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An approach to abstract argumentation with recursive attack and support ☆

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ABSTRACT

This work introduces the *Attack–Support Argumentation Framework (ASAF)*, an approach to abstract argumentation that allows for the representation and combination of attack and support relations. This framework extends the Argumentation Framework with Recursive Attacks (AFRA) in two ways. Firstly, it adds a *support relation* enabling to express support for arguments; this support can also be given to attacks, and to the support relation itself. Secondly, it extends AFRA's attack relation by allowing attacks to the aforementioned support relation. Moreover, since the support relation of the ASAF has a necessity interpretation, the ASAF also extends the Argumentation Framework with Necessities (AFN). Thus, the ASAF provides a unified framework for representing attack and support for arguments, as well as attack and support for the attack and support relations at any level.

1. Introduction

Argumentation is an attractive and effective paradigm for conceptualizing common-sense reasoning [6,8,29]. Briefly, argumentation is a form of reasoning where a piece of information (*claim*) is accepted or rejected after considering the reasons (*arguments*) for and against that acceptance providing a reasoning mechanism capable of handling contradictory, incomplete and/or uncertain information. Several approaches were proposed to model argumentation: on an abstract basis [20], using classical logics [7], or using logic programming [22]. Additionally, the argumentation process has been employed in various applications and domains such as decision making and negotiation [3,9], and multi-agent systems [28,2].

☆ This article is an extended version of [18].

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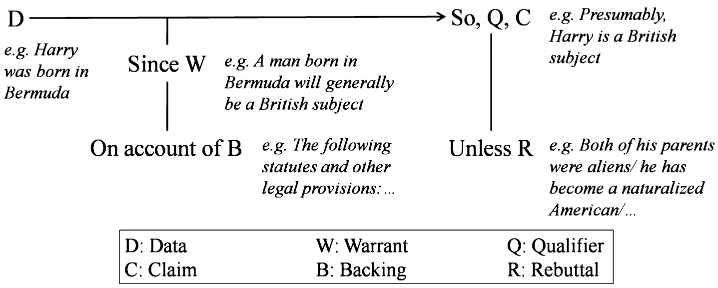

Fig. 1. Toulmin's scheme for the layout of arguments, including the famous "*Harry is a British subject*" example [30].

One of the reasons that argumentation is so useful is that it can handle conflicts due to inconsistent information, and inconsistency naturally arises in multi-agent systems since, among other reasons, different agents represent different views of the world [11]. Such conflicts are captured with the notion of *attack* between arguments. On the other hand, we can also identify situations where there exists a positive interaction between arguments, that is, situations in which arguments *support* each other.

The notion of support has been present in the literature of argumentation since its foundation. In [30] Toulmin proposed a model for the layout of arguments, shown in Fig. 1, that distinguishes between data, claim, warrant, backing, rebuttal and qualifier. Given Toulmin's scheme, we can identify two kinds of interactions among its elements. First, in addition to the data supporting the claim, the backing provides support for the warrant. Second, the presence of a rebuttal leads to the rejection of the claim through an attack on the argument.

Following the seminal work by Dung [20] later studies on argumentation put aside the notion of support to focus on the notion of attack. Several extensions of Dung's frameworks were proposed over the years, including the consideration of attacks to the attack relation [24,4,5]. Notwithstanding this, in the last decade, the study of the notion of support regained attention among the researchers. Several approaches were proposed, where different interpretations for the notion of support are considered [19]. The Bipolar Argumentation Frameworks [1,16] were the first to extend Dung's work by incorporating a general support relation between arguments. Then, other interpretations of support such as evidential support [27], deductive support [10,32] necessary support [25,26,12] and a backing relation [17] were addressed by different argumentation formalisms.

This substantial body of research shows that having attack and support relations between arguments is relevant; furthermore, adding attacks to attacks has also been proved useful, for instance, to express preferences over conflicting arguments [24,4,5]. On the other hand, as shown in [10,32], allowing attacks to the support relation gives the possibility of overriding the acceptability constraints imposed by the support relation.

In this work we will introduce the Attack–Support Argumentation Framework (ASAF), an abstract argumentation framework considering recursive attack and support between arguments, as well as the combination of the attack and support relations. Thus, we propose a novel unified framework for representing attack and support for arguments, as well as attack and support for the attack and support relations at any level. We will use a necessity interpretation of support [25,26,12] which, as will become clearer in the following sections, allows to capture the intuitions behind the combination of the attack and support relations.

Briefly, according to [25,26,12], the necessity interpretation of support establishes that if $\mathcal{A}$ is *necessary* for $\mathcal{B}$ then: if $\mathcal{B}$ is accepted, then $\mathcal{A}$ is also accepted; and if $\mathcal{A}$ is not accepted, then $\mathcal{B}$ is not accepted either. Taking this into account, by having an argument supporting an attack (respectively, a support), the supporting argument will provide conditions under which the attack (respectively, the support) makes sense. Similarly, by having an argument that attacks an attack (respectively, a support), the attacking argument

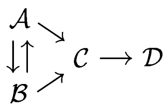

Fig. 2. Abstract argumentation framework AF_1 from Example 1.

will provide reasons against that attack (respectively, support) by expressing conditions under which it does not make sense.

The rest of this work is organized as follows. Section 2 briefly presents the background notions needed for the formalization of the ASAF; specifically, from Dung's Abstract Argumentation Frameworks [20], the Argumentation Frameworks with Recursive Attacks [4,5] and the Argumentation Frameworks with Necessities [25,26,12]. Then, in Section 3 we present some examples that motivate the existence of recursive support, as well as the combination of the attack and support relations. Section 4 formally introduces the Attack–Support Argumentation Framework (ASAF). Section 5 introduces some formal results regarding the ASAF. Section 6 discusses related work. Finally, Section 7 presents conclusions and future lines of work.

2. Background

In this section we will introduce some background material needed for the formalization of our proposed framework. First, we will present some basic notions from Dung's approach to abstract argumentation [20]; then, we will briefly review the Argumentation Framework with Recursive Attacks (AFRA) [4,5], which extends Dung's approach by considering attacks to the attack relation. Finally, a short presentation of the Argumentation Framework with Necessities (AFN) [25,26,12] will be included.

2.1. Abstract Argumentation Framework

Next, we briefly present the necessary elements of the Abstract Argumentation Framework proposed in [20].

Definition 1 *(AF).* An *Abstract Argumentation Framework (AF)* is a pair $\langle \mathbb{A}, \mathbb{R} \rangle$, where $\mathbb{A}$ is a set of arguments and $\mathbb{R} \subseteq \mathbb{A} \times \mathbb{A}$ is an attack relation.

The attack relation between two arguments $\mathcal{A}$ and $\mathcal{B}$ denotes the fact that these arguments cannot be accepted simultaneously since they are conflicting. An argument $\mathcal{A}$ *attacks* an argument $\mathcal{B}$ iff $(\mathcal{A}, \mathcal{B}) \in \mathbb{R}$, and it is noted as $\mathcal{A} \to \mathcal{B}$. To illustrate this, let us consider the following example.

Example 1. Let $AF_1 = \langle \mathbb{A}_1, \mathbb{R}_1 \rangle$ be the abstract argumentation framework depicted in Fig. 2, where:

$$\mathbb{A}_1 = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\}$$

$$\mathbb{R}_1 = \{(\mathcal{A}, \mathcal{B}), (\mathcal{B}, \mathcal{A}), (\mathcal{A}, \mathcal{C}), (\mathcal{B}, \mathcal{C}), (\mathcal{C}, \mathcal{D})\}$$

In this case, arguments $\mathcal{A}$ and $\mathcal{B}$ attack each other, arguments $\mathcal{A}$ and $\mathcal{B}$ attack argument $\mathcal{C}$, and argument $\mathcal{C}$ attacks argument $\mathcal{D}$.

The author in [20] then defines the acceptability of arguments and the admissible sets of the framework as follows.

Definition 2 *(Conflict-freeness, acceptability, admissibility).* Let $AF = \langle \mathbb{A}, \mathbb{R} \rangle$ be an abstract argumentation framework and $S \subseteq \mathbb{A}$. Then:

- S is *conflict-free* iff $\nexists \mathcal{A}, \mathcal{B} \in S$ s.t. $(\mathcal{A}, \mathcal{B}) \in \mathbb{R}$.
- $\mathcal{A}$ is *acceptable* w.r.t. S iff $\forall \mathcal{B} \in \mathbb{A}$: if $(\mathcal{B}, \mathcal{A}) \in \mathbb{R}$ then $\exists \mathcal{C} \in S$ s.t. $(\mathcal{C}, \mathcal{B}) \in \mathbb{R}$.
- If S is conflict-free, then S is an *admissible set* of AF iff each argument in S is acceptable w.r.t. S.

Intuitively, an argument $\mathcal{A}$ is acceptable w.r.t. S if for any argument $\mathcal{B}$ that attacks $\mathcal{A}$, there is an argument $\mathcal{C}$ in S that attacks $\mathcal{B}$, in which case $\mathcal{C}$ is said to defend $\mathcal{A}$. An admissible set S can then be interpreted as a coherent defendable position. For instance, given the AF_1 from Example 1, argument $\mathcal{D}$ is acceptable w.r.t. the sets $\{\mathcal{A}\}$, $\{\mathcal{B}\}$ and $\{\mathcal{A}, \mathcal{B}\}$; however, only the first two of these are admissible sets because the last one is not conflict-free. Then, starting from the notion of admissibility, Dung defines the acceptability semantics of the framework.

Definition 3 *(AF extensions).* Let $AF = \langle \mathbb{A}, \mathbb{R} \rangle$ be an abstract argumentation framework and $S \subseteq \mathbb{A}$ a conflict-free set of arguments. Then:

- S is a *complete extension* of AF iff all arguments acceptable w.r.t. S belong to S.
- S is a *preferred extension* of AF iff it is a maximal (w.r.t. $\subseteq$) admissible set (*i.e.*, a maximal complete extension).
- S is a *stable extension* of AF iff it attacks all arguments in $\mathbb{A} \backslash S$.
- S is the *grounded extension* of AF iff it is the smallest (w.r.t. $\subseteq$) complete extension.

Although in the literature there exist other semantics for AFs (*e.g.*, *semi-stable* [15] or *ideal* [21]), in this paper we will only address the four semantics presented in Definition 3, which were originally proposed in [20]. For instance, the complete extensions of the abstract argumentation framework AF_1 from Example 1 are $\emptyset$, $\{\mathcal{A}, \mathcal{D}\}$ and $\{\mathcal{B}, \mathcal{D}\}$; the preferred and stable extensions are $\{\mathcal{A}, \mathcal{D}\}$ and $\{\mathcal{B}, \mathcal{D}\}$; and the grounded extension is $\emptyset$.

2.2. Argumentation Framework with Recursive Attacks

Next, we briefly review the Argumentation Framework with Recursive Attacks (AFRA) [4,5], which extends Dung's approach by allowing attacks to the attack relation.

Definition 4 *(AFRA).* An *Argumentation Framework with Recursive Attacks (AFRA)* is a pair $\langle \mathbb{A}, \mathbb{R} \rangle$ where:

- $\mathbb{A}$ is a set of arguments; and
- $\mathbb{R}$ is a set of attacks, namely pairs $(\mathcal{A}, X)$ s.t. $\mathcal{A} \in \mathbb{A}$ and either $X \in \mathbb{A}$ or $X \in \mathbb{R}$.

Given an attack $\alpha = (\mathcal{A}, X) \in \mathbb{R}$, $\mathcal{A}$ is called the source of α, denoted as $\mathsf{src}(\alpha) = \mathcal{A}$, and X is called the target of α, denoted as $\mathsf{trg}(\alpha) = X$.

Defeat to an argument or an attack in an AFRA is determined by analyzing the recursive attack relation. In addition, defeats on arguments are also propagated to the attacks they originate.

Definition 5 *(Defeat in AFRA).* Let $\langle \mathbb{A}, \mathbb{R} \rangle$ be an AFRA, $\alpha, \beta \in \mathbb{R}$ and $X \in \mathbb{A} \cup \mathbb{R}$:

- α *directly defeats* X iff $\mathsf{trg}(\alpha) = X$.
- α *indirectly defeats* β iff $\mathsf{trg}(\alpha) = X$, and $X = \mathsf{src}(\beta)$.

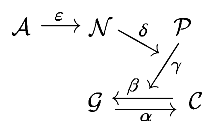

Fig. 3. AFRA Γ_2 from Example 2.

A graphical representation for AFRAs can be provided, where an attack $\alpha = (\mathcal{A}, X)$ is denoted as $\mathcal{A} \xrightarrow{\alpha} X$. To illustrate this, let us consider the following example inspired in [4], adapted to account for the criticism pointed out in [23].

Example 2. Suppose Bob is making decisions regarding his Christmas vacation. He always buys last minute offers, and he knows there are deals for traveling to Gstaad ($\mathcal{G}$) and Cuba ($\mathcal{C}$), two places he would like to visit for several reasons (landscape, nightlife, etc.). In particular, some friends told him that the beaches in Cuba are really nice and that there is a good ski resort in Gstaad. When possible, Bob prefers to go skiing ($\mathcal{P}$); however, the weather service reports that it has not snowed in Gstaad ($\mathcal{N}$). Notwithstanding this, Bob is informed that the ski resort in Gstaad has a good amount of artificial snow that makes it possible to ski ($\mathcal{A}$).

This situation can be represented by the AFRA $\Gamma_2 = \langle \mathbb{A}_2, \mathbb{R}_2 \rangle$, where:

$$\mathbb{A}_2 = \{\mathcal{G}, \mathcal{C}, \mathcal{P}, \mathcal{N}, \mathcal{A}\} \qquad \mathbb{R}_2 = \{\alpha, \beta, \gamma, \delta, \varepsilon\}$$

with $\alpha = (\mathcal{G}, \mathcal{C})$, $\beta = (\mathcal{C}, \mathcal{G})$, $\gamma = (\mathcal{P}, \beta)$, $\delta = (\mathcal{N}, \gamma)$, and $\varepsilon = (\mathcal{A}, \mathcal{N})$. A graphical representation of Γ_2 is provided in Fig. 3.

In particular, given Γ_2, we have that ε directly defeats $\mathcal{N}$ and indirectly defeats δ; δ directly defeats γ; γ directly defeats β; β directly defeats $\mathcal{G}$ and indirectly defeats α; α directly defeats $\mathcal{C}$ and indirectly defeats β.

In [4,5], the authors show that there exists a correspondence between an AFRA and an AF. Thus, by expressing an AFRA in terms of an AF, they are able to reuse the properties and theoretical results available for Dung's frameworks.

Definition 6 *(AF associated to an AFRA).* Let $\Gamma = \langle \mathbb{A}, \mathbb{R} \rangle$ be an AFRA. The *AF associated to* Γ is $\widetilde{\Gamma} = \langle \widetilde{\mathbb{A}}, \widetilde{\mathbb{R}} \rangle$, defined as follows:

- $\widetilde{\mathbb{A}} = \mathbb{A} \cup \mathbb{R}$; and
- $\widetilde{\mathbb{R}} = \{(X, Y) \mid X, Y \in \mathbb{A} \cup \mathbb{R}$ and X directly or indirectly defeats $Y\}$.

Given an AFRA Γ and its associated AF $\widetilde{\Gamma}$, the authors in [5] provide a bijective correspondence between the extensions of Γ (defined in terms of a characteristic function) and the extensions of $\widetilde{\Gamma}$. In particular, they show that E is an extension of Γ under a semantics $\mathsf{s} \in \{$*complete*, *preferred*, *stable*, *grounded*, *semi-stable*, *ideal*$\}$ iff E is an extension of $\widetilde{\Gamma}$ under the same semantics.[1] In addition note that, by Definition 6, attacks in the AFRA correspond to arguments in its associated AF. Therefore, they may appear in the extensions of the associated AF (and thus, in the extensions of the AFRA) because they have not been made ineffective by another attack. To illustrate this, let us consider the following example.

Example 3. Let Γ_2 be the AFRA from Example 2. The AF associated to Γ_2 is $\widetilde{\Gamma_2} = \langle \widetilde{\mathbb{A}_2}, \widetilde{\mathbb{R}_2} \rangle$, where:

$$\widetilde{\mathbb{A}_2} = \{\mathcal{G}, \mathcal{C}, \mathcal{P}, \mathcal{N}, \mathcal{A}, \alpha, \beta, \gamma, \delta, \varepsilon\}$$

[1] As mentioned before, differently from [5], in this work we will only consider the *complete*, *preferred*, *stable* and *grounded* semantics.

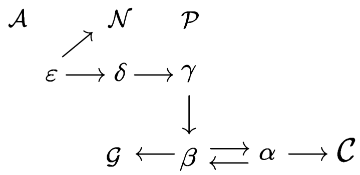

Fig. 4. AF $\widetilde{\Gamma_2}$, corresponding to Example 3.

$$\widetilde{\mathbb{R}_2} = \{(\varepsilon, \mathcal{N}), (\varepsilon, \delta), (\delta, \gamma), (\gamma, \beta), (\beta, \mathcal{G}), (\beta, \alpha), (\alpha, \beta), (\alpha, \mathcal{C})\}$$

A graphical representation of $\widetilde{\Gamma_2}$ is included in Fig. 4. According to the preferred semantics (see Definition 3), the only preferred extension of $\widetilde{\Gamma_2}$ is $\{\mathcal{A}, \mathcal{P}, \mathcal{G}, \varepsilon, \gamma, \alpha\}$. As a result, since the extensions of $\widetilde{\Gamma_2}$ under the preferred semantics coincide with the extensions of Γ_2 under the same semantics, the only preferred extension of Γ_2 is $\{\mathcal{A}, \mathcal{P}, \mathcal{G}, \varepsilon, \gamma, \alpha\}$.

Finally, observe that given an attack $\mathcal{A} \xrightarrow{\alpha} X$ in the AFRA, the associated AF does not provide any explicit connection between $\mathcal{A}$ and α. For instance, given the AFRA Γ_2 from Example 2, the associated AF $\widetilde{\Gamma_2}$ (depicted in Example 3) provides no link between argument $\mathcal{N}$ and the attack δ it originates. Notwithstanding this, the connection between an argument in the AFRA and the attacks it originates is captured by the semantics through the existence of indirect defeats, thus propagating attacks on arguments to the attacks they originate. Then, given the AFRA Γ_2 and its associated AF $\widetilde{\Gamma_2}$ from Example 3, for instance, the attack $\mathcal{A} \xrightarrow{\varepsilon} \mathcal{N}$ affects the attack δ originated by $\mathcal{N}$, which is captured in $\widetilde{\Gamma_2}$ through the attack from ε to δ.

2.3. Argumentation Framework with Necessities

Here, we will give a brief presentation of the Argumentation Framework with Necessities (AFN). In [25,26,12] the authors extend Dung's framework by incorporating a support relation with a particular interpretation and thus, imposing some acceptability constraints on the arguments it relates.

Definition 7 *(AFN).* An *Argumentation Framework with Necessities (AFN)* is a tuple $\langle \mathbb{A}, \mathbb{R}, \mathbb{N} \rangle$ where $\mathbb{A}$ is a set of arguments, $\mathbb{R} \subseteq \mathbb{A} \times \mathbb{A}$ is an attack relation, and $\mathbb{N} \subseteq \mathbb{A} \times \mathbb{A}$ is an irreflexive and transitive necessity relation.

The attack relation $\mathbb{R}$ of the AFN is the same as in Dung's argumentation frameworks. On the other hand, the support relation $\mathbb{N}$ has a necessity interpretation, establishing some acceptability constraints on the arguments it relates: if $\mathcal{A}\mathbb{N}\mathcal{B}$ it holds that if $\mathcal{B}$ is accepted, then $\mathcal{A}$ is also accepted and, conversely, if $\mathcal{A}$ is not accepted then $\mathcal{B}$ is not accepted either. Then, the authors propose the notion of *extended attack*, which arises from the coexistence of the original attack and support relations.

Definition 8 *(Extended attack).* Let $\langle \mathbb{A}, \mathbb{R}, \mathbb{N} \rangle$ be an AFN and $\mathcal{A}, \mathcal{B} \in \mathbb{A}$. There is an *extended attack* from $\mathcal{A}$ to $\mathcal{B}$, noted as $\mathcal{A}\mathbb{R}^+\mathcal{B}$, iff $\exists \mathcal{C} \in \mathbb{A}$ s.t. $\mathcal{A}\mathbb{R}\mathcal{C}\mathbb{N}\mathcal{B}$. The direct attack $\mathcal{A}\mathbb{R}\mathcal{B}$ can be considered as a particular case of extended attack.

An AFN can be graphically represented by a directed multi-graph, where nodes are arguments and there are two kinds of edges: $\mathcal{A} \longrightarrow \mathcal{B}$, denoting $\mathcal{A}\,\mathbb{R}\,\mathcal{B}$ (attack), and $\mathcal{A} \Longrightarrow \mathcal{B}$, denoting $\mathcal{A}\,\mathbb{N}\,\mathcal{B}$ (necessity). To illustrate this, let us consider the following example.

Example 4. Let $AFN_4 = \langle \mathbb{A}_4, \mathbb{R}_4, \mathbb{N}_4 \rangle$ be an argumentation framework with necessities, where:

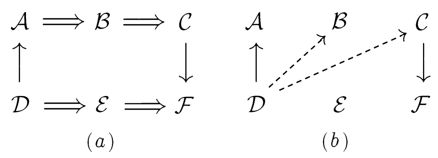

Fig. 5. (a) AFN_4 from Example 4 and (b) the extended attacks it originates.

$$\mathbb{A}_4 = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}\}$$

$$\mathbb{R}_4 = \big\{(\mathcal{D}, \mathcal{A}), (\mathcal{C}, \mathcal{F})\big\}$$

$$\mathbb{N}_4 = \big\{(\mathcal{A}, \mathcal{B}), (\mathcal{B}, \mathcal{C}), (\mathcal{A}, \mathcal{C}), (\mathcal{D}, \mathcal{E}), (\mathcal{E}, \mathcal{F}), (\mathcal{D}, \mathcal{F})\big\}$$

A graphical representation of AFN_4 is provided in Fig. 5(a).

Note that, since the necessity relation is transitive, $\mathcal{A}$ is necessary for $\mathcal{C}$, and $\mathcal{D}$ is necessary for $\mathcal{F}$. However, to simplify the graphical representation, the necessities obtained by transitivity are not explicitly shown, but they can be visualized by following the support paths on the graph illustrated in Fig. 5(a). On the other hand, Fig. 5(b) summarizes the extended attacks obtained from AFN_4, which are depicted using dashed arrows (except for those that are, in particular, direct attacks). Given that $\mathcal{D}$ attacks $\mathcal{A}$, and $\mathcal{A}$ is necessary for $\mathcal{B}$ and $\mathcal{C}$, there are extended attacks from $\mathcal{D}$ to $\mathcal{B}$ and $\mathcal{C}$.

In [26] the authors show that every AFN has an associated AF, as characterized by the following definition. Then, they show that the extensions of an AFN and its associated AF coincide.

Definition 9 *(AF associated to an AFN).* Given an AFN $\langle \mathbb{A}, \mathbb{R}, \mathbb{N} \rangle$, its *associated AF* is $\langle \mathbb{A}, \mathbb{R}^+ \rangle$.

Note that the AF associated to an AFN takes the extended attack relation $\mathbb{R}^+$ into account. Then, since direct attacks are a particular case of extended attacks (see Definition 8), the associated AF contemplates both direct and extended attacks on the arguments of the original AFN. For instance, given the AFN_4 from Example 4 its associated AF is $AF_4 = \langle \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}\}, \{(\mathcal{D}, \mathcal{A}), (\mathcal{D}, \mathcal{B}), (\mathcal{D}, \mathcal{C}), (\mathcal{C}, \mathcal{F})\} \rangle$. Then, by Definition 3, the only preferred extension of AF_4 is $\{\mathcal{D}, \mathcal{E}, \mathcal{F}\}$. As a result, since the extensions of AFN_4 and AF_4 coincide, $\{\mathcal{D}, \mathcal{E}, \mathcal{F}\}$ is also the only preferred extension of AFN_4.

3. Motivation

As mentioned in the introduction, there is a vast amount of research that motivates the existence of attack and support relations in the context of abstract argumentation. In particular, the AFRA provides the means for representing attacks to attacks, capability that has been proved useful, for instance, to express preferences between conflicting arguments. On the other hand, the AFN incorporates a necessary support relation, expressing positive interactions between arguments. Next, we will discuss why it is interesting to allow for the representation of recursive support, as well as support for attacks and vice-versa, where the support relation has a necessity interpretation.

To start with, let us consider the scenario introduced in Example 2, where Bob is deciding about a destination for his Christmas vacation. Arguments $\mathcal{G}$ and $\mathcal{C}$ are conflicting in the AFRA Γ_2 because they correspond to last minute offers for going to two different places (Gstaad and Cuba) at the same time; otherwise, Bob could be able to go to both Gstaad *and* Cuba. Then, we can have an argument $\mathcal{SD}$ that expresses this situation, namely, that the last minute offers for going to Gstaad and Cuba are for the same

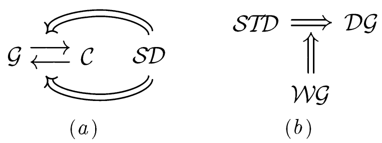

Fig. 6. (a) Support to an attack and (b) support to a support.

date or time-period. In addition, we would need to represent the fact that arguments $\mathcal{G}$ and $\mathcal{C}$ are conflicting only in the situation described by $\mathcal{SD}$. This can be captured by having a necessity relation between $\mathcal{SD}$ and the attacks between $\mathcal{G}$ and $\mathcal{C}$, as depicted in Fig. 6(a). In this case, the constraints imposed by the necessity interpretation of support allow to capture the intuitions behind the conflicts between $\mathcal{G}$ and $\mathcal{C}$: if the conflicts hold, then it means that the offers for going to Gstaad and Cuba were for the same date and, conversely, if the offers correspond to different dates, then there is no conflict between $\mathcal{G}$ and $\mathcal{C}$ since Bob could be able to go to both places.

Continuing with the scenario presented in Example 2, let us now suppose that Bob has decided to go to Gstaad for Christmas, and he wants to rent a car during his stay. We can represent this through an argument $\mathcal{DG}$ stating that since Bob will rent a car during his vacation, he will be driving in Gstaad. Then, since there will possibly be snow in Gstaad, Bob should put some snow traction device on the car in order to drive safely. This could be represented by an argument $\mathcal{STD}$ expressing that since Bob will be renting a car in Gstaad he should put some snow traction device on it. Then, we could have a necessity support between $\mathcal{STD}$ and $\mathcal{DG}$ ($\mathcal{STD} \Longrightarrow \mathcal{DG}$) expressing that having the snow traction device on the car is necessary for Bob's driving in Gstaad.

We can note that the necessity between $\mathcal{STD}$ and $\mathcal{DG}$ is only reasonable because Bob is going to Gstaad on Christmas, therefore, during winter. If he were going to Gstaad on summer, he will no longer be required to put a snow traction device on the car. We could represent this restriction by having an argument $\mathcal{WG}$ expressing that Bob is going to Gstaad in winter (for his Christmas vacation). In addition, we could represent the fact that being in Gstaad during winter provides the conditions under which the necessity from $\mathcal{STD}$ to $\mathcal{DG}$ makes sense. In order to do this, we add a necessity from $\mathcal{WG}$ to the support between $\mathcal{STD}$ and $\mathcal{DG}$, as depicted in Fig. 6(b). Here, the constraints imposed by the necessity interpretation of support allow to capture the intuition that Bob should put a snow traction device on the car before driving during his stay in Gstadd in winter. If the necessity between $\mathcal{STD}$ and $\mathcal{DG}$ holds, then it means that Bob is driving in Gstaad during winter and, conversely, if Bob is not in Gstaad during winter, then he would not be required to put a snow traction device before driving.

Let us now suppose that the weather report informs that it has not snowed in Gstaad for the last month and the forecast says it will not snow in the upcoming weeks; this could be represented, similarly to Example 2, by an argument $\mathcal{N}$. Then, the weather report provides a reason to believe that there will be no snow on the roads in Gstaad and therefore, Bob will not be driving on snow during his stay. As a result, we have that argument $\mathcal{N}$ does not attack the fact that Bob will be driving in Gstaad during winter, nor attacks the fact that being in winter provides a reason for putting a snow traction device on the car before driving. Instead, argument $\mathcal{N}$ provides a reason against Bob's need to put a snow traction device on the car before driving in Gstaad. This is represented by an attack from $\mathcal{N}$ to the necessity between $\mathcal{STD}$ and $\mathcal{DG}$, as depicted in Fig. 7. In that case, the fact that Bob will be in Gstaad during winter still provides a reason for putting a snow traction device on the car before driving there. On the other hand, the information from the weather report (which states that there will be no snow in Gstaad during Bob's stay) provides a reason against Bob's need to put a snow traction device on the car before driving in Gstaad. Then, the attack from $\mathcal{N}$ makes the support between $\mathcal{STD}$ and $\mathcal{DG}$ ineffective, overriding the constraints imposed by the necessity relation. As a result, in this new scenario, Bob will not be required to put a snow traction device on the car in order to drive safely in Gstaad.

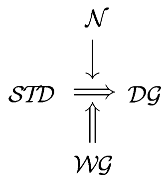

Fig. 7. Fig. 6(b) extended by incorporating an attack to a support.

The difference between the scenarios depicted in Figs. 6(*b*) and 7 becomes more evident when considering an argument $\mathcal{X}$ that attacks $\mathcal{STD}$ (*i.e.*, $\mathcal{X}$ provides a reason against putting a snow traction device on the car). In the case of Fig. 6(b), since the necessity between $\mathcal{STD}$ and $\mathcal{DG}$ holds, the attack from $\mathcal{X}$ will also affect $\mathcal{DG}$. Thus, Bob would not be able to drive safely in Gstaad since he's not putting a snow traction device on the car. On the other hand, given the situation depicted in Fig. 7, the necessity between $\mathcal{STD}$ and $\mathcal{DG}$ is made ineffective by the attack from $\mathcal{N}$. Therefore, the attack from $\mathcal{X}$ to $\mathcal{STD}$ does not affect $\mathcal{DG}$. As a result, in this case, Bob will be able to drive safely in Gstaad even though he does not put a snow traction device on the car.

Given the preceding examples, we can see that using recursive support as well as combined support and attack relations provides an interesting and useful representation approach. The following section formalizes these intuitions in the context of an abstract argumentation framework called *Attack–Support Argumentation Framework*.

4. Attack–Support Argumentation Framework

In this section we will present the Attack–Support Argumentation Framework (ASAF). Following the formalization of the ASAF, we will introduce an approach for computing the acceptability of arguments which consists on mapping the ASAF into an associated AFN and then, into its corresponding AF.

4.1. Defining the framework

Having presented the required background material, now we will introduce the Attack–Support Argumentation Framework (ASAF), a novel approach that extends the AFRA [4,5] in two ways: it incorporates a support relation enabling to express support for arguments, for attacks, and for the support relation itself; and it extends the AFRA's attack relation by allowing attacks to the support relation. Moreover, since the support relation of the ASAF has a necessity interpretation, the ASAF also extends the AFN [25,26,12]: it allows for recursive attack and support, as well as attacks on the support relation and vice-versa.

Definition 10 *(ASAF).* An *Attack–Support Argumentation Framework* is a tuple $\langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ where $\mathbb{A}$ is a set of arguments, $\mathbb{R} \subseteq \mathbb{A} \times (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ is an attack relation, $\mathbb{S} \subseteq \mathbb{A} \times (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ is an irreflexive and transitive support relation, and the attack and support relations are such that $\mathbb{R} \cap \mathbb{S} = \emptyset$.

The attack relation in an ASAF extends the attack relation of the AFRA to incorporate attacks to the support relation. Similarly, it extends the attack relation of the AFN to allow for recursive attacks, as well as attacks to the support relation. The support relation in an ASAF has a necessity interpretation (see Section 2) and, therefore, it imposes some acceptability constraints on the elements it relates. Notwithstanding this, it is important to note that, although the support relation of an ASAF has the same interpretation as the support relation of an AFN, these relations are different: the necessity support relation of the ASAF extends the support relation of the AFN to incorporate recursive support as well as the possibility to provide support for attacks.

As mentioned before, attacks and supports in an ASAF can be themselves attacked. Thus, for instance, an attack from an argument $\mathcal{A}$ to a support from $\mathcal{B}$ to X will be represented by a pair $(\mathcal{A}, (\mathcal{B}, X))$ in the attack relation $\mathbb{R}$ of the ASAF, where the pair $(\mathcal{B}, X)$ belongs to the support relation $\mathbb{S}$ of the ASAF. In cases like this one, to simplify the notation, we will denote the attack originated by $\mathcal{A}$ with the pair $(\mathcal{A}, \alpha)$, where $\alpha = (\mathcal{B}, X)$.

Given the combination of attack and support relations in an ASAF, we need to provide a way to identify attacks and supports unequivocally. This is because, when an argument $\mathcal{A}$ is attacking (respectively, supporting) a pair $\alpha = (\mathcal{B}, X)$, we need to know whether $\mathcal{A}$ is attacking (respectively, supporting) an attack from $\mathcal{B}$ to X or a support from $\mathcal{B}$ to X. If $\alpha = (\mathcal{B}, X)$ is such that it may correspond to *both* an attack and a support from $\mathcal{B}$ to X, when $\mathcal{A}$ attacks (respectively, supports) α it would be impossible to determine whether $\mathcal{A}$ is attacking (respectively, supporting) the attack from $\mathcal{B}$ to X or the support from $\mathcal{B}$ to X. Therefore, we require the attack and support relations of the ASAF to be disjoint (*i.e.*, $\mathbb{R} \cap \mathbb{S} = \emptyset$).

Similarly to [4,5], given an attack $\alpha = (\mathcal{A}, X) \in \mathbb{R}$, $\mathcal{A}$ is called the source of α, denoted as $\mathsf{src}(\alpha) = \mathcal{A}$, and X is called the target of α, denoted as $\mathsf{trg}(\alpha) = X$. Analogously, given a support $\beta = (\mathcal{B}, Y) \in \mathbb{S}$, $\mathcal{B}$ is called the source of β, denoted as $\mathsf{src}(\beta) = \mathcal{B}$, and Y is called the target of β, denoted as $\mathsf{trg}(\beta) = Y$.

An ASAF can be graphically represented using a graph-like notation: an argument $\mathcal{A} \in \mathbb{A}$ will be denoted as a node in the graph, an attack $\alpha = (\mathcal{A}, X) \in \mathbb{R}$ will be denoted as $\mathcal{A} \xrightarrow{\alpha} X$, and a support $\beta = (\mathcal{B}, Y) \in \mathbb{S}$ will be denoted as $\mathcal{B} \stackrel{\beta}{\Longrightarrow} Y$. To simplify the notation, the attack from an argument $\mathcal{C}$ to an attack or a support $\alpha = (\mathcal{A}, X)$ will be denoted as $(\mathcal{C}, \alpha)$. Similarly, the support from an argument $\mathcal{D}$ to an attack or a support $\beta = (\mathcal{B}, Y)$ will be denoted as $(\mathcal{D}, \beta)$. Since, as mentioned before, the attack and support relations of an ASAF are disjoint, a pair $\gamma = (\mathcal{E}, Z)$ in the attack relation *or* the support relation will be unequivocally identified by γ. Thus, when referring to γ, it will be possible to determine the attack or support it represents. To illustrate this, let us consider the following example.

Example 5. Let us consider the scenario presented in Section 3, where Bob has decided to spend his Christmas vacation in Gstaad, and he wants to rent a car during his stay. In addition, Bob has information from a weather report for Gstaad. This could be represented by the following arguments:

- $\mathcal{DG}$: which expresses that since Bob is renting a car during his Christmas vacation, he will be driving in Gstaad.
- $\mathcal{STD}$: which expresses that since Bob is renting a car during his stay in Gstaad, he should put a snow traction device on it.
- $\mathcal{WG}$: which expresses that Bob is going to Gstaad in winter, since he is going there to spend his Christmas vacation.
- $\mathcal{N}$: which expresses information from a weather report saying that it has not snowed in Gstaad for the last month and, according to this forecast, it will not snow in the upcoming weeks.

These arguments and their interactions can be represented by the ASAF $\Delta_5 = \langle \mathbb{A}_5, \mathbb{R}_5, \mathbb{S}_5 \rangle$, where:

$$\mathbb{A}_5 = \{\mathcal{DG}, \mathcal{STD}, \mathcal{WG}, \mathcal{N}\} \qquad \mathbb{R}_5 = \{\gamma\} \qquad \mathbb{S}_5 = \{\alpha, \beta\}$$

with $\alpha = (\mathcal{STD}, \mathcal{DG})$, $\beta = (\mathcal{WG}, \alpha)$ and $\gamma = (\mathcal{N}, \alpha)$.

A graphical representation of Δ_5 is included in Fig. 8. In this case, the argument stating that Bob is going to Gstaad during winter provides the conditions under which he would be required to put a snow traction device on his car before driving there. On the other hand, the information from the weather report attacks the fact that Bob should put a snow traction device on his car in order to drive safely in Gstaad. Thus,

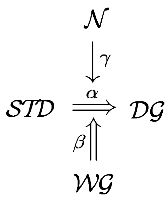

Fig. 8. ASAF Δ_5 from Example 5.

the attack γ makes the support α ineffective, overriding the constraints it imposes on the acceptability of arguments $\mathcal{STD}$ and $\mathcal{DG}$. In that way, Bob would be able to drive safely in Gstaad even though he has not installed a snow traction device on his car.

4.2. Calculating acceptability

Acceptability of arguments in an ASAF is calculated following an extension-based approach. To achieve this, we will provide a transformation from an ASAF into an AF [20] performed in two steps: first, we will transform the ASAF into an AFN; second, we will obtain the AF associated to the AFN (see Definition 9). Finally, this AF will be the Abstract Argumentation Framework associated to the ASAF, and the extensions of the ASAF will be obtained in terms of the extensions of its associated AF.

4.2.1. Translating an ASAF into an AFN

In this section we will present the transformation from an ASAF into its associated AFN. First, we will explain the intuitions behind this transformation and then, we will proceed to its formalization. Briefly, for each argument $\mathcal{A}$ in the ASAF there will be an argument $\mathcal{A}$ in the associated AFN. In addition, following the spirit of [4,5] and [10], the attack and support relations in the ASAF will be jointly represented by arguments, attacks, and supports in the associated AFN.

The transformation proposed in this section is such that each attack and support in the ASAF will be partly represented in its associated AFN using a special kind of argument: for each attack in the ASAF there will be an attack-argument in the associated AFN, and for each support in the ASAF there will be two support-arguments in the associated AFN. Consequently, since all arguments of the associated AFN will be arguments of its associated AF (see Definition 9), they may appear in the corresponding extensions. Thus, we need to determine what it means to accept an attack/support-argument.

Let us first consider the case of attack-arguments. Given an attack $\alpha = (\mathcal{A}, X)$ in the ASAF, there will be an attack-argument α in the associated AFN and therefore, α will be an argument of the associated AF. Hence, if α belongs to an extension E of the associated AF, then the attack from $\mathcal{A}$ to X will be active, implying that if $\mathcal{A}$ is accepted (*i.e.*, $\mathcal{A} \in \mathsf{E}$), then X is not accepted (*i.e.*, $X \notin \mathsf{E}$).

Let us now analyze the case of support-arguments. Since the support relation of an ASAF has a necessity interpretation, a support $\beta = (\mathcal{B}, Y)$ encodes the following constraints: if Y is accepted, then $\mathcal{B}$ is accepted and, conversely, if $\mathcal{B}$ is not accepted, then Y is not accepted either. We will refer to these, respectively, as the *positive constraint* and the *negative constraint*. It should be noted that these constraints are mutually exclusive in the following sense: if the conditions for the positive constraint are met, then the conditions stated by the negative constraint cannot hold, too (that is, because an argument cannot be simultaneously "accepted" and "not accepted").

Given a support $\beta = (\mathcal{B}, Y)$ in the ASAF, there will be two arguments in the associated AFN (and thus, in the associated AF): β^+ and β^- which, respectively, correspond to the positive and negative constraint of β. Hence, for instance, if β^- belongs to an extension E of the associated AF, then it means that the support between $\mathcal{B}$ and Y is active and satisfies the negative constraint, implying that if $\mathcal{B}$ is not accepted (*i.e.*, $\mathcal{B} \notin \mathsf{E}$), then Y is not accepted either (*i.e.*, $Y \notin \mathsf{E}$).

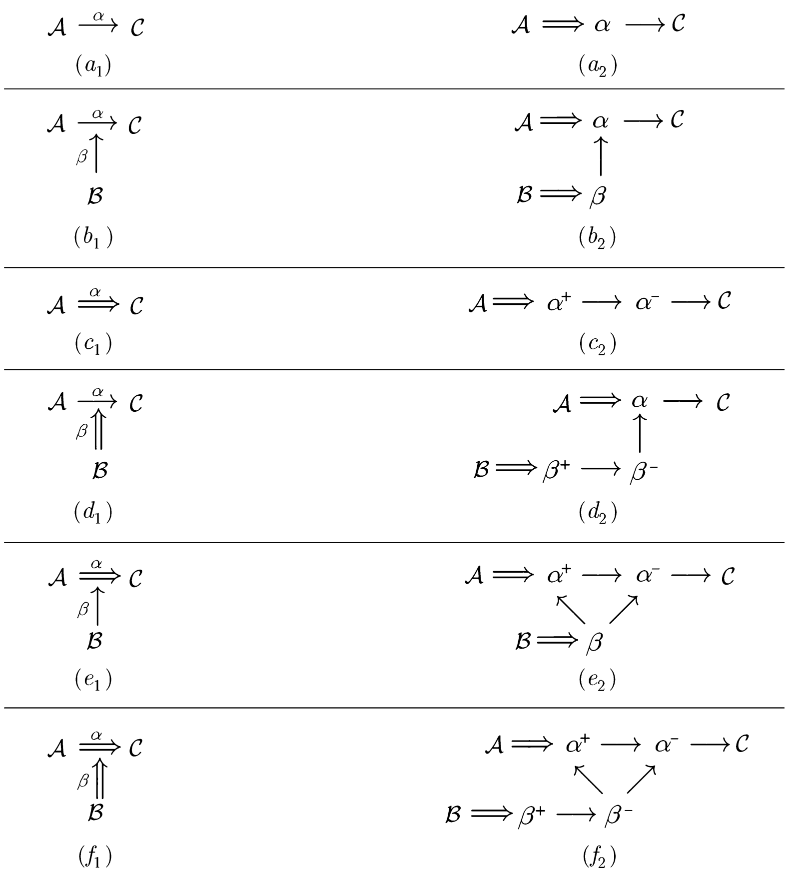

Fig. 9. Different cases of the translation from an ASAF into its associated AFN.

In addition to the attack/support-arguments described above, the attack and support relations of an ASAF will be represented in its associated AFN through a combination of attacks and supports. In the following, we will consider the different cases of attack and support that can occur within an ASAF, showing the corresponding attacks and supports they originate in the associated AFN.

First, let us consider the case where an argument attacks another in the ASAF, as depicted in Fig. 9(a_1). The attack α in the ASAF will result in a sequence of a support followed by an attack in the associated AFN, as depicted in Fig. 9(a_2). In particular, the support $\mathcal{A} \Longrightarrow \alpha$ in the associated AFN represents that $\mathcal{A}$ is necessary for the attack it originates since, as will be shown by Proposition 1, attacks on arguments affect the attacks they originate. On the other hand, the attack $\alpha \longrightarrow \mathcal{C}$ represents the attack from $\mathcal{A}$ to $\mathcal{C}$ itself. Analogously, an attack to an attack in the ASAF, as depicted in Fig. 9(b_1), is directly translated by adding an attack to the corresponding attack-argument of the associated AFN, as depicted in Fig. 9(b_2).

Let us now consider the case of a support between arguments in the ASAF, as depicted in Fig. 9(c_1). In this case, the support α in the ASAF is transformed into a sequence of a support followed by two attacks in the associated AFN, as depicted in Fig. 9(c_2). In particular, as mentioned before, a support α in the ASAF leads to considering two support-arguments α^+ and α^- in the associated AFN, which respectively correspond to the positive and negative constraint of α. Then, the attack $\alpha^+ \longrightarrow \alpha^-$ in the associated AFN represents that the positive and negative constraints are mutually exclusive: if α^+ is accepted, then it means that if $\mathcal{C}$ is accepted, then $\mathcal{A}$ is also accepted; therefore, if this positive constraint holds, the negative constraint α^- will not hold (since it is not the case that $\mathcal{A}$ is not accepted). Hence, the attack $\alpha^- \longrightarrow \mathcal{C}$ represents that if the conditions established by the negative constraint hold, then $\mathcal{A}$ is not accepted and thus, $\mathcal{C}$ should not be accepted either. Finally, the support $\mathcal{A} \Longrightarrow \alpha^+$ propagates attacks on $\mathcal{A}$ to α^+ through an extended attack (see Definition 8), reinstating the support-argument α^- corresponding to the negative constraint.

Similarly to the support for an argument, the support for an attack in the ASAF, as depicted in Fig. 9(d_1), is translated directly by considering the corresponding attack-argument in the associated AFN, as shown in Fig. 9(d_2). In this case, the support from $\mathcal{B}$ to α in the ASAF expresses that $\mathcal{B}$ is necessary for the attack from $\mathcal{A}$ to $\mathcal{C}$ to hold. Therefore, the non-acceptability of $\mathcal{B}$ in the associated AFN will lead to the non-acceptability of α, meaning that the attack from $\mathcal{A}$ to $\mathcal{C}$ is not active.

Consider now the case of an attack to a support in the ASAF, as depicted in Fig. 9(e_1). The attacks from β to α^+ and α^- in the associated AFN, as depicted in Fig. 9(e_2), express that if the attack from $\mathcal{B}$ to α in the ASAF is active, then the support from $\mathcal{A}$ to $\mathcal{C}$ does not hold, overriding the positive and negative constraints of α.

Finally, recursive support in the ASAF, as depicted in Fig. 9(f_1), is translated by considering the corresponding support-arguments in the associated AFN. In this case, β^- attacks both support-arguments α^+ and α^- in the associated AFN, as depicted in Fig. 9(f_2). This is because $\mathcal{B}$ is necessary for the support from $\mathcal{A}$ to $\mathcal{C}$ to hold. Therefore, if $\mathcal{B}$ is not accepted (which makes the support-argument β^- accepted), then α no longer holds and thus, its positive and negative constraints are overridden.

All these intuitions are formalized by the following definition, which characterizes the transformation from an ASAF into its associated AFN.

Definition 11 *(AFN associated to an ASAF).* Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF. The *AFN associated to* Δ is $\Delta_{AFN} = \langle \mathbb{A}_{AFN}, \mathbb{R}_{AFN}, \mathbb{N}_{AFN} \rangle$, where:

- $\mathbb{A}_{AFN} = \mathbb{A} \cup \{\alpha \mid \alpha = (\mathcal{A}, X) \in \mathbb{R}\} \cup \{\beta^+, \beta^- \mid \beta = (\mathcal{B}, Y) \in \mathbb{S}\}$.
- $\mathbb{R}_{AFN}$ and $\mathbb{N}_{AFN}$ are such that:
  1. If $\alpha = (\mathcal{A}, X) \in \mathbb{R}$ and $X \in \mathbb{A} \cup \mathbb{R}$, then:
     - $(\mathcal{A}, \alpha) \in \mathbb{N}_{AFN}$; and
     - $(\alpha, X) \in \mathbb{R}_{AFN}$.
  2. If $\beta = (\mathcal{B}, Y) \in \mathbb{S}$ and $Y \in \mathbb{A} \cup \mathbb{R}$, then:
     - $(\mathcal{B}, \beta^+) \in \mathbb{N}_{AFN}$;
     - $(\beta^+, \beta^-) \in \mathbb{R}_{AFN}$; and
     - $(\beta^-, Y) \in \mathbb{R}_{AFN}$.
  3. If $\alpha = (\mathcal{A}, \beta) \in \mathbb{R}$, where $\beta = (\mathcal{B}, Y) \in \mathbb{S}$, then:
     - $(\mathcal{A}, \alpha) \in \mathbb{N}_{AFN}$;
     - $(\alpha, \beta^+) \in \mathbb{R}_{AFN}$; and
     - $(\alpha, \beta^-) \in \mathbb{R}_{AFN}$.
  4. If $\beta = (\mathcal{B}, \gamma) \in \mathbb{S}$, where $\gamma = (\mathcal{C}, Z) \in \mathbb{S}$, then:
     - $(\mathcal{B}, \beta^+) \in \mathbb{N}_{AFN}$;
     - $(\beta^+, \beta^-) \in \mathbb{R}_{AFN}$;
     - $(\beta^-, \gamma^+) \in \mathbb{R}_{AFN}$; and
     - $(\beta^-, \gamma^-) \in \mathbb{R}_{AFN}$.

Note that the attacks to arguments or to attacks in the ASAF are considered together in Definition 11. This is because arguments and attacks in the ASAF are represented by a single argument in the associated AFN. Likewise, supports to arguments or to attacks are also considered together in Definition 11.

4.2.2. Obtaining the extensions of the ASAF

Provided the translation from an ASAF into its associated AFN, as presented in Section 4.2.1, we will next introduce a mechanism for obtaining the extensions of an ASAF by using its associated AFN. Recall

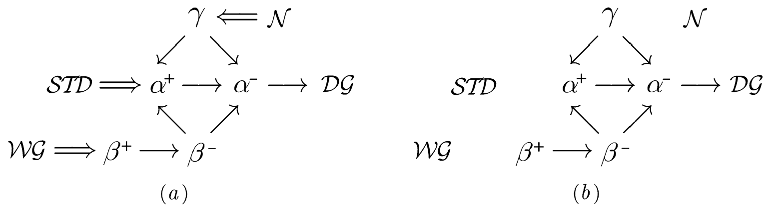

Fig. 10. (a) AFN Δ_{AFN_5} and (b) AF Δ_{AF_5}, corresponding to Example 6.

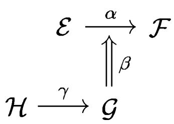

Fig. 11. ASAF Δ_7 from Example 7.

that, as presented in Section 2.3, given an AFN it is possible to obtain its associated AF (see Definition 9). Therefore, given an ASAF and its associated AFN, we can obtain the AF associated to the ASAF, as defined next.

Definition 12 *(AF associated to an ASAF).* Let Δ be an ASAF and Δ_{AFN} the AFN associated to Δ. The *AF associated to* Δ is Δ_{AF}, where Δ_{AF} is the AF associated to Δ_{AFN}.

To illustrate this, let us consider the following two examples.

Example 6. Let Δ_5 be the ASAF from Example 5. The AFN associated to Δ_5 is $\Delta_{AFN_5} = \langle \mathbb{A}_{AFN5}, \mathbb{R}_{AFN5}, \mathbb{N}_{AFN5} \rangle$, where:

$$\mathbb{A}_{AFN5} = \{\mathcal{DG}, \mathcal{STD}, \mathcal{WG}, \mathcal{N}, \alpha^+, \alpha^-, \beta^+, \beta^-, \gamma\}$$

$$\mathbb{R}_{AFN5} = \{(\gamma, \alpha^+), (\gamma, \alpha^-), (\alpha^+, \alpha^-), (\alpha^-, \mathcal{DG}), (\beta^+, \beta^-), (\beta^-, \alpha^+), (\beta^-, \alpha^-)\}$$

$$\mathbb{N}_{AFN5} = \{(\mathcal{N}, \gamma), (\mathcal{STD}, \alpha^+), (\mathcal{WG}, \beta^+)\}$$

Then, the AF associated to Δ_5 is $\Delta_{AF_5} = \langle \mathbb{A}_{AFN5}, \mathbb{R}_{AFN5} \rangle$. The graphical representation of Δ_{AFN_5} is included in Fig. 10(a), and the graphical representation of Δ_{AF_5} is provided in Fig. 10(b).

It can be noted that there are no extended attacks between the arguments of Δ_{AFN_5} and thus, Δ_{AF_5} is characterized by the arguments and the attacks of Δ_{AFN_5}. In particular, the attacks from γ to α^+ and α^- give the possibility of $\mathcal{DG}$ being accepted even though $\mathcal{STD}$ is not accepted. In other words, the attacks from γ to α^+ and α^- in Δ_{AF_5} correspond to the attack from $\mathcal{N}$ to the necessity between $\mathcal{STD}$ and $\mathcal{DG}$ in Δ_5, meaning that Bob would be able to drive safely in Gstaad even though he does not put a snow traction device in his car.

Example 7. Let us consider the ASAF $\Delta_7 = \langle \mathbb{A}_7, \mathbb{R}_7, \mathbb{S}_7 \rangle$, where:

$$\mathbb{A}_7 = \{\mathcal{E}, \mathcal{F}, \mathcal{G}, \mathcal{H}\} \qquad \mathbb{R}_7 = \{\alpha, \gamma\} \qquad \mathbb{S}_7 = \{\beta\}$$

with $\alpha = (\mathcal{E}, \mathcal{F})$, $\beta = (\mathcal{G}, \alpha)$ and $\gamma = (\mathcal{H}, \mathcal{G})$. A graphical representation of Δ_7 is given in Fig. 11.

The AFN associated to Δ_7 is $\Delta_{AFN_7} = \langle \mathbb{A}_{AFN7}, \mathbb{R}_{AFN7}, \mathbb{N}_{AFN7} \rangle$, where:

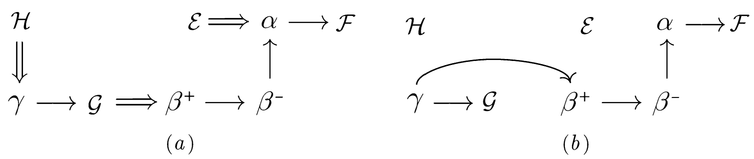

Fig. 12. (a) AFN Δ_{AFN_7} associated to the ASAF Δ_7 and (b) AF Δ_{AF_7} associated to Δ_7, corresponding to Example 7.

$$\mathbb{A}_{AFN7} = \{\mathcal{E}, \mathcal{F}, \mathcal{G}, \mathcal{H}, \alpha, \beta^+, \beta^-, \gamma\}$$

$$\mathbb{R}_{AFN7} = \{(\gamma, \mathcal{G}), (\beta^+, \beta^-), (\beta^-, \alpha), (\alpha, \mathcal{F})\}$$

$$\mathbb{N}_{AFN7} = \{(\mathcal{H}, \gamma), (\mathcal{G}, \beta^+), (\mathcal{E}, \alpha)\}$$

Then, the AF associated to Δ_7 is $\Delta_{AF_7} = \langle \mathbb{A}_{AFN7}, \mathbb{R}_{AFN7}^+ \rangle$, where $\mathbb{R}_{AFN7}^+ = \mathbb{R}_{AFN7} \cup \{(\gamma, \beta^+)\}$. The graphical representation of Δ_{AFN_7} is included in Fig. 12(a), and the graphical representation of Δ_{AF_7} is given in Fig. 12(b).

In this case, the attack α from $\mathcal{E}$ to $\mathcal{F}$ in Δ_7 is supported by $\mathcal{G}$. Thus, since $\mathcal{H}$ attacks $\mathcal{G}$, it affects the support from $\mathcal{G}$ to α. This is captured in Δ_{AFN_7} by an extended attack from γ to β^+ (and thus, by the attack from γ to β^+ in Δ_{AF_7}), which represents the fact that the attack from $\mathcal{H}$ to $\mathcal{G}$ in Δ_7 provides a reason against the positive constraint associated to β.

As mentioned in Section 2.3, the extensions of an AFN coincide with the extensions of its associated AF. On the other hand, by Definition 12, we can obtain the AF associated to an ASAF by considering its associated AFN. Then, we can obtain the extensions of an ASAF in terms of the extensions of its associated AF.

Given an ASAF Δ and its associated AF Δ_{AF}, arguments in Δ_{AF} will correspond to arguments from Δ, or support-arguments and attack-arguments obtained through the transformation from Definition 11. Thus, by Definition 3, extensions of Δ_{AF} will be sets of arguments from Δ_{AF}, and they will possibly contain elements that are neither arguments, attacks nor supports of Δ. Hence, to obtain the extensions of Δ we would need to map the attack/support-arguments belonging to the extensions of Δ_{AF} into their corresponding attacks and supports from Δ. This is formalized by the following definition.

Definition 13 *(Mapping function).* Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\Delta_{AF} = \langle \mathbb{A}_{AF}, \mathbb{R}_{AF} \rangle$ its associated AF. Given $S \subseteq \mathbb{A}_{AF}$, we define a *mapping function* $\mathsf{Map} : 2^{\mathbb{A}_{AF}} \longmapsto 2^{\mathbb{A} \cup \mathbb{R} \cup \mathbb{S}}$, such that $\mathsf{Map}(S) = S_\Delta$ and for every $x \in S$ it holds that:

- If $x \in \mathbb{A}$, then $x \in S_\Delta$.
- If $x = (\mathcal{A}, Y) \in \mathbb{R}$, then $x \in S_\Delta$.
- If $x = \beta^+$ s.t. $\beta = (\mathcal{B}, Z) \in \mathbb{S}$, then $\beta \in S_\Delta$.
- If $x = \beta^-$ s.t. $\beta = (\mathcal{B}, Z) \in \mathbb{S}$, then $\beta \in S_\Delta$.

Given an ASAF Δ and its associated AF Δ_{AF}, the mapping function maps a set of arguments of Δ_{AF} into the corresponding set of arguments, attacks and supports of Δ. In this way, we can obtain the extensions of an ASAF by applying the mapping function to the extensions of its associated AF. As mentioned before, in the following, when referring to an extension E under a semantics s, we will be referring to an extension under the *complete*, *preferred*, *stable*, or *grounded* semantics (see Definition 3).

Definition 14 *(ASAF extensions).* Let Δ be an ASAF and Δ_{AF} its associated AF. If E is an extension of Δ_{AF} under a semantics s, then E_Δ is an extension of Δ under the semantics s, where $\mathsf{Map}(\mathsf{E}) = \mathsf{E}_\Delta$.

Given an extension E_{Δ} of an ASAF Δ, if $X \in \mathsf{E}_{\Delta}$ we say that X is *accepted* w.r.t. E_{Δ}; otherwise, we say that X is *not accepted* w.r.t. E_{Δ}.

Similarly to [4,5], an extension of an ASAF may contain, in addition to arguments of the ASAF, elements of the attack and support relations. The presence of attacks and supports in the extensions of an ASAF will indicate that they are *active*. In particular, given an extension E_{Δ} of the ASAF and a support $\beta = (\mathcal{B}, Y)$ such that $\beta \in \mathsf{E}_{\Delta}$, we can determine which constraint holds (whether the positive or the negative constraint) by looking at the acceptance of argument $\mathcal{B}$: if $\mathcal{B} \in \mathsf{E}_{\Delta}$, then the positive constraint holds; otherwise, the negative constraint holds.

For instance, let us consider the ASAF Δ_5 from Example 5 and its associated AF Δ_{AF_5} from Example 6. Following the preferred semantics, the only preferred extension of Δ_5 is $\mathsf{E}_{\Delta_5} = \{\mathcal{STD}, \mathcal{WG}, \mathcal{N}, \mathcal{DG}, \beta, \gamma\}$. The presence of γ in E_{Δ_5} means that the attack from $\mathcal{N}$ to α is active. Consequently, α does not belong to E_{Δ_5}. On the other hand, the presence of β and $\mathcal{WG}$ in E_{Δ_5} represents that the support from $\mathcal{WG}$ to α is active and, in particular, the positive constraint holds.

Let us now consider the ASAF Δ_7 from Example 7 whose associated AF is Δ_{AF_7}. According to the preferred semantics, the only preferred extension of Δ_7 is $\mathsf{E}_{\Delta_7} = \{\mathcal{E}, \mathcal{F}, \mathcal{H}, \beta, \gamma\}$. In this case, the presence of γ in E_{Δ_7} means that the attack from $\mathcal{H}$ to $\mathcal{G}$ is active and thus, $\mathcal{G}$ is not accepted. As a result, since β belongs to E_{Δ_7} and $\mathcal{G}$ does not, the support is active and, in particular, the negative constraint holds. Note that, in this case, α does not belong to E_{Δ_7} not because of being made ineffective by another attack, but because of being left without the required support. Although this situation is similar to that of indirect defeat in the AFRA [5], it originates from the fact that the negative constraint of the support relation holds. That is, given that the negative constraint of β holds, since $\mathcal{G}$ is not accepted, α cannot be accepted either.

5. ASAF semantical results

This section presents some results regarding the ASAF. In particular, we will show that the transformation from an ASAF into its associated AF is sound, in the sense that the constraints imposed by the attack and support relations of the ASAF are captured within its associated AF. We will also identify some properties of the ASAF's extensions (and their corresponding extensions in the associated AF), which establish the characteristics that the extensions should have in order to be considered *coherent*. Finally, we will show that the ASAF extends both the AFRA and the AFN. On the one hand, we will show that an ASAF with an empty support relation is equivalent to an AFRA. On the other hand, we will show that an ASAF where attacks and supports occur only at the argument level is equivalent to an AFN.

The results in this section hold for complete, preferred, stable and grounded semantics. That is, they are based on complete semantics and extend to other semantics that select some of the complete extensions.[2] Thus, as mentioned before, when referring to a given semantics s, it will be such that $\mathsf{s} \in \{$*complete*, *preferred*, *stable*, *grounded*$\}$. In addition, for the sake of readability, the proofs for all propositions in this section are included in an appendix.

Given an ASAF Δ, we will say that an attack $\alpha = (\mathcal{A}, X)$ is an *active attack* with respect to an extension E of Δ under a semantics s iff α belongs to E. Similarly, we will say that a support $\beta = (\mathcal{B}, Y)$ is an *active support* with respect to E under s iff β belongs to E. If an attack or a support is not active, we will say that it is *inactive*. For instance, given the ASAF Δ_7 from Example 7, γ is an active attack, α is an inactive attack, and β is an active support with respect to the preferred extension $\mathsf{E}_7 = \{\mathcal{E}, \mathcal{F}, \mathcal{H}, \beta, \gamma\}$.

It is important to remark that, given an attack $\alpha = (\mathcal{A}, X)$ in an ASAF Δ, the acceptance of $\mathcal{A}$ affects the activation of α, as shown by the following proposition.

[2] While the results in this section also apply to other semantics based on complete extensions (such as *ideal semantics* [21], where the ideal extension corresponds to the intersection of all preferred extensions), we will focus on the four classical semantics presented in [20] (*i.e.*, *complete*, *preferred*, *stable* and *grounded* semantics).

Proposition 1. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\alpha = (\mathcal{A}, X) \in \mathbb{R}$. If $\mathcal{A}$ does not belong to an extension E_Δ of Δ under a semantics s, then α does not belong to E_Δ either.*

Note that this proposition captures the behavior modeled by the indirect defeat of the AFRA (see Definition 5). In other words, it expresses that the attacks on arguments are propagated to the attacks they originate.

The following proposition shows that, given an ASAF Δ, its associated AF Δ_{AF} satisfies the acceptability constraints imposed by the attack relation of Δ. That is, if there exists an active attack from $\mathcal{A}$ to X w.r.t. an extension of Δ, then X cannot belong to that extension.

Proposition 2. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\alpha = (\mathcal{A}, X) \in \mathbb{R}$. If α belongs to an extension E_Δ of Δ under a semantics s, then X does not belong to E_Δ.*

Similarly to Proposition 2, the following proposition shows that, given an ASAF Δ, its associated AF Δ_{AF} satisfies the acceptability constraints imposed by the support relation of Δ. That is, if there exists an active support from $\mathcal{B}$ to Y w.r.t. an extension of Δ, then it holds that: *i*) if Y is accepted then $\mathcal{B}$ is also accepted and, conversely, *ii*) if $\mathcal{B}$ is not accepted then Y is not accepted either.

Proposition 3. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\beta = (\mathcal{B}, Y) \in \mathbb{S}$. If β belongs to an extension E_Δ of Δ under a semantics s, then it holds that:*

i) *if $Y \in \mathsf{E}_\Delta$, then $\mathcal{B} \in \mathsf{E}_\Delta$; and*
ii) *if $\mathcal{B} \notin \mathsf{E}_\Delta$, then $Y \notin \mathsf{E}_\Delta$.*

The following definition characterizes some features that are desirable for any set of accepted arguments of an ASAF. Recall that, as presented in Section 4.2, the extensions of an ASAF are obtained in terms of the extensions of its associated AF. Thus, the extensions of the ASAF may contain supports or attacks, which result from the presence of support-arguments or attack-arguments in the extensions of its associated AF. In particular, since the constraints imposed by the support relation of the ASAF are mutually exclusive it cannot be the case that, given a support β in the ASAF, an extension of the associated AF contains both support-arguments β^+ and β^-. On the other hand, if neither β^+ nor β^- belong to an extension of the associated AF, then it must be the case that the support β in the ASAF is attacked or being left without the required support. These intuitions characterize desirable features for the extensions of the associated AF (*i.e.*, sets of arguments of the associated AF), and are formalized through the notion of *coherent set*.

Definition 15 *(Coherent set).* Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\Delta_{AF} = \langle \mathbb{A}_{AF}, \mathbb{R}_{AF} \rangle$ its associated AF. A set $S \subseteq \mathbb{A}_{AF}$ is a *coherent set* iff $\forall \beta \in \mathbb{S}$ it holds that:

i) if $\beta^+ \in S$, then $\mathsf{src}(\beta) \in S$, $\beta^- \notin S$, and $\nexists \alpha \in S$ s.t. $(\alpha, \beta^+) \in \mathbb{R}_{AF}$;
ii) if $\beta^- \in S$, then $\beta^+ \notin S$, $\mathsf{src}(\beta) \notin S$, $\mathsf{trg}(\beta) \notin S$, $\exists \alpha \in S$ s.t. $(\alpha, \beta^+) \in \mathbb{R}_{AF}$ and $(\alpha, \mathsf{src}(\beta)) \in \mathbb{R}_{AF}$, and $\nexists \gamma \in S$ s.t. $(\gamma, \beta^-) \in \mathbb{R}_{AF}$; and
iii) if $\beta^+, \beta^- \notin S$, then $\exists \alpha \in \mathbb{A}_{AF}$ s.t. $(\alpha, \beta^+) \in \mathbb{R}_{AF}$, $(\alpha, \beta^-) \in \mathbb{R}_{AF}$ and $\{\alpha\} \cup S$ is conflict-free.

Next we show that, given an ASAF Δ, all extensions of its associated AF Δ_{AF} are coherent sets.

Proposition 4. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\Delta_{AF} = \langle \mathbb{A}_{AF}, \mathbb{R}_{AF} \rangle$ its associated AF. If E_Δ is an extension of Δ under a semantics s, then E is a coherent set, where $\mathsf{E}_\Delta = \mathsf{Map}(\mathsf{E})$.*

The following proposition shows that the ASAF extends the formalism of the AFRA. That is, considering an ASAF with an empty support relation is equivalent to considering an AFRA that has the same arguments and attack relation as the ASAF.

Proposition 5. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \emptyset \rangle$ be an ASAF and $\Gamma = \langle \mathbb{A}, \mathbb{R} \rangle$ an AFRA. E_Δ is an extension of Δ under a semantics s iff E_Δ is an extension of Γ under the semantics s.*

Similarly to Proposition 5, the following proposition shows that the ASAF extends the formalism of the AFN. That is, an ASAF with no recursion (where attacks and supports occur only at the argument level) is equivalent to an AFN.

Proposition 6. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF s.t. $\forall (\mathcal{A}, \mathcal{B}) \in \mathbb{R} \cup \mathbb{S} : \mathcal{A}, \mathcal{B} \in \mathbb{A}$, and let $\Phi = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an AFN. It holds that:*

i) *if E_Δ is an extension of Δ under a semantics s, then there exists an extension E_Φ of Φ under the semantics s s.t. $\mathsf{E}_\Phi = \{\mathcal{A} \in \mathsf{E}_\Delta | \ \mathcal{A} \in \mathbb{A}\}$; and*
ii) *if E_Φ is an extension of Φ under a semantics s, then there exists an extension E_Δ of Δ under the semantics s s.t. $\mathsf{E}_\Phi = \{\mathcal{A} \in \mathsf{E}_\Delta | \ \mathcal{A} \in \mathbb{A}\}$.*

6. Related work and discussion

The approach introduced in this paper draws on the work by [4,5] and [25,26,12]. Specifically, the Attack–Support Argumentation Framework (ASAF) proposed here extends the Argumentation Framework with Recursive Attack (AFRA) by incorporating a support relation, as well as allowing for attacks on the support relation. Similarly, the ASAF extends the Argumentation Framework with Necessities (AFN) by allowing for recursive support and support for attacks, as well as incorporating the above mentioned features into the attack relation.

One of the first approaches in the literature that simultaneously accounts for the representation of support and attack relations is DefLog [31]. Briefly, its logical language has two connectives $\times$ and $\rightsquigarrow$. The dialectical negation $\times S$ of a statement S expresses that S is defeated. The primitive implication $\rightsquigarrow$ is a binary connective used to express that one statement supports another and validates *modus ponens*. In DefLog is possible to combine and nest the connectives $\times$ and $\rightsquigarrow$ to obtain more complex statements, allowing to represent attacks to supports or attacks, as well as support to supports or attacks. However, DefLog is a sentence-based approach, whereas the ASAF proposed in this paper is an argument-based theory. In addition, the semantics of DefLog's primitive implication differs from the semantics of the support relations used in abstract argumentation approaches. In particular, the support relation of the ASAF encodes a necessity between the elements it relates, whereas the primitive implication in DefLog is used to obtain new sentences through the use of *modus ponens*.

Starting from [1,16], the study of the notion of support in abstract argumentation frameworks regained attention among the researchers. Several formalizations were proposed in the literature, where different interpretations of support such as evidential [27], deductive [10,32], necessity [25,26,12] and backing [17] were considered. In particular, the support relation of the ASAF proposed in this work follows the necessity interpretation of the Argumentation Frameworks with Necessities. It is important to note that, as shown in [19], although the notions of necessity and sub-argument are related (in the sense that sub-arguments are necessary for their super-arguments), they correspond to different concepts.

The study of attacks to attacks in abstract argumentation frameworks was addressed by several approaches such as [24], [4,5] and [10,32]. However, the work by [4,5] was the only to consider a recursive attack relation that enables attacks to attacks at any level. On the other hand, the meta-argumentation approach of [10,32] also accounted for a support relation between arguments, allowing for the representation of attacks to the support relation. However, this interaction was fixed, thus not being able to combine and nest the attack and support relations at any level.

In argumentation theory it is usually assumed that the arguments always hold since argumentation frameworks represent a snapshot of the arguments and relations involved on the reasoning process. However, alternative approaches like [27] consider that arguments should be backed up by evidence. Evidential reasoning involves determining which arguments are applicable based on some evidence. In that way, the Evidential Argumentation System (EAS) proposed in [27] intends to capture a particular notion: an argument cannot be accepted unless it is supported by evidence.

An argument in an EAS will be accepted if it is defended against attacks coming from arguments supported by evidence. However, there is an additional requirement that accepted arguments must satisfy: they should be backed up by evidence. That is, they must be supported through a chain of arguments, each of them itself supported; at the beginning of this chain of arguments there is a special argument η that represents support from the environment (*i.e.*, the existence of supporting evidence). There is a clear difference between the approach by [27] and other approaches to support in abstract argumentation. Given an Evidential Argumentation System with no attacks among arguments, it can be the case that some arguments are not accepted. Specifically, arguments that are not backed up by evidence will not be justified in the EAS.

Given the characteristics of the support relation in an EAS, if we wanted to represent an EAS by using an ASAF we would have to find a way to express the notion of "support by evidence" using the necessity support relation. On the one hand, we would need to incorporate the special argument η denoting support from the environment into the ASAF. On the other hand, as mentioned before, arguments that are not supported by evidence (*i.e.*, supported by the special argument η) in the EAS should not be accepted. Thus, one could think of including, for every argument $\mathcal{A}$ supported by η in the EAS, the pair $(\eta, \mathcal{A})$ in the support relation $\mathbb{S}$ of the ASAF. However, this does not prevent arguments not being supported by η from belonging to the extensions of the ASAF. This is because, as formalized in Section 4.2, arguments in the ASAF can be accepted even though they have no supporting arguments.

In [14] the authors first introduced the Abstract Dialectical Frameworks (ADFs). Briefly, an ADF is a directed graph, whose nodes represent arguments which can be accepted or not, and the links between the nodes represent dependencies. Each argument $\mathcal{X}$ in the graph is associated with an acceptance condition $C(\mathcal{X})$ which is some propositional function whose truth status is determined by the corresponding values of the acceptance conditions for those arguments $\mathcal{Y}$ such that $(\mathcal{Y}, \mathcal{X})$ is link in the ADF (*i.e.*, $\mathcal{Y}$ is a parent of $\mathcal{X}$).

Regarding the notion of support, the authors in [14] state that the acceptance conditions in the ADFs are more flexible than the constraints described by specific interpretations of support such as deductive, necessary, or evidential support. However, as pointed out by the authors in [10], ADFs might not be able to capture the acceptability constraints imposed by specific interpretations of support, such as deductive support. A similar situation would occur with the necessity interpretation of support used in this paper. For instance, if $\mathcal{B}$ supports $\mathcal{A}$ (*i.e.*, $\mathcal{B}$ is necessary for $\mathcal{A}$) and $\mathcal{C}$ attacks $\mathcal{B}$, then there exists an extended attack from $\mathcal{C}$ to $\mathcal{A}$. When defining an ADF as in [14], it is necessary to specify explicitly the links between nodes. Thus, an ADF that considers the arguments mentioned above, as well as the attack and support relations between them, would have the nodes $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$, and the links $(\mathcal{B}, \mathcal{A})$ (support) and $(\mathcal{C}, \mathcal{B})$ (attack). Given this situation, no acceptance condition in the ADF will be capable of ensuring that $\mathcal{A}$ is not accepted whenever $\mathcal{C}$ is accepted (as expressed by the extended attack from $\mathcal{C}$ to $\mathcal{A}$), since there would be no link between $\mathcal{C}$ and $\mathcal{A}$ in the ADF.

In [13] the authors propose a revisited approach to ADFs in which they tacitly assume that the acceptance conditions specify the parents a node depends on implicitly. Then, they do not require to give the links in the graph explicitly and thus, an ADF can be simply specified by determining the nodes and their associated acceptance conditions. Hence, the links between nodes are inferred from the corresponding acceptance conditions. By applying the revisited approach to the situation depicted above, the ADF would be able to model that $\mathcal{A}$ is not accepted whenever $\mathcal{C}$ is accepted (as expressed by the extended attack from $\mathcal{C}$ to $\mathcal{A}$) by formulating the acceptance condition accordingly. However, this would mean that there is a link between $\mathcal{A}$ and $\mathcal{C}$ in the ADF, corresponding to a different situation from the one in which the only links between nodes were $(\mathcal{B}, \mathcal{A})$ (support) and $(\mathcal{C}, \mathcal{B})$ (attack).

7. Conclusions and future work

In this work we have introduced the Attack–Support Argumentation Framework (ASAF), which provides a unified setting for representing attack and support for arguments, as well as attack and support for the attack and support relations at any level. The ASAF extends the Argumentation Framework with Recursive Attack (AFRA) [4,5] in two ways: it incorporates a support relation enabling to express support for arguments, for attacks, and for the support relation itself; and it extends the attack relation by allowing attacks to the support relation. Moreover, the support relation of the ASAF has a necessity interpretation and thus, the ASAF also extends the Argumentation Framework with Necessities (AFN) [25,26,12]: it allows for recursive attack and support, as well as attack (respectively, support) on the support (respectively, attack) relation.

Section 4 shows how to obtain the extensions of an ASAF by translating the ASAF into its associated Abstract Argumentation Framework (AF) [20]. To achieve this, the ASAF is first translated into an AFN, and then this associated AFN is translated into the corresponding AF. In particular, the transformation takes advantage from the fact that both ASAF and AFN make use of a support relation with a necessity interpretation. Then, in Section 5, it was shown that the transformation from an ASAF into its associated AF is sound, in the sense that the AF satisfies the acceptability constraints imposed by the attack and support relations of the ASAF. Also, it was proven that the extensions of the associated AF satisfy some desired features for any set of accepted arguments; namely, that they are coherent sets. Finally, it was shown that the ASAF extends both the AFRA and the AFN. On the one hand, it was shown that an ASAF with an empty support relation is equivalent to an AFRA. On the other hand, it was shown that an ASAF where attack and support occurs only at the argument level is equivalent to an AFN.

From the existing works in the literature that address attack and support in the context of abstract argumentation frameworks, we can note that none of the existing approaches so far have addressed the possibility of allowing recursive support at any level. Moreover, they have neither considered all the alternatives for combining and nesting the attack and support relations, as considered in the ASAF. As mentioned before, the meta-argumentation approach of [10,32] only allows for attacks to the support relation at one level. Therefore, we believe that the ASAF constitutes a novel approach to representing recursive attack and support in abstract argumentation.

It is worth noticing that, although in this work we propose an approach for obtaining the extensions of the ASAF in terms of the extensions of its associated AF, as future work we aim to provide a full characterization of the semantics at the level of the ASAF. Then, given such a characterization, we will study how the different approaches for computing the acceptability of arguments in the ASAF relate to one another. In particular, it is to expect that these approaches are equivalent in the sense that they lead to obtaining the same sets of extensions.

If we were to define the semantics at the level of the ASAF, we would need to characterize the notion of acceptability, which accounts for the different kinds of defeat that can occur within the framework. On the one hand, similarly to the AFRA, we would need to consider *indirect defeats*, which propagate attacks on arguments to the attacks they originate. On the other hand, given the necessity interpretation of the support relation in the ASAF, similarly to the AFN we would need to consider the *extended defeat*. Moreover, since the attack and support relations of the ASAF are such that it is possible to provide support for attacks (and vice-versa), the coexistence of indirect defeat and extended defeat would lead to considering additional kinds of defeat. As a result, the characterization of the semantic notions at the level of the ASAF would require to consider a wide range of cases. This would directly affect the study of the relationship between the different approaches for obtaining the extensions of the ASAF, making it more complex than the one performed in [5] for the AFRA.

As mentioned before, the results in this work concern the complete, preferred, stable and grounded semantics. However, as future work we aim to extend these results to other semantics, such as *semi-stable* [15] or *ideal* [21]. Finally, as another topic for future work, we are interested in studying the characterization of recursive attack and support relations within the context of structured argumentation systems.

Appendix A

For proving the following propositions we will use some existing results for Dung's abstract argumentation frameworks [20]. It was shown in [20] that any preferred, stable or grounded extension of an abstract argumentation framework AF is also a complete extension of AF. Then, it was shown that every complete extension E of AF coincides with a fixed point of the characteristic function F_{AF}[3] (*i.e.*, $F_{AF}(\mathsf{E}) = \mathsf{E}$). Therefore, given an abstract argumentation framework AF and an extension E of AF under a semantics s,[4] if $X \notin \mathsf{E}$, then X is not acceptable w.r.t. E.

Proposition 1. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\alpha = (\mathcal{A}, X) \in \mathbb{R}$. If $\mathcal{A}$ does not belong to an extension E_Δ of Δ under a semantics s, then α does not belong to E_Δ either.*

Proof. By Definition 11, there exists an AFN Δ_{AFN} associated to Δ. By Definition 9, there exists an AF Δ_{AF} associated to Δ_{AFN}. Then, by Definition 12, Δ_{AF} is the AF associated to Δ. Since $\mathcal{A} \notin \mathsf{E}_\Delta$, by Definitions 13 and 14 $\mathcal{A} \notin \mathsf{E}$, where E is an extension of Δ_{AF} under s such that $\mathsf{Map}(\mathsf{E}) = \mathsf{E}_\Delta$. Then, there exists an argument Y in Δ_{AF} with an attack from Y to $\mathcal{A}$ in Δ_{AF} and there is no $Z \in \mathsf{E}$ such that Z attacks Y in Δ_{AF} (*i.e.*, $\mathcal{A}$ is not acceptable w.r.t. E). Then, since Y attacks $\mathcal{A}$ in Δ_{AF}, by Definition 9, there exists a direct or extended attack from Y to $\mathcal{A}$ in Δ_{AFN}. By Definition 11, the attack $\alpha = (\mathcal{A}, X)$ in Δ is represented by a sequence $\mathcal{A} \Longrightarrow \alpha \longrightarrow X$ in Δ_{AFN}. Therefore, by Definition 8, there will be an extended attack from Y to α in Δ_{AFN}. As a result, by Definition 9, there will be an attack from Y to α in Δ_{AF}. If E does not defend $\mathcal{A}$ against Y, then it cannot defend α from Y either; hence, α is not acceptable w.r.t. E and thus, it does not belong to E. Finally, by Definitions 13 and 14, α does not belong to E_Δ. □

Proposition 2. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\alpha = (\mathcal{A}, X) \in \mathbb{R}$. If α belongs to an extension E_Δ of Δ under a semantics s, then X does not belong to E_Δ.*

Proof. By Definition 11, there exists an AFN Δ_{AFN} associated to Δ. By Definition 9, there exists an AF Δ_{AF} associated to Δ_{AFN}. Then, by Definition 12, Δ_{AF} is the AF associated to Δ. Since $\alpha \in \mathsf{E}_\Delta$, by Definitions 13 and 14 $\alpha \in \mathsf{E}$, where E is an extension of Δ_{AF} under s such that $\mathsf{Map}(\mathsf{E}) = \mathsf{E}_\Delta$. Now we have two cases: *a*) $X \in \mathbb{A} \cup \mathbb{R}$ or *b*) $X \in \mathbb{S}$.

a) If $X \in \mathbb{A} \cup \mathbb{R}$, by Definition 11, the attack $\alpha = (\mathcal{A}, X)$ is translated as a sequence $\mathcal{A} \Longrightarrow \alpha \longrightarrow X$ in Δ_{AFN}. Then, by Definition 9 there is also an attack $\alpha \longrightarrow X$ in Δ_{AF}. Since by hypothesis $\alpha \in \mathsf{E}$, all arguments attacked by α in Δ_{AF} do not belong to E; in particular, $X \notin \mathsf{E}$. Then, by Definitions 13 and 14, $X \notin \mathsf{E}_\Delta$.

[3] Briefly, given an abstract argumentation framework $AF = \langle \mathbb{A}, \mathbb{R} \rangle$ and a set of arguments $S \subseteq \mathbb{A}$, the characteristic function $F_{AF} : 2^{\mathbb{A}} \mapsto 2^{\mathbb{A}}$ is defined as $F_{AF}(S) = \{\mathcal{A} \mid \mathcal{A} \text{ is acceptable w.r.t. } S\}$.

[4] As mentioned before, when we refer to any semantics s in this paper it means $\mathsf{s} \in \{complete, preferred, stable, grounded\}$.

b) If $X \in \mathbb{S}$, by Definition 11, the attack $\alpha = (\mathcal{A}, X)$ is translated as $\mathcal{A} \Longrightarrow \alpha \longrightarrow X^+$ and $\alpha \longrightarrow X^-$ in Δ_{AFN}. Then, by Definition 9, there are also attacks $\alpha \longrightarrow X^+$ and $\alpha \longrightarrow X^-$ in Δ_{AF}. Since by hypothesis $\alpha \in \mathsf{E}$, all arguments attacked by α do not belong to E; in particular, $X^+ \notin \mathsf{E}$ and $X^- \notin \mathsf{E}$. Then, by Definitions 13 and 14, $X \notin \mathsf{E}_\Delta$. □

Proposition 3. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\beta = (\mathcal{B}, Y) \in \mathbb{S}$. If β belongs to an extension E_Δ of Δ under a semantics s, then it holds that:*

i) *if $Y \in \mathsf{E}_\Delta$, then $\mathcal{B} \in \mathsf{E}_\Delta$; and*
ii) *if $\mathcal{B} \notin \mathsf{E}_\Delta$, then $Y \notin \mathsf{E}_\Delta$.*

Proof. Since *i*) and *ii*) are logically equivalent, it suffices to prove one of them.

ii) By Definition 11, there exists an AFN Δ_{AFN} associated to Δ. By Definition 9, there exists an AF Δ_{AF} associated to Δ_{AFN}. Then, by Definition 12, Δ_{AF} is the AF associated to Δ. Since $\beta \in \mathsf{E}_\Delta$, by Definitions 13 and 14 either $\beta^+ \in \mathsf{E}$ or $\beta^- \in \mathsf{E}$, where E is an extension of Δ_{AF} under s such that $\mathsf{Map}(\mathsf{E}) = \mathsf{E}_\Delta$. It cannot be the case that both $\beta^+ \in \mathsf{E}$ and $\beta^- \in \mathsf{E}$ because by Definitions 11 and 12, there is an attack $\beta^+ \longrightarrow \beta^-$ in Δ_{AF}. If $\mathcal{B} \notin \mathsf{E}_\Delta$, by Definitions 13 and 14 $\mathcal{B} \notin \mathsf{E}$. Then, there exists an attack from X to $\mathcal{B}$ in Δ_{AF}, and there is no $Z \in \mathsf{E}$ such that Z attacks X in Δ_{AF} (*i.e.*, $\mathcal{B}$ is not acceptable w.r.t. E). Since X attacks $\mathcal{B}$ in Δ_{AF}, by Definition 9, there exists a direct or extended attack from X to $\mathcal{B}$ in Δ_{AFN}. Now we have two cases: *a*) $Y \in \mathbb{A} \cup \mathbb{R}$ or *b*) $Y \in \mathbb{S}$.

a) By Definition 11, the support $\beta = (\mathcal{B}, Y)$ in Δ is represented as $\mathcal{B} \Longrightarrow \beta^+ \longrightarrow \beta^- \longrightarrow Y$ in Δ_{AFN}. Thus, by Definition 8 there is an extended attack from X to β^+ in Δ_{AFN}. By Definition 9, there are also attacks $X \longrightarrow \beta^+$ and $\beta^- \longrightarrow Y$ in Δ_{AF}. If E does not defend $\mathcal{B}$ against X, then it cannot defend β^+ from X either. Hence, β^+ is not acceptable w.r.t. E and thus $\beta^+ \notin \mathsf{E}$. Therefore, it must be the case that $\beta^- \in \mathsf{E}$. As a result, every argument attacked by β^- in Δ_{AF} will not belong to E; in particular, $Y \notin \mathsf{E}$. Finally, by Definitions 13 and 14, $Y \notin \mathsf{E}_\Delta$.

b) By Definition 11, the support $\beta = (\mathcal{B}, Y)$ in Δ is represented as $\mathcal{B} \Longrightarrow \beta^+ \longrightarrow \beta^- \longrightarrow Y^+$ and $\beta^- \longrightarrow Y^-$ in Δ_{AFN}. Thus, by Definition 8 there is an extended attack from X to β^+ in Δ_{AFN}. By Definition 9, there are also attacks $X \longrightarrow \beta^+$, $\beta^- \longrightarrow Y^+$ and $\beta^- \longrightarrow Y^-$ in Δ_{AF}. If E does not defend $\mathcal{B}$ against X, then it cannot defend β^+ from X either. Hence, β^+ is not acceptable w.r.t. E and thus $\beta^+ \notin \mathsf{E}$. Consequently, it must be the case that $\beta^- \in \mathsf{E}$. Finally, neither Y^+ nor Y^- will belong to E since they are attacked by β^- in Δ_{AF}; hence, by Definitions 13 and 14 $Y \notin \mathsf{E}_\Delta$. □

Proposition 4. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\Delta_{AF} = \langle \mathbb{A}_{AF}, \mathbb{R}_{AF} \rangle$ its associated AF. If E_Δ is an extension of Δ under a semantics s, then E is a coherent set, where $\mathsf{E}_\Delta = \mathsf{Map}(\mathsf{E})$.*

Proof. If E_Δ is an extension of Δ under the semantics s, then, by Definition 14, E is an extension of Δ_{AF} under the semantics s such that $\mathsf{Map}(\mathsf{E}) = \mathsf{E}_\Delta$. If E is a coherent set, then it must satisfy the conditions *i*), *ii*) and *iii*) in Definition 15:

i) Suppose $\beta^+ \in \mathsf{E}$. Since, by Definition 3, E is conflict-free $\nexists \alpha \in \mathsf{E}$ s.t. $(\alpha, \beta^+) \in \mathbb{R}_{AF}$. Similarly, since by Definition 12 $(\beta^+, \beta^-) \in \mathbb{R}_{AF}$, $\beta^- \notin \mathsf{E}$. Suppose by contradiction that $\mathsf{src}(\beta) \notin \mathsf{E}$. Then, $\mathsf{src}(\beta)$ would not be acceptable w.r.t. E and there would exist $\gamma \in \mathbb{A}_{AF}$ s.t. $(\gamma, \mathsf{src}(\beta)) \in \mathbb{R}_{AF}$ and E does not defend $\mathsf{src}(\beta)$ against γ. By Definition 12 for every attack $(\gamma, \mathsf{src}(\beta)) \in \mathbb{R}_{AF}$ there is also an attack $(\gamma, \beta^+) \in \mathbb{R}_{AF}$. Hence, if E does not defend $\mathsf{src}(\beta)$ against γ, it cannot defend β^+ against γ either, which contradicts the hypothesis that $\beta^+ \in \mathsf{E}$. As a result, it must be the case that $\mathsf{src}(\beta) \in \mathsf{E}$.

ii) Suppose $\beta^- \in \mathsf{E}$. By Definition 3, β^- is acceptable w.r.t. E and thus, $\forall\delta \in \mathbb{A}_{AF}$ s.t. $(\delta, \beta^-) \in \mathbb{R}_{AF}$, $\exists\alpha \in \mathsf{E}$ s.t. $(\alpha, \delta) \in \mathbb{R}_{AF}$. In particular, $(\alpha, \beta^+) \in \mathbb{R}_{AF}$ since, by Definition 12, $(\beta^+, \beta^-) \in \mathbb{R}_{AF}$. By Definition 12 all attacks on β^+ are also attacks on $\mathsf{src}(\beta)$ or attacks on β^-. Since $\beta^-, \alpha \in \mathsf{E}$ and E is conflict-free, α must be an attacker of the first kind and thus $(\alpha, \mathsf{src}(\beta)) \in \mathbb{R}_{AF}$; consequently, $\beta^+ \notin \mathsf{E}$ and $\mathsf{src}(\beta) \notin \mathsf{E}$. Moreover, since E is conflict-free and, by hypothesis, $\beta^- \in \mathsf{E}$, $\nexists\gamma \in \mathsf{E}$ s.t. $(\gamma, \beta^-) \in \mathbb{R}_{AF}$ or $(\beta^-, \gamma) \in \mathbb{R}_{AF}$. As a result, since by Definitions 11 and 9 $(\beta^-, \mathsf{trg}(\beta)) \in \mathbb{R}_{AF}$, it must be the case that $\mathsf{trg}(\beta) \notin \mathsf{E}$.

iii) Suppose $\beta^+, \beta^- \notin \mathsf{E}$. Then, β^+ and β^- will not be acceptable w.r.t. E and thus, $\exists\alpha, \gamma \in \mathbb{A}_{AF}$ s.t. $(\alpha, \beta^-) \in \mathbb{R}_{AF}$, $(\gamma, \beta^+) \in \mathbb{R}$ and $\nexists\pi, \lambda \in \mathsf{E}$ s.t. $(\pi, \gamma) \in \mathbb{R}_{AF}$ and $(\lambda, \alpha) \in \mathbb{R}$. By Definition 12 all attacks on β^- are attacks from β^+ or attacks that are also attacks on β^+. Then, since by hypothesis $\beta^+ \notin \mathsf{E}$, it must be the case that α is an attacker of the second kind. Therefore, α is s.t. $(\alpha, \beta^-) \in \mathbb{R}_{AF}$ and $(\alpha, \beta^+) \in \mathbb{R}_{AF}$. Moreover, $\nexists\delta \in \mathsf{E}$ s.t. $(\alpha, \delta) \in \mathbb{R}_{AF}$ because otherwise δ would not be acceptable w.r.t. E, contradicting the hypothesis that E is an extension of Δ_{AF} under the semantics s. As a result, $\{\alpha\} \cup \mathsf{E}$ is conflict-free. □

Proposition 5. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \emptyset \rangle$ be an ASAF and $\Gamma = \langle \mathbb{A}, \mathbb{R} \rangle$ an AFRA. E_Δ is an extension of Δ under a semantics s iff E_Δ is an extension of Γ under the semantics s.*

Proof. By Definition 11, there exists an AFN Δ_{AFN} associated to Δ. Then, by Definition 9, there exists an AF Δ_{AF} associated to Δ_{AFN} which, by Definition 12, is the AF associated to Δ. On the other hand, by Definition 6 there exists an AF $\widetilde{\Gamma}$ associated to Γ. By [5, Proposition 12], the extensions of Γ under a semantics s coincide with the extensions of its associated AF $\widetilde{\Gamma}$ under the semantics s. Thus, we prove the equivalence by showing that Δ_{AF} and $\widetilde{\Gamma}$ coincide. To achieve this, we perform a case-by-case analysis of the transformations from Γ and Δ into their associated AFs.

Given $\Delta_{AF} = \langle \mathbb{A}_{AF}, \mathbb{R}_{AF} \rangle$ and $\widetilde{\Gamma} = \langle \widetilde{\mathbb{A}}, \widetilde{\mathbb{R}} \rangle$, let us now consider the following cases:

- Let $\mathcal{A} \in \mathbb{A}$:
  Δ_{AF}: By Definitions 11 and 9, Δ_{AF} is such that $\mathcal{A} \in \mathbb{A}_{AF}$.
  $\widetilde{\Gamma}$: By Definition 6, $\widetilde{\Gamma}$ is such that $\mathcal{A} \in \widetilde{\mathbb{A}}$.
- Let $\alpha = (\mathcal{A}, X) \in \mathbb{R}$:
  Δ_{AF}: Since the support relation of Δ is empty, then we have two cases: *a*) $X \in \mathbb{A}$ or *b*) $X \in \mathbb{R}$.
  - *a*) If $X \in \mathbb{A}$, by Definition 11 the attack $\alpha = (\mathcal{A}, X)$ is represented by $\mathcal{A} \Longrightarrow \alpha \longrightarrow X$ in Δ_{AFN}. Then, by Definition 9, Δ_{AF} is such that $\alpha \in \mathbb{A}_{AF}$ and $(\alpha, X) \in \mathbb{R}_{AF}$.
    In addition, if X is such that $\exists\beta = (X, Y) \in \mathbb{R}$, by Definition 11 the attack β is represented by $X \Longrightarrow \beta \longrightarrow Y$ in Δ_{AFN}. Then, by Definition 8, there will be an extended attack from α to β in Δ_{AFN}. Therefore, by Definition 9, Δ_{AF} will be such that $(\alpha, \beta) \in \mathbb{R}_{AF}$.
  - *b*) If $X \in \mathbb{R}$, by Definition 11 the attack $\alpha = (\mathcal{A}, X)$ is represented by $\mathcal{A} \Longrightarrow \alpha \longrightarrow X$ in Δ_{AFN}. Then, by Definition 9, Δ_{AF} is such that $\alpha \in \mathbb{A}_{AF}$ and $(\alpha, X) \in \mathbb{R}_{AF}$.

  $\widetilde{\Gamma}$: We have two cases: *a*) $X \in \mathbb{A}$ or *b*) $X \in \mathbb{R}$.
  - *a*) If $X \in \mathbb{A}$, by Definition 5, α directly defeats X. Then, by Definition 6, $\widetilde{\Gamma}$ is such that $\alpha \in \widetilde{\mathbb{A}}$ and $(\alpha, X) \in \widetilde{\mathbb{R}}$.
    In addition, if X is such that $\exists\beta = (X, Y) \in \mathbb{R}$, by Definition 5, α indirectly defeats β. Therefore, by Definition 6, $\widetilde{\Gamma}$ is such that $(\alpha, \beta) \in \widetilde{\mathbb{R}}$.
  - *b*) If $X \in \mathbb{R}$, by Definition 5, α directly defeats X. Then, by Definition 6, $\widetilde{\Gamma}$ is such that $\alpha \in \widetilde{\mathbb{A}}$ and $(\alpha, X) \in \widetilde{\mathbb{R}}$.

When considering the translation of arguments and attacks of Δ and Γ, the same arguments and attacks are added into their associated AFs Δ_{AF} and $\widetilde{\Gamma}$. As a result, Δ_{AF} and $\widetilde{\Gamma}$ coincide and thus, they lead to the same extensions under a semantics s. □

Proposition 6. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF s.t. $\forall (\mathcal{A}, \mathcal{B}) \in \mathbb{R} \cup \mathbb{S}$: $\mathcal{A}, \mathcal{B} \in \mathbb{A}$, and let $\Phi = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an AFN. It holds that:*

i) if E_Δ is an extension of Δ under a semantics s, then there exists an extension E_Φ of Φ under the semantics s s.t. $\mathsf{E}_\Phi = \{\mathcal{A} \in \mathsf{E}_\Delta | \ \mathcal{A} \in \mathbb{A}\}$; and
ii) if E_Φ is an extension of Φ under a semantics s, then there exists an extension E_Δ of Δ under the semantics s s.t. $\mathsf{E}_\Phi = \{\mathcal{A} \in \mathsf{E}_\Delta | \ \mathcal{A} \in \mathbb{A}\}$.

Proof. Let $\Delta_{AF} = \langle \mathbb{A}_{AF}, \mathbb{R}_{AF} \rangle$ be the AF associated to Δ and $\Phi_{AF} = \langle \mathbb{A}_\Phi, \mathbb{R}_\Phi \rangle$ the AF associated to Φ. We prove the proposition by showing that the attack relations of Δ_{AF} and Φ_{AF} are equivalent in the sense that they lead to the same acceptability constraints on the arguments from Δ and Φ.

Let us first consider the mapping from arguments Δ and Φ into arguments in Δ_{AF} and Φ_{AF}:

- Let $\mathcal{A} \in \mathbb{A}$:
  Φ_{AF}: By Definition 9, Φ_{AF} is s.t. $\mathcal{A} \in \mathbb{A}_\Phi$.
  Δ_{AF}: By Definitions 11 and 9, Δ_{AF} is s.t. $\mathcal{A} \in \mathbb{A}_{AF}$.

Let us now consider the following cases, which correspond to situations that lead to attacks in Φ_{AF}:

- Let $\alpha = (\mathcal{A}, \mathcal{B}) \in \mathbb{R}$:
  Φ_{AF}: By Definition 9, Φ_{AF} is s.t. $\alpha = (\mathcal{A}, \mathcal{B}) \in \mathbb{R}_\Phi$. Since by Definition 3 E_Φ is conflict-free, if $\mathcal{A} \in \mathsf{E}_\Phi$, then $\mathcal{B} \notin \mathsf{E}_\Phi$.
  Δ_{AF}: By Definition 11, the attack $\alpha = (\mathcal{A}, \mathcal{B})$ is represented by $\mathcal{A} \Longrightarrow \alpha \longrightarrow \mathcal{B}$ in Δ_{AFN} (*i.e.*, the AFN associated to Δ). Then, by Definition 9, Δ_{AF} is s.t. $\alpha \in \mathbb{A}_{AF}$ and $(\alpha, \mathcal{B}) \in \mathbb{R}_{AF}$. Since by Definition 3 E_Δ is conflict-free, if $\alpha \in \mathsf{E}_\Delta$, then $\mathcal{B} \notin \mathsf{E}_\Delta$. By Propositions 2 and 1, if $\alpha \in \mathsf{E}_\Delta$, then $\mathcal{A} \in \mathsf{E}_\Delta$. As a result, since all attacks and supports in Δ occur at the argument level, if $\mathcal{A} \in \mathsf{E}_\Delta$, then $\mathcal{B} \notin \mathsf{E}_\Delta$.
- Let $\alpha = (\mathcal{A}, \mathcal{B}) \in \mathbb{R}$ and $\beta = (\mathcal{B}, \mathcal{C}) \in \mathbb{S}$:
  Φ_{AF}: By Definition 8, there is an extended attack from $\mathcal{A}$ to $\mathcal{C}$ in Φ. Then, by Definition 9, Φ_{AF} is s.t. $(\mathcal{A}, \mathcal{C}) \in \mathbb{R}_\Phi$. As a result, since by Definition 3 E_Φ is conflict-free, if $\mathcal{A} \in \mathsf{E}_\Phi$, then $\mathcal{C} \notin \mathsf{E}_\Phi$.
  Δ_{AF}: By Definition 11, the support $\beta = (\mathcal{B}, \mathcal{C})$ is represented by $\mathcal{B} \Longrightarrow \beta^+ \longrightarrow \beta^- \longrightarrow \mathcal{C}$ in Δ_{AFN} (*i.e.*, the AFN associated to Δ). Then, by Definition 9, Δ_{AF} is s.t. $\beta^+, \beta^- \in \mathbb{A}_{AF}$, $(\beta^+, \beta^-) \in \mathbb{R}_{AF}$ and $(\beta^-, \mathcal{C}) \in \mathbb{R}_{AF}$. Since by Definition 11 $\alpha \longrightarrow \mathcal{B}$ in Δ_{AFN}, then by Definition 8 there exists an extended attack from α to β^+ in Δ_{AFN}. Therefore, by Definition 9, Δ_{AF} is s.t. $(\alpha, \beta^+) \in \mathbb{R}_{AF}$. As a result, given that α reinstates β^- in Δ_{AF}, which in turn attacks $\mathcal{C}$, if $\alpha \in \mathsf{E}_\Delta$, then $\mathcal{C} \notin \mathsf{E}_\Delta$ (since E_Δ is an extension of Δ under a semantics $\mathsf{s} \in \{$*complete*, *preferred*, *stable*, *grounded*$\}$). Moreover, since all attacks and supports in Δ occur at the argument level, by Propositions 2 and 1, if $\mathcal{A} \in \mathsf{E}_\Delta$, then $\mathcal{C} \notin \mathsf{E}_\Delta$.

As a result, every argument $\mathcal{A} \in \mathbb{A}$ belonging to an extension E_Δ of Δ under the semantics s will also belong to an extension E_Φ of Φ under the same semantics and vice-versa; thus, proving *i*) and *ii*). □

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