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Bipolarity in temporal argumentation frameworks ☆

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A B S T R A C T

A *Timed Argumentation Framework* (*TAF*) is a formalism where arguments are only valid for consideration during specific intervals of time, called availability intervals, which are defined for every individual argument. The original proposal is based on a single abstract notion of attack between arguments that remains static and permanent in time. Thus, in general, when identifying the set of acceptable arguments, the outcome associated with a *TAF* will vary over time.

Here, we are introducing an extension of *TAF* adding the capability of modeling a support relation between arguments. In this sense, the resulting framework provides a suitable model for different time-dependent issues; thus, the main contribution of this work is to provide an enhanced framework for modeling a positive (support) and negative (attack) interaction which varies over time, features that are highly relevant in many real-world situations. This addition leads to a *Timed Bipolar Argumentation Framework* (*T-BAF*), where classical argument extensions can be defined, aiming at advancing in the integration of temporal argumentation in different application domains.

1. Introduction

Commonsense reasoning presents many characteristics that have been explored by philosophers first and by Artificial Intelligence research community more recently. Argumentation is one of these aspects which models a human mechanism for decision making that explores reasons in favor and against different pieces of potential beliefs, or claims, supported by some form of reasoning from a set of premises; the final purpose of this process is to ascertain if a particular claim is acceptable [1–3]. Research in the area of argumentation has introduced several argument-based formalisms for dealing with applications in many practical areas, i.e., legal reasoning, autonomous agents, and multi-agent systems. In such environments, an agent may use argumentation to perform individual reasoning to reach a resolution over contradictory evidence or to decide between conflicting goals, while multiple agents may use dialectical argumentation to identify and settle differences interacting via diverse processes such as negotiation, persuasion, or joint deliberation. Many of such accounts of

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argumentation are based on Dung's foundational work characterizing *Abstract Argumentation Frameworks* (*AF*) [4,5] where arguments are considered as atomic entities and their interaction is represented solely through an attack relation.

A salient feature of commonsense reasoning is that in many cases the use of temporal information is important, requiring the representation of time because the notion of "change" is relevant in the modeling of the argumentation capabilities of intelligent agents [6,7]. In particular, in [8–10] a novel framework called *Timed Abstract Framework* (*TAF*) was proposed combining arguments and temporal notions, where arguments are relevant only during a period of time, which is called its availability interval. This framework maintains a high level of abstraction in an effort to capture intuitions related with the dynamic interplay of arguments as they become available and cease to be so. The notion of *availability interval* conceptualizes an interval of time in which the argument can be legally used for the particular purpose of an argumentation process; thus, a timed-argument has a limited influence in the system that will be given by the temporal context in which this argument can be taken into account. Using an expanded concept of admissibility that considers time, a skeptical timed interval-based semantics will be introduced for *TAFs*. As arguments may get attacked during a certain period of time, the notion of defense is also time-dependent, requiring a proper adaptation of classical acceptability; furthermore, algorithms for the characterization of defenses between timed-arguments are presented, and used to specify the acceptability status of an argument varying over time [10,9].

In the original abstract argumentation frameworks [5], only a conflict interaction between arguments is considered; however, in recent years, studies on argumentation have shown that a support interaction may also exist between arguments, and this intuitions represents relations in real world situations. Several formal approaches have been considered such as deductive support, necessary support, and evidential support [11–14], where an abstract argumentative framework is enhanced with the capability to model not only the negative interaction of attacks between arguments, but also a positive relation of support is considered. In particular, a simple abstract formalization of argument support was provided in the framework proposed by Cayrol and Lagasquie-Schiex in [11], called *Bipolar Argumentation Framework* (*BAF*), where they extend Dung's notion of acceptability by distinguishing two independent forms of interaction between arguments: support and attack. Besides the classical semantic consequences of attack, new semantic considerations are introduced that relies on the support to an attack and the attack to a support.

Here, we provide a timed argumentation framework seeking to analyze the effect of attacks and supports in a dynamic situation, obtaining a specialized form of *BAF* where the resulting framework provides a suitable model for different time-dependent issues. The main contribution of this move is to provide an enhanced framework for modeling a positive (support) and negative (attack) interaction varying over time, both of which are relevant in many real-world situations, advancing in the integration of temporal argumentation in different, time-related application domains and contributing to the successful integration of argumentation in different artificial intelligence applications, such as Knowledge Representation, Autonomous Agents in Decision Support Systems, and others of similar importance. Next, in order to state the relevance of our formalization, we examine a classical example of bipolar argumentation case introduced in [15] about editorial publishing. Our formalism helps to represent a model that analyzes the temporal effects, as follows:

Let us consider the following scenario where an Editor is evaluating the presentation of an important note related to a public person $\mathtt{P}$*. For that, the Chief Editorial Writer considers the following arguments which contemplate the importance and legality of the note.*

$\mathcal{I}$: *Information* $\mathtt{I}$ *concerning person* $\mathtt{P}$ *should be published.*
$\mathcal{P}$: *Information* $\mathtt{I}$ *is private so,* $\mathtt{P}$ *denies publication.*
$\mathcal{S}$: $\mathtt{I}$ *is an important information concerning* $\mathtt{P}$*'s son.*
$\mathcal{M}$: $\mathtt{P}$ *is the prime minister so, everything related to* $\mathtt{P}$ *is public.*

Some of these arguments are controversial, as is the case of the conflict between arguments $\mathcal{P}$ *and* $\mathcal{I}$*, and between arguments* $\mathcal{M}$ *and* $\mathcal{P}$*. On the other hand, there is a relation between arguments* $\mathcal{P}$ *and* $\mathcal{S}$*, which clearly is not of conflict. Moreover,* $\mathcal{S}$ *provides a new piece of information enforcing argument* $\mathcal{P}$*.*

This is an appropriate example to introduce positive and negative argument relations; nevertheless, it does not *explicitly* consider the evolution of time. By its own nature, argumentation is a *process* in which arguments are issued as time progresses, thus it is interesting to evaluate the arguments and conflicts at different stages of such a process. Also, as the context where argumentation occurs evolves in time, some arguments may become useless, invalid, or even irrelevant. In the example above, argument $\mathcal{M}$ results valid only in the period when $\mathtt{P}$ holds the Prime Minister office, i.e., such an argument is irrelevant to a dialectical analysis taking place after $\mathtt{P}$ has left the PM office. If $\mathtt{P}$ is Prime Minister from 2010 to 2014, then argument $\mathcal{M}$ is pertinent only in the interval of time $[2010, 2014]$. To get a more accurate description of the situation, we improve the argument representation with temporal information about the periods of time in which this argument is relevant. Consider now the following, time-enriched scenario about the publishing problem above:

Arguments $\mathcal{I}$ *and* $\mathcal{P}$ *can be both considered as general information applicable at any moment, a sort of editorial rules; however, argument* $\mathcal{M}$ *is available only during the period of time when* $\mathtt{P}$ *is prime minister. Before that time span, argument* $\mathcal{M}$ *does not apply, and after* $\mathtt{P}$ *leaves the Primer Minister Office, the information could be less relevant for publication. Then, a new prime minister* $\mathtt{P}_2$

Argument	*Temporal availability*
$\mathcal{I}$	$[0, \infty)$
$\mathcal{P}$	$[0, \infty)$
$\mathcal{S}$	[2013, *Apr* – 2013, *Oct*]
$\mathcal{T}$	[2012, *Feb* – 2012, *Jun*]
$\mathcal{M}$	[2012, *Oct* – 2014, *Oct*]
$\mathcal{N}$	[2010, *Jun* – 2012, *Oct*)

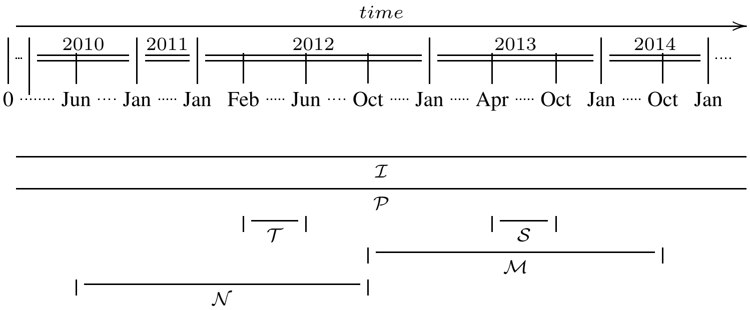

Fig. 1. Availability distribution for the arguments.

may be a more important public person than P*, and at least for media purposes the information about* P *may be disregarded. Consider now that the Chief Editorial Writer analyzes a more complex scenario, taking into account additional, temporal information to make a proper evaluation represented as the periods of time where* P *and* $\mathtt{P}_2$ *hold office as well as the birth dates of their children. This is specified with intervals of time as follows:*

$\mathcal{I}$: *Information* I *concerning person* P *should be published.* $\longrightarrow [0, \infty]$
$\mathcal{P}$: *Information* I *is private so,* P *denies publication.* $\longrightarrow [0, \infty]$
$\mathcal{S}$: I *is an important information concerning* $\mathtt{P}_2$*'s son.* $\longrightarrow$ [2013, *Apr* – 2013, *Oct*]
$\mathcal{T}$: I *is an important information concerning* $\mathtt{P}_2$*'s son.* $\longrightarrow$ [2012, *Feb* – 2012, *Jun*]
$\mathcal{M}$: P *is the prime minister, everything related to* P *is public.* $\longrightarrow$ [2012, *Oct* – 2014, *Oct*]
$\mathcal{N}$: $\mathtt{P}_2$ *is the prime minister, everything related to* $\mathtt{P}_2$ *is public.* $\longrightarrow$ [2010, *Jun* – 2012, *Oct*)

Arguments and its "availability" are summarized in Fig. 1. The attack relation between arguments $\mathcal{I}$ *and* $\mathcal{P}$ *is available in any moment of time, while the conflict between arguments* $\mathcal{M}$ *and* $\mathcal{P}$ *is active only in the time interval where* $\mathcal{M}$ *is available,* [2012, *Oct* – 2014, *Oct*]*. On the other hand, argument* $\mathcal{M}$ *reinforces argument* $\mathcal{I}$ *in the time interval* [2012, *Oct* – 2014, *Oct*]*, giving additional information about the person* P*.*

In this example the time dimension is necessary to create a proper argumentation model that describes the evolution of the argumentative discussion. Using this representation, we can analyze the different relationships between the arguments from a new perspective, such as the specification of time intervals where an argument is accepted, the determination of moments in which an argument is strengthened by its supports (providing more information about a particular point) or to establish *when* the supporting arguments provide extra conflict points for the supported argument. Furthermore, the proposed extension of the representation allows the study of certain temporal properties associated with the arguments, such as their acceptability status over time.

This work is organized as follows: Section 2 presents a brief review of the classical bipolar abstract argumentation frameworks which allows the representation of support and conflict defined over arguments. In Section 3, we present the intuition followed to model time related time aspects in the argumentative process. Then, in Section 4, we introduce a concrete extension of the bipolar argumentation formalism where the temporal notion associated to the arguments is taken into account, and different temporal acceptability semantic process are presented. Later, in Section 5, we present a real world example where the *T-BAF*'s notions are applied to analyze a dynamic argumentation model. Finally, Sections 6 and 7 are devoted to explore the pertinent related works, concluding remarks, and the discussion of interesting issues that remain to be explored.

2. Bipolar abstract argumentation

The basic idea behind argumentation is to construct arguments for and against a conclusion, analyze the general scenario, and then select the *acceptable* ones. Arguments have different roles in front of each other. One might then say that arguments are presented in a "bipolar" way since arguments in favor of a conclusion can be considered as positive while arguments against the conclusion as negative ones. Based on these intuition, when representing the essential mechanism of argumentation the notion of bipolarity is a natural one. Abstracting away from the inner structure of the arguments,

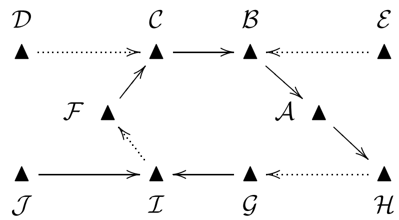

Fig. 2. Bipolar argumentation graph.

the Abstract Bipolar Argumentation Framework proposed by Cayrol and Lagasquie-Schiex in [11], extend Dung's notion of acceptability distinguishing two independent forms of interaction between arguments: support and attack. This new relation is assumed to be totally independent of the attack relation (i.e. it is not defined using the attack relation) providing a positive relation between arguments.

Definition 1 *(Bipolar Argumentation Framework).* A Bipolar Argumentation Framework (*BAF*) is a 3-tuple $\Theta = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s \rangle$, where $\mathtt{Arg}$ is a set of arguments, R_a and R_s are disjoint binary relations on $\mathtt{Arg}$ called attack relation and support relation, respectively.

Cayrol and Lagasquie-Schiex extended the notion of graph presented by Dung in [5] by adding the representation of support between arguments. Thus, we use nodes in the graph to represent the elements in $\mathtt{Arg}$ and two types of arcs: one for represent the attack relation (depicted as plain arrows), and the other to represent the support relation (depicted as squid arrows). This argumentative model provides a starting point to analyze an argumentative discussion enriched by the bipolarity of human reasoning. This notion is defined as follows.

In order to consider the interaction between supporting and attacking arguments, Cayrol and Lagasquie-Schiex in [11] introduce the notions of *supported* and *secondary* attack which combine a sequence of supports with a direct attack. This notion is presented in the following definition.

Definition 2 *(Supported and Secondary Attack).* Let $\Theta = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s \rangle$ be a *BAF*, and $\mathcal{A}, \mathcal{B} \in \mathtt{Arg}$ two arguments.

- A supported attack from $\mathcal{A}$ to $\mathcal{B}$ is a sequence $\mathcal{A}_1$ R_1 ... R_{n-1} $\mathcal{A}_n$, $n \geq 3$, with $\mathcal{A}_1 = \mathcal{A}$ and $\mathcal{A}_n = \mathcal{B}$, $\forall i = 1 \ldots n-2$, $\mathrm{R}_i = \mathrm{R}_s$ and $\mathrm{R}_{n-1} = \mathrm{R}_a$.
- A secondary attack from $\mathcal{A}$ to $\mathcal{B}$ is a sequence $\mathcal{A}_1$ R_1 ... R_{n-1} $\mathcal{A}_n$, $n \geq 3$, with $\mathcal{A}_1 = \mathcal{A}$ and $\mathcal{A}_n = \mathcal{B}$, such that $\mathrm{R}_1 = \mathrm{R}_a$ and $\forall i = 2 \ldots n-1$, $\mathrm{R}_i = \mathrm{R}_s$.

The authors in [11] state that, by extension, a sequence reduced to two arguments $\mathcal{A}$ R_a $\mathcal{B}$ (that is, a *direct* attack $\mathcal{A} \rightarrow \mathcal{B}$) is also considered as a supported attack from $\mathcal{A}$ to $\mathcal{B}$. The following example shows the intuitions presented in these definitions, showing the different kind of attacks in a bipolar environment.

Example 1. Given a *BAF* $\Theta = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s \rangle$, where:

$\mathtt{Arg} = \{\mathcal{A}; \mathcal{B}; \mathcal{C}; \mathcal{D}; \mathcal{E}; \mathcal{F}; \mathcal{G}; \mathcal{H}; \mathcal{I}; \mathcal{J}\}$,
$\mathrm{R}_a = \{(\mathcal{B}, \mathcal{A}); (\mathcal{A}, \mathcal{H}); (\mathcal{C}, \mathcal{B}); (\mathcal{G}, \mathcal{I}); (\mathcal{J}, \mathcal{I}); (\mathcal{F}, \mathcal{C})\}$, and
$\mathrm{R}_s = \{(\mathcal{D}, \mathcal{C}); (\mathcal{H}, \mathcal{G}); (\mathcal{I}, \mathcal{F}); (\mathcal{E}, \mathcal{B})\}$.

The bipolar argumentation framework Θ is shown in Fig. 2. Argument $\mathcal{J}$ is a direct attacker of $\mathcal{I}$ and $\mathcal{H}$ support-attacks $\mathcal{I}$, since $\mathcal{H}$ supports G, and $\mathcal{I}$ is attacked by $\mathcal{G}$; in addition, $\mathcal{J}$ and $\mathcal{G}$ secondary-attack $\mathcal{F}$, because $\mathcal{I}$ supports $\mathcal{F}$, which is attacked by $\mathcal{J}$ and $\mathcal{G}$. However, there exists a secondary attacks from $\mathcal{A}$ to $\mathcal{G}$ through $\mathcal{H}$, and then $\mathcal{G}$ is attacked; also argument $\mathcal{B}$ is a direct attacker of $\mathcal{A}$, but $\mathcal{D}$ support-attacks $\mathcal{B}$ through $\mathcal{C}$. Note that the secondary attacks from $\mathcal{G}$ to $\mathcal{F}$ is threatened by $\mathcal{A}$, which attacks $\mathcal{H}$, a support of $\mathcal{G}$.

Cayrol and Lagasquie-Schiex in [11] argued that a set of arguments presented by an agent that is involved in a dispute must exhibit coherence, in the sense of not being contradictory. The coherence of a set of arguments is analyzed *internally* (a set of arguments in which an argument attacks another in the same set is not acceptable), and *externally* (a set of arguments which contains both a supporter and an attacker for the same argument is not acceptable). The internal coherence is captured extending the definition of *conflict-free set* proposed in [5], and external coherence is captured with the notion of *safe* set.

Definition 3 *(Conflict-free and Safe).* Let $\Phi = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s \rangle$ be a *BAF*, and $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments.

– $\mathtt{S}$ is *Conflict-free* iff $\nexists \mathcal{A}, \mathcal{B} \in \mathtt{S}$ such that there is a supported or a secondary attack from $\mathcal{A}$ to $\mathcal{B}$.
– $\mathtt{S}$ is *Safe* iff $\nexists \mathcal{A} \in \mathtt{Arg}$ and $\nexists \mathcal{B}, \mathcal{C} \in \mathtt{S}$ such that there is a supported or a secondary attack from $\mathcal{B}$ to $\mathcal{A}$, and either there is a sequence of support from $\mathcal{C}$ to $\mathcal{A}$, or $\mathcal{A} \in \mathtt{S}$.

The notion of conflict-freeness in the above definition requires to take supported and secondary attacks into account, becoming a more restrictive definition than the classical version of conflict-freeness proposed by Dung. In addition, Cayrol and Lagasquie-Schiex show that the notion of safety is powerful enough to encompass the notion of conflict-freeness (i.e., if a set is safe, then it is also conflict-free). Another requirement has been considered in [11], which concerns only the support relation, namely the *closure under* $\mathtt{R}_s$.

Definition 4 *(Closure in BAF).* Let $\Phi = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a *BAF*. $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments. $\mathtt{S}$ is closed under $\mathtt{R}_s$ iff $\forall\ \mathcal{A} \in \mathtt{S}$, $\forall\ \mathcal{B} \in \mathtt{Arg}$ if $\mathcal{A}\ \mathtt{R}_s\ \mathcal{B}$ then $\mathcal{B} \in \mathtt{S}$.

Example 2 *(Continued Example 1).* The set $\mathtt{S}_1 = \{\mathcal{I}; \mathcal{F}; \mathcal{D}; \mathcal{B}; \mathcal{E}\}$ is conflict free but not safe, since $\mathcal{I}$ support-attacks $\mathcal{C}$ through $\mathcal{F}$, and $\mathcal{D}$ supports $\mathcal{C}$. Hence, set $\mathtt{S}_1$ supports and attacks argument $\mathcal{C}$. The set $\mathtt{S}_2 = \{\mathcal{J}; \mathcal{C}; \mathcal{D}; \mathcal{A}\}$ is conflict free and closed by $\mathtt{R}_s$, then it is safe.

The notion of closure proposes maximality in the support relation: if an argument $\mathcal{A}$ is supported by members of the set $\mathtt{S} \subseteq \mathtt{Arg}$, then $\mathcal{A}$ belongs to $\mathtt{S}$. Although it is simply the mathematical concept of closure of a set applied to the support relation, it is useful as a link between some semantic notions:

– If $\mathtt{S}$ is conflict-free and closed, then it is safe.
– If $\mathtt{S}$ is stable and safe, then it is closed.

Based on the previous concepts, Cayrol and Lagasquie-Schiex in [11] extend the notions of defense for an argument with respect to a set of arguments, where they take into account the relations of support and attack between arguments.

Definition 5 *(Defense of $\mathcal{A}$ from $\mathcal{B}$ by $\mathtt{S}$).* Let $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments, and $\mathcal{A} \in \mathtt{Arg}$ be an argument. $\mathtt{S}$ defends collectively A iff $\forall\ \mathcal{B} \in \mathtt{Arg}$ if $\mathcal{B}$ is a (supported or secondary) attacker of $\mathcal{A}$ then $\exists\ \mathcal{C} \in S$ such that $\mathcal{C}$ is a supported or secondary attacker of $\mathcal{B}$. In this case, it can be interpreted that $\mathcal{C}$ defends $\mathcal{A}$ from $\mathcal{B}$.

The authors proposed three different definitions for admissibility, one being weaker than the other. The most weak notion is based on Dung's admissibility; later, they extended the notion of d-admissibility by taking into account external coherence. Finally, external coherence is strengthened by requiring admissible sets to be closed for $\mathtt{R}_s$.

Definition 6 *(Admissibility in BAF).* Let $\Phi = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a *BAF*. Let $S \subseteq \mathtt{Arg}$ be a set of arguments. The admissibility of a set $\mathtt{S}$ is defined as follows:

– $\mathtt{S}$ is d-admissible if $\mathtt{S}$ is conflict-free and defends all its elements.
– $\mathtt{S}$ is s-admissible if $\mathtt{S}$ is safe and defends all its elements.
– $\mathtt{S}$ is c-admissible if $\mathtt{S}$ conflict-free, closed for $\mathtt{R}_s$ and defends all its elements.

Example 3 *(Continued Example 1).* The set $\mathtt{S}_1 = \{\mathcal{J}; \mathcal{C};\ \mathcal{D}; \mathcal{A}; \mathcal{E}\}$ is d-admissible, since it is conflict-free and it defends all its elements; however, it is not s-admissible, because $\mathcal{C}$ and $\mathcal{E}$ belong to $\mathtt{S}_1$, where $\mathcal{C}$ is a direct attacker of $\mathcal{B}$ and $\mathcal{E}$ is a support of $\mathcal{B}$, and hence $\mathtt{S}_1$ is not safe. It is important to note that, if a set of arguments is not an s-admissible set, then it is not a c-admissible set; therefore $\mathtt{S}_1$ is not c-admissible. The set $\mathtt{S}_2 = \{\mathcal{J}; \mathcal{C}; \mathcal{D}; \mathcal{A}\}$ is s-admissible, since it is safe and defends all its elements; in addition, it is closed for $\mathtt{R}_s$, and then $\mathtt{S}_2$ is c-admissible.

From the notions of coherence, admissibility, and extending the propositions introduced in [5], Cayrol and Lagasquie-Schiex in [11] proposed different new semantics for the acceptability of arguments.

Definition 7 *(Stable extension).* Let $\Phi = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a *BAF*. Let $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments. The set $\mathtt{S}$ is a stable extension of Φ if $\mathtt{S}$ is conflict-free and for all $\mathcal{A} \notin \mathtt{S}$, there is a supported or a secondary of $\mathcal{A}$ in $\mathtt{S}$.

Definition 8 *(Preferred extension).* Let $\Phi = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a *BAF*. Let $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments. $\mathtt{S}$ is a d-preferred (resp. s-preferred, c-preferred) extension if $\mathtt{S}$ is maximal (for set-inclusion) among the d-admissible (resp. s-admissible, c-admissible) subsets of $\mathtt{Arg}$.

Example 4 *(Continued Example 1).* The set of arguments $\mathtt{S}_1 = \{\mathcal{J}; \mathcal{C}; \mathcal{D}; \mathcal{A}; \mathcal{E}\}$ is the stable extension, since there exist an attacker for the arguments $\mathcal{I}$, $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$ (as we explained in Example 1). However, as we see in Example 3, this extension

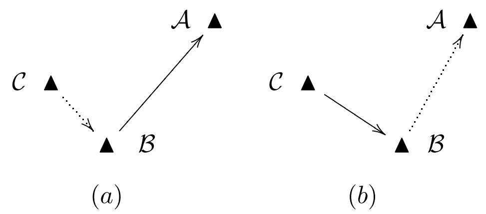

Fig. 3. Argument relations.

is not safe. In consequence, there are not s-stable nor c-stable extensions. On the other hand, based on Definition 8, it can be stated that $\mathtt{S}_1$ is a maximal d-admissible set, so $\mathtt{S}_1$ is a d-preferred extension; The set $\mathtt{S}_2 = \{\mathcal{J}; \mathcal{C}; \mathcal{D}; \mathcal{A}\}$ is a maximal s-admissible sets, so $\mathtt{S}_2$ is an s-preferred extensions; and $\mathtt{S}_2$ is a maximal c-admissible set, therefore $\mathtt{S}_2$ is a c-preferred extension.

In the following section the support relation is considered among attacks in a timed context. Later, time-dependent semantics are presented.

3. Towards a temporal argumentation framework

Our interest is to enrich bipolar argumentation frameworks with a time-based notion of argument interaction. The focus is put on an abstract notion of *availability* of arguments, which is a metaphor for a dynamic relative importance. Throughout this paper we mainly use the term "available", meaning that an argument will be considered just for a specific interval of time. However, availability can be interpreted in different ways. It may be the period of time in which an argument is relevant or strong enough or appropriate or any other suitable notion of *relative importance* among arguments. The premise is that this availability is not persistent. In such a dynamic scenario, attack and support may be sporadic and then proper time-based semantics need to be elaborated.

Consider the following situation. Let $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ be three arguments such that $\mathcal{B}\ \mathtt{R}_a\ \mathcal{A}$ and $\mathcal{C}\ \mathtt{R}_s\ \mathcal{B}$, as shown in Fig. 3(a). This is a minimal example of supported attack. In the classical definition of bipolar argumentation framework, the set $\mathtt{S} = \{\mathcal{C}, \mathcal{B}\}$ is conflict-free. When considering availability of arguments, different conflict-free situations may arise. Suppose at moment t_1 arguments $\mathcal{C}$ and $\mathcal{A}$ are available while $\mathcal{B}$ is not. Then the set $\mathtt{S}_1 = \{\mathcal{C}, \mathcal{A}\}$ is conflict-free, since the attacker of $\mathcal{A}$ is not available i.e. not relevant or strong at this particular moment. Suppose later at moment t_2 argument $\mathcal{B}$ becomes available. Then $\mathtt{S}_1$ is no longer conflict-free since $\mathcal{C}$ supports a (now available) attacker of $\mathcal{A}$. Suppose later at moment t_3 argument $\mathcal{B}$ is not available again. Then set S_1 regains its conflict-free quality. Hence, a set of arguments in a timed context is not a conflict-free set by itself, but regarding certain moments in time. The set $\mathtt{S}_1$ is conflict-free in t_1 and in t_3, and more generally speaking, in intervals of time in which availability of related arguments does not change.

In a similar fashion, consider the scenario of Fig. 3(b), where $\mathcal{B}\ \mathtt{R}_s\ \mathcal{A}$ and $\mathcal{C}\ \mathtt{R}_a\ \mathcal{B}$. Suppose at moment t_1 arguments $\mathcal{B}$ and $\mathcal{A}$ are available while $\mathcal{C}$ is not. Then the set $\mathtt{S}_1 = \{\mathcal{A}, \mathcal{B}\}$ is conflict-free. Suppose at moment t_2 arguments $\mathcal{C}$ and $\mathcal{A}$ are available, while $\mathcal{B}$ is not. Then, the set $\mathtt{S}_1 = \{\mathcal{C}, \mathcal{A}\}$ is conflict-free. Now, suppose that all the arguments are available at time t_3, then there is a conflict underlying in $\mathtt{S}_1$. In this case, argument $\mathcal{B}$ provides a support to $\mathcal{A}$, but in some moments of time $\mathcal{B}$ is attacked by the argument $\mathcal{C}$ providing a conflict for $\mathcal{A}$.

One may wonder if the situation in which $\mathcal{A}$ is available, but not $\mathcal{B}$, makes sense, since an argument is available without its supporting arguments. This actually depends on the interpretation of the notion of support. There are two main views of this relation, that we will call *necessary support* and *independent support*. In the first case, the supporting argument is needed to justify the existence of the supported argument. This is the case, for instance, of the subargument relation. It is not possible to accept an argument without accepting its subarguments since there is structural dependency between them. In some logic-based models of arguments a subargument T_1 is often a subtree of a larger derivation tree T. Accepting the bigger tree T as a tentative piece of reasoning naturally implies the acceptance of T_1. Note that according to the *necessary support* notion, the supporting arguments must be available whenever the supported argument is. On the other hand, in the case of *independent support*, the supporting argument is an optional, independent advocacy for the supported argument. This is related to the concept of *accrual of reasoning*, were the number of supporting arguments simply influences the strength of the supported one. Here, the periods of availability of both arguments are independent of each other. In this work we do not impose additional restrictions regarding the availability of arguments and its supports and then our approach is compatible with the interpretation of independent support.

An interesting aspect of timed argumentation is that the concept of *argument extension* must be revised. Now the question is not *whether* an argument is accepted (or rejected), but *when*. Hence, a semantic analysis of the status of an argument must refer to intervals of time. We define a specific structure for this notion called *t-profile*, to be presented later. In a dynamic environment, the set of conflict-free sets changes through time. Thus, the notion of acceptability in a bipolar argumentation scenario must be adapted when properly considered in a timed context. In this sense, we reformulate the attacks and supports notions defined in the classical bipolar argumentation framework, modeling the positive and negative effect of the arguments over time. In the following section the formal model of Timed Bipolar Argumentation Framework is introduced and the corresponding argument semantics are presented.

4. Modeling temporal argumentation with T-BAF

The *Timed Bipolar Argumentation Framework* (*T-BAF*) is a formalism where arguments are valid only during specific intervals of time, called *availability intervals*. Attacks and supports are not persistent in time, since they are considered only when the involved arguments are *available*. Thus, when identifying the set of acceptable arguments, the outcome associated with a *T-BAF* may vary in time.

To represent time, we assume that a correspondence was defined between the timeline and the set of real numbers. A time interval, representing a period of time without interruptions, will be defined as follows. For legibility reasons we use a different symbol as separator in the definition of intervals: "−" instead of the traditional ",".

Definition 9 *(Time Interval).* A time interval I represents a continuous period of time, identified by a pair of time-points. The initial time-point is called the startpoint of I, and the final time-point is called the endpoint of I. The intervals can be:

Closed: defines a period of time that includes the definition points (startpoint and endpoint). Closed intervals are noted as $[a-b]$.

Open: defines a period of time without the start and endpoint. Open intervals are noted as $(a-b)$.

Semi-Closed: the periods of time includes one of the definition points but not both. Depending on which one is included, they are noted as $(a-b]$ (includes the endpoint) or $[a-b)$ (includes the startpoint).

As it is usual, any of the previous intervals is considered empty if $b<a$. The interval $[a-a]$ represents the time-point $\{a\}$. For infinite endpoints, we use the symbol $+\infty$ and $-\infty$, as in $[a-+\infty)$ or $(-\infty-a]$ respectively, to indicate that there is no upper or lower bound for the interval respectively. An interval containing this symbol will always be closed by ")" or "(" respectively. Since it will be necessary to group different intervals, we introduce the notion of *time intervals set*.

Definition 10 *(Time Intervals Set).* A time intervals set, or just intervals set, is a finite set $\mathcal{T}$ of time intervals.

Some semantic elaborations are focused on the maximality of intervals. In this work, often two subsequent intervals may be joined and considered as one interval. When convenient, we will use the set of sets notation for time intervals sets. Concretely, a time interval set $\mathcal{T}$ will be denoted as the set of all disjoint and $\subseteq$-maximal individual intervals included in the set. For instance, we will use $\{(1-3],\ [4.5-8)\}$ to denote the time interval set $(1-3]\ \cup\ [4.5-8)$.

The notion of Timed Bipolar Argumentation Framework (T-BAF) is formally defined as follows. This framework extends the BAF of Cayrol and Lagasquie-Schiex by incorporating the time-based evolution of the framework through the concept of availability of arguments.

Definition 11 *(Timed Bipolar Argumentation Framework).* A Timed Bipolar Argumentation framework (or simply *T-BAF*) is a triple $\Omega=\langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$, where $\texttt{Arg}$ is a set of arguments, $\texttt{R}_a$ is a binary relation defined over $\texttt{Arg}$ (representing attack), $\texttt{R}_s$ is a binary relation defined over $\texttt{Arg}$ (representing support), and $\texttt{Av}:\texttt{Arg}\longrightarrow \wp(\mathbb{R})$ is an availability function for timed arguments, such that $\texttt{Av}(\mathcal{A})$ is the set of availability intervals of an argument $\mathcal{A}$.

Note that since arguments are only available during a certain period of time (the availability interval), it is rational to think that relationships between arguments are relevant only when the arguments involved are available at the same time.

Example 5. The corresponding timed bipolar argumentation framework of the introductory example is $\Omega_{intro} = \langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$ where

$\texttt{Arg}=\{\mathcal{I},\mathcal{P},\mathcal{S},\mathcal{T},\mathcal{M},\mathcal{N}\}$,
$\texttt{R}_a=\{(\mathcal{P},\mathcal{I}),(\mathcal{M},\mathcal{P})\}$,
$\texttt{R}_s=\{(\mathcal{S},\mathcal{P})\}$, and
$\texttt{Av}=\{\langle\mathcal{I},\{[0,\infty]\}\rangle;\ \langle\mathcal{P},\{[0,\infty]\}\rangle;\ \langle\mathcal{S},\{[201304,201309]\}\rangle;$
$\langle\mathcal{T},\{[201202,201206]\}\rangle;\ \langle\mathcal{M},\{[201209,201409]\}\rangle;$
$\langle\mathcal{N},\{[201006,201209)\}\rangle\}$.

Months and years are encoded to integers to preserve order.

Some definitions are needed towards the formalization of the notion of *acceptability* of arguments in *T-BAF*, which is a time-based adaptation of the acceptability notions presented in Section 2 for *BAF*, with some new intuitions. First, we present the notion of *t-profile*, binding an argument to a set of time intervals. This set represents intervals for special semantic consideration of the corresponding argument. It is a structure that formalizes the phrase "*this argument, in those intervals*". It is not necessarily the total availability of the argument as it is defined in the framework, so the reference has a special meaning when applied in appropriate contexts. T-profiles constitute a fundamental component for the formalization of time-based acceptability, since it is the basic unit of timed reference for an argument.

Definition 12 *(T-Profile).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*. A timed argument profile for $\mathcal{A}$ in Ω, or just *t-profile* for $\mathcal{A}$, is a pair $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ where $\mathcal{A} \in \mathtt{Arg}$ and $\mathcal{T}_\mathcal{A}$ is a set of time intervals where $\mathcal{A}$ is available, i.e., $\mathcal{T}_\mathcal{A} \subseteq \mathtt{Av}(\mathcal{A})$. The t-profile $\langle \mathcal{A}, \mathtt{Av}(\mathcal{A})\rangle$ is called the basic t-profile of $\mathcal{A}$.

The basic t-profile of an argument $\mathcal{X}$ may be interpreted as the reference "$\mathcal{X}$, whenever it is available". Note that, as discussed previously, this argument may be attacked and defended as time evolves. At some moment argument $\mathcal{X}$ may even be free of attacks, and be certainly attacked just a moment before. Hence, when asking about the acceptance of $\mathcal{X}$ according to a semantic notion $\mathcal{S}$, the basic t-profile will be probably fragmented, capturing the relevant moments in its own history. Since argument extensions are a collective construction based on arguments and interactions, several t-profiles will be considered.

Definition 13 *(Collection of T-Profiles).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*. The set $\mathsf{S} = \{\langle \mathcal{X}_1, \mathcal{T}_{\mathcal{X}_1}\rangle, \langle \mathcal{X}_2, \mathcal{T}_{\mathcal{X}_2}\rangle, \cdots, \langle \mathcal{X}_n, \mathcal{T}_{\mathcal{X}_n}\rangle\}$ is a collection of *t-profiles* iff it verifies the following conditions:

i) $\mathcal{X}_i \neq \mathcal{X}_j$ for all i, j such that $i \neq j$, $1 \leq i, j \leq n$.
ii) $\mathcal{T}_{\mathcal{X}_i} \neq \emptyset$, for all i such that $1 \leq i \leq n$.

Since arguments interact with each other and every argument will be related to several intervals of time, it is necessary to introduce some basic relations. The following definitions present the intersection and the inclusion of t-profiles, denoted as *t-intersections* and *t-inclusions*.

Definition 14 *(t-intersection).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*. Let S_1 and S_2 be two collections of *t-profiles*. We define the t-intersection of S_1 and S_2, denoted $\mathsf{S}_1 \cap_t \mathsf{S}_2$, as the collection of t-profiles such that:

$$\mathsf{S}_1 \cap_t \mathsf{S}_2 = \{(X, \mathcal{T}_\mathcal{X} \cap \mathcal{T}'_\mathcal{X}) \mid \langle \mathcal{X}, \mathcal{T}_\mathcal{X}\rangle \in \mathsf{S}_1, \langle \mathcal{X}, \mathcal{T}'_\mathcal{X}\rangle \in \mathsf{S}_2, \text{and}\, \mathcal{T}_\mathcal{X} \cap \mathcal{T}'_\mathcal{X} \neq \emptyset\}$$

Definition 15 *(t-inclusion).* Let S_1 and S_2 be two collections of *t-profiles*. We say that S_1 is *t-included* in S_2, denoted as $\mathsf{S}_1 \subseteq_t \mathsf{S}_2$, if for any t-profile $\langle \mathcal{X}, \mathcal{T}_\mathcal{X}\rangle \in \mathsf{S}_1$ there exists a t-profile $\langle \mathcal{X}, \mathcal{T}'_\mathcal{X}\rangle \in \mathsf{S}_2$ such that $\mathcal{T}_\mathcal{X} \subseteq \mathcal{T}'_\mathcal{X}$.

In *T-BAF*, given a collection of t-profiles, a sequence of t-profiles from the existing relations between the arguments that are involved in them.

Definition 16 *(Sequence of T-profiles).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*. Let $\mathsf{S} = \{\ \langle \mathcal{X}_1, \mathcal{T}_{\mathcal{X}_1}\rangle, \langle \mathcal{X}_2, \mathcal{T}_{\mathcal{X}_2}\rangle, \cdots, \langle \mathcal{X}_n, \mathcal{T}_{\mathcal{X}_n}\rangle\ \}$ be a collection of t-profiles where $\mathcal{X}_i \in Args$, $1 \leq i \leq n$. We say that S is a sequence of t-profiles if $\forall i = 1 \ldots n-1$ $(\mathcal{X}_i, \mathcal{X}_{i+1}) \in \mathrm{R}_s$ or $(\mathcal{X}_i, \mathcal{X}_{i+1}) \in \mathrm{R}_a$, where $\bigcap_{i=1}^{n} \mathcal{T}_{\mathcal{X}_i} \neq \emptyset$.

The following definitions reformulate previous *BAF* formalizations considering t-profiles instead of arguments. First, we will define the central notions of direct, supported and secondary attack over time in *T-BAF*.

Definition 17 *(Direct Attack over Time).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*, S the associated collection of t-profiles, and $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ and $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$ two t-profiles of S. We say that there exists a direct attack from $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ to $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$ iff $(\mathcal{A}, \mathcal{B}) \in \mathrm{R}_a$ and not exits a t-profile $\langle \mathcal{C}, \mathcal{T}_\mathcal{C}\rangle$ in S verifying that $(\mathcal{C}, \mathcal{A}) \in \mathrm{R}_s$. The time interval in which $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ direct attacks $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$, denoted as $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{B})}$, is defined as $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{B})} = \mathcal{T}_\mathcal{A} \cap \mathcal{T}_\mathcal{B}$.

Definition 18 *(Supported Attack over Time).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*, S the associated collection of t-profiles, $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ and $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$ two t-profiles of S, and $\langle \mathcal{A}_1, \mathcal{T}_{\mathcal{A}_1}\rangle$ $\langle \mathcal{A}_2, \mathcal{T}_{\mathcal{A}_2}\rangle$ $\cdots$ $\langle \mathcal{A}_{n-1}, \mathcal{T}_{\mathcal{A}_{n-1}}\rangle$ $\langle \mathcal{A}_n, \mathcal{T}_{\mathcal{A}_n}\rangle$ be a sequence of t-profiles in S, with $n \geq 3$, $\langle \mathcal{A}_1, \mathcal{T}_{\mathcal{A}_1}\rangle = \langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ and $\langle \mathcal{A}_n, \mathcal{T}_{\mathcal{A}_n}\rangle = \langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$, such that $\forall i = 1...n-2$ $(A_i, A_{i+1}) \in \mathrm{R}_s$ and $(A_{n-1}, A_n) \in \mathrm{R}_a$. The time interval in which $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ supported attacks $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$, denoted as $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{B})}$, is defined as $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{B})} = \bigcap_{i=1}^{n} \mathcal{T}_{\mathcal{A}_i}$.

Definition 19 *(Secondary Attack over Time).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*, S the associated collection of t-profiles, $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ and $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$ two t-profiles of S, and $\langle \mathcal{A}_1, \mathcal{T}_{\mathcal{A}_1}\rangle$ $\langle \mathcal{A}_2, \mathcal{T}_{\mathcal{A}_2}\rangle$ $\cdots$ $\langle \mathcal{A}_{n-1}, \mathcal{T}_{\mathcal{A}_{n-1}}\rangle$ $\langle \mathcal{A}_n, \mathcal{T}_{\mathcal{A}_n}\rangle$ be a sequence of t-profiles in S, with $n \geq 3$, $\langle \mathcal{A}_1, \mathcal{T}_{\mathcal{A}_1}\rangle = \langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ and $\langle \mathcal{A}_n, \mathcal{T}_{\mathcal{A}_n}\rangle = \langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$, such that $(A_1, A_2) \in \mathrm{R}_a$ and $\forall i = 2...n-1$, $(A_i, A_{i+1}) \in \mathrm{R}_s$. The time interval in which $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ secondary attacks $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$, denoted as $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{B})}$, is defined as $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{B})} = \bigcap_{i=1}^{n} \mathcal{T}_{\mathcal{A}_i}$.

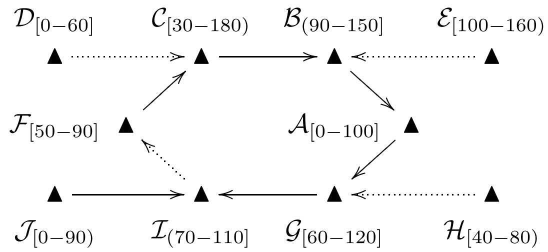

Fig. 4. Bipolar argumentation graph with timed availability.

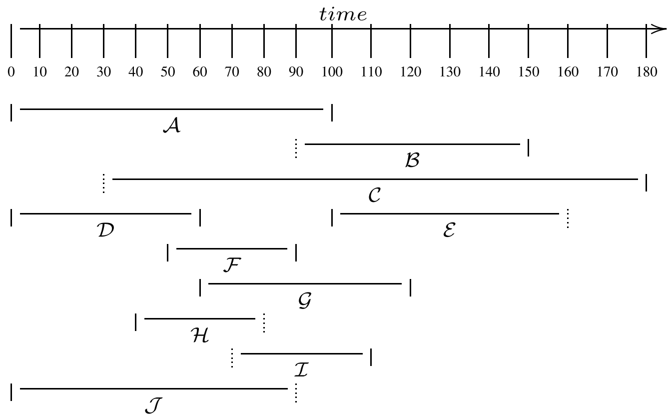

Fig. 5. Temporal distribution.

The following example shows the modelization of availability applied to the arguments presented in Example 1.

Example 6. Given a *T-BAF* $\Omega = \langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$, where:

$\texttt{Arg} = \{\mathcal{A}; \mathcal{B}; \mathcal{C}; \mathcal{D}; \mathcal{E}; \mathcal{F}; \mathcal{G}; \mathcal{H}; \mathcal{I}; \mathcal{J}\}$,
$\texttt{R}_a = \{(\mathcal{B}, \mathcal{A}); (\mathcal{A}, \mathcal{H}); (\mathcal{C}, \mathcal{B}); (\mathcal{G}, \mathcal{I}); (\mathcal{J}, \mathcal{I}); (\mathcal{F}, \mathcal{C})\}$,
$\texttt{R}_s = \{(\mathcal{D}, \mathcal{C}); (\mathcal{H}, \mathcal{G}); (\mathcal{I}, \mathcal{F}); (\mathcal{E}, \mathcal{B})\}$, and
$\texttt{Av} = \{\langle \mathcal{A}, \{[0-100]\}\rangle; \langle \mathcal{B}, \{(90-150]\}\rangle; \langle \mathcal{C}, \{[30-180]\}\rangle; \langle \mathcal{D}, \{[0-60]\}\rangle;$
$\langle \mathcal{E}, \{[100-160)\}\rangle; \langle \mathcal{F}, \{[50-90]\}\rangle; \langle \mathcal{G}, \{[60-120]\}\rangle; \langle \mathcal{H}, \{[40-80]\}\rangle;$
$\langle \mathcal{I}, \{(70-110]\}\rangle; \langle \mathcal{J}, \{[0-90)\}\rangle\}$.

The timed bipolar argumentation framework Ω is shown in the graph of Fig. 4, and the temporal distribution is summarized in Fig. 5. Argument $\mathcal{J}$ is a direct attacker of $\mathcal{I}$ in the intervals $\mathcal{T}^{Dir}_{(\mathcal{J}\text{-}\mathcal{I})} = \mathcal{T}_{\mathcal{J}} \cap \mathcal{T}_{\mathcal{I}} = \{(70-90)\}$, and also argument $\mathcal{J}$ secondary-attacks $\mathcal{F}$ in intervals $\mathcal{T}^{Sup}_{(\mathcal{J}\text{-}\mathcal{F})} = \mathcal{T}_{\mathcal{J}} \cap \mathcal{T}_{\mathcal{I}} \cap \mathcal{T}_{\mathcal{F}} = \{(70-90)\}$. Note that argument $\mathcal{D}$ seems to support-attack $\mathcal{B}$ through $\mathcal{C}$, but this is not true since the time interval $\mathcal{T}^{Sup}_{(\mathcal{D}\text{-}\mathcal{B})} = \{\emptyset\}$, where $\mathcal{T}^{Sup}_{(\mathcal{D}\text{-}\mathcal{B})} = \mathcal{T}_{\mathcal{A}} \cap \mathcal{T}_{\mathcal{C}} \cap \mathcal{T}_{\mathcal{B}} = \{\emptyset\}$. On the other hand, there exists a support-attack from $\mathcal{E}$ to $\mathcal{A}$ through $\mathcal{B}$ in the time interval $\mathcal{T}^{Sup}_{(\mathcal{E}\text{-}\mathcal{A})} = \mathcal{T}_{\mathcal{E}} \cap \mathcal{T}_{\mathcal{B}} \cap \mathcal{T}_{\mathcal{A}} = \{[100-100]\}$.

Once defined the relations of attack over time using the t-profiles, the notions of conflict-freeness and safeness presented in *BAF* can be adapted here, now considering time references, as follows.

Definition 20 *(Conflict-free and Safe).* Let $\Omega = \langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$ be a *T-BAF*, and S be a collection of t-profiles defined for Ω.

- S is *Conflict-free* iff $\nexists \langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle, \langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle \in \mathsf{S}$ such that $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$ or $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$ or $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$.
- S is *Safe* iff $\nexists \langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle, \langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle \in \mathsf{S}$ and $\nexists \langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle$ where $\langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle$ is a valid Ω's t-profile such that $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$ or $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$ or $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$, and either there is a sequence of support from $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ to $\langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle$, or $\langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle \in \mathsf{S}$.

In addition, another requirement has been considered in [11], which concerns only the support relation, namely the *closure under* $\texttt{R}_s$. This is presented in a timed context as follows.

Definition 21 *(Closure in T-BAF).* Let $\Omega = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*, and S be a collection of t-profiles defined for Ω. The set S is closed under $\mathtt{R}_s$ iff $\forall\ \langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle \in \mathsf{S}$, $\forall\ \langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ where $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ is a valid Ω's t-profile: if $(\mathcal{A}, \mathcal{B}) \in \mathtt{R}_s$ where $\mathcal{T}_{\mathcal{A}} \cap \mathcal{T}_{\mathcal{B}} \neq \emptyset$ then at least $\langle \mathcal{B}, \mathcal{T}_{\mathcal{A}} \cap \mathcal{T}_{\mathcal{B}}\rangle \in \mathsf{S}$.

Example 7. The collection $\mathsf{S}_1 = \{\langle \mathcal{A}, \{[0-100]\}\rangle; \langle \mathcal{C}, \{[30-50), (90-180)\}\rangle; \langle \mathcal{D}, \{[0-60]\}\rangle; \langle \mathcal{E}, \{[100-160)\}\rangle; \langle \mathcal{F}, \{[50-90]\}\rangle; \langle \mathcal{G}, \{[80-120]\}\rangle; \langle \mathcal{J}, \{[0-90)\}\rangle\}$ is conflict-free but not safe, since the argument $\mathcal{D}$ supports $\mathcal{C}$ and $\mathcal{F}$ attacks $\mathcal{C}$ in the time interval set $\{[50-60]\}$; On another case, the argument $\mathcal{B}$ is supported and attacked by $\mathcal{E}$ and $\mathcal{C}$, respectively, in the time interval set $\{[100-150]\}$. The collection $\mathsf{S}_2 = \{\langle \mathcal{A}, \{[0-100]\}\rangle; \langle \mathcal{D}, \{[0-50)\}\rangle; \langle \mathcal{C}, \{[30-50), (90-100), (150-180)\}\rangle; \langle \mathcal{E}, \{(150-160)\}\rangle; \langle \mathcal{F}, \{(60-90]\}\rangle; \langle \mathcal{G}, \{[80-120]\}\rangle; \langle \mathcal{J}, \{[0-90)\}\rangle\}$ is conflict-free and safe.

The timed notion of closure proposes maximality in the support relation only when both arguments are available: if an argument $\mathcal{A}$ is supported by members of the collection S in a specific period of time $\mathcal{T}_{\mathcal{A}}$, then $\mathcal{A}$ is required to belong to S in precisely $\mathcal{T}_{\mathcal{A}}$. This is the mathematical concept of closure applied to the support relation in a timed context, and it is useful as a link between some semantic notions, as introduced in the following proposition.

Proposition 1. *Let* S *be a collection of t-profiles:*

- *If* S *is conflict-free and closed for* $\mathtt{R}_s$ *then* S *is safe.*
- *If* S *is safe then* S *is conflict-free.*
- *If* S *is safe, then any collection* $\mathsf{S}' \subseteq_t \mathsf{S}$ *is safe.*

The next definitions reformulate *BAF* notions under a timed context, by considering t-profiles instead of arguments. We define the *defense* of an argument over time, taking into account the corresponding direct, support and secondary attacks as follows.

Definition 22 *(Defense of $\mathcal{A}$ from $\mathcal{B}$ by a collection* S*).* Let $\Omega = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*, and S be a collection of t-profiles. Let $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$ and $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ be two t-profiles in S, where $\mathcal{B}$ attacks $\mathcal{A}$ through a support, secondary or direct attack (i.e., $\mathcal{T}^{Sec}_{(\mathcal{B}\text{-}\mathcal{A})} \neq \emptyset$ and/or $\mathcal{T}^{Sup}_{(\mathcal{B}\text{-}\mathcal{A})} \neq \emptyset$ and/or $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$). The defense t-profile of $\mathcal{A}$ from $\mathcal{B}$ with respect to S, denoted as $\mathcal{T}^{\mathcal{B}}_{(\mathcal{A}|\mathsf{S})}$, is defined as follows:

$$\mathcal{T}^{\mathcal{B}}_{(\mathcal{A}|\mathsf{S})} =_{def} \mathtt{Av}(\mathcal{A}) \cap (Sup(\mathcal{B}, \mathsf{S}) \cup Sec(\mathcal{B}, \mathsf{S}) \cup Dir(\mathcal{B}, \mathsf{S})) \text{ where}$$

$$Dir(\mathcal{B}, \mathsf{S}) =_{def} \bigcup_{\{\mathcal{X} \mid \langle \mathcal{X}, \mathcal{T}_{\mathcal{X}}\rangle \in \mathsf{S},\ \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{B})} \neq \emptyset\}} \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{B})},$$
$$Sup(\mathcal{B}, \mathsf{S}) =_{def} \bigcup_{\{\mathcal{X} \mid \langle \mathcal{X}, \mathcal{T}_{\mathcal{X}}\rangle \in \mathsf{S},\ \mathcal{T}^{Sup}_{(\mathcal{X}\text{-}\mathcal{B})} \neq \emptyset\}} \mathcal{T}^{Sup}_{(\mathcal{X}\text{-}\mathcal{B})}, \text{ and}$$
$$Sec(\mathcal{B}, \mathsf{S}) =_{def} \bigcup_{\{\mathcal{X} \mid \langle \mathcal{X}, \mathcal{T}_{\mathcal{X}}\rangle \in \mathsf{S},\ \mathcal{T}^{Sec}_{(\mathcal{X}\text{-}\mathcal{B})} \neq \emptyset\}} \mathcal{T}^{Sec}_{(\mathcal{X}\text{-}\mathcal{B})}.$$

Intuitively, $\mathcal{A}$ is defended from the attack of $\mathcal{B}$ in those intervals where the attacker $\mathcal{B}$ is available but it is attacked by an argument $\mathcal{C}$ in the collection S. Also, if $\mathcal{B}$ is not available, then any set of arguments defends $\mathcal{A}$.

Acceptability also requires adaptation according to the time dimension. An acceptable t-profile $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$ with respect to a set S of t-profiles represents the intervals of time in which argument $\mathcal{A}$ is acceptable with respect to S. This means only those moments in which actually a defense occurs, or there is a lack of attackers. This is formalized in the following definition.

Definition 23 *(Acceptable t-profile of $\mathcal{A}$ w.r.t.* S*).* Let $\Omega = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*. The acceptable t-profile for $\mathcal{A}$ w.r.t. S, denoted as $\mathcal{T}_{(\mathcal{A}|\mathsf{S})}$, is defined as follows:

$$\mathcal{T}_{(\mathcal{A}|\mathsf{S})} =_{def} \bigcap_{\mathcal{B} \in H} \left(\mathtt{Av}(\mathcal{A}) \setminus \left(\mathcal{T}^{Sup}_{(\mathcal{B}\text{-}\mathcal{A})} \cup \mathcal{T}^{Sec}_{(\mathcal{B}\text{-}\mathcal{A})} \cup \mathcal{T}^{Dir}_{(\mathcal{B}\text{-}\mathcal{A})}\right)\right) \cup \mathcal{T}^{\mathcal{B}}_{(\mathcal{A}|\mathsf{S})}$$

where $H = \{\mathcal{X} \mid \mathcal{T}^{Sup}_{(\mathcal{X}\text{-}\mathcal{A})} \neq \emptyset \ \vee\ \mathcal{T}^{Sec}_{(\mathcal{X}\text{-}\mathcal{A})} \neq \emptyset \ \vee\ \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{A})} \neq \emptyset\}$ and $\mathcal{T}^{\mathcal{B}}_{(\mathcal{A}|\mathsf{S})}$ is the time interval where $\mathcal{A}$ is defended of its attacker $\mathcal{B}$ by S. Then, the intersection of all time intervals in which $\mathcal{A}$ is defended from each of its attackers by the collection S, is the time interval where $\mathcal{A}$ is available and is acceptable with respect to S.

Example 8. In the framework of Example 6, the acceptable t-profile of $\mathcal{I}$ from a collection $\mathsf{S}_3 = \{\langle \mathcal{A}, \{[0-100]\}\rangle$ is:

$$\mathcal{T}_{(\mathcal{I}|\mathsf{S}_3)} = \left(\mathtt{Av}(\mathcal{I}) \setminus \left(\mathcal{T}^{Sec}_{(\mathcal{H}\text{-}\mathcal{I})} \cup \mathcal{T}^{Dir}_{(\mathcal{G}\text{-}\mathcal{I})}\right)\right) \cup \left(\mathcal{T}^{\mathcal{G}}_{(\mathcal{A}|\mathsf{S}_3)} \cup \mathcal{T}^{\mathcal{H}}_{(\mathcal{A}|\mathsf{S}_3)}\right) =$$
$$= (70-110] \setminus ((70-80) \cup (70-110])) \cup ([40-80] \cup [60-100]) = [40-100]$$

Having an adapted notion of acceptability, timed-admissibility can be defined. Here, we adapt the three different definitions for admissibility proposed by Cayrol and Lagasquie-Schiex in [11], considering the new version of conflict-freeness and safety.

Definition 24 *(Admissibility in T-BAF).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*. We say that S defends all its elements if and only if $\forall \mathcal{A} \in \mathrm{S}$, if $\exists \mathcal{B} \in \mathtt{Arg}$ such that $\mathcal{B}$ attacks $\mathcal{A}$ (Definition 22). Let S be a collection of t-profiles. The admissibility of a collection S is defined as follows:

- S is td-admissible if S is conflict-free and defends all its elements.
- S is ts-admissible if S is safe and defends all its elements.
- S is tc-admissible if S conflict-free, closed for R_s and defends all its elements.

Example 9. The collection $\mathrm{S}_4 = \{\langle \mathcal{A}, \{[0-100)\}\rangle; \langle \mathcal{C}, \{[30-50), (70-180)\}\rangle; \langle \mathcal{D}, \{[0-60]\}\rangle; \langle \mathcal{E}, \{[100-160)\}\rangle; \langle \mathcal{F}, \{[50-70]\}\rangle; \langle \mathcal{G}, \{(80-120]\}\rangle; \langle \mathcal{J}, \{[0-90)\}\rangle\}$ is td-admissible since it is conflict-free and defends all its elements over time. However, S_4 is not a ts-admissible or tc-admissible collection of t-profiles. $\mathrm{S}_5 = \{\langle \mathcal{A}, \{[0-100]\}\rangle; \langle \mathcal{C}, \{[30-50), (70-180)\}\rangle; \langle \mathcal{D}, \{[0-60]\}\rangle; \langle \mathcal{E}, \{(150-160)\}\rangle; \langle \mathcal{F}, \{[50-70]\}\rangle; \langle \mathcal{G}, \{(80-120]\}\rangle; \langle \mathcal{J}, \{[0-90)\}\rangle\}$ is ts-admissible because its safe and defends all its elements. In addition, S_5 is closed for R_s, then it is tc-admissible.

Proposition 2. *Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a T-BAF, then:*

- *A ts-admissible extension is t-included in a td-admissible extension.*
- *A tc-admissible extension is t-included in a ts-admissible extension.*

Now we can define the corresponding classic argument semantics for *T-BAF*. First, we present the stable extension with a dynamic intuition. Then we introduce an adapted, timed version of preferred extension. Each one has a special property based on the admissibility notion.

Definition 25 *(Stable extension over Time).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*. Let S be a collection of t-profiles, such that for all $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle \notin \mathrm{S}$, it verifies that $\mathcal{T}_\mathcal{A} \setminus (\bigcup \mathcal{T}^{Sec}_{(\mathcal{B}\text{-}\mathcal{A})} \cup \bigcup \mathcal{T}^{Sup}_{(\mathcal{B}\text{-}\mathcal{A})} \cup \bigcup \mathcal{T}^{Dir}_{(\mathcal{B}\text{-}\mathcal{A})}) = \emptyset$ for all $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle \in \mathrm{S}$. Then, S is a td-stable (resp. ts-stable extension, tc-stable extension) extension of Ω if S is conflict-free (resp. safe, closed under R_s).

Note that stableness does not imply closure. A collection S may be stable, attacking a t-profile $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ not in S. At the same time, $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ may be supported by a t-profile in S in the intervals time $\mathcal{T}_\mathcal{A}$. Hence, it is not closed. However, if a collection S is stable and safe, then it is closed. Since it is safe, it does not support and attack a specific t-profile at the same time. It means that attacked t-profiles outside S are not supported by S. Then, every t-profile supported by S is in S. Also, if S is stable and not safe, then S is not closed. These relations are expressed in the following proposition.

Proposition 3. *Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a T-BAF, then:*

- *A td-stable extension is not necessarily closed under R_s.*
- *A ts-stable extension is also a td-stable extension.*
- *A tc-stable extension is also a ts-stable extension.*

In addition, since the main difference between tc-stable and td-stable is that the first one grants closure under R_s and the second one does not, then the following relation can be established.

Corollary 1. *A tc-stable extension is strictly included in a td-stable extension.*

As expected, a preferred-like extension can be defined using these varied admissible notions. Again, it can be graded according the attacks involved. This is formalized as follows, and the relation between these semantics is stated next.

Definition 26 *(Preferred extension over Time).* Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ be a *T-BAF*. Let S be a collection of t-profiles. S is a td-preferred (resp. ts-preferred, tc-preferred) extension if S is maximal (for set-t-inclusion) among the td-admissible (resp. ts-admissible, tc-admissible).

In our example, the set of arguments S_4 is the td-stable extension, since there exist an attacker for each t-profile that does not belong to S_4. However, S_4 is not a ts-stable or tc-stable extensions, since S_4 is not safe or closed under R_s, respectively. In addition, S_4 is a td-preferred extension since it is the maximal td-admissible collection of t-profiles that defends all of its elements. On the other hand, S_5 is a ts-preferred extension because it is the maximal ts-admissible collection of t-profiles that defends all of its elements. Also, S_5 is closed by R_s, then it is tc-preferred extension.

Proposition 4. *Let* $\Omega = \langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$ *be a T-BAF, then:*

- *A ts-preferred extension is t-included in a td-preferred extension.*
- *A tc-preferred extension is t-included in a ts-preferred extension.*
- *A ts-preferred extension closed under* $\texttt{R}_s$ *is also a tc-preferred.*

The relations between the temporal preferred extensions and the temporal stable extensions are stated in the following proposition. Note that these are consistent with the classical *BAF*.

Proposition 5. *Let* $\Omega = \langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$ *be a T-BAF, then:*

- *A td-stable extension is t-included in a td-preferred extension.*
- *A ts-stable extension is t-included in a ts-preferred extension.*
- *A ts-stable extension is t-included in a tc-preferred extension.*

Given a *T-BAF* $\Omega = \langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$ and an argument $\mathcal{A} \in \texttt{Arg}$, we use $t\text{-}PR_d(\mathcal{A})$, $t\text{-}PR_s(\mathcal{A})$, $t\text{-}PR_c(\mathcal{A})$, $t\text{-}ES_d(\mathcal{A})$, $t\text{-}ES_s(\mathcal{A})$, and $t\text{-}ES_c(\mathcal{A})$ to denote the set of intervals on which $\mathcal{A}$ is acceptable in Ω according to td-preferred, ts-preferred, tc-preferred, td-stable, ts-stable and tc-stable semantics respectively, using the skeptical approach where it corresponds. The following theorem establishes a connection between acceptability in our extended temporal framework T-BAF and acceptability in Cayrol and Lagasquie-Schiex's frameworks.

Theorem 1. *Let* $\Omega = \langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$ *be a T-BAF and let* α *represent a point in time. Let* $\Theta'_\alpha = \langle \texttt{Arg}'_\alpha, \texttt{R}_a^\alpha, \texttt{R}_s^\alpha\rangle$ *be a bipolar abstract framework obtained from* Ω *in the following way:*

$$\texttt{Arg}'_\alpha = \{\mathcal{A} \in \texttt{Arg} \mid \alpha \in \mathcal{T}_\mathcal{A}\},$$
$$\texttt{R}_a^\alpha = \{(\mathcal{A}, \mathcal{B}) \in \texttt{R}_a \mid \alpha \in \texttt{Av}(\mathcal{A}) \cap \texttt{Av}(\mathcal{B})\} \text{ and}$$
$$\texttt{R}_s^\alpha = \{(\mathcal{A}, \mathcal{B}) \in \texttt{R}_s \mid \alpha \in \texttt{Av}(\mathcal{A}) \cap \texttt{Av}(\mathcal{B})\}.$$

Let S *be a collection of t-profiles in* Ω *and* $\mathsf{S}'_\alpha = \{\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle \in \mathsf{S} \mid \alpha \in \mathcal{T}_\mathcal{A}\}$. *It holds that, if* S *is a td-preferred extension (resp. ts-preferred, tc-preferred, td-stable, ts-stable and tc-stable) w.r.t.* Ω, *then* S'_α *is a d-preferred extension (resp. s-preferred, c-preferred, d-stable, s-stable and c-stable) w.r.t.* Θ'_α.

Intuitively, the BAF Θ'_α represents a *snapshot* of the T-BAF framework Ω at the time point α, where arguments and attacks in Θ'_α are those available at α in Ω. Then, this theorem states that a T-BAF "frozen" at a particular moment is essentially a classical, bipolar argumentation framework. Hence, the semantics of Θ'_α and Ω coincide.

In addition, we formally establish that two conflictive arguments cannot be available at the same time when belonging to the same extension in a given semantics.

Proposition 6. *Let* $\Omega = \langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$ *be a T-BAF, and* $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ *and* $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$ *be two t-profiles, where* $\mathcal{B}$ *attacks* $\mathcal{A}$ *through a direct, support or secondary attacks, then it holds that:*

- $t\text{-}ES_d(\mathcal{A}) \cap t\text{-}ES_d(\mathcal{B}) = \emptyset$;
- $t\text{-}ES_s(\mathcal{A}) \cap t\text{-}ES_s(\mathcal{B}) = \emptyset$;
- $t\text{-}ES_c(\mathcal{A}) \cap t\text{-}ES_c(\mathcal{B}) = \emptyset$;
- $t\text{-}PR_d(\mathcal{A}) \cap t\text{-}PR_d(\mathcal{B}) = \emptyset$;
- $t\text{-}PR_s(\mathcal{A}) \cap t\text{-}PR_s(\mathcal{B}) = \emptyset$; *and*
- $t\text{-}PR_c(\mathcal{A}) \cap t\text{-}PR_c(\mathcal{B}) = \emptyset$.

It is important to remark that when there are no arguments involved in a support relation, the acceptability semantics coincides with the classical timed argumentation framework, as shown in the following theorem.

Theorem 2. *Let* $\Omega = \langle \texttt{Arg}, \texttt{R}_a, \texttt{R}_s, \texttt{Av}\rangle$ *be a T-BAF and let* $\Phi = \langle \texttt{Arg}, \texttt{R}_a, \texttt{Av}\rangle$ *be a* TAF. *If* $\texttt{R}_s = \emptyset$ *then any* $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ *acceptable with respect to a set* S *in* Ω *is acceptable in* Φ *with respect to the same set* S.

Thus, the T-BAF is an extension of TAF that considers support relation, but it is consistent with the original TAF semantics. Theorem 1 and Theorem 2 show that the novel framework proposed in this work, can be reduced to a classical formalism when time and support relation are ignored respectively.

In the following section we evaluate an application example in detail and later we discuss related and future work.

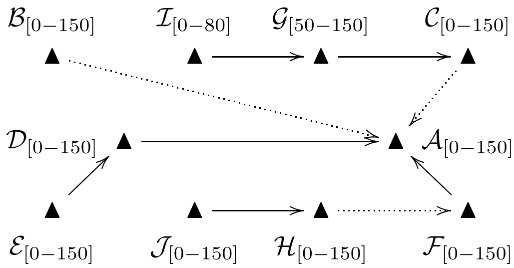

Fig. 6. Bipolar argumentation graph with timed availability.

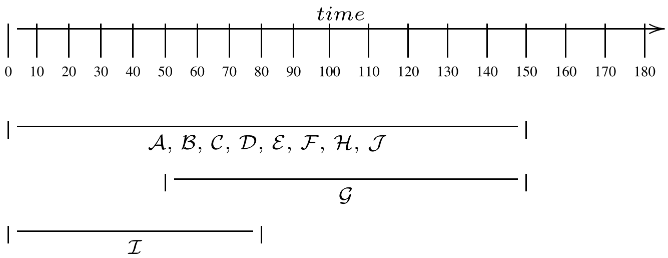

Fig. 7. Temporal distribution.

5. Application example

As stated before, the aim of this work is to increase the representational capability of *BAF*s, by the addition of a temporal dimension towards a model of dynamic argumentation discussion. In order to illustrate this direction in the context of agents systems, we discuss here an example where the formalism provides a better characterization of the overall situation.

Consider the following scenario where an agent is looking for an apartment to rent. As expected, while considering a candidate she analyzes different arguments for and against renting such an apartment. These arguments are subject to availability or relevance in time. The task is to determine in the present (time 0) if the property is a good option in the future, counting with 150 days to make such a decision. The arguments and the availability intervals follows.

$\mathcal{A}$ *She should rent it, since the apartment has a good location as it is near of her work.* $[0-150]$
$\mathcal{B}$ *The apartment is located in a well illuminated and safe area.* $[0-150]$
$\mathcal{C}$ *The property is in a quiet area, because most of the neighbors are retirees and peaceful people.* $[0-150]$
$\mathcal{D}$ *The apartment is small; therefore, she should not rent it.* $[0-150]$
$\mathcal{E}$ *Despite the apartment size, the spaces are well distributed.* $[0-150]$
$\mathcal{F}$ *She should not rent it, since the apartment seems to have humidity problems.* $[0-150]$
$\mathcal{G}$ *A nightclub is open in the area, so it is not a quite zone at night.* $[50-150]$
$\mathcal{H}$ *The humidity problems are difficult and costly to resolve.* $[0-150]$
$\mathcal{I}$ *Laws forbid the opening of a nightclub in this urban area.* $[0-80]$
$\mathcal{J}$ *The person responsible for maintenance is committed to fixing the humidity problem at a low cost.* $[0-150]$

Argument $\mathcal{G}$ refers to the presence of a night club, but this is now under construction and it will be ready in 50 days and hence the availability interval is $[50-150]$. Argument $\mathcal{I}$ refers to a local law about night clubs, which is in the process of being revised by the Town Hall in 80 days, hence the interval is $[0-80]$. The corresponding *T-BAF* $\Omega = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s, \mathtt{Av}\rangle$ is defined as (see Figs. 6 and 7):

$$\mathtt{Arg} = \{\mathcal{A}; \mathcal{B}; \mathcal{C}; \mathcal{D}; \mathcal{E}; \mathcal{F}; \mathcal{G}; \mathcal{H}; \mathcal{I}; \mathcal{J}\},$$

$$\mathtt{R}_a = \{(\mathcal{B}, \mathcal{A}); (\mathcal{C}, \mathcal{A}); (\mathcal{H}, \mathcal{F})\},$$

$$\mathtt{R}_s = \{(\mathcal{E}, \mathcal{D}); (\mathcal{D}, \mathcal{A}); (\mathcal{I}, \mathcal{G}); (\mathcal{G}, \mathcal{C}); (\mathcal{F}, \mathcal{A}); (\mathcal{J}, \mathcal{H})\}, \text{ and}$$

$$\mathtt{Av} = \{\langle \mathcal{A}, \{[0-150]\}\rangle; \langle \mathcal{B}, \{[0-150]\}\rangle; \langle \mathcal{C}, \{[0-150]\}\rangle; \langle \mathcal{D}, \{[0-150]\}\rangle;$$
$$\langle \mathcal{E}, \{[0-150]\}\rangle; \langle \mathcal{F}, \{[0-150]\}\rangle; \langle \mathcal{G}, \{[50-150]\}\rangle; \langle \mathcal{H}, \{[0-150]\}\rangle;$$
$$\langle \mathcal{I}, \{[0-80]\}\rangle; \langle \mathcal{J}, \{[0-150]\}\rangle\}.$$

Let's analyze a couple of collections of t-profiles in order to determine which of them are conflict-free and safe. On one hand, we have the collection $\mathsf{S}_1 = \{\langle \mathcal{C}, \{[0-80]\}\rangle; \langle \mathcal{G}, \{(80-150]\}\rangle; \langle \mathcal{E}, \{[0-80)\}\rangle; \langle \mathcal{F}, \{[0-150]\}\rangle\}$ which is conflict-free but not safe, since the argument $\mathcal{C}$ supports $\mathcal{A}$ and $\mathcal{F}$ attacks $\mathcal{A}$ in the time interval set $\{[0-80]\}$. On the other hand, the collection $\mathsf{S}_2 = \{\langle\mathcal{C}, \{[0-80]\}\rangle; \langle\mathcal{G}, \{(80-150]\}\rangle; \langle\mathcal{E}, \{[0-80)\}\rangle; \langle\mathcal{F}, \{(80-150]\}\rangle; \langle\mathcal{A}, \{[0-80]\}\rangle\}$ is conflict-free and safe, since the argument $\mathcal{C}$ that supports $\mathcal{A}$ is available when $\mathcal{F}$ (that attacks $\mathcal{A}$) is not ($\mathcal{C}$ is available in the time interval $[0-80]$, while $\mathcal{F}$ is available in the interval $(80-150]$).

Let's determine the collection of t-profile that represents a t-stable extension. In this case, the collection $\mathsf{S}_3 = \{\langle\mathcal{C}, \{[0-80]\}\rangle; \langle\mathcal{G}, \{(80-150]\}\rangle; \langle\mathcal{E}, \{[0-150]\}\rangle; \langle\mathcal{A}, \{[0-80]\}\rangle; \langle\mathcal{I}, \{[0-80]\}\rangle; \langle\mathcal{J}, \{[0-150]\}\rangle; \langle\mathcal{B}, \{[0-150]\}\rangle\}$ is a safe t-stable extension, since it is conflict-free and attacks all the t-profiles not considered in S_3. In this case these t-profiles are:

$\langle\mathcal{C}, \{(80-150]\}\rangle$	support defeated by $\langle\mathcal{G}, \{[80-150]\}\rangle$
$\langle\mathcal{G}, \{[50-80]\}\rangle$	support defeated by $\langle\mathcal{I}, \{[0-80]\}\rangle$
$\langle\mathcal{D}, \{[0-50]\}\rangle$	support defeated by $\langle\mathcal{E}, \{[0-150]\}\rangle$
$\langle\mathcal{H}, \{[0-150]\}\rangle$	support defeated by $\langle\mathcal{J}, \{[0-150]\}\rangle$
$\langle\mathcal{F}, \{[0-150]\}\rangle$	secondary defeated by $\langle\mathcal{J}, \{[0-150]\}\rangle$ and $\langle\mathcal{H}, \{[0-150]\}\rangle$
$\langle\mathcal{A}, \{(80-150]\}\rangle$	secondary defeated by $\langle\mathcal{G}, \{[80-150]\}\rangle$ and $\langle\mathcal{C}, \{(80-150]\}\rangle$

Applying Proposition 3 that relates through t-inclusion the td-preferred, ts-preferred and tc-preferred extensions it is possible to state that the collection of t-profiles S_3, which is a safe and t-stable, is also td-preferred, ts-preferred and tc-preferred. We only need to show that this extension is td-preferred. This means that S_3 should be maximal and td-acceptable (i.e. conflict-free and must defends all its elements). We have already shown that S_3 is conflict-free and by attacking all the t-profiles that do not belong to S_3 we can assure its maximality in acceptability. In this particular case, S_3 is a safe t-stable, td-preferred, ts-preferred and tc-preferred extension. This situation occurs because the bipolar argumentation Ω does not include cycles. The graph representing our example is acyclic *in every moment of time*. As we see in the abstract example, the difference is produced by cycles of support and attacks, as it was proved elsewhere [5] for classical, non-bipolar argumentation frameworks.

The association of temporal information to arguments towards a refined, timed reasoning process is presented in diverse domains of application, such as recommender systems [16,17], multi-agents systems [18,19], legal reasoning [20,21], Defeasible Reasoning in Social Nets [22,23], among others. The intuition behind this idea is that all the domains that represent real-world situations are affected by the passage of time by the occurrence of certain events. We discuss related proposals in the next section.

6. Related work

As discussed in the introduction, reasoning about time is an important concern in commonsense reasoning. Thus, its consideration becomes relevant when modeling argumentation capabilities of intelligent agents [3]. There have been recent advances in modeling time in argumentation frameworks. Mann and Hunter in [24] propose a calculus for representing temporal knowledge, which is defined in terms of propositional logic. The use of this calculus is then considered with respect to argumentation, where an argument is defined in the standard way: an argument is a pair constituted by a minimally consistent subset of a database entailing its conclusion. Briefly speaking, the authors discuss a way of encoding temporal information into propositional logic, and examined its impact in a coherence argumentation system. The central idea is that they draw heavily on temporal knowledge due to their day-to-day nature – what is true today may well not be true tomorrow – as well as their inclusion of information concerning periods of time. In order to represent time variable, the authors propose a calculus built upon the ideas of Allen's interval logic [25] using abstract intervals, and in keeping with the desire for a practical system, they restrict the system to using specific timepoints. This work is related to the works proposed by Hunter in [26] and Augusto and Simari in [7]. Hunter's system is based on maximally consistent subsets of the knowledge base, which are now not normally regarded as representative of arguments, while Augusto and Simari's contribution is based upon a many sorted logic with defeasible formula, and hence also falls into a different category of argumentation, and the use of many sorted logic raises similar concerns to that of using first order logic. These formalisms are not abstract, since they provide a well-specified language to construct arguments and produce conflicts. In order to use T-BAF as an abstract model for concrete systems, it is necessary to define an equivalence between (a) abstract arguments and constructions in the underlying language, (b) abstract attacks and the definition of conflict using elements in the language and (c) abstract support and the corresponding notion of concrete support, if applies. However, availability as the evolution of argument relevance through time is not captured in these systems. In our work we are focused in the dynamics of argumentation more than the modelization of time.

Barringer et al. present two important approaches that share elements of our research. In the first one [27], the authors present a temporal argumentation approach, where they extend Dung networks using temporal and modal language formulas to represent the structure associated to the arguments. To do that, they use the concept of usability of arguments defined as a function that determines if an argument is usable or not in a given context, changing this status over time based on the change in a dynamics context. In addition, they improved the representational capability of the formalism by using the ability of modal logic to represent accessibility between different argumentative networks; in this way, the modal operator is treated as a fibring operator to obtain a result for another argumentation network context, and then apply it to the local argumentation network context. In the second [28], they study the relationships of support and attack between arguments through a numerical argumentation network, where both the strength of the arguments and the strength that carry the attack and support between them is considered. This work pays close attention to the relations of support and attack between arguments, and to the treatment of cycles in an argumentative network. Furthermore, they offer different motivations for modeling domains in which the strengths can be time-dependent, presenting a brief explanation of how to deal with this issue in a numerical argumentation network. We identify some key differences with these research lines. On one hand, in [27] only a negative relation between arguments is considered over a time line, while in T-BAF it is possible to represent two kind of relation between arguments: a negative (attack) and a positive one (support). Also, in T-BAF we redefine the acceptability process by considering the coherence associated to an acceptable set of arguments. In this sense, a more refined study of the arguments involved in a specific argumentation line is performed. However, in T-BAF the structure of the arguments is abstracted away; Thus, the availability associated to an argument is given by an abstract function, while in [27] is obtained using the temporal information associated to its structure. On the other hand, the framework proposed in [28] and T-BAF allows the representation of the same classes of relations between arguments. However, the difference arises in the manner in which the semantic process is carried out to determine the set of acceptable arguments. In T-BAF the coherence of a set of arguments is analyzed internally (a set of arguments in which an argument attacks another in the same set is not acceptable) and externally (a set of arguments which contains both a supporter and an attacker for the same argument is not acceptable), while in the temporal framework proposed [28] only the internal coherence is considered.

Pardo and Godo in [29] and Budán et al. in [30], explored the possibility of expressing the uncertainty or reliability of temporal rules and events, and how this features may change over time. In the first one [29], the authors propose an argumentation-based defeasible logic, called *t-DeLP*, that focuses on forward temporal reasoning for causal inference. They extend the language of the *DeLP* [31] logical framework by associating temporal parameters to literals. As usual, a dialectical procedure determines which arguments are undefeated, and hence which literals are warranted, or defeasibly follow from the program. *t-DeLP*, though, slightly differs from *DeLP* in order to accommodate temporal aspects, like the persistence of facts. The output of a t-DeLP program is a set of warranted literals, which is first shown to be non-contradictory and be closed under sub-arguments. This basic framework is then modified to deal with programs whose strict rules encode mutex constraints. The resulting framework is shown to satisfy stronger logical properties like indirect consistency and closure. Regarding our proposal, the elements presented in t-DeLP can be *interpreted* in timed frameworks. The attack relation established in t-DeLP (called *defeat* relation, an evaluated attack) can be mapped to a unique abstract attack relation in T-BAF. The subargument relation in t-DeLP should be represented by a support relation with a constraint in availability intervals: a subargument must be available whenever a superargument is. Availability is reserved to the persistence of facts, which has a direct consequence in the construction of arguments. Then, the semantics notion presented in T-BAF can be applied to compute the acceptance of arguments in t-DeLP. In [30], the authors present a different extension of *DeLP* introducing the possibility of formalizing arguments and the corresponding defeat relations among them by combining both temporal criteria and belief strength criteria. This extension is based on the *Extended Temporal Argumentation Framework* (*E-TAF*) [32] which has the capability of modeling different time-dependent properties associated with arguments. Briefly speaking, this extension of *DeLP* incorporate the representation of temporal availability and strength factors of arguments varying overtime, associating these characteristics with the DeLP language elements. The strength factors are used to model different more concrete measures such as reliability, priorities, among others; the information is propagated to the level of arguments, and then the *E-TAF* definitions are applied establishing their temporal acceptability. These elements composing DeLP can be interpreted in a general abstract framework, T-BAF. However, *E*-TAF works on time on a higher level of granularity, and our formalism is not able to represent attributes associated with arguments that vary in time, and how these attributes affect the temporal intervals in which an argument attacks another argument. Thus, once a semantics is fixed, all the accepted arguments in the proposed DeLP extension should be also accepted in at least one extension of T-BAF.

Finally, Budán et al. in [32], provide an enhanced framework for modeling special features of argumentation varying over time, which are relevant in many real-world situations. There, a novel approach integrates features from two separate directions in argumentation. On one hand, they consider time as a distinctive element, and provide the mechanisms to associate time intervals to attacks. On the other hand, they considered the structure of arguments as a way of abstracting away the structural parts of arguments and their interrelationships (sub-argument, conflict, etc.). Consequently, the authors added structure to E-TAF to formalize the notion of valuation for an argument varying on time. The resulting framework E-TAF* incorporates the ability of representing temporal availability and valuation factors associated with the abstract entities from which the argumental structures are built. This information is then propagated to the level of argumental structures using it to define temporal availability of attacks. Briefly speaking, the abstract entity (simple arguments) defined in the E-TAF* can be mapped to the abstract arguments defined in T-BAF. Then, an argumentation model defined in E-TAF* can be adapted to T-BAF in order to apply more refined semantics procedures considering the notions of safe and closed under support. However, the capability representation given by E-TAF* allow us to represent the attributes associated to this entity varying over the time, while in T-BAF an arguments has the same strength in its corresponding time availability. Then, in future works we extend T-BAF to supplies this breach considering the attributes associated to the arguments varying over time.

7. Conclusions and future work

We have expanded temporal argumentation frameworks (*TAF*) to include an argument support relation, as in bipolar argumentation frameworks. In this formalization, arguments are only valid for consideration (*available* or *relevant*) in a given period of time, which is defined for every individual argument. Hence, support and defeat relation are sporadic and proper argument semantics are defined. We adapted admissibility-based extensions for bipolar scenarios to the context of timed argumentation, providing new formalizations of argument semantics with time involved. We have proven that this extension can be consistently reduced to classical bipolar frameworks and timed abstract frameworks when respectively time or the support relation are not taken into account.

Future work presents different possibilities. In recent lines of research some semantics elaborations are presented in which there is a level of *conflict tolerance* [33]. These semantics characterize argument extensions where conflictive arguments may co-exist in the same extension set. The semantic extensions presented in our work are based on the premise of being conflict-free, in the classical sense. This is the usual method when presenting a new argumentation framework. Nevertheless, an interesting issue in argumentation named conflict tolerance is perhaps natural and should be analyzed. On another hand, we view temporal information as an additional dimension that can be applied to several argumentation models. We are interested in the formalization of other timed argument relations, especially the ones defined in the backing–undercutting argumentation framework of [34]. Also, we will investigate how the approach could be developed by considering a timed version of Caminada labeling, where an argument has a particular label for a specified period of time. Besides interval-based semantics defined in the present work, we are also interested in new integrations of timed notions in argumentation, such as temporal modal logic [35,28]. We are developing a framework combining the representation capabilities of *BAF* with an algebra of argumentation labels [36] to represent timed features of arguments in dynamic domains.

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Appendix A. Proofs

Proposition 1. *Let* S *be a collection of t-profiles:*

- *If* S *is conflict-free and closed for* R_s *then* S *is safe.*
- *If* S *is safe then* S *is conflict-free.*
- *If* S *is safe, then any collection* $\mathsf{S}' \subseteq_t \mathsf{S}$ *is safe.*

Proof. We will separate the proof in two parts according to the statements giving in the proposition.

- *If* S *is conflict-free and closed for* R_s *then* S *is safe*: Since by the hypothesis the collection of t-profiles S is conflict-free and closed for R_s, then there exist no t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}) \in \mathsf{S}$, t-profile $(\mathcal{Y}, \mathcal{T}_{(\mathcal{Y}|\mathsf{S})}) \in \mathsf{S}$ and t-profile $(\mathcal{Z}, \mathcal{T}_{\mathcal{Z}} \setminus \mathcal{T}_{(\mathcal{Z}|\mathsf{S})})$ such that $(\mathcal{Y}, \mathcal{T}_{(\mathcal{Y}|\mathsf{S})})$ has a support sequence from $(\mathcal{Z}, \mathcal{T}_{\mathcal{Z}} \setminus \mathcal{T}_{(\mathcal{Z}|\mathsf{S})})$ in the time intervals $\mathcal{T}_{(\mathcal{Y}|\mathsf{S})} \cap (\mathcal{T}_{\mathcal{Z}} \setminus \mathcal{T}_{(\mathcal{Z}|\mathsf{S})})$, and there should not exist attacks (secondary, supported or direct attacks) from $(\mathcal{Z}, \mathcal{T}_{\mathcal{Z}} \setminus \mathcal{T}_{(\mathcal{Z}|\mathsf{S})})$ to $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})})$ in the time interval $\mathcal{T}_{(\mathcal{X}|\mathsf{S})} \cap \mathcal{T}_{(\mathcal{Y}|\mathsf{S})} \cap (\mathcal{T}_{\mathcal{Z}} \setminus \mathcal{T}_{(\mathcal{Z}|\mathsf{S})})$. So, S maintains an internal and external coherence satisfying the safe property.
- *If* S *is safe then* S *is conflict-free:* Suppose that there is a set S safe but not conflict-free. In order to be Safe, then $\nexists\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle, \langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle \in \mathsf{S}$ and $\nexists\langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle$ where $\langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle$ is a valid Ω's t-profile such that $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$ or $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$ or $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$, and either there is a sequence of support from $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ to $\langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle$, or $\langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle \in \mathsf{S}$. If S is not conflict-free, then there exists a t-profile in S such that $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$ or $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$ or $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$. Contradiction.
- *If* S *is safe, then any collection* $\mathsf{S}' \subseteq_t \mathsf{S}$ *is safe*: If S is safe then, by definition, it can't be the case that a t-profile $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$, that belongs to the collection S, is supported for any t-profile $\langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle$ that belongs to S and attacks (by supported, secondary or direct attacks) a t-profile $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle \in C$, both at the same time. More formally $\nexists\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle, \langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle \in \mathsf{S}$ and $\nexists\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle, \langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle \in \mathsf{S}$ such that $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$ supported attacks $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ with a $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$ or $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$ secondary attacks $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ with a $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$ or $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$ direct attacks $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ with a $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$, and either there is a sequence of support from $\langle \mathcal{C}, \mathcal{T}_{\mathcal{C}}\rangle$ to $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$, or $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle \in \mathsf{S}$. Suppose that there is a collection $\mathsf{S}' \subseteq_t \mathsf{S}$ that is not conflict-free. Then, $\exists\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle, \langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle \in \mathsf{S}'$ such that $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$ supported attacks $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ with a $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$ or $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$ secondary attacks $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ with a $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$ or $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}}\rangle$ direct attacks $\langle \mathcal{B}, \mathcal{T}_{\mathcal{B}}\rangle$ with a $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$. This leads us to a contradiction, since $\mathsf{S}' \subseteq_t \mathsf{S}$ defining that for any t-profile $\langle \mathcal{X}, \mathcal{T}'_{\mathcal{X}}\rangle \in \mathsf{S}'$ there exists a t-profile $\langle \mathcal{X}, \mathcal{T}_{\mathcal{X}}\rangle \in \mathsf{S}$ such that $\mathcal{T}'_{\mathcal{X}} \subseteq \mathcal{T}_{\mathcal{X}}$ where $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{B})} = \emptyset$, $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{B})} = \emptyset$, and $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{B})} = \emptyset$ for any pairs of t-profile in S. $\square$

Proposition 2. *Let $\Omega = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s, \mathtt{Av}\rangle$ be a T-BAF, then:*

- *A ts-admissible extension is t-included in a td-admissible extension.*
- *A tc-admissible extension is t-included in a ts-admissible extension.*

Proof. We will separate the proof in two parts according to the statements giving in the proposition.

- *A ts-admissible extension is t-included in a td-admissible extension*: Suppose there is a collection of t-profiles S that is ts-admissible but not td-admissible. By definition if a collection of t-profile S is a ts-admissible extension, then S is safe and defends all its elements. If S is not td-admissible then it is not conflict-free or it fails in attacking all its elements. This leads us to a contradiction that arises from the supposition. S can't defend and fails attacking all its elements at the same time, and cannot be safe and not conflict-free (see Proposition 1). So all collections of t-profiles that are td-admissible are also ts-admissible.
- *A tc-admissible extension is t-included in a ts-admissible extension:* Supose there is a collection of t-profiles S that is tc-admissible but not ts-admissible. By definition if a collection of t-profile S is a tc-admissible extension, then S is conflict-free, closed under $\mathtt{R}_s$, and defends all its elements. If S is not ts-admissible then it is not safe or it fails in attacking all its elements. This leads us to a contradiction that arises from the supposition. S can't defend and fails attacking all its elements at the same time, and cannot be safe and conflict-free and closed under $\mathtt{R}_s$ (see Proposition 1). So all collections of t-profiles that are tc-admissible are also ts-admissible. □

Proposition 3. *Let $\Omega = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s, \mathtt{Av}\rangle$ be a T-BAF, then:*

- *A td-stable extension does not imply closure under $\mathtt{R}_s$.*
- *A ts-stable extension is also a td-stable extension.*
- *A ts-stable extension is also a tc-stable extension.*

Proof. We will separate the proof in the two parts of the proposition corresponding to each of the two relations.

i) A td-stable extension does not imply closure under $\mathtt{R}_s$. A collection S may be td-stable, attacking a t-profile $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ not in S. At the same time, $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ may be supported by a t-profile in S in the intervals time $\mathcal{T}_\mathcal{A}$. Hence, S cannot be closed under $\mathtt{R}_s$ preserving the conflict-free condition imposed by the supposition that S is a td-stable extension.
ii) A ts-stable extension is also a td-stable extension. Suppose there is a ts-stable extension that is not td-stable. In order to verify this the extension must be safe but not conflict-free. Since by Proposition 1 any set S safe is conflict-free, we arrive to a contradiction.
iii) A ts-stable extension is also a tc-stable extension. If a collection S is ts-stable (stable extension satisfying the safety property), then it does not support and attack a specific t-profile at the same time. It means that attacked t-profiles outside S are not supported by S. Then, every t-profile supported by S is in S. Thus, S is closed under $\mathtt{R}_s$ satisfying the property of all tc-stable extension. □

Proposition 4. *Let $\Omega = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s, \mathtt{Av}\rangle$ be a T-BAF, then:*

- *A ts-preferred extension is t-included in a td-preferred extension.*
- *A tc-preferred extension is t-included in a ts-preferred extension.*
- *A ts-preferred extension closed under $\mathtt{R}_s$ is also a tc-preferred.*

Proof. We will separate the proof in the three parts of the proposition corresponding to each of the three relations.

i) *A ts-preferred extension is t-included in a td-preferred extension*: Let's assume that S is a ts-preferred extension, S′ is a td-preferred extension, and $\mathsf{S}' \subsetneq_t \mathsf{S}$. However, S is maximal w.r.t. set t-inclusion among the ts-admissible, while S′ is maximal w.r.t. set t-inclusion among the td-admissible. In addition, by Proposition 2, we know that td-admissible extension is t-included in a ts-admissible extension. So, this contradicts the assumption that S is a ts-preferred extension or that it exists $\mathsf{S}' \subsetneq_t \mathsf{S}$ with S′ being a td-preferred extension. So a td-preferred extension is t-included in a ts-preferred extension.
ii) *A tc-preferred extension is t-included in a ts-preferred extension*: Let's assume that S is a tc-preferred extension, S′ is a ts-preferred extension, and $\mathsf{S}' \subsetneq_t \mathsf{S}$. However, S is maximal w.r.t. set t-inclusion among the tc-admissible, while S′ is maximal w.r.t. set t-inclusion among the ts-admissible. In addition, by Proposition 2, we know that ts-admissible extension is t-included in a tc-admissible extension. So, this contradicts the assumption that S is a tc-preferred extension or that it exists $\mathsf{S}' \subsetneq_t \mathsf{S}$ with S′ being a ts-preferred extension. So a ts-preferred extension is t-included in a tc-preferred extension.

iii) *A ts-preferred extension closed under* R_s *is also a tc-preferred*: Let's assume that S is a ts-preferred extension closed under R_s. This means that S is safe, is closed under R_s, and defends all its elements. In addition, we know that if S is safe, then S is conflict free by Definition 20. In this sense, S is conflict free, closed under support and defends all its elements corresponding with the tc-admissible definition. In addition, S is the maximal set w.r.t. set inclusion. Consequently, S is a tc-preferred extension. □

Proposition 5. *Let* $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ *be a T-BAF, then:*

- *A td-stable extension is t-included in a td-preferred extension.*
- *A ts-stable extension is t-included in a ts-preferred extension.*
- *A tc-stable extension is t-included in a tc-preferred extension.*

Proof. We will separate the proof in the four parts of the proposition corresponding to each of the four relations.

i) *A td-stable extension is t-included in a td-preferred extension*: Let's assume that S' is a td-stable extension, S is a td-preferred extension, and $\mathsf{S}' \nsubseteq_t \mathsf{S}$. Then, there should exist at least a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S}')}) \in \mathsf{S}'$ and a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}) \in \mathsf{S}$ such that $\mathcal{T}_{(\mathcal{X}|\mathsf{S}')} \nsubseteq \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$. In particular, there exists a time point α verifying that $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S}')}$ and $\alpha \notin \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$. In this sense, α is a time point where argument $\mathcal{X}$ is defended in S' but it is not in S. Contradiction, S' is a td-stable extension attacking all the t-profiles that do not belong to it, and S' is a maximal (w.r.t. t-inclusion) td-admissible collection of t-profile where each t-profile in S' is acceptable w.r.t. S'. Then, if the argument $\mathcal{X}$ is defended by S' in α, it is also defended by S in such time point.

ii) *A ts-stable extension is t-included in a ts-preferred extension*: Let's assume that S' is a ts-stable extension, S is a ts-preferred extension, and $\mathsf{S}' \nsubseteq_t \mathsf{S}$. Then, there should exist at least a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S}')}) \in \mathsf{S}'$ and a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}) \in \mathsf{S}$ such that $\mathcal{T}_{(\mathcal{X}|\mathsf{S}')} \nsubseteq \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$. In particular, there exists a time point α verifying that $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S}')}$ and $\alpha \notin \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$. In this sense, α is a time point where argument $\mathcal{X}$ is defended in S' but it is not in S. Contradiction, S' is a ts-stable extension attacking all the t-profiles that do not belong to it, and S' is a maximal (w.r.t. t-inclusion) ts-admissible collection of t-profile where each t-profile in S' is acceptable w.r.t. S'. Then, if the argument $\mathcal{X}$ is defended by S' in α, it is also defended by S in such time point.

iii) *A tc-stable extension is t-included in a tc-preferred extension*: Let's assume that S' is a tc-stable extension, S is a tc-preferred extension, and $\mathsf{S}' \nsubseteq_t \mathsf{S}$. Then, there should exist at least a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S}')}) \in \mathsf{S}'$ and a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}) \in \mathsf{S}$ such that $\mathcal{T}_{(\mathcal{X}|\mathsf{S}')} \nsubseteq \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$. In particular, there must exist a time point α verifying that $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S}')}$ and $\alpha \notin \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$. In this sense, α is a time point where argument $\mathcal{X}$ is defended in S' but it is not in S. Contradiction, S' is a tc-stable extension attacking all the t-profiles that do not belong to it, and S' is a maximal (w.r.t. t-inclusion) tc-admissible collection of t-profile where each t-profile in S' is acceptable w.r.t. S'. Then, if the argument $\mathcal{X}$ is defended by S' in α, it is also defended by S in such time point. □

Theorem 1. *Let* $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av}\rangle$ *be a T-BAF and let* α *represent a point in time. Let* $\Theta'_\alpha = \langle \mathtt{Arg}'_\alpha, \mathrm{R}^\alpha_a, \mathrm{R}^\alpha_s\rangle$ *be a bipolar abstract framework obtained from* Ω *in the following way:*

$\mathtt{Arg}'_\alpha = \{\mathcal{A} \in \mathtt{Arg} \mid \alpha \in \mathcal{T}_\mathcal{A}\}$,
$\mathrm{R}^\alpha_a = \{(\mathcal{A}, \mathcal{B}) \in \mathrm{R}_a \mid \alpha \in \mathtt{Av}(\mathcal{A}) \cap \mathtt{Av}(\mathcal{B})\}$ *and*
$\mathrm{R}^\alpha_s = \{(\mathcal{A}, \mathcal{B}) \in \mathrm{R}_s \mid \alpha \in \mathtt{Av}(\mathcal{A}) \cap \mathtt{Av}(\mathcal{B})\}$.

Let S *be a collection of t-profiles in* Ω *and* $\mathsf{S}'_\alpha = \{\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle \in \mathsf{S} \mid \alpha \in \mathcal{T}_\mathcal{A}\}$. *It holds that, if* S *is a td-preferred extension (resp. ts-preferred, tc-preferred, td-stable, ts-stable and tc-stable) w.r.t.* Ω, *then* S'_α *is a d-preferred extension (resp. s-preferred, c-preferred, d-stable, s-stable and c-stable) w.r.t.* Θ'_α.

Proof. We will separate the proof in the six parts of the theorem corresponding to each of the six semantics. First, we will prove the general property of conflict-freeness that any extension corresponding to any semantics should satisfy (the safe property require that a collection of t-profile has the conflict-free property):

If S is an extension w.r.t. Ω, then S'_α is conflict-free w.r.t. Θ'_α.

Let us assume that S'_α is not a conflict-free set of arguments. In that case, there should exist two arguments $X, Y \in C'_\alpha$ such that X support attacks Y or X secondary attacks Y through a sequence of arguments $\mathcal{X}\ \mathrm{R}_1 \ldots \mathrm{R}_n\ \mathcal{Y}$ or X is a direct attacker of Y. From the definition of S'_α, we know that there are $(X, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}), (Y, \mathcal{T}_{(\mathcal{Y}|\mathsf{S})}) \in \mathsf{S}$, such that $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})} \cap \mathcal{T}_{(\mathcal{Y}|\mathsf{S})}$, and $\alpha \in \mathcal{T}^{Sup}_{(\mathcal{X}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{X}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{Y})}$. Consequently, S is not an at-conflict-free set contradicting our initial assumption that S is an extension, and this contradiction comes from assuming that S'_α is not a conflict-free set.

We will now proceed under the assumption that S'_α is a conflict-free set for the four semantics mentioned.

a) If S is a td-preferred extension w.r.t. Ω, then S'_α is a d-preferred extension w.r.t. Θ'_α.

Let S be a td-preferred extension for Ω, and let S'_α be a set of arguments such that $\mathsf{S}'_\alpha = \{\mathcal{X} \mid (\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})})) \in \mathsf{S}$ and $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})}\}$, and let us assume that S'_α is not a d-preferred extension of Θ'_α. For this to be the case, knowing S'_α is conflict-free, at least one of the two conditions required for d-preferred semantics should fail, namely:

i) S'_α should be a d-admissible set. Let us assume that S'_α does not satisfy that condition. In this case, there should exist two arguments $\mathcal{X}, \mathcal{Y} \in \mathtt{Arg}'_\alpha$, such that $\mathcal{Y} \notin \mathsf{S}'_\alpha$, $\mathcal{X} \in \mathsf{S}'_\alpha$, $\mathcal{Y}$ support attacks $\mathcal{X}$ or $\mathcal{Y}$ secondary attacks $\mathcal{X}$ or $\mathcal{Y}$ direct attacks $\mathcal{X}$, and there should not exist an argument $\mathcal{Z} \in \mathtt{Arg}'_\alpha$ verifying that $\mathcal{Z} \in \mathsf{S}'_\alpha$ and $\mathcal{Z}$ support attacks $\mathcal{Y}$ or $\mathcal{Z}$ secondary attacks $\mathcal{Y}$ or $\mathcal{Z}$ direct attacks $\mathcal{Y}$. From the definition of S'_α, we know that there is a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}) \in \mathsf{S}$ where $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$, a t-profile $(\mathcal{Y}, \mathcal{T}_{\mathcal{Y}})$ such that $\mathcal{T}^{Sup}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Sec}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Dir}_{(\mathcal{Y}\text{-}\mathcal{X})} \neq \emptyset$ with $\alpha \in \mathcal{T}^{Sup}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Sec}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Dir}_{(\mathcal{Y}\text{-}\mathcal{X})}$, and does not exist a t-profile $(\mathcal{Z}, \mathcal{T}_{(\mathcal{Z}|\mathsf{S})}) \in \mathsf{S}$ such that $\mathcal{T}^{Sup}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{Z}\text{-}\mathcal{Y})} \neq \emptyset$ with $\alpha \in \mathcal{T}^{Sup}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{Z}\text{-}\mathcal{Y})}$ verifying that $(\mathcal{T}^{Sup}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Sec}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Dir}_{(\mathcal{Y}\text{-}\mathcal{X})}) \cap (\mathcal{T}^{Sup}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{Z}\text{-}\mathcal{Y})}) \neq \emptyset$. But S is a td-preferred extension, and therefore it should satisfies dt-admissibility, in particular that C defends all its elements in the time intervals in which each t-profiles belong to it (contradiction).

ii) S'_α should be a maximal set w.r.t. inclusion. Let us assume the contrary, then there exists a set S''_α such that $\mathsf{S}'_\alpha \subsetneq \mathsf{S}''_\alpha$ and it satisfies conflict-freeness and admissibility. Let $\mathsf{S}_m = \mathsf{S} \cup \{(\mathcal{X}, \alpha) \mid \mathcal{X} \in \mathsf{S}''_\alpha$ and $\mathcal{X} \notin \mathsf{S}'_\alpha\}$. Note that $\mathsf{S} \subsetneq_t \mathsf{S}_m$ (by construction). Also, S_m is td-admissible. Contradiction, since S is a td-preferred extension and therefore it is the maximal collection of t-profiles w.r.t. t-inclusion which is td-admissible. □

b) If S is a ts-preferred extension w.r.t. Ω, then S'_α is an s-preferred extension w.r.t. Θ'_α.

Let S be a ts-preferred extension for Ω, and let S'_α be a set of arguments such that $\mathsf{S}'_\alpha = \{\mathcal{X} \mid (\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})})) \in \mathsf{S}$ and $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})}\}$, and let us assume that S'_α is not an s-preferred extension of Θ'_α. For this to be the case, knowing S'_α is conflict-free, at least one of the two conditions required for s-preferred semantics should fail, namely:

i) S'_α should be an s-admissible set. Let us assume that S'_α does not satisfy that condition. In this case, two situations can arise: (a) there should exist two arguments $\mathcal{X}, \mathcal{Y} \in \mathtt{Arg}'_\alpha$, such that $\mathcal{Y} \notin \mathsf{S}'_\alpha$, $\mathcal{X} \in \mathsf{S}'_\alpha$, $\mathcal{Y}$ support attacks $\mathcal{X}$ or $\mathcal{Y}$ secondary attacks $\mathcal{X}$ or $\mathcal{Y}$ direct attacks $\mathcal{X}$, and there should not exist an argument $\mathcal{Z} \in \mathtt{Arg}'_\alpha$ verifying that $\mathcal{Z} \in \mathsf{S}'_\alpha$ and $\mathcal{Z}$ support attacks $\mathcal{Y}$ or $\mathcal{Z}$ secondary attacks $\mathcal{Y}$ or $\mathcal{Z}$ direct attacks $\mathcal{Y}$. From the definition of S'_α, we know that there is a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}) \in \mathsf{S}$ where $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$, a t-profile $(\mathcal{Y}, \mathcal{T}_{\mathcal{Y}})$ such that $\mathcal{T}^{Sup}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Sec}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Dir}_{(\mathcal{Y}\text{-}\mathcal{X})} \neq \emptyset$ and $\alpha \in \mathcal{T}^{Sup}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Sec}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Dir}_{(\mathcal{Y}\text{-}\mathcal{X})}$, and does not exist a t-profile $(\mathcal{Z}, \mathcal{T}_{(\mathcal{Z}|\mathsf{S})}) \in \mathsf{S}$ such that $\mathcal{T}^{Sup}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{Z}\text{-}\mathcal{Y})} \neq \emptyset$ with $\alpha \in \mathcal{T}^{Sup}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{Z}\text{-}\mathcal{Y})}$ verifying that $(\mathcal{T}^{Sup}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Sec}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Dir}_{(\mathcal{Y}\text{-}\mathcal{X})}) \cap (\mathcal{T}^{Sup}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{Z}\text{-}\mathcal{Y})}) \neq \emptyset$. But S is a td-preferred extension, and therefore it should satisfies st-admissibility, in particular that S defends all its elements in the time intervals in which each t-profiles belong to it (contradiction); or (b) there should exist three arguments $\mathcal{X}, \mathcal{Y}, \mathcal{Z} \in \mathtt{Arg}'_\alpha$, such that $\mathcal{Z} \notin \mathsf{S}'_\alpha$, $\mathcal{X} \in \mathsf{S}'_\alpha$, $\mathcal{Y} \in \mathsf{S}'_\alpha$, $\mathcal{X}$ support attacks $\mathcal{Z}$ or $\mathcal{X}$ secondary attacks $\mathcal{Z}$ or $\mathcal{X}$ direct attacks $\mathcal{Z}$, and there exist a sequence of support from $\mathcal{Y}$ to $\mathcal{Z}$. From the definition of S'_α, we know that there is a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}) \in \mathsf{S}$ where $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$, a t-profile $(\mathcal{Y}, \mathcal{T}_{(\mathcal{Y}|\mathsf{S})}) \in \mathsf{S}$ where $\alpha \in \mathcal{T}_{(\mathcal{Y}|\mathsf{S})}$, and a t-profile $(\mathcal{Z}, \mathcal{T}_{\mathcal{Z}})$ such that $\mathcal{T}^{Sup}_{(\mathcal{X}\text{-}\mathcal{Z})} \cup \mathcal{T}^{Sec}_{(\mathcal{X}\text{-}\mathcal{Z})} \cup \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{Z})} \neq \emptyset$ with $\alpha \in \mathcal{T}^{Sup}_{(\mathcal{X}\text{-}\mathcal{Z})} \cup \mathcal{T}^{Sec}_{(\mathcal{X}\text{-}\mathcal{Z})} \cup \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{Z})}$ and $\mathcal{T}_{(\mathcal{Y}|\mathsf{S})} \cap (\mathcal{T}^{Sup}_{(\mathcal{X}\text{-}\mathcal{Z})} \cup \mathcal{T}^{Sec}_{(\mathcal{X}\text{-}\mathcal{Z})} \cup \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{Z})}) \neq \emptyset$ with $\alpha \in \mathcal{T}_{(\mathcal{Y}|\mathsf{S})} \cap (\mathcal{T}^{Sup}_{(\mathcal{X}\text{-}\mathcal{Z})} \cup \mathcal{T}^{Sec}_{(\mathcal{X}\text{-}\mathcal{Z})} \cup \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{Z})})$. But S is a td-preferred extension, and therefore it should satisfies st-admissibility, in particular the safe condition (contradiction);

ii) S'_α should be a maximal set w.r.t. inclusion. Let us assume the contrary, then there exists a set S''_α such that $\mathsf{S}'_\alpha \subsetneq \mathsf{S}''_\alpha$ and it satisfies conflict-freeness and admissibility. Let $\mathsf{S}_m = \mathsf{S} \cup \{(\mathcal{X}, \alpha) \mid \mathcal{X} \in \mathsf{S}''_\alpha$ and $\mathcal{X} \notin \mathsf{S}'_\alpha\}$. Note that $\mathsf{S} \subsetneq_t \mathsf{S}_m$ (by construction). Also, S_m is td-admissible. Contradiction, since S is a td-preferred extension and therefore it is the maximal collection of t-profiles w.r.t. t-inclusion which is td-admissible. □

c) If S is a tc-preferred extension w.r.t. Ω, then S'_α is a c-preferred extension w.r.t. Θ'_α.

Let S be a tc-preferred extension for Ω, and let S'_α be a set of arguments such that $\mathsf{S}'_\alpha = \{\mathcal{X} \mid (\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})})) \in \mathsf{S}$ and $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})}\}$, and let us assume that S'_α is not a c-preferred extension of Θ'_α. For this to be the case, knowing S'_α is conflict-free, at least one of the three conditions required for c-preferred semantics should fail, namely:

i) S'_α should be a c-admissible set. Let us assume that S'_α does not satisfy that condition. In this case, two situations can arise: (a) there should exist two arguments $\mathcal{X}, \mathcal{Y} \in \mathtt{Arg}'_\alpha$, such that $\mathcal{Y} \notin \mathsf{S}'_\alpha$, $\mathcal{X} \in \mathsf{S}'_\alpha$, $\mathcal{Y}$ support attacks $\mathcal{X}$ or $\mathcal{Y}$ secondary attacks $\mathcal{X}$ or $\mathcal{Y}$ direct attacks $\mathcal{X}$, and there should not exist an argument $\mathcal{Z} \in \mathtt{Arg}'_\alpha$ verifying that $\mathcal{Z} \in \mathsf{S}'_\alpha$ and $\mathcal{Z}$ support attacks $\mathcal{Y}$ or $\mathcal{Z}$ secondary attacks $\mathcal{Y}$. From the definition of S'_α, we know that there is a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}) \in \mathsf{S}$ where $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$, a t-profile $(\mathcal{Y}, \mathcal{T}_{\mathcal{Y}})$ such that $\mathcal{T}^{Sup}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Sec}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Dir}_{(\mathcal{Y}\text{-}\mathcal{X})} \neq \emptyset$ with $\alpha \in \mathcal{T}^{Sup}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Sec}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup$

$\mathcal{T}^{Dir}_{(\mathcal{Y}\text{-}\mathcal{X})}$, and does not exist a t-profile $(\mathcal{Z}, \mathcal{T}_{(\mathcal{Z}|\mathsf{S})}) \in \mathsf{S}$ such that $\mathcal{T}^{Sup}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{Z}\text{-}\mathcal{Y})} \neq \emptyset$ with $\alpha \in \mathcal{T}^{Sup}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{Z}\text{-}\mathcal{Y})}$ verifying that $(\mathcal{T}^{Sup}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Sec}_{(\mathcal{Y}\text{-}\mathcal{X})} \cup \mathcal{T}^{Dir}_{(\mathcal{Y}\text{-}\mathcal{X})}) \cap (\mathcal{T}^{Sup}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Sec}_{(\mathcal{Z}\text{-}\mathcal{Y})} \cup \mathcal{T}^{Dir}_{(\mathcal{Z}\text{-}\mathcal{Y})}) \neq \emptyset$. But S is a tc-preferred extension, and therefore it should satisfies tc-admissibility, in particular that S defends all its elements in the time intervals in which each t-profiles belong to it (contradiction); or (b) there should exist two arguments $\mathcal{X}, \mathcal{Y} \in \mathtt{Arg}'_\alpha$, such that there exist a support sequence from $\mathcal{X}$ to $\mathcal{Y}$, $\mathcal{Y} \notin \mathsf{S}'_\alpha$, $\mathcal{X} \in \mathsf{S}'_\alpha$, and not exist an argument $\mathcal{Z} \in \mathtt{Arg}'_\alpha$ verifying that $\mathcal{Z}$ support attacks $\mathcal{Y}$ or $\mathcal{Z}$ secondary attacks $\mathcal{Y}$ or $\mathcal{Z}$ direct attacks $\mathcal{Y}$. From the definition of S'_α, we know that there is a t-profile $(\mathcal{X}, \mathcal{T}_{(\mathcal{X}|\mathsf{S})}) \in \mathsf{S}$ where $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})}$ and a t-profile $(\mathcal{Y}, \mathcal{T}_\mathcal{Y})$ such that $\mathcal{T}_{(\mathcal{X}|\mathsf{S})} \cap \mathcal{T}_\mathcal{Y} \neq \emptyset$ with $\alpha \in \mathcal{T}_{(\mathcal{X}|\mathsf{S})} \cap \mathcal{T}_\mathcal{Y}$. But S is a tc-preferred extension, and therefore it should satisfies tc-admissibility, in particular that S is closed under $\mathtt{R}_s$ (contradiction).

ii) S'_α should be a maximal set w.r.t. inclusion. Let us assume the contrary, then there exists a set S''_α such that $\mathsf{S}'_\alpha \subsetneq \mathsf{S}''_\alpha$ and it satisfies conflict-freeness and admissibility. Let $\mathsf{S}_m = \mathsf{S} \cup \{(\mathcal{X}, \alpha) \mid \mathcal{X} \in \mathsf{S}''_\alpha$ and $\mathcal{X} \notin \mathsf{S}'_\alpha\}$. Note that $\mathsf{S} \subsetneq_t \mathsf{S}_m$ (by construction). Also, S_m is td-admissible. Contradiction, since S is a td-preferred extension and therefore it is the maximal collection of t-profiles w.r.t. t-inclusion which is td-admissible.

d) If S is a td-stable extension w.r.t. Ω, then S'_α is a stable extension w.r.t. Θ'_α.

Let S be a td-stable extension for Ω, and let S'_α be a set of arguments such that $\mathsf{S}'_\alpha = \{\mathcal{X} \mid (\mathcal{X}, \mathcal{T}_\mathcal{X}) \in \mathsf{S}$ and $\alpha \in \mathcal{T}_\mathcal{X}\}$, and let us assume that S'_α is not a stable extension of Θ'_α. For this to be the case, knowing S'_α is conflict-free, the following condition required for stable semantics should fail: S'_α should attack (secondary or support or direct attacks) all arguments that do not belong to it. Let us assume that the condition fails; then, there exist at least an argument $\mathcal{X} \in \mathtt{Arg}'_\alpha \setminus \mathsf{S}'_\alpha$ that is not attacked (secondary or support or direct attacks) by any argument in S'_α. Consequently, there exists a t-profile $(\mathcal{X}, \mathcal{T}_\mathcal{X}) \notin \mathsf{S}$, where $\mathsf{S} = \{(\mathcal{Y}_i, \mathcal{T}_{(\mathcal{Y}_i,\mathsf{S})}) \mid 1 \leq i \leq n\}$ such that $\alpha \in \mathcal{T}_\mathcal{X}$ and therefore and therefore $\mathcal{T}_\mathcal{X} \setminus \bigcup_{i=1}^n \mathcal{T}_{(\mathcal{Y}_i,\mathsf{S})} \neq \emptyset$ since it contains at least the time point α. But this is not possible since S is a td-stable extension, thus S attacks all the t-profiles that do not belong to that td-stable extension, in particular this is true for $(\mathcal{X}, \mathcal{T}_\mathcal{X})$. This is a contradiction that arises from our assumption that S'_α does not attack all arguments that are outside of it.

e) If S is a ts-stable extension w.r.t. Ω, then S'_α is a stable extension w.r.t. Θ'_α. By Proposition 3 and item d), this point is satisfied.

f) If S is a tc-stable extension w.r.t. Ω, then S'_α is a stable extension w.r.t. Θ'_α. By Corollary 1 and item d), this point is satisfied. □

Proposition 6. *Let $\Omega = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s, \mathtt{Av}\rangle$ be a T-BAF, and $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ and $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$ be two t-profiles, where $\mathcal{B}$ attacks $\mathcal{A}$ through a support or secondary or direct attacks, then it holds that:*

- $t\text{-}ES_d(\mathcal{A}) \cap t\text{-}ES_d(\mathcal{B}) = \emptyset$;
- $t\text{-}ES_s(\mathcal{A}) \cap t\text{-}ES_s(\mathcal{B}) = \emptyset$;
- $t\text{-}ES_c(\mathcal{A}) \cap t\text{-}ES_c(\mathcal{B}) = \emptyset$;
- $t\text{-}PR_d(\mathcal{A}) \cap t\text{-}PR_d(\mathcal{B}) = \emptyset$;
- $t\text{-}PR_s(\mathcal{A}) \cap t\text{-}PR_s(\mathcal{B}) = \emptyset$; *and*
- $t\text{-}PR_c(\mathcal{A}) \cap t\text{-}PR_c(\mathcal{B}) = \emptyset$.

Proof. Let S be the td-stable extension for Ω. Let $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$, $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$ be two t-profiles in S. Since S is the td-stable extension for Ω, then S is conflict-free and for all $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle \notin \mathsf{S}$, verifies that $\mathcal{T}_\mathcal{A} \setminus (\bigcup \mathcal{T}^{Sec}_{(\mathcal{B}\text{-}\mathcal{A})} \cup \bigcup \mathcal{T}^{Sup}_{(\mathcal{B}\text{-}\mathcal{A})}) = \emptyset$ for all $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle \in \mathsf{S}$. For definition of conflict-free there is no t-profiles $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle, \langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle \in \mathsf{S}$ such that $(\mathcal{A}, \mathcal{B}) \in \mathtt{R}_a$ and $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$ or $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$ or $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{B})} \neq \emptyset$. Therefore, $t\text{-}ES(\mathcal{A}) \cap t\text{-}ES(\mathcal{B}) = \emptyset$.

The proof of the result is based on the property that states that the collection S is conflict free. This is an implicit requirement for the td-stable extension, the tc-stable extension, the tc-preferred extension, and the td-preferred through the notion of td-admissible (S is td-admissible if S is conflict-free and defends all its elements), meaning there cannot be a conflict between two elements of S. On another hand, the ts-stable extension and the ts-preferred extension established that the collection S must satisfy the internal and external coherence, satisfying the conflict-free condition (S is *Safe* iff $\nexists \langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle, \langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle \in \mathsf{S}$ and $\nexists \langle \mathcal{C}, \mathcal{T}_\mathcal{C}\rangle$ where $\langle \mathcal{C}, \mathcal{T}_\mathcal{C}\rangle$ is a valid Ω's t-profile such that $\mathcal{T}^{Sup}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$ or $\mathcal{T}^{Sec}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$ or $\mathcal{T}^{Dir}_{(\mathcal{A}\text{-}\mathcal{C})} \neq \emptyset$, and either there is a sequence of support from $\langle \mathcal{B}, \mathcal{T}_\mathcal{B}\rangle$ to $\langle \mathcal{C}, \mathcal{T}_\mathcal{C}\rangle$, or $\langle \mathcal{C}, \mathcal{T}_\mathcal{C}\rangle \in \mathsf{S}$). Consequently, the proof for the other four extensions are analogous. □

Theorem 2. *Let $\Omega = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s, \mathtt{Av}\rangle$ be a T-BAF and let $\Phi = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{Av}\rangle$ be a* TAF. *If $\mathtt{R}_s = \emptyset$ then any $\langle \mathcal{A}, \mathcal{T}_\mathcal{A}\rangle$ acceptable with respect to a set* S *in Ω is acceptable in Φ with respect to the same set* S.

Proof. Let $\Omega = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s, \mathtt{Av} \rangle$ be a *T-BAF* and $\Phi = \langle \mathtt{Arg}, \mathrm{R}_a, \mathtt{Av} \rangle$ be a *TAF*. The set of arguments is the same. The same attack relations are defined in both frameworks and the availability function is the same. Also there are not support relations defined in Ω, i.e. $\mathrm{R}_s = \emptyset$.

Suppose there is a t-profile $\langle \mathcal{A}, \mathcal{T}_{\mathcal{A}} \rangle$ that is acceptable with respect to a set S in Ω but not in Φ. There must be at least one moment in $\mathcal{T}_{\mathcal{A}}$ where $\mathcal{A}$ is acceptable with respect to S in Ω but not in Φ. Since $\mathrm{R}_s = \emptyset$, then according to Definition 23 the maximal acceptable t-profile for $\mathcal{A}$ with respect to S is defined in *T-BAF* as:

$$\bigcap\nolimits_{\mathcal{B}:\mathcal{T}^{Dir}_{(\mathcal{B}\text{-}\mathcal{A})} \neq \emptyset} \left(\mathtt{Av}(\mathcal{A}) \setminus \mathcal{T}^{Dir}_{(\mathcal{B}\text{-}\mathcal{A})} \right) \cup \mathcal{T}^{\mathcal{B}}_{(\mathcal{A}|\mathsf{S})} \tag{A.1}$$

recall that $\mathcal{T}^{\mathcal{B}}_{(\mathcal{A}|\mathsf{S})}$ with $\mathrm{R}_s = \emptyset$ is

$$\mathtt{Av}(\mathcal{A}) \cap \left(\bigcup \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{B})} : \langle \mathcal{X}, \mathcal{T}_{\mathcal{X}} \rangle \in \mathsf{S},\ \mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{B})} \neq \emptyset \right)$$

On the other hand, in *TAF*, the maximal acceptable t-profile for an argument with respect to a set S is defined as:

$$\bigcap\nolimits_{\mathcal{B}:(B,A)\in \mathrm{R}_a} (\mathtt{Av}(\mathcal{A}) \setminus \mathtt{Av}(\mathcal{B})) \bigcup\nolimits_{\langle \mathcal{X}, \mathcal{T}_{\mathcal{X}} \rangle \in \mathsf{S}, (\mathcal{X},\mathcal{B}) \in \mathrm{R}_a} \left[\mathtt{Av}(\mathcal{A}) \cap \left(\bigcup \mathcal{T}_{\mathcal{X}} \cap \mathtt{Av}(\mathcal{B}) \right) \right] \tag{A.2}$$

Note that, according to Definition 17, $\mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{B})} = \mathcal{T}_{\mathcal{X}} \cap \mathcal{T}_{\mathcal{B}}$, but in intersection (A.1), the basic t-profile of argument $\mathcal{B}$ must be taken into account, and then here $\mathcal{T}^{Dir}_{(\mathcal{X}\text{-}\mathcal{B})} = \mathcal{T}_{\mathcal{X}} \cap \mathtt{Av}(\mathcal{B})$ as in intersection (A.2). Hence, both sets are defined as the intersection of $\mathtt{Av}(\mathcal{A})$ with the moments in which S provides a defense, i.e. the union of moments where each attacker and the corresponding defenders from S coexist. Then there exists no t-profile for $\mathcal{A}$ that is accepted in Ω but not in Φ. □

References

[1] T.J.M. Bench-Capon, P.E. Dunne, Argumentation in artificial intelligence, Artif. Intell. 171 (10–15) (2007) 619–641.
[2] P. Besnard, A. Hunter, Elements of Argumentation, vol. 47, MIT Press, Cambridge, 2008.
[3] I. Rahwan, G.R. Simari, Argumentation in Artificial Intelligence, Springer, 2009.
[4] P.M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning and logic programming, in: R. Bajcsy (Ed.), IJCAI, Morgan Kaufmann, 1993, pp. 852–859.
[5] P.M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning and logic programming and *n*-person games, Artif. Intell. 77 (1995) 321–357.
[6] J.C. Augusto, G.R. Simari, A temporal argumentative system, AI Commun. 12 (4) (1999) 237–257.
[7] J.C. Augusto, G.R. Simari, Temporal defeasible reasoning, Knowl. Inf. Syst. 3 (3) (2001) 287–318.
[8] M.L. Cobo, D.C. Martínez, G.R. Simari, Acceptability in timed frameworks with intermittent arguments, in: L.S. Iliadis, I. Maglogiannis, H. Papadopoulos (Eds.), EANN/AIAI (2), in: IFIP Publications, vol. 364, Springer, 2011, pp. 202–211.
[9] M.L. Cobo, D.C. Martinez, G.R. Simari, An approach to timed abstract argumentation, in: Proc. Int. Workshop of Non-Monotonic Reasoning, 2010, pp. 16–23.
[10] M.L. Cobo, D.C. Martínez, G.R. Simari, On admissibility in timed abstract argumentation frameworks, in: H. Coelho, R. Studer, M. Wooldridge (Eds.), ECAI, in: Front. Artif. Intell. Appl., vol. 215, IOS Press, 2010, pp. 1007–1008.
[11] C. Cayrol, M.-C. Lagasquie-Schiex, On the acceptability of arguments in bipolar argumentation frameworks, in: Symbolic and Quantitative Approaches to Reasoning with Uncertainty, Springer, 2005, pp. 378–389.
[12] S. Polberg, N. Oren, Revisiting support in abstract argumentation systems, in: Computational Models of Argument: Proceedings of COMMA 2014, vol. 266, 2014, p. 369.
[13] A. Cohen, S. Gottifredi, A.J. García, G.R. Simari, A survey of different approaches to support in argumentation systems, Knowl. Eng. Rev. 29 (2014) 513–550.
[14] C. Cayrol, M.-C. Lagasquie-Schiex, An axiomatic approach to support in argumentation, in: International Workshop on Theory and Applications of Formal Argumentation, Springer, 2015, pp. 74–91.
[15] L. Amgoud, C. Cayrol, M.-C. Lagasquie-Schiex, P. Livet, On bipolarity in argumentation frameworks, Int. J. Intell. Syst. 23 (10) (2008) 1062–1093.
[16] C.E. Briguez, M. Capobianco, A.G. Maguitman, A theoretical framework for trust-based news recommender systems and its implementation using defeasible argumentation, Int. J. Artif. Intell. Tools 22 (4) (2013).
[17] C.E. Briguez, M.C. Budán, C.A.D. Deagustini, A.G. Maguitman, M. Capobianco, G.R. Simari, Argument-based mixed recommenders and their application to movie suggestion, Expert Syst. Appl. 41 (14) (2014) 6467–6482.
[18] M. Beer, M. D'inverno, M. Luck, N. Jennings, C. Preist, M. Schroeder, Negotiation in multi-agent systems, Knowl. Eng. Rev. 14 (03) (1999) 285–289.
[19] I. Rahwan, Guest editorial: argumentation in multi-agent systems, Auton. Agents Multi-Agent Syst. 11 (2) (2005) 115–125.
[20] J. Hage, A theory of legal reasoning and a logic to match, Artif. Intell. Law 4 (3–4) (1996) 157–368.
[21] H. Prakken, Logical Tools for Modelling Legal Arguments, Ph.D. thesis, Vrije University, Amsterdam, Jan. 1993.
[22] J. Leite, J. Martins, Social abstract argumentation, in: IJCAI 2011, Proceedings of the 22nd International Joint Conference on Artificial Intelligence, Barcelona, Catalonia, Spain, July 16–22, 2011, 2011, pp. 2287–2292.
[23] K. Grosse, M.P. González, C.I. Chesñevar, A.G. Maguitman, Integrating argumentation and sentiment analysis for mining opinions from twitter, AI Commun. 28 (3) (2015) 387–401.
[24] N. Mann, A. Hunter, Argumentation using temporal knowledge, in: Proceedings of the 2nd International Conference on Computational Models of Argument, COMMA 2008, 2008, pp. 204–215.
[25] J.F. Allen, Maintaining knowledge about temporal intervals, Commun. ACM 26 (11) (1983) 832–843.
[26] A. Hunter, Ramification analysis using causal mapping, Data Knowl. Eng. 32 (1) (2000) 1–27.
[27] H. Barringer, D.M. Gabbay, J. Woods, Modal and temporal argumentation networks, Argument Comput. 3 (2–3) (2012) 203–227.
[28] H. Barringer, D. Gabbay, J. Woods, Temporal, numerical and meta-level dynamics in argumentation networks, Argument Comput. 3 (2–3) (2012) 143–202.
[29] P. Pardo, L. Godo, t-DeLP: an argumentation-based temporal defeasible logic programming framework, Ann. Math. Artif. Intell. 69 (1) (2013) 3–35.
[30] M.C. Budán, M.G. Lucero, C.I. Chesñevar, G.R. Simari, An approach to argumentation considering attacks through time, in: Scalable Uncertainty Management, Springer, 2012, pp. 99–112.

[31] A.J. García, G.R. Simari, Defeasible logic programming: an argumentative approach, Theory Pract. Log. Program. 4 (1) (2004) 95–138.
[32] M.C. Budán, M.G. Lucero, C. Chesñevar, G.R. Simari, Modeling time and valuation in structured argumentation frameworks, Inf. Sci. 290 (2015) 22–44.
[33] O. Arieli, Conflict-tolerant semantics for argumentation frameworks, in: L.F. del Cerro, A. Herzig, J. Mengin (Eds.), Logics in Artificial Intelligence – Proceedings of the 13th European Conference, JELIA 2012, Toulouse, France, in: Lect. Notes Comput. Sci., vol. 7519, Springer, 2012, pp. 28–40.
[34] A. Cohen, A.J. García, G.R. Simari, Backing and undercutting in abstract argumentation frameworks, in: Proceedings of the 7th International Symposium on Foundations of Information and Knowledge Systems, FoIKS 2012, Kiel, Germany, 2012, pp. 107–123.
[35] D.M. Gabbay, A. Kurucz, F. Wolter, M. Zakharyaschev, Many-Dimensional Modal Logics: Theory and Applications, 1st edition, Stud. Logic Found. Math., vol. 148, North-Holland, 2003.
[36] M.C. Budán, M.J.G. Lucero, G.R. Simari, Modeling reliability varying over time through a labeled argumentative framework, in: WL4AI-2013, 2013, pp. 26–33.