Optics Letters

Extreme events and crises observed in an all-solid-state laser with modulation of losses

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Received 26 April 2016; revised 27 May 2016; accepted 30 May 2016; posted 9 June 2016 (Doc. ID 264013); published 24 June 2016

We report observations of extreme events (or dissipative optical rogue waves) in a laser with a modulated parameter (cavity losses). Experimental data supporting the hypothesis that these events are related with multi-stability and external crises is presented. It is also shown that the time separation between a pulse and an extreme event can be predicted more accurately than that between pulses of average intensity, in agreement with the theoretical description and opening the road to the prediction and control of extreme optical events.

OCIS codes: (140.1540) Chaos; (190.3100) Instabilities and chaos.

http://dx.doi.org/10.1364/OL.41.003010

In recent years several optical systems have been used as test benches to study rogue or rare events [1–6]. Such extreme events (EE) have been also called "optical rogue waves" or "dissipative rogue waves" for their high amplitude, much higher than average. Furthermore, it is recognized that optical rogue waves may be the consequence of different dynamics. They can be relatively frequent in linear systems, induced by random boundary conditions and the possibility of multiple interference [7]. They have been predicted and observed also in nonlinear dynamical systems, for example, after a modulational instability [8–11]. In other words, EE appear in complex systems, and most of them can be studied only through statistical methods and described by models based on partial differential equations. These models can be 1D plus time [12,13] or full 2D plus time [14].

Nevertheless, some systems predicted to display EE can be described by ordinary differential equations. One of the simplest systems in this class is the laser with a modulated parameter [15–18]. The emergence of EE in such a system was studied theoretically [15] for both CO_2 and solid-state lasers, and associated to an *external crisis*, i.e., the collision of an unstable orbit belonging to one of the branches of the multistable system, with a chaotic attractor generated in a different solution branch. The collision produces an abrupt increase of the region that can be reached by the system in the phase space, and hence, leads to the emergence of EE. A key theoretical prediction is that as the control parameter gets farther from the bifurcation point corresponding to the crisis EE vanish. EE would be then associated to the neighborhood (in parameter space) of an external crisis.

In this Letter, we report the first observation of EE in a laser (an all-solid-state laser) with a modulated parameter. We also present experimental evidence in support of this mechanism as one of the possible origins of optical rogue waves. Furthermore, we study the predictability of the formation of such EE both theoretically and experimentally. We show that their emergence can be predicted with larger anticipation than for any other pulse in the chaotic attractor, hence providing hope for predicting and controlling them.

The quantitative definition of an EE usually is: (a) amplitude higher than twice the "significant intensity" $I_{1/3}$ [8,9,16], which is the average calculated among the set of the third-highest events in the series. An event is then considered "extreme" if its *abnormality index* $\mathrm{AI} = I_{\mathrm{event}}/I_{1/3}$ is larger than 2 [17]. An alternative is (b) amplitude higher than four times the average plus the standard deviation. These two definitions can be a coincidence, depending on the form of the distribution. Here we use the second alternative. Such choice ensures that the intensity of the selected pulse is much larger than most of the other ones in the chaotic attractor.

Our experiment is shown in Fig. 1. A $\mathrm{Nd{:}YVO_4}$ laser crystal 3 mm × 3 mm × 1 mm, 1% Nd doped, is pumped by a 2W CW diode laser at 808 nm. A standard V-shaped cavity produces a waist near the output (plane) mirror, where a small aperture electro-optic modulator (EOM) is placed. The EOM axes are oriented at 45 degrees of the main polarization axis of the field (which is the c-axis of the $\mathrm{Nd{:}YVO_4}$ crystal, see below), so that it introduces a polarization rotation proportional to the voltage V_{mod} provided by a signal generator and voltage amplifier. Be aware that the voltage values indicated in the figures are the output of the signal generator; the actual

0146-9592/16/133010-03 Journal

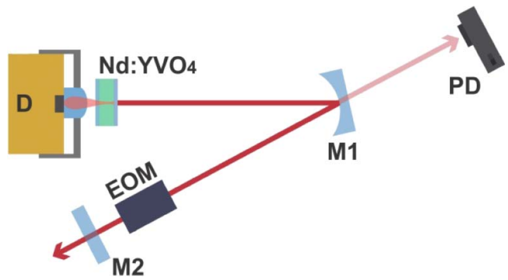

Fig. 1. Scheme of the experiment. D, pump diode; Nd:YVO_4, laser slab; M1, HR folding mirror ROC = 25 cm; M2, output coupler; $R = 83\%$; EOM, electro-optical modulator; PD, fast photodiode.

voltage applied to the EOM is 50 times larger. The active medium Nd:YVO_4 has a strong polarization-dependent gain, with the highest gain parallel to its c-axis. The effect of the modulated voltage applied to the EOM is hence a small modulated change in cavity losses. The EOM is driven at a sinusoidal frequency f near the frequency of the relaxation oscillations, which is measured here to be 111 ± 1 kHz. The laser dynamics are detected with a fast photodiode illuminated by a leak beam at the curved mirror. Time series are recorded in a storage oscilloscope. The typical output power is 200 mW for a pumping power $P = 1.5$ W. The output beam has a nearly TEM_{00} transversal shape, with $M^2 = 1.10 \pm 0.05$.

Depending on the values of the control parameters P, V_{mod}, and f, the laser output is observed to be CW, periodically pulsed, and chaotically pulsed with, or without, EE. With $f \approx$ 100 kHz and low values of V_{mod}, pulsed regimes of different periodicities are observed to coexist. Uniform pulsing, 2-period, 3-period, and until 6-period are observed to be stable. Higher 8- and 10-period are also observed, but are rather unstable. Passing from one pulsed regime to the other is possible by blocking and unblocking the cavity, or by tapping one of the mirrors. Transitions among the different regimes occur spontaneously too, in a time range of a minute, probably caused by uncontrolled mechanical perturbations. The pulses last between 0.8 and 1.2 μs. These observations mean that the system is multistable, in agreement with the theoretical predictions [15].

As V_{mod} is increased bifurcation cascades are observed, and the output becomes chaotic. The zoom of a typical chaotic oscilloscope trace is displayed in Fig. 2. The peaks of each pulse compose a time series. The dimension of embedding (dE) of the series is determined by the false nearest neighbor method and, if dE has a clearly defined value, the Lyapunov exponents are then calculated. All the recorded time series are processed in

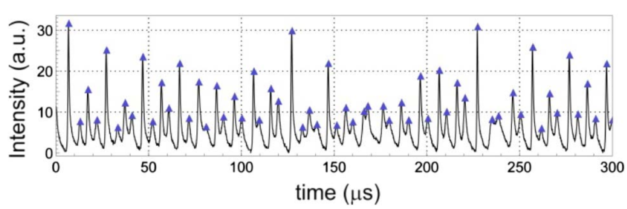

Fig. 2. Section of the chaotic time series recorded for $V_{\mathrm{mod}} = 1.7$ V, $f = 100$ kHz, $P = 1.024$ W. The triangles indicate the position of the maxima of the pulses. These maxima compose a time series, which is numerically analyzed.

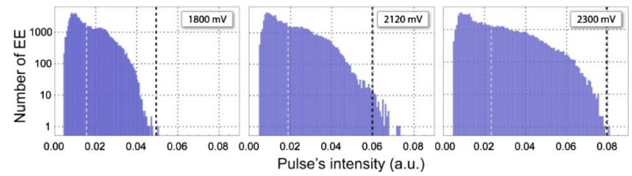

Fig. 3. Histograms of the pulse amplitudes for increasing loss modulation amplitude. The dotted line indicates the EE threshold for each distribution.

this way [19]. Yet, only the series displaying a well-defined value of dE, and no defined value for the surrogate series, are considered valid. The free available software packet TISEAN is used for these purposes. For the complete time series of this example (89602 pulses) we obtain dE = 5, and the Lyapunov exponents $\lambda_i = +0.32$, $+0.11$, -0.08, -0.30, -0.76. This regime is hence dissipative and chaotic.

Inside the chaotic regime, EE occur for some values of the parameters. In Fig. 3, histograms of the distribution of the pulses' amplitudes are shown as V_{mod} is varied. The regime is always chaotic, but it displays no EE at 1.80 V. It does at 2.12 V but does not at 2.30 V. This is the typical behavior predicted for this system at an external crisis [15].

In Fig. 4, the number of observed EE is plotted as V_{mod} is scanned through the critical point. The number increases as the crisis is approached; it reaches a maximum and then decreases after the system has climbed to the next "step" of higher intensity. Note that the average output intensity increases monotonically. The agreement with the theoretical prediction (Fig. 8 in Ref. [15]) is remarkable.

We observe this to be a robust result, for the actual laser is not described exactly by the theory. The Nd:YVO_4 laser is linearly polarized in most cases, but it may deviate from this regime if the losses are too high. Single-mode oscillation is assumed in the theory, but this is not the case in this laser, which oscillates over many longitudinal modes. In other words, the actual laser has more degrees of freedom than theoretically assumed. Yet, in spite of these differences the agreement between the observed and the predicted variation of the number of EE remains.

In both the experiments and the theory on semiconductor laser with injected signal [20], and also in the semiconductor laser with optical feedback in the short-cavity regime [11], EE are more predictable than the average chaotic pulse. The same feature is observed here. In the upper part of Fig. 5, the traces for the 74 EE of the time series corresponding to the peak in

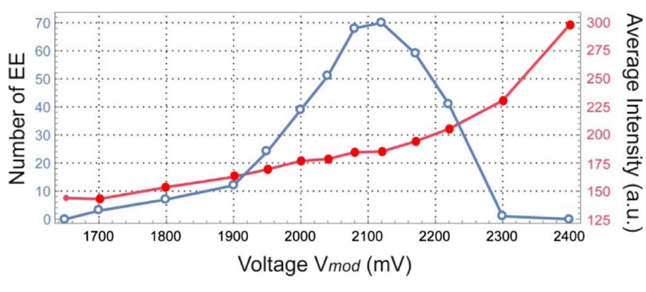

Fig. 4. Number of EE (open circles, blue online) and average intensity (full circles, red online) as a function of the modulation amplitude as the crisis is traversed, $f = 100$ kHz, $P = 1.024$ W.

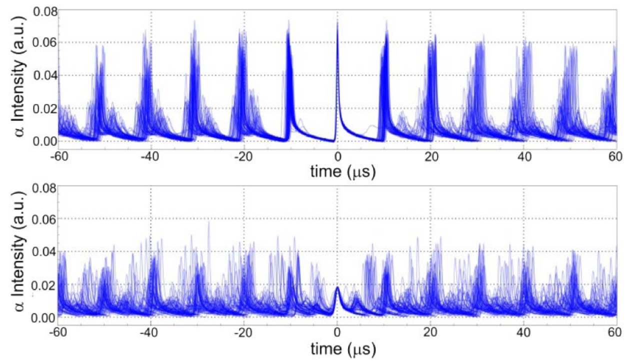

Fig. 5. Superposition of 74 EE (up) and the same number of average pulses (down), both extracted from the same experimental time series, which corresponds to the maximum in Fig. 4.

Fig. 4 are superposed, taking as central reference the EE. The pulse neighbors to the EE occur at about the same time and have nearly the same intensity. Naturally, they "blur" as the time distance to the EE increases. For a pulse whose maximum intensity is nearly the average value instead (lower part of Fig. 5) the characteristics of the pulse neighbors become unpredictable almost immediately. This result suggests that the evolution of EE occurs into a defined manifold in phase space. It gives hope of predicting EE with enough time to implement the necessary control to avoid them.

In Fig. 6 we display the result of the same study of Fig. 5; however, we use data from the numerical integration of the laser-rate equations of the theoretical model developed in [15], using parameter values compatible with the experimental ones. In the notation of [15], $\gamma = 3.5 \times 10^{-4}$, $\beta = 1.67 \times 10^{-3}$, $A = 1.03$, $k_0 = 3 \times 10^7\ \mathrm{s}^{-1}$. The horizontal scale is given in the number of modulation periods. In this case, 180 traces are superposed. The agreement with the experimental result is noticeable.

The maximum Lyapunov exponent in the experimental time series in Fig. 5 is $\lambda = 0.42$, in units of the inverse of the average period time between pulses. This means that the temporal limitation of predictability is ≈ 2.4 pulses. This result roughly agrees with what can be estimated from Figs. 5 and 6.

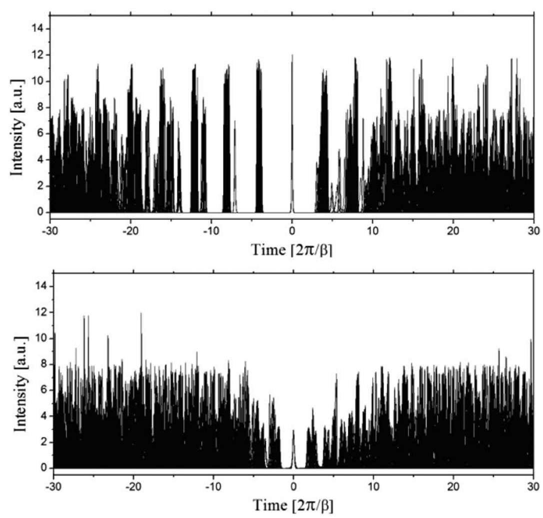

Fig. 6. Superposition of 180 EE (up) and the same number of average pulses (down), both extracted from the same numerical time series, obtained from the theoretical model in [15].

In conclusion, we show that:

(i) Experiments in a Class B laser with modulated losses display a chaotic regime, which can develop optical rogue waves, or EE.

(ii) The variation of the number of EE and the average intensity as the parameter is scanned are compatible with the predictions of the consequence of traversing a bifurcation, called external crisis.

(iii) The predictability of an EE is larger than that of an average pulse. This suggests that the evolution of the system previous to an EE is always the same, hence giving hope to developing a protocol able to predict, and eventually control, the emergence of EE in this type of laser system. This protocol may, hopefully, be later adapted and extended to other types of EE.

We stress that the model in Ref. [15] does not account for the possible polarization shift of the laser or the many longitudinal-mode oscillations. In spite of this limitation, it can describe the essential properties of the system, as it is verified in this series of experiments. An enlargement of that model is currently in progress.

Funding. Air Force Office of Scientific Research (AFOSR) (FA9550-13-1-0120); Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET) (PIP2011-077).

REFERENCES

1. C. Kharif, E. Pelinovsky, and A. Slunyaev, *Rogue Waves in the Ocean* (Springer, 2009).
2. D. Solli, C. Ropers, P. Koonath, and B. Jalali, Nature **450**, 1054 (2007).
3. C. Finot, K. Hammami, J. Fatome, J. Dudley, and G. Millot, IEEE J. Quantum. Electron. **46**, 205 (2010).
4. A. Montina, U. Bortolozzo, S. Residori, and F. T. Arecchi, Phys. Rev. Lett. **103**, 173901 (2009).
5. C. Bonatto, M. Feyereisen, S. Barland, M. Giudici, C. Masoller, J. Ríos Leite, and J. Tredicce, Phys. Rev. Lett. **107**, 053901 (2011).
6. A. Pisarchik, R. Jaimes-Reátegui, R. Sevilla-Escoboza, G. Huerta-Cuéllar, and M. Taki, Phys. Rev. Lett. **107**, 274101 (2011).
7. J. Soto-Crespo, P. Grelu, and N. Akhmediev, Phys. Rev. E **84**, 016604 (2011).
8. A. Zaviyalov, O. Egorov, R. Iliew, and F. Lederer, Phys. Rev. A **85**, 013828 (2012).
9. C. Lecaplain, P. Grelu, J. Soto-Crespo, and N. Akhmediev, Phys. Rev. Lett. **108**, 233901 (2012).
10. A. Runge, C. Aguergaray, N. Broderick, and M. Erkintalo, Opt. Lett. **39**, 319 (2014).
11. C. Bonazzola, A. Hnilo, M. Kovalsky, and J. Tredicce, J. Opt. **15**, 064004 (2013).
12. M. Kovalsky, A. Hnilo, and J. Tredicce, Opt. Lett. **36**, 4449 (2011).
13. J. Dudley, F. Dias, M. Erkintalo, and G. Genty, Nat. Photonics **8**, 755 (2014).
14. C. Gibson, A. Yao, and G. Oppo, Phys. Rev. Lett. **116**, 043903 (2016).
15. C. Metayer, A. Serres, E. Rosero, W. Barbosa, F. de Aguiar, J. RiosLeite, and J. Tredicce, Opt. Express **22**, 19850 (2014).
16. M. Erkintalo, G. Genty, and J. Dudley, Eur. Phys. J. Spec. Top. **185**, 135 (2010).
17. H. Svedrup and W. Munk, "Wind, sea and swell. Theory of relations for forecasting," US Navy Hidrographic office Pub.601 (1947).
18. R. Meucci, A. Poggi, F. Arecchi, and J. Tredicce, Opt. Commun. **65**, 151 (1988).
19. M. Kovalsky and A. Hnilo, Opt. Lett. **35**, 3498 (2010).
20. J. Reinoso, J. Zamora-Munt, and C. Masoller, Phys. Rev. E **87**, 062913 (2013).