labels will inform on how the arguments have been affected by the process. The algebra will be based on an ordered set allowing the comparison of the labels, and this set is characterized in the most abstract way to accommodate the different requirement of a particular application.

Definition 1 *(Algebra of argumentation labels).* An algebra of argumentation labels is a 6-tuple which has the form $\mathsf{A} = \langle A, \leq, \odot, \oplus, \ominus, \top, \bot \rangle$ where:

- A is a set of labels called the *domain of labels*.
- $\leq$ is a partial order relation on A (that is, a reflexive, antisymmetric, and transitive relation).
- $\odot : A \times A \to A$ is called a *support operation* and satisfies that:
  - $\odot$ is a *commutative* operation: for all $\alpha, \beta \in A, \alpha \odot \beta = \beta \odot \alpha$.
  - $\odot$ is *monotone*: for all $\alpha, \beta, \gamma \in A$, if $\alpha \leq \beta$, then $\alpha \odot \gamma \leq \beta \odot \gamma$.
  - $\odot$ is *associative*: for all $\alpha, \beta, \gamma \in A$, $\alpha \odot (\beta \odot \gamma) = (\alpha \odot \beta) \odot \gamma$.
- $\oplus : A \times A \to A$ is called an *aggregation operation* and satisfies that:
  - $\oplus$ is a *commutative* operation: for all $\alpha, \beta \in A, \alpha \oplus \beta = \beta \oplus \alpha$.
  - $\oplus$ is *monotone*: for all $\alpha, \beta, \gamma \in A$, if $\alpha \leq \beta$, then $\alpha \oplus \gamma \leq \beta \oplus \gamma$.
  - $\oplus$ is *associative*: for all $\alpha, \beta, \gamma \in A$, $\alpha \oplus (\beta \oplus \gamma) = (\alpha \oplus \beta) \oplus \gamma$.
- $\ominus : A \times A \to A$ is called a *conflict operation* and satisfies that:
  - For all $\alpha, \beta \in A$, $\alpha \ominus \beta \leq \alpha$.
  - For all $\alpha \in A$, $\alpha \ominus \alpha = \bot$.
  - *Right neutral element*: for all $\alpha \in A, \alpha \ominus \bot = \alpha$.
  - For all $\alpha, \beta \in A$, if $\alpha \ominus \beta = \bot$ and $\beta \ominus \alpha = \bot$, then $\alpha = \beta$.
  - For all $\alpha, \beta \in A$, $(\alpha \oplus \beta) < \top$, then $((\alpha \oplus \beta) \ominus \beta) = \alpha$.
- $\top$ is the greatest element of A, while $\bot$ is the least one. Furthermore:
  - $\top$ is the *neutral element* for $\odot$: for all $\alpha \in A$, $\alpha \odot \top = \alpha$.
  - $\bot$ is the neutral element for $\oplus$: for all $\alpha \in A$, $\alpha \oplus \bot = \alpha$.

The operation $\odot$ is used to determine the valuation of an argument based on the valuations of the arguments supporting it. It is clear that one wants this dependency to be invariant of the order in which the supporting arguments are considered, and therefore the operation is both commutative and associative. Monotonicity is also easy to justify: If an argument is supported by stronger arguments, its valuation must be higher than that of one with lesser valuations for its supporters. Finally, an argument should not have a valuation higher than the lesser one of its supporters (weakest-link principle); these conditions can be summarized by saying that $\odot$ is a *T-norm* (see [30,31]).

The operation $\oplus$ determines the valuation of an argument based on the valuations of various sources of support for it. Thus, an argument that can be supported by independent sources can accrue or aggregate the valuations of those sources. The most natural way of doing this would be to directly add the valuations, if that operation is available. The operation $\oplus$ has some of the properties of the addition instead. It is then a commutative and associative operation, with a neutral element: arguments with the least possible valuation $\bot$ do not add to the accrual. These conditions on $\oplus$ may be summarized saying that it is a *T-conorm*.

The conditions on the operation $\ominus$ say that the conflict operator acts with respect to the aggregation operation somewhat as the subtraction acts with respect to the addition of real numbers. Note that if an argument has a label with valuation $\alpha \oplus \beta$ because it has accrued the valuations of other arguments, and then it is attacked by an argument with valuation β, and $\alpha \oplus \beta < \top$ its valuation becomes reduced to α. Thus, the conflict operation is in some sense an inverse of the accrual operation. In the case that $\alpha \oplus \beta = \top$, some information is lost and for that reason $\ominus$ is not an inverse of the accrual in all cases.

We will show how labels composed of elements from the algebra of argumentation labels allow to represent the different features concerning arguments, and provide assistance, by taking this information into account, in the process of determining argument acceptability. Note that, by using the information attached to the arguments it is possible to provide different degrees of acceptability. For example, as was proposed by Cayrol and Lagasquie-Schiex in [7], when an attacked argument is not defeated (but just weakened), the argument that is effectively attacked can be less acceptable than one that is not; in a different approach, Elvang-Goransson et al. in [14] associate an uncertainty degree to arguments defining different acceptability classes, and assigning a linguistic qualifier to a claim.

There are many possible examples of algebras of argumentation labels, and it is important to determine the most appropriated one to use in each case. This is a methodological question involving the semantics of the domain that could be tackled by devising experiments using examples where the desired conclusion is well known, or by performing tests using the cognitive evaluation of human subjects to approximate their assessment of the valuations obtained after their interactions. We have chosen functions ranging over partially ordered sets with additional operations as our representation. A full discussion of the generality of this choice for representing uncertain information can be found in [35].

4. Labeled argumentation framework

It is useful to attach additional information about their effectiveness and acceptability to arguments; for instance, arguments could be built from an agent's knowledge, each of them having a reliability measure of the information source attached. In the end, the agent determines the decision, recommendation, or action to be taken based on the most reliable information.

In this section we will focus on the development a formalism, called *Labeled Argumentation Framework* (LAF), that will allow us to represent arguments, their internal structure, the interactions between arguments, and certain special features of the arguments through argumentation labels. The argument interactions of support, conflict, and aggregation, have associated operations in the algebra of argumentation labels; moreover, these operations allow the system to propagate and combine argumentation labels to obtain the final label attached to each argument. Using this information, we can establish the acceptability status of arguments providing additional information, such as degree of justification, restrictions on justification, and explanation.

Having additional information associated with the arguments, it is possible use this information to establish when an argument is better than another; furthermore, the representation power of argumentation labels enables us to set a threshold, which will be used to determine which arguments are strong enough to be accepted, *e.g.*, in application domains where decisions are critical or high-risk, we should only accept those arguments that satisfy the conditions imposed by the threshold.

As we have mentioned, it is necessary to use a knowledge representation framework to enable modeling of the argumentation structures and the relationships that exist among them. For this purpose, we will use the *Argument Interchange Format* (AIF) which is composed of a set of argument-related concepts used to unify the representation of different argumentation formalisms and schemes. In this formalism, arguments are represented as a set of nodes in a directed graph, called an *argument network* (see [10] for full details), where it is possible to visualize the relationships that exist among argument structures.

Definition 2 *(Labeled argumentation framework).* A Labeled Argumentation Framework (LAF) is a 5-tuple of the form $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ where:

- $\mathcal{L}$ is a logical language for knowledge representation (claims) about the domain of discourse. We assume that the connectives of this language include one distinguished symbol "$\sim$" denoting strong negation,[1] so that $\sim\sim\varphi$ is considered equivalent to φ for every formula φ in $\mathcal{L}$. Thus, we can assume that no subexpression of the form "$\sim\sim\varphi$" appears in the formulas of the language, yet the set of formulas in $\mathcal{L}$ is closed with respect to "$\sim$".
- $\mathcal{R}$ is a set of (domain independent) inference rules $R_1, R_2, \ldots, R_n$ defined in terms of $\mathcal{L}$ (*i.e.*, with premises and conclusion in $\mathcal{L}$).
- $\mathcal{K}$ is the knowledge base, a set of formulas of $\mathcal{L}$ describing knowledge about the domain of discourse.
- $\mathcal{A}$ is a set of algebras of argumentation labels (Definition 1) $\mathsf{A}_1, \mathsf{A}_2, \ldots, \mathsf{A}_n$, one for each feature that will be represented by labels.
- $\mathcal{F}$ is a function that assigns to each element of $\mathcal{K}$, an n-tuple of elements[2] in the algebras $\mathsf{A}_i, i = 1, \ldots, n$. This is, $\mathcal{F}\colon \mathcal{K} \longrightarrow \mathsf{A}_1 \times \mathsf{A}_2 \times \cdots \times \mathsf{A}_n$.

It is important to understand that the use of two or more consecutive "$\sim$" in $\mathcal{L}$ is not allowed in order to simplify the definition of conflict between claims; this does not limit its expressive power or generality of the representation. We denote with $\overline{\varphi}$ the negation of a formula in $\mathcal{L}$, so $\overline{\varphi}$ is $\sim\varphi$ and $\overline{\sim\varphi}$ is simply φ.

The language $\mathcal{L}$ is used to specify knowledge about a particular domain. The inferences that can be made from the knowledge expressed in $\mathcal{L}$ will be specified by inference rules which represent domain-independent patterns of reasoning, such as deductive inference rules (*modus ponens*, *modus tollens*, *etc.*) or defeasible inference rules (*defeasible modus ponens*, *defeasible modus tollens*, *etc.*).

Example 1. Let $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ be a LAF, where:

- $\mathcal{L}$ is the language of first-order classical logic, with the proviso made in Definition 2 on how we write negations. Over this language, we consider only the ground formulas (Herbrand base).

[1] We refer to the term strong negation as the concept of constructible falsity introduced by Nelson in [26] and presented in the form of an axiomatic system by Vorob'ev in [34].

[2] When no confusion can occur we will follow the usual convention of mentioning elements in an algebra instead of referring to elements in the corresponding carrier set of that algebra.

– $\mathcal{R} = \{dMP\}$, where the inference rule dMP is defined as follows:

$$dMP\colon \frac{P_1, \ldots, P_n \quad C \mathrel{-\!\!\prec} P_1, \ldots, P_n}{C} \text{ (Defeasible Modus Ponens)}$$

The symbol "$-\!\!\prec$" represents a defeasible rule which are ordered pairs $C \mathrel{-\!\!\prec} P_1, \ldots, P_n$ whose first component, C is a claim, and whose second component, $P_1, \ldots, P_n$, is a finite non-empty set of claims premises [32,19,20].

– $\mathcal{A} = \{\mathsf{A}, \mathsf{B}\}$ the set of algebras of argumentation labels where:
  – A is an algebra of argumentation labels representing the reliability degree attached to arguments. The domain of labels A is the real interval $[0, 1]$ and represents a normalized reliability valuation, where $\top = 1$ is the maximum reliability valuation and neutral element for $\odot$, while $\bot = 0$ is the minimum reliability valuation.
  Let $\alpha, \beta \in A$ be two labels, the operators of support, conflict and aggregation over labels representing the reliability valuations associated with arguments, are specified as follows:

Reliability attribute	
$\alpha \odot \beta = min(\alpha, \beta)$	The support operator reflects that an argument is as reliable as its weakest support, based on the weakest link rule.
$\alpha \oplus \beta = \alpha + \beta - \alpha\beta$	The aggregation operator states that if there are more than an argument for a conclusion, its reliability valuation is the sum of the reliability of the arguments supporting it with a penalty term on this aggregation.
$\alpha \ominus \beta = \begin{cases} \frac{\alpha - \beta}{1 - \beta} & \text{if } \alpha \geq \beta, \beta \neq 1 \\ 0 & \text{otherwise.} \end{cases}$	This conflict operator reflects that the reliability valuation of a conclusion is weakened by the reliability of its contrary.

  - B is an algebra of argumentation labels representing an accuracy measurement attached to arguments. The domain of labels B is again the real interval $[0, 1]$ and represents a normalized accuracy valuation.
  The operations of the algebra B are specified as follows:

Accuracy attribute	
$\alpha \odot \beta = min(\alpha, \beta)$	As in A.
$\alpha \oplus \beta = min(\alpha + \beta, 1)$	The aggregation operation reflects the idea that if we have more than one argument for a conclusion, its accuracy valuation is the sum of the accuracy valuations of the arguments that support it.
$\alpha \ominus \beta = \begin{cases} \alpha - \beta & \text{if } \alpha \geq \beta \\ 0 & \text{otherwise.} \end{cases}$	This conflict operation reflects that valuation of a conclusion is weakened by the accuracy valuation of its contrary.

– $\mathcal{K}$ is the following knowledge base showing next to each claim the reliability and accuracy valuation associated by $\mathcal{F}$. These valuations are indicated between brackets, where the first element represent the reliability valuation and the second the accuracy valuation, using a colon to separate it of a claim. In particular, the valuations attached to rules represent the reliability and accuracy of the connection between the antecedent and consequent of the rule. Strict and defeasible rules are ground. However, following the usual convention [23], some examples will use "schematic rules" with variables; to distinguish variables from other elements of a schematic rule, we will denote variables with an initial uppercase letter. We display below the set of claims forming $\mathcal{K}$:

$$\left\{\begin{array}{l}
\mathtt{r}_1 : \mathtt{invest(X)} \prec \sim\mathtt{depreciate(X)}, \mathtt{safetyInvest(X)} : [1, 1] \\
\mathtt{r}_2 : \mathtt{invest(X)} \prec \mathtt{goodArea(X)} : [0.6, 0.8] \\
\mathtt{r}_3 : \mathtt{goodArea(X)} \prec \mathtt{quietArea(X)}, \mathtt{safeArea(X)} : [0.8, 1] \\
\mathtt{r}_4 : \sim\mathtt{invest(X)} \prec \mathtt{maintCost(X)} : [0.7, 0.5] \\
\mathtt{r}_5 : \mathtt{invest(X)} \prec \sim\mathtt{maintCost(X)}, \sim\mathtt{deteriorate(X)} : [1, 1] \\
\mathtt{r}_6 : \sim\mathtt{invest(X)} \prec \mathtt{expensive(X)}, \mathtt{unpredictable(X)} : [1, 1] \\
\mathtt{r}_7 : \mathtt{expensive(X)} \prec \mathtt{placeStore(X)} : [0.5, 0.5] \\
\mathtt{r}_8 : \mathtt{undepictable(X)} \prec \mathtt{globalEconomy(X)} : [1, 1]
\end{array}\right\}$$

$$\left\{\begin{array}{ll}
\sim\mathtt{depreciate(land)} : [0.5, 0.8] & \sim\mathtt{maintCost(gold)} : [0.9, 0.9] \\
\mathtt{safetyInvest(land)} : [0.5, 0.9] & \sim\mathtt{deteriorate(gold)} : [0.8, 0.9] \\
\mathtt{quietArea(land)} : [0.9, 0.8] & \mathtt{placeStore(gold)} : [0.5, 0.8] \\
\mathtt{safeArea(land)} : [0.7, 0.8] & \mathtt{globalEconommy(gold)} : [0.8, 1] \\
\mathtt{maintCost(land)} : [0.3, 0.7] &
\end{array}\right\}$$

Next, we present the notion of argumentation graph which will be used to represent the argumentative analysis derived from a *LAF*. We assume that there are no two nodes in a given graph which are named with the same sentence of $\mathcal{L}$, so we will use the naming sentence to refer to the I-node in the graph.

Definition 3 *(Argumentation graph).* Let $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ be a LAF. Its associated argumentation graph is the digraph $G_\Phi = (N, E)$, where N is the set of nodes and E is the set of the edges constructed as follows:

i) each element $X \in \mathcal{K}$ or derived from $\mathcal{K}$ through $\mathcal{R}$, is represented for an I-node $X \in N$.
ii) for each application of an inference rule defined in Φ, there exists an RA-node $R \in N$ such that:
   – the inputs are all I-nodes: $P_1, \dots, P_m \in N$ representing the premises. These include the defeasible rules in $\mathcal{K}$ that may be applied;
   – the output is an I-node $C \in N$ representing the conclusion.
iii) if both X and $\overline{X}$ are in N, then there exists a CA-node with edges going to and coming from both I-nodes X and $\overline{X}$.
iv) for all $X \in N$ there exists no path from X to X in G_Φ that does not pass through a CA-node. This is, G_Φ without the CA-nodes is an acyclic graph.

The condition iv) forbids cycles other than the mutual attacks between nodes. This seems to be too restrictive, but it must be noted that RA-node cycles are mostly generated by fallacious specifications.

Associated with a LAF Φ we have the set of arguments Arg_Φ that can be built from $\mathcal{K}$ and $\mathcal{R}$, *i.e.*, an argument can be represented by linking a set of I-nodes denoting premises to an I-node denoting a conclusion via a particular RA-node.

Definition 4 *(Simple argument).* Let $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ be a LAF. Let $G_\Phi = (N, E)$ be its derived argumentative graph as in Definition 3. A simple argument is a tuple $\mathbb{A} = (P, R, C)$ where $P = \{P_1, \dots, P_m\} \subseteq I$ is a set of premises, $C \in I$ is a conclusion and $R \in RA$ represents an application of an inference rule, such that for all premises $P_i \in P$ there exist edges (P_i, R) and $(R, C) \in E$ connecting the premises to the conclusion. We say that each of the premises *supports* the conclusion (Fig. 1).

In LAF, a simple argument $\mathbb{A}$ supports an argument $\mathbb{B}$ if the conclusion of the argument $\mathbb{A}$ is part of the premises that support the conclusion of the argument $\mathbb{B}$. Then, simple arguments can be aggregated into a more complex argument structures called arguments.

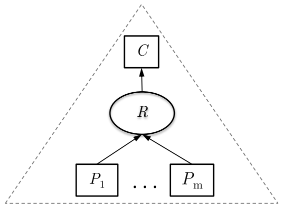

Fig. 1. Representation of a simple argument.

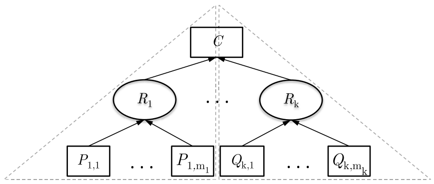

Fig. 2. Representation of argument accrual.

Also, it is possible that two or more arguments share the same conclusion. This corresponds to with the notion of argument accrual developed in [33,28,25], where the strength of the shared conclusion is the aggregation of the strengths of each individual argument supporting it.

Definition 5 *(Argument accrual).* Let $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ be a LAF, and let $G_\Phi = (N, E)$ be its associated argumentative graph. Let Arg_Φ be a set of argument, and let Arg_Φ^Q be set of argument supporting the same conclusion Q, where $Arg_\Phi^Q \subseteq Arg_\Phi$. We define the argument accrual for Q from the set Arg_Φ^Q as the union of these arguments, after identifying their roots (Fig. 2).

Example 2. From the knowledge base $\mathcal{K}$ presented in the Example 1, we get the argumentation graph in Fig. 3, where arguments are surrounded by dotted lines. This argumentation graph can be analyzed in two groups.

In the upper group, on the right, there are two arguments composing an argument accrual for the literal `invest(land)`. In turn, the argument with premises `goodArea(land)` and $\mathtt{r}_2$ is supported with another argument increasing the information about land investments; on the left, there is an argument for the claim ∼`invest(land)` supported by the premises ∼`maintCost(land)` and $\mathtt{r}_4$. In the lower group, on the right, there is an argument for the literal `invest(gold)` with premises ∼`maintCost(gold)`, ∼`deteriorate(gold)` and $\mathtt{r}_5$; on the left, there is an argument for the claim ∼`invest(gold)`, supported by two arguments, one with premises `placeStore(gold)` and $\mathtt{r}_7$, the other with premises `globalEconomy(gold)` and $\mathtt{r}_8$. The CA-nodes link complementary literals.

Once we obtain a representation of Arg_Φ through the argumentative graph G_Φ, we proceed to attach a label to each I-node in G_Φ representing the valuations with the extra information we want to represent. For each feature represented in the label, we assign two valuations to each I-node, from each of the algebras in $\mathcal{A}$. For a given feature, the valuations from the algebra A_i that appear in the label of the I-node X will be denominated μ_i^X and δ_i^X, where μ_i^X represents the *aggregated valuation* assigned to X, obtained through the accrual and support operations defined in the algebra A_i, while δ_i^X is the *weakened valuation*, obtained through the conflict operation, if applicable. The labeling process is captured in the following definition.

Definition 6 *(Labeled argumentation graph).* Let $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ be a LAF, and G_Φ be the corresponding argumentation graph. Let A_i be one of the algebras in $\mathcal{A}$, representing a feature to be associated to each I-node X. A labeled argumentation graph is an assignment of two valuations in each of the algebras to

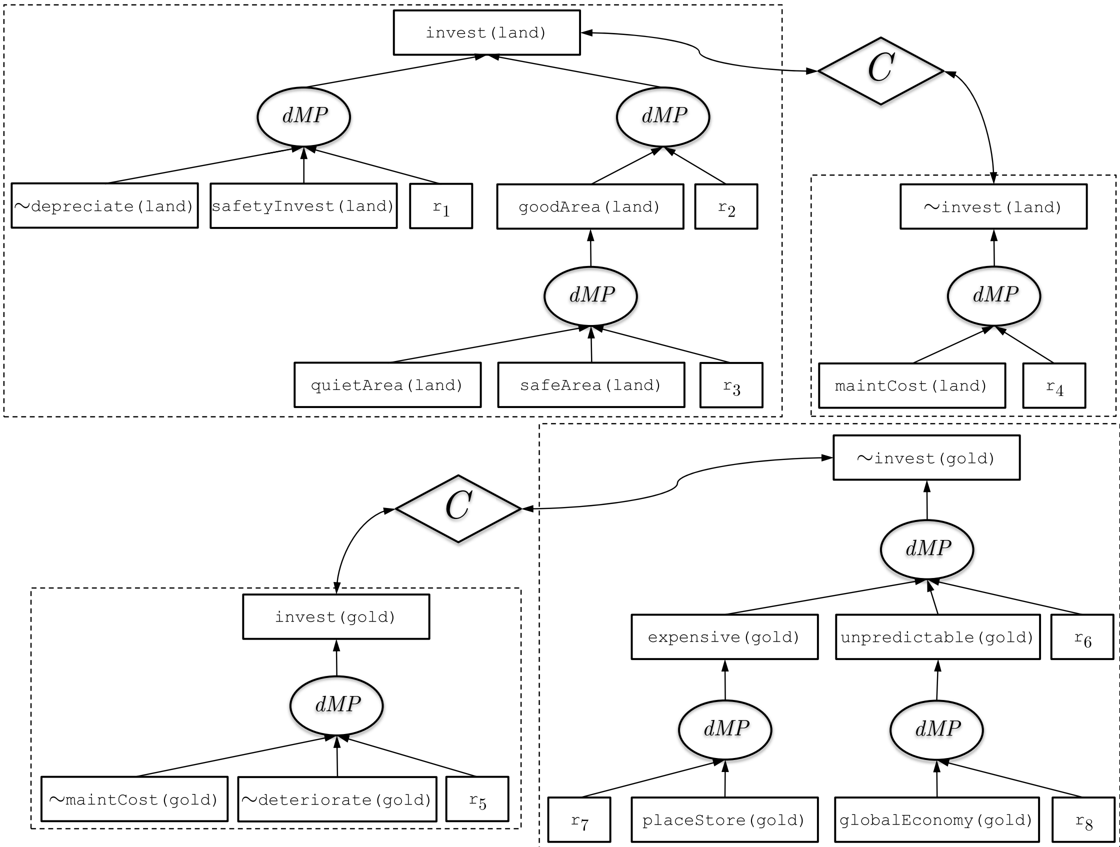

Fig. 3. Representation of an argumentation graph.

each I-node of the graph, denoted as μ_i^X and δ_i^X, where μ_i^X, $\delta_i^X \in \mathsf{A}_i$, are such that μ_i^X accounts for the aggregation of the reasons supporting the claim X, and δ_i^X displays the state of the claim after taking conflict into account. Thus, if X is an I-node, then:

i) If X has no inputs, then X has a label that corresponds to it as an element of $\mathcal{K}$, thus we define $\mu_i^X = \mathcal{F}(X)$.

ii) If X has input from a CA-node representing conflict with an I-node $\overline{X}$, then:

$$\delta_i^X = \mu_i^X \ominus \ \mu_i^{\overline{X}}.$$

If there doesn't exist an I-node $\overline{X}$, then

$$\delta_i^X = \mu_i^X.$$

iii) If X has inputs from the RA-nodes $R_1, \ldots, R_k$, where each R_s has premises $X_1^{R_s}, \ldots, X_{n_s}^{R_s}$, then:

$$\mu_i^X = \oplus_{s=1}^{k} (\odot_{t=1}^{n_s} \delta_i^{X_t^{R_s}}).$$

To get this equation, we first use the support operation applied to the weakened valuations assigned to the premises of each of the rules R_s that form the body of an argument supporting X, and then we calculate the accrual of all these arguments.

The *label* of an I-node X is then an n-tuple of pairs of valuations:

$$((\mu_1^X, \delta_1^X), (\mu_2^X, \delta_2^X), \ldots, (\mu_n^X, \delta_n^X)).$$

The properties mentioned in the following proposition follow directly from the definition of μ_i^X and δ_i^X.

Proposition 1. *Let $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ be a LAF, and G_Φ be its argumentation graph, and let X be an I-node in G_Φ. Then the labels μ_i^X and δ_i^X satisfy:*

- *For all X and i, $\delta_i^X \leq \mu_i^X$.*
- *If for some X and i, $\mu_i^X = \bot$, it follows that $\delta_i^X = \bot$ as well.*
- *If for some X and i, $\mu_i^{\overline{X}} = \bot$, then $\delta_i^X = \mu_i^X$.*

The following underlying principles can be directly mapped to the principles laid out in [7] and are satisfied by all the valuations defined according to our scheme:

Property 1 *(Underlying principles). The valuations given by Definition 6 respect the following principles:*

P_1 *The weakened valuation is equal to the accrued valuation for the arguments without attackers, and for an attacked but undefeated argument the weakened valuation is less than the accrued valuation, if the attacking arguments are strong enough to weaken it.*

P_2 *The weakened valuation for an argument depends, in a non/increasing manner, on the accrued valuation of the attacking argument.*

P_3 *The valuations of arguments supporting a claim X contribute to increase the accrued valuation of X, and for this reason they increase the strength of the direct attack to the argument that supports the opposite claim $\overline{X}$.*

Once the I-nodes are labeled, we can consider their acceptability status. First, we present a classic version of this notion, where the acceptability status for an argument is either *accepted* or *rejected*. Then, we use the information attached to the arguments in order to provide different degrees of acceptability.

Definition 7 *(Acceptability status).* Let $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ be a LAF, and G_Φ be the corresponding argumentation graph. Let X be an I-node in G_Φ. For each of the algebras A_i in $\mathcal{A}$, representing a feature to be associated to each I-node X, with $\bot_i$ the neutral element for $\oplus_i$. Then, the I-node X has assigned one of two possible acceptability status accordingly to their associated labels:

- Accepted iff $\delta_i^X \neq \bot_i$ for each $1 \leq i \leq n$.
- Rejected iff $\delta_i^X = \bot_i$ for any $1 \leq i \leq n$.

The following proposition is immediate from the definitions above.

Proposition 2. *Let Φ be a LAF, and G_Φ be the corresponding argumentation graph, then the acceptability status of G_Φ is uniquely determined.*

Definition 8 *(Acceptability degree status).* Let $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ be a LAF, and G_Φ be the corresponding argumentation graph. Let X be an I-node in G_Φ. For each of the algebras A_i in $\mathcal{A}$, representing a feature to be associated to each I-node X, with $\bot_i$ the neutral element for the operation $\oplus_i$, the I-node X has assigned one of the following possible acceptability status:

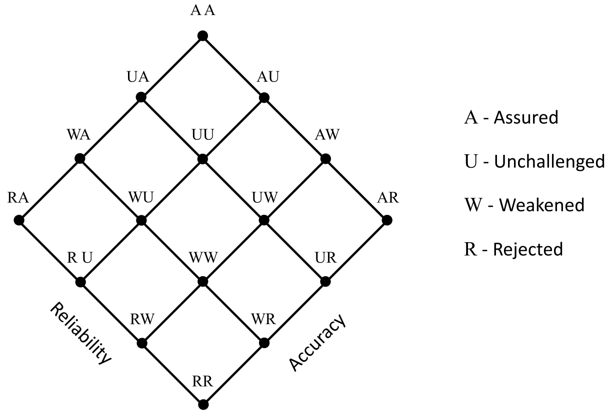

Fig. 4. Status degree for a claim based on two attributes.

- Assured iff $\delta_i^X = \top_i$.
- Unchallenged iff $\mu_i^X = \delta_i^X$.
- Weakened iff $\bot < \delta_i^X < \mu_i^X$.
- Rejected iff $\delta_i^X = \bot_i$.

Finally, for each claim, we form a vector with the acceptability of that claim with respect to each of the attributes, and take the least degree of those that appear in the vector as the acceptability degree for the claim as whole.

Note that the status of *"Assured"* can only be granted to a claim when $\delta_i^X = \top_i$ for every $1 \leq i \leq n$. In the other end of the spectrum, it is enough that there exists one i such that $\delta_i^X = \bot_i$ to rule a claim X as *"Rejected"*. In Fig. 4, we illustrate Definition 8, where we take into account Example 1, and the attributes associated to a claim, in order to determine the overall status for that claim.

From the acceptability status assigned to a specific claim it is possible to obtain the acceptability status associated with an argument, *i.e.*, if the claim that represents the conclusion of the argument is marked as accepted, then the argument is accepted respectively.

Example 3. Going back to Example 2, we calculate the label for each I-node (or claim) of the argumentation graph, according to Definition 6. First, we calculate μ_1, δ_1, representing the reliability valuation associated with each I-node.

$$\mu_1^{\texttt{invest(land)}} = (\delta_1^{\sim\texttt{depreciate(land)}} \odot \delta_1^{\texttt{safetyInvest(land)}} \odot \delta_1^{\texttt{r}_1}) \oplus (\delta_1^{\texttt{goodArea(land)}} \odot \delta_1^{\texttt{r}_2})$$
$$= min((min(0.5, 0.5), 1) \oplus min(0.8, 1)) = 0.5 \oplus 0.8 = 0.5 + 0.8 - 0.5 * 0.8 = 0.9$$
$$\mu_1^{\texttt{goodArea(land)}} = \delta_1^{\texttt{quietArea(land)}} \odot \delta_1^{\texttt{safeArea(land)}} \odot \delta_1^{\texttt{r}_3} = min(0.9, 0.8, 1) = 0.8$$
$$\mu_1^{\sim\texttt{invest(land)}} = \delta_1^{\texttt{maintCost(land)}} \odot \delta_1^{\texttt{r}_4} = min(0.3, 0.6) = 0.3$$
$$\delta_1^{\texttt{invest(land)}} = \mu_1^{\texttt{invest(land)}} \ominus \mu_1^{\sim\texttt{invest(land)}} = \frac{0.9 - 0.3}{1 - 0.3} = 0.85$$
$$\delta_1^{\sim\texttt{invest(land)}} = 0$$
$$\mu_1^{\texttt{invest(gold)}} = \delta_1^{\sim\texttt{maintCost(gold)}} \odot \delta_1^{\sim\texttt{deteriorate(gold)}} \odot \delta_1^{\texttt{r}_5} = min(0.9, 0.8, 1) = 0.8$$
$$\mu_1^{\sim\texttt{invest(gold)}} = \delta_1^{\texttt{expensive(gold)}} \odot \delta_1^{\texttt{unpredictable(gold)}} \odot \delta_1^{\texttt{r}_6} = min(0.5, 0.8, 1) = 0.5$$
$$\mu_1^{\texttt{expensive(gold)}} = \delta_1^{\texttt{placeStore(gold)}} \odot \delta_1^{\texttt{r}_7} = min(0.5, 0.5) = 0.5$$
$$\mu_1^{\texttt{unpredictable(gold)}} = \delta_1^{\texttt{globalEconomy(gold)}} \odot \delta_1^{\texttt{r}_8} = min(0.8, 1) = 0.8$$

Next, we calculate the functions μ_2, δ_2, representing the accuracy valuation associated with each I-node.

$$\mu_2^{\texttt{invest(land)}} = (\delta_2^{\sim\texttt{depreciate(land)}} \odot \delta_2^{\texttt{safetyInvest(land)}} \odot \delta_2^{\texttt{r}_1}) \oplus (\delta_2^{\texttt{goodArea(land)}} \odot \delta_2^{\texttt{r}_2})$$
$$= min(0.8, 0.5, 1) \oplus min(0.8, 1)) = 0.5 \oplus 0.8 = min(0.5 + 0.8, 1) = 1$$
$$\mu_2^{\texttt{goodArea(land)}} = \delta_2^{\texttt{quietArea(land)}} \odot \delta_2^{\texttt{safeArea(land)}} \odot \delta_2^{\texttt{r}_3} = min(0.8, 0.8, 1) = 0.8$$
$$\mu_2^{\sim\texttt{invest(land)}} = \delta_2^{\texttt{maintCost(land)}} \odot \delta_2^{\texttt{r}_4} = min(0.7, 0.5) = 0.5$$
$$\delta_2^{\texttt{invest(land)}} = \mu_2^{\texttt{invest(land)}} \ominus \mu_2^{\sim\texttt{invest(land)}} = 1 - 0.5 = 0.5$$
$$\delta_2^{\sim\texttt{invest(land)}} = 0$$
$$\mu_2^{\texttt{invest(gold)}} = \delta_2^{\sim\texttt{maintCost(gold)}} \odot \delta_2^{\sim\texttt{deteriorate(gold)}} \odot \delta_2^{\texttt{r}_5} = min(0.9, 0.9, 1) = 0.9$$
$$\mu_2^{\sim\texttt{invest(gold)}} = \delta_2^{\texttt{expensive(gold)}} \odot \delta_2^{\texttt{unpredictable(gold)}} \odot \delta_2^{\texttt{r}_6} = min(0.5, 1, 1) = 0.5$$
$$\mu_2^{\texttt{expensive(gold)}} = \delta_2^{\texttt{placeStore(gold)}} \odot \delta_2^{\texttt{r}_7} = min(0.8, 0.5) = 0.5$$
$$\mu_2^{\texttt{unpredictable(gold)}} = \delta_2^{\texttt{globalEconomy(gold)}} \odot \delta_2^{\texttt{r}_8} = min(1, 1) = 1$$
$$\delta_2^{\texttt{invest(gold)}} = \mu_2^{\texttt{invest(gold)}} \ominus \mu_2^{\sim\texttt{invest(land)}} = 0.9 - 0.5 = 0.4$$
$$\delta_2^{\sim\texttt{invest(gold)}} = 0$$

On one hand, based on the classical acceptability status, the sets of accepted and rejected claims are:

- $S^A = \{$invest(land), ∼depreciate(land), safetyInvest(land), goofArea(land), quietArea(land), safeArea(land), maintCost(land), invest(gold), expensive(gold), ∼maintCost(gold), ∼deteriorate(gold), unpredictable(gold), globalEconomy(gold), placeStore(gold)$\}$
- $S^R = \{$∼invest(land), ∼invest(gold)$\}$

On the other hand, based on the acceptability degree status, the claims are classified as follows:

- $S^A = \emptyset$
- $S^U = \{$∼depreciate(land), safetyInvest(land), goofArea(land), quietArea(land), safeArea(land), maintCost(land), expensive(gold), ∼maintCost(gold), ∼deteriorate(gold), unpredictable(gold), globalEconomy(gold), placeStore(gold)$\}$
- $S^W = \{$invest(land), invest(gold)$\}$
- $S^R = \{$∼invest(land), ∼invest(gold)$\}$

Once the acceptability status of the arguments in favor and against investing in gold bullion is analyzed, (or in favor and against investing in land), different scenarios may occur. If the argument in favor of investing in gold bullion is defeated by the argument against it, and the argument in favor of investing in land is strong enough to withstand its counterargument, then the agent should invest in land. In the opposite situation, when the argument in favor of investing in gold bullion is undefeated, and the argument in favor of investing in land is defeated by its opposition, then the agent should invest in gold. In a third case, both arguments (invest in land and invest in gold bullion) are defeated, so the agent should not invest. Finally, if both arguments are strong enough to withstand the attacks of their opponent arguments, then a conflict in the strict sense does not exist, since the arguments are not contradictory. However, the domain imposes

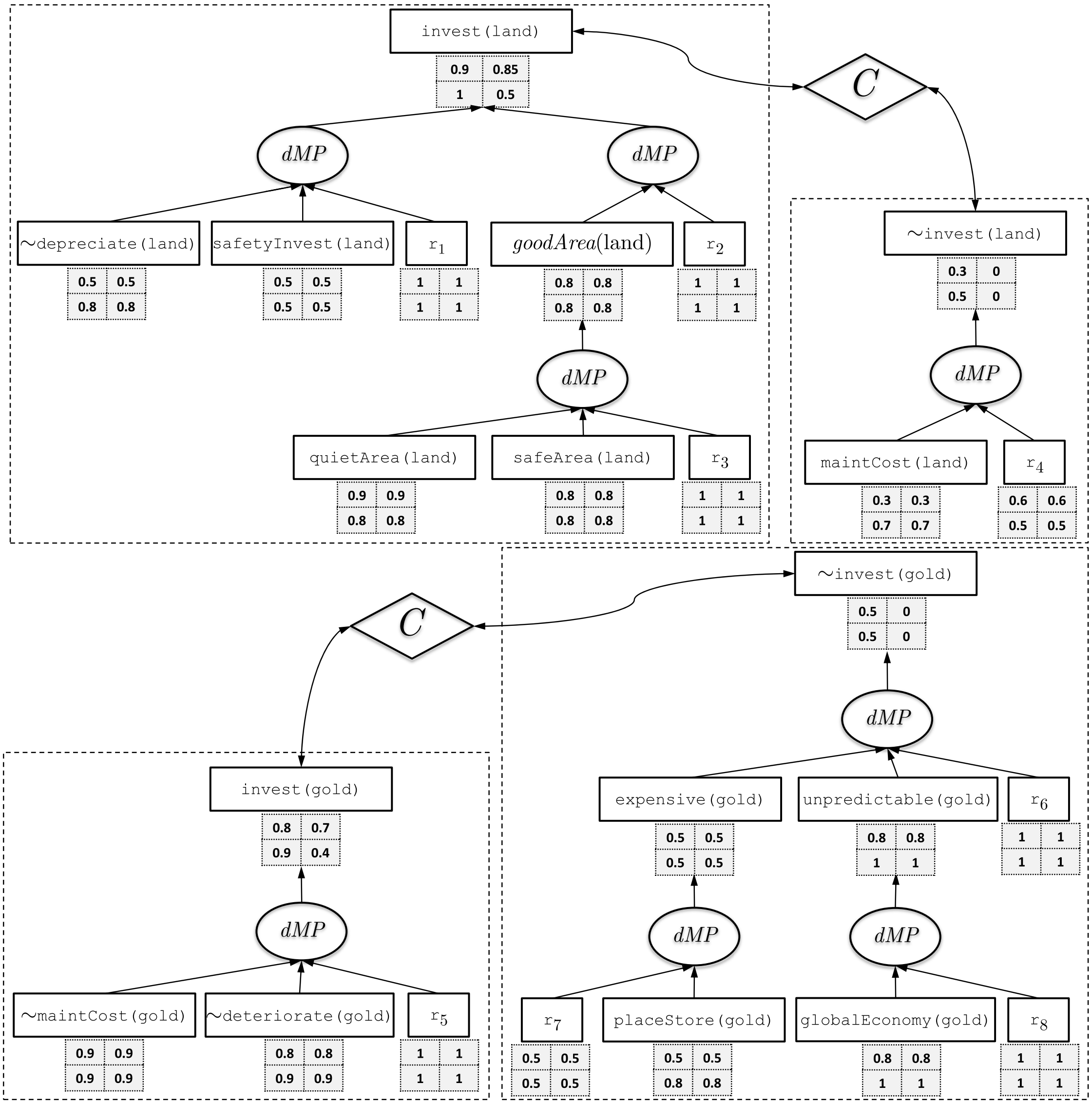

Fig. 5. Representation of labeled argumentation graph.

the additional constraint that the agent only has enough money to make one investment. To solve these special situation, we will define when an argument is better than other based on their features. This notion is formalized below.

Definition 9 *(Better argument).* Let $\Phi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ be a LAF, G_Φ be the corresponding argumentation graph, and Arg_Φ the set of arguments. Let $\mathbb{A}, \mathbb{B}$ be two arguments and $\mathbb{A}, \mathbb{B} \in Arg_\Phi$. Let X and Y be two I-nodes that represent the conclusion for the arguments $\mathbb{A}$ and $\mathbb{B}$ respectively. Then, $\mathbb{A}$ is better than $\mathbb{B}$ iff $\delta_i^X > \delta_i^Y$ for each $1 \leq i \leq n$.

Example 4. Going back to Example 3, under the condition "the agent only has enough money to make one investment", based on the information depicted in Fig. 5 and applying Definition 9, we conclude that the accrual argument supporting the conclusion `invest(land)` is better than the argument supporting the conclusion `invest(gold)`, since

$$\delta_1^{\texttt{invest(land)}} = 0.85 > 0.5 = \delta_1^{\texttt{invest(gold)}}$$

and

$$\delta_2^{\texttt{invest(land)}} = 0.7 > 0.4 = \delta_2^{\texttt{invest(gold)}}$$

As mentioned previously, in certain real-world applications agents need to take decisions to meet their goals with requirements that depend of the domain. For that, we set a threshold that determines the minimum acceptable valuation that an argument must satisfy to be a part of the justification for a recommendation or decision. To do this we must verify that each piece of knowledge that is part of an argument satisfies the set of thresholds proposed. Using this form of comparing argument's labels, we can extend LAF into E-LAF considering a threshold in the form of a tuple that allows to decide whether each label is good enough. Formally:

Definition 10 *(E-LAF).* An Extended Labeled Argumentation Framework is a 6-tuple $\Psi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F}, \boldsymbol{\tau} \rangle$, where $\langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F} \rangle$ is a labeled argumentation framework, and $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_n) \in \mathsf{A}_1 \times \cdots \times \mathsf{A}_n$ will be called *threshold*, where τ_i is a distinguished element in A_i, $i = 1, \ldots, n$, that refers to the particular threshold of the corresponding label.

Definition 11 *(Labeled argumentation graph with threshold).* Let $\Psi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F}, \boldsymbol{\tau} \rangle$ be an E-LAF, and G_Ψ be the corresponding argumentation graph. Let $\tau = (\tau_1, \ldots, \tau_n)$ be a threshold. A *labeled argumentation graph with threshold* is a labeled argumentation graph where the labels μ_i^X and δ_i^X satisfy:

i) If X has no inputs, then X has a label that corresponds to it as an element of $\mathcal{K}$, thus we define:
   - $\mu_i^X = \mathcal{F}(X)$ if $\mathcal{F}(X) \geq \tau_i$,
   - $\mu_i^X = \perp$ otherwise.

ii) If X has input from a CA-node C representing conflict with an I-node $\overline{X}$, then:
   - $\delta_i^X = \mu_i^X \ominus \ \mu_i^{\overline{X}}$ if $\mu_i^X \ominus \ \mu_i^{\overline{X}} \geq \tau_i$,
   - $\delta_i^X = \perp$ otherwise.

   If an I-node X has no corresponding $\overline{X}$, then $\delta_i^X = \mu_i^X$.

iii) If X has inputs from the RA-nodes $R_1, \ldots, R_k$, where each R_s has premises $X_1^{R_s}, \ldots, X_n^{R_s}$, then:
   - $\mu_i^X = \oplus_{s=1}^k (\odot_{t=1}^n \delta_i^{X_t^{R_s}})$ if $\oplus_{s=1}^k (\odot_{t=1}^n \delta_i^{X_t^{R_s}}) \geq \tau_i$,
   - $\mu_i^X = \perp$ otherwise.

The following proposition follows directly from the definition.

Proposition 3. *Let $\Psi = \langle \mathcal{L}, \mathcal{R}, \mathcal{K}, \mathcal{A}, \mathcal{F}, \boldsymbol{\tau} \rangle$ be an E-LAF, where $\tau = (\tau_1, \ldots, \tau_n)$ is the threshold. Let G_Ψ be the corresponding labeled argumentation graph, and X an I-node in G_Ψ. Then, the labels μ_i^X and δ_i^X satisfy that if $\delta_i^X \neq \perp_i$, then $\delta_i^X \geq \tau_i$ and $\mu_i^X \geq \tau_i$, for $i = 1, \ldots, n$.*

Example 5. We analyze the Example 3, using the described marking process and defining a threshold $\tau_1 = 0.6 \in \mathsf{A}$ and $\tau_2 = 0.7 \in \mathsf{B}$ (see Fig. 6). In this case, under the restrictions imposed by the thresholds τ_1 and τ_2, the following situations obtain. In the upper group, on the right, one of two arguments composing an argument accrual for the literal `invest(land)` is defeated, because the reliability of the literals ∼`depreciate(land)` and `safetyInvest(land)` are below τ_1 and the accuracy measure of `safetyInvest(land)` is below τ_2; on the left, the argument for the claim ∼`invest(land)` supported by the premises ∼`maintCost(land)` and $\texttt{r}_4$ is defeated, because ∼`maintCost(land)` does not satisfy τ_1 and $\texttt{r}_4$ does not satisfy τ_2. In the lower group, on the left there is an argument for the claim ∼`invest(gold)`, supported for two arguments, one of them with premises `placeStore(gold)` and $\texttt{r}_7$, with `placeStore(gold)` not satisfying τ_1, and $\texttt{r}_7$ not satisfying τ_1 nor τ_2.

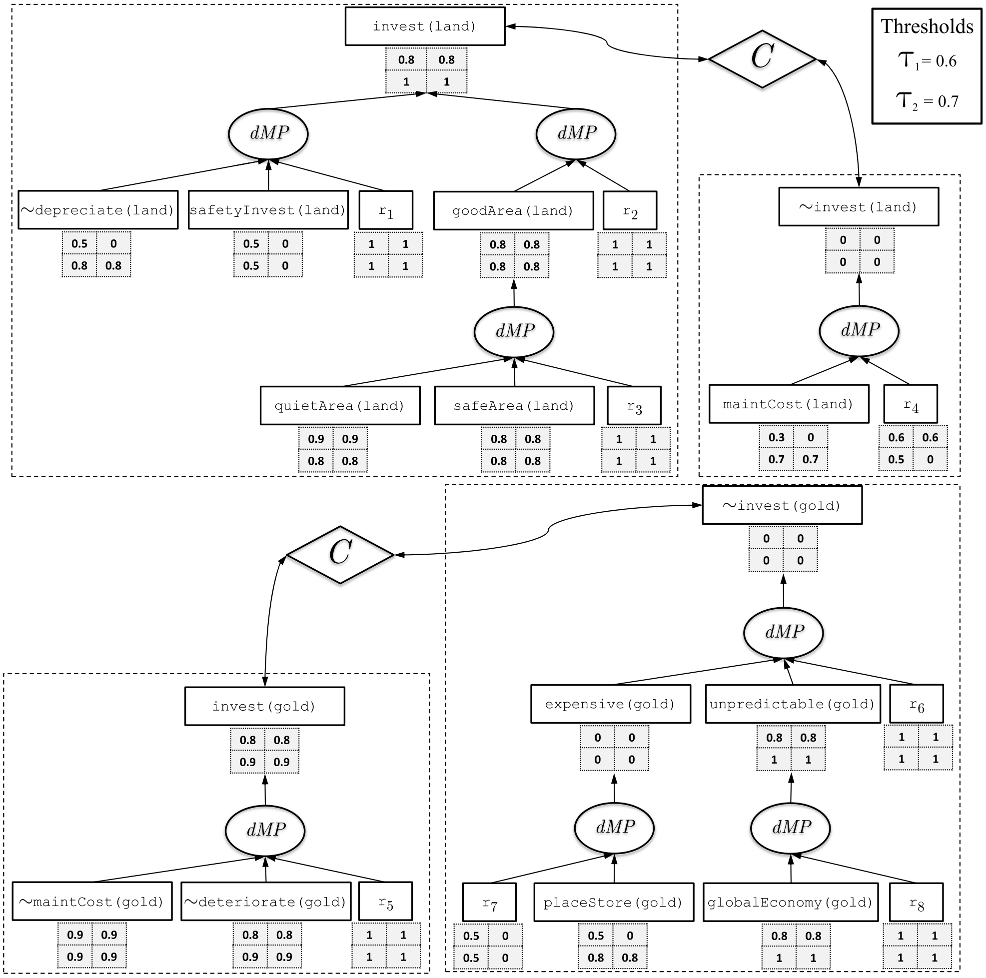

Fig. 6. Representation of marked argumentation graph with threshold.

Based on the classical acceptability status, the sets of accepted and rejected claims are:

- $S^A = \{$invest(land), goofArea(land), quietArea(land), safeArea(land), ~maintCost(gold), ~deteriorate(gold), unpredictable(gold), globalEconomy(gold)$\}$
- $S^R = \{$~depreciate(land), safetyInvest(land), maintCost(land), invest(gold), expensive(gold), placeStore(gold), ~invest(land), ~invest(gold)$\}$

On the other hand, based on the acceptability degree status, the claims are classified as follows:

- $S^A = S^W = \emptyset$
- $S^U = \{$invest(land), goofArea(land), quietArea(land), safeArea(land), ~maintCost(gold), ~deteriorate(gold), unpredictable(gold), invest(gold), globalEconomy(gold)$\}$
- $S^R = \{$~depreciate(land), safetyInvest(land), maintCost(land), ~invest(land), ~invest(gold), expensive(gold), placeStore(gold)$\}$

Finally, based on Definition 9, we conclude that the accrued argument supporting the conclusion invest(land) is better than the argument supporting the conclusion invest(gold), since $\delta_2^{\texttt{invest(land)}} =$

$1 > 0.9 = \delta_2^{\texttt{invest(gold)}}$. Note that, based on the reliability degree associated to the claims invest(land) and invest(gold), both options have the same strength. However, the claim invest(land) has a higher accuracy degree than invest(gold), and for that reason it is the best option in this model.

In summary, the valuations associated to the arguments through labels provide us with the means to model peculiarities of an application domain. In a model, the arguments are related in different ways, and these relationships influence the information attached to them, so we presented a general scheme to propagate such information through the argumentative graph. Thus, we get more details about the arguments, which in turn reveal different degrees of acceptability for them. In addition, there are certain domains in which the arguments must satisfy specific conditions to be taken into account; for example, knowledge used for decision making must satisfy a specific degree of reliability; to do that, in the second part of our formalism, we proposed a tool to model these situations.

5. Discussion and related work

Dov Gabbay's groundbreaking work on Labeled Deductive Systems [15,16], has provided a clear and direct motivation for this work. The introduction of a flexible and rigorous formalism to tackle complex problems using logical frameworks that include labeled deduction capabilities has permitted to address research problems in areas such as temporal logics, database query languages, and defeasible reasoning systems. In labeled deduction, the formulas are replaced by labeled formulas, expressed as $L : \phi$, where L represents a label associated with the logical formula ϕ. Labels are used to carry additional information that enrich the representation language. The intuitions attached to labels may vary accordingly with the system modeling needs. The idea of structuring labels as an algebra was present from the very inception of labeled systems [15].

In [16], Gabbay's proposal was applied to argumentation systems. There, the authors proposed a framework with the main purpose of formally characterizing and comparing different argument-based inference mechanisms through a unified framework; in particular, two non-monotonic inference operators were used to model argument construction and dialectical analysis in the form of warrant. Labels were used in the framework to represent arguments and dialectical trees. Our proposal shares with those works the characteristic of also involving the use of labels together with an algebra of argumentation labels. Nevertheless, our intention is focused on pursuing a different goal; we are not to trying to unify the presentation of different logics and formally compare them, but to *extend* the representational capabilities of argumentation frameworks by allowing them to handle additional domain specific information. Certainly, it can be argued that due to the extreme generality of Gabbay's framework, somehow it could also be instantiated in some way to achieve this purpose, but we have aimed in our proposal to provide a concrete framework in the context of the Argument Interchange Format, showing how to propagate labels in the specific case of the argument interactions of aggregation, support, and conflict.

Dov Gabbay in [17], concerned with a different problem, proposes a numerical approach to the problem of merging of argumentation networks; in this work, he considers an augmented network containing the arguments and attacks of all the networks to be merged. Then, agents put forward their vote on the components of the network depending on how they perceive these components locally, where a vote means reinforcement in the sense that the more a component appears locally, the more it is represented globally. In addition, he presents a way to calculate the values of arguments in the weighted augmented network, and discern how the attacks to an argument affect its initial support value; finally, he presents a threshold for acceptance to determine the acceptability of an argument based on its weight. In the work presented here, we have been focusing in arguments with structure and with the way the integration of these arguments affect acceptability; thus our proposal also involves the assignment of valuations to arguments, and a way of propagating these valuations through an argumentation graph. However, in the presented framework, it is possible to associate more than one attribute to the arguments; also, we used an abstract algebraic structure in which we can perform the operations of aggregation, support, and conflict of arguments depending the relationship that exist among them and we determine the valuation associated with an argument trough its internal structure. In addition, the conflict operator models situations in which an undefeated argument is weakened when counter-arguments exist; we used the valuation associated with arguments for determining when an argument is better than another, and we used a threshold valuation throughout the process of propagation of labels to establish the conditions that an argument must satisfy to be considered as accepted.

Elvang-Goransson, Krause, and Fox in [14] analyze the fact that in an inconsistent database non-trivial arguments may be constructed for and against a specific proposition; under this situation, the problem arises when determining which conclusion must be accepted. In this work, they define a particular concept of acceptability, which is used to reflect the different acceptability levels associated with an argument; then, they argue that "the more acceptable an argument, the more confident we are in it". Additionally, they define acceptability classes to assign linguistic qualifiers to the arguments. This work has similarities with our proposal, starting from a knowledge base from which it is possible to find the parts that compose an argument. Then, the relationships that exist among the arguments are analyzed, and the acceptability class that they belong to is determined; however, they do not take into account the domain dependent characteristics associated with the arguments. In another related work, Krause et al. in [22] hold that "arguments have the form of logical proof, but they do not have the force of logical proof"; thus, they present a concrete formal model for practical reasoning in which a structured argument rather than some measure is used for describing uncertainty, *i.e.*, the degree of confidence in a proposition is obtained by analyzing the structure of the arguments relevant to it. In this way, this formalism is focused on the representation of uncertainty, and proposes a way to calculate the aggregation of reasons for a certain proposition. In our formalism, depending on the application domain, we introduce a general framework in which it is possible to instantiate each of the elements to represent various attributes associated with the arguments; in addition, the formalism not only allows the aggregation of the valuations associated with a particular conclusion, but also provides ways to effect the weakening of the valuations for a conclusion produced by the existence of reasons against it.

Cayrol and Lagasquie-Schiex in [7], describe the argumentation process as a process which is divided into two steps: a valuation of the relative strength of the arguments and the selection of the most acceptable among them. Their focus is on defining a gradual valuation of arguments based on their interactions, and then establish a graded concept of acceptability of arguments. They argue that an argument is all the more acceptable if it can be preferred to its attackers. They propose a domain of argument valuations, and define aggregation and reduction operators; however, they do not consider the argument structure, and the evaluations of the arguments are solely based on their interaction. In our work, we determine the valuation of arguments through their internal structure considering the different interactions among them, propagating the valuations associated to the arguments using the operations defined in the algebra of argumentation labels. It is important to note that, unlike the proposal of Cayrol and Lagasquie-Schiex, the operations assigned to each relation among arguments are defined by the user providing the possibility of considering the domain of the problem. Also, we provide the ability of assigning more than one valuation to the arguments depending on the features we want to model. Finally, after analyzing all the interactions among arguments we obtain final valuations assigned to each argument; then, through these valuations the acceptability status (*assured*, *unchallenged*, *weakened*, and *rejected*) of the arguments is obtained.

T.J.M. Bench-Capon and J.L. Pollock have introduced systems that currently have great influence over research in argumentation, we will discuss them in turn. Bench-Capon [3] argues in his research that oftentimes it is impossible to conclusively demonstrate in the context of disagreement that either party is wrong, particularly in situations involving practical reasoning. The fundamental role of argument in such cases is to persuade rather than to prove, demonstrate, or refute.

Considering the intuitions of these research lines, we formalized an argumentative framework, additionally integrating AIF into the system. Labels provide a way of representing salient characteristics of the arguments, generalizing the notion of value. It is important to note that the characteristics or properties associated with an argument could vary over time and that they can be affected by various characteristics that influence the real world; for instance, the reliability of a given source [6,5]. Using this framework, we established argument acceptability, where the final labels propagated to the accepted arguments provide additional acceptability information, such as degree of justification, restrictions on justification, and others.

6. Conclusions

Giving an argumentation formalism more representational capabilities can enhance its use in different applications that require different elements to support conclusions. For instance, in an agent implementation, it would be beneficial to establish a measure of the success obtained by reaching a given objective; or, in the domain of recommender systems, it is interesting to provide recommendations together with an uncertainty measure or a reliability degree associated with it.

Our work has focused on the development of a *Labeled Argumentation Framework*, (LAF) combining the KR capabilities provided by the *Argument Interchange Format* (*AIF*) [10] together with the management of labels by an algebra developed to that end. We have associated operations in an algebra of argumentation labels to three different types of argument interactions, allowing to propagate information in the argumentation graph. From the algorithm used to label an argumentation graph, it is possible to determine the acceptability of arguments and the resulting extra data associated to them. A peculiarity of the conflict operation defined in the algebra is that it allows the weakening of arguments contributing to a better representation of application domains.

In the example presented, we used the reliability of the source and a measure of the accuracy associated with the arguments supporting decision making. We defined when an argument is better than other based on these features, and we extend *LAF* into *E-LAF* to consider a threshold, using it to specify constraints arguments need to satisfy to be part of a justification for a recommendation.

Finally, currently we are developing an implementation of LAF extending the existing system DeLP [20,18] which has been taken as a basis.[3] Currently, as part of a computationally oriented sequel of the work presented here, we are analyzing the complexity of the algorithms for labeled argumentation graphs and labeled argumentation graphs with threshold—where the support, conflict and accrued relations are taken into account—in the context of DeLP. The resulting implementation will be exercised in different domains requiring to model extra information associated with the arguments, taking as motivation studies and analysis of P-DeLP [11,2,1].

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