Cities and Methods from Complexity Science

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DOI: 10.1007/s11424-016-6084-2
Received: 25 April 2016

Abstract The dynamics of human society is now been studying in the context of the artificial environment created by cities. In this work, the authors describe some of the formal methods used in complexity science to study urban systems. The authors discuss some of the important quantitative approaches on cities paying attention to some of the deepest controversies in present scientific studies. The authors will stress the importance of a transdisciplinary approach when studying this type of cooperative social environments.

Keywords Cities, complexity, environment.

1 Introduction

Human population used to live disaggregated. This scenario changed in 2008, when more than half of the planet's population started to concentrate in cities. Estimations indicate that the total number of people living in this artificial environment will swell to almost 5 billion by the year 2030[1]. The acceleration of this evolution is astonishing: Given that urban areas have existed for more than 10 thousand years[2]. Roughly speaking, still in 1950 less than thirty percent of the population lived in cities, while nowadays it is over fifty percent. Moreover, the expectations for the year 2050, only 35 years ahead, will reach more than the seventy percent.

1.1 Evolutionary Accidents

The exodus of human population from rural areas to cities, poses one, if not the most important question and challenge since human beings became urban, after the Neolithic revolution around ten thousand years ago[3].

It is obvious that the future effects of this collective cooperative behaviour demands urgent understanding and appropriated public policies to decrease the demand of energy, the increase of pollution, and all the problems associated with socio-economic inequalities.

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◇ *This paper was recommended for publication by Editor-in-Chief GAO Xiao-Shan.*

Besides all this problematic secondary effects, cities can also be seen as the place where individuals, families, companies, etc., gather to take advantage of proximity opportunities, diversity, and marketplace competition. In this sense cities can be thought as an important place where human cooperation can and must be enhanced in the near future, thanks to intricated networks operating in different topics and levels.

There are many ways to approach the understanding of the evolution of these particular cooperative human systems. Some try to explain the specific spatial configuration of the economy for a given city - observed today - as the outcome of its historical evolutionary process[4]. These efforts, amongst others, show how the evolution of cities can be thought of as a continuous evolutionary system crossing some catastrophic bifurcations (after the work by the French mathematician René Thom circa 1960), sometimes driven by a number of accidents usually proposed by a very short-viewed, contingent and merchandised public policy. These accidents are certainly in the same spirit of the frozen accidents –events that shape evolution history– discussed by the Nobel price scientist Murray Gell-Mann[5].

A very interesting approach to understand the trends on energy demands and diverse aspects of human cooperation/production, as for example: Patents, employment, road surface, creation of green spaces, etc., is to look at the scaling relations for urban indicators[6]. In this way, using a large data set, functionality curves similar to the ones in ecological environments can be constructed. For example, the metabolic rate can be used as a metaphor for the energy demand of a given city. Knowing the functionality of the energy demand versus, for example, the city size, predictions of the optimal density of the city can be made, given specific constrains that could be present as: Goods, energy transportation, geographical location, etc. The practical implications of these findings highlight the importance of measuring and understanding the drivers of economic and population growth in cities across the entire social system.

The practical implications on these lines of research are basically that urban planning must be informed of the state of art of our scientific knowledge on this subject, and public policies must be applied in complete resonance with it. We subscribe to the idea that cities can, and must, be planned using a deep transdisciplinary view, with the mentioned scientific knowledge as one of the key components for that endeavor. We consider cities as evolutionary adaptive systems, but we also believe that some of the tragic evolutionary accidents, as the typical short-viewed contingent and merchandised decisions on urban design, must be avoided based on historical experience and scientific knowledge.

This manuscript is intended for the general public. In the following sections we will pay attention to the complex system approach in general, and its application to the study of urban systems. We will list some of the most important complex systems methods that can be used. Next we will discuss some quantitative approaches on cities and some of the deepest controversies in present scientific studies. Finally, we will finish mentioning the importance of a transdisciplinary approach when studying this type of cooperative social environments.

2 A New Paradigm

Nowadays there is no precise, and therefore unique, definition of complex systems and complexity science, a relative new brach of science. Most researchers working in this area agree on some of the essential properties a system has to possess to be called complex[7–9]. Let's mention now some of these characteristics. A complex system: Consists on a large number of agents interacting usually via simple rules; and exhibits the so-called emergence: A hard–to–predict collective behavior which does not result from the existence of a central controller, i.e., it is self-organized[10]. A good discussion on these characteristics (that any complex system should exhibits) and the mathematical models that could be used as an approximation for them, can be found in Nicolis and Nicolis[11], 2007. The basic idea exposed there, with which we personally agree, is that non linear–behavior is a necessary condition for a complex behavior, and its signature is the multiplicity of different states that the system can achieve[12–14].

2.1 The Entangled Nature of Human Interaction

Cities are without any doubt complex objects. There is a tradition on thinking the city and urban spaces in this way. We can mention the work of Jacob - and all the lines of research derived from her work - where she defines cities as problems with organized complexity[15].

Cities present several characteristics that make them perfect archetypical examples where complex system methodology can be applied. They are certainly the product of the action of several individuals, from less than a hundred to millions, cooperating and interacting in networks of space-time entangled nature. Among them we find that social, transport and energy flow networks and interactions of companies with government, etc., are the intrinsic skeleton of a city. Therefore, the different structures that emerge in cities are the product of the collective behavior occurring in these networks.

It is also worth mentioning some of the particular characteristics that a city has. They are diverse and interconnected. The diversity of them can be seen in, for example: Professions[16], wealth, culture[17, 18], geographical diversity[19], etc. From this point of view a city is an extremely rich environment that generates new products, ideas and innovations. Using a metaphor, a city is like a complex chemical reactor, where many different components interact at different levels and scales, in an intricate network fashion, with several feedback loops. In this sense the city of the future should be diverse, to increase the potentialities of the structures living in it: From individuals to organizations, to produce innovations, something of urgent need[6].

2.2 Formal Methods

First, we would like to stress that some of the methods presented here, which are the usual formal tools in complexity science, have a very long tradition in Physics and Mathematics. The complex system approach has a unique way of using all these traditional methodologies, exploring nature in a quantitative manner, always approaching the question with a deep inter/transdisciplinary view.

Non–linear science: Many body systems, such as complex-human societies, and in general all the interactions occurring in cities, are highly non-linear. This basically means that in this type of systems abrupt transitions can be observed, i.e., the state of the system changes dramatically upon small perturbations. For instance: It can collapse, go extinct or thrive[20]. In some cases, multiple possible stable solutions can arise; and also, unpredictability in both, space and time, which in deterministic systems, is known as classical chaos[14].

Scaling: Widom B, awarded with the Boltzmann Medal in 1998, noted: "Scaling laws are the expression of physical principles in the mathematical language of homogeneous function". That is, given a relation of a mathematical function, when scaling the argument by a constant factor it causes only a proportionate scaling of the function itself[21]. The importance of this simple relation is the following: The equivalence of power laws with a particular scaling exponent sometimes can have a deeper origin in the dynamical processes that generates this behavior at a microscopical level. In complex systems science it is very usual to find this particular characteristic. Scaling laws appear in: Biological systems, for example the relation between metabolic rate and the size of an organism; fractals; social interactions; cities, an example can be the total road length as a function of population size; etc.

Bifurcation theory: As we mentioned before, non-linear behavior is the usual type of dynamics observed in social systems. Considering public policy, it is importance to realise that final stable solutions, equilibrium points or final states in societies, can change drastically and sometimes in an irreversible way, when some of the parameters, that drive the evolution of the system, reach some particular value[22].

Pattern formation: Patterns appear everywhere in nature, they where mathematically formulated for the first time in the context of morphogenesis[23]. Spatio-temporal patterns can be observed in chemical reactions or in living systems such bacteria cultures or alike[24], it is simple to see the connection with cities, and some researches are explore this idea. In a recent work[25] some of these ideas has been applied to Global Pattern Formation and Ethnic/Cultural Violence.

Network theory: It is one of the fundamental tools to study human cooperation, and social interaction in general[26, 27]. Social events, like the Arab Spring in 2011[28], or the Ebola outbreak in 2014[29] highlight the importance of this quantitative methodology tool when studying social systems. The ubiquity of networks in cities is surprising. Among other (the list is boundless): Social networks, with connections between individuals, organizations, etc.; transport networks inside cities or between cities, as for example airplane networks; the World Wide Web; food webs in urban environments; etc.

Game theory: Game theory, or the theory of social dilemmas, focuses on how a group of individuals interact using strategic decision making[30]. This analytic theory is of vital importance when studying cases of cooperation or defection in human interactions.

Information theory: In complex systems, information theory has been used in connection with a theory propoced by Shannon[31], and developed by Jaynes, who did a giant step showing the the correspondence between statistical mechanics and information theory[32]. Jaynes's work paid much attention to a general physical principle: The Maximum Entropy Principle, or MaxEnt. In present researches, MaxEnt is been used to understand several distributions appearing in biology and ecology, from a complex system approach. Size distribution, range distribution, energy distribution, etc., are the typical analytic predictions that one can make using this tool. In a very recent effort we used MaxEnt and Super Statistics applied to social and urban systems, but this work is still in progress.

Complexity measures: This type of statistical measure is still understudied[33]. The importance of them can be understood when thinking that systems with the same level of complexity (defined in some way) may share universal properties. Complexity measures can be used to recognise patterns appearing in many natural systems. The applicability to social and urban systems is still in progress.

Cellular automata: Giving its flexibility to produce simple models, the application of Cellular Automata to natural phenomena is very large. One can use it to understand pattern formation in biology, epidemiology or models to simulate urban dynamics through the local actions[34], etc.

Agent-based modeling: Agent-based modeling can be thought as the natural evolution of cellular automata models. This can be considered as a bottom-up approach, due to the fact that the properties observed in the system as a whole (i.e., emergent properties) are the result of the interactions of the microscopic components of the system. The benefits of using agent-based modeling are[35]: It captures emergent phenomena, this is because in principle emergent phenomena comes from microscopic interactions, or individual entities; and also in many cases agent-based modeling provides the most natural way to describe dynamics and rules of the system, focusing on the individual rules of the agents. For these reasons agents-based modeling is an amazing tool when projecting the different scenarios that urban systems can have. Some of the areas where agent-based modeling is being applied are: Diffusion of innovation and adoption, operational risk, organization design, stock market, flows (as for example traffic or evacuation), etc.

Data mining: Nowadays there is much data available: From scientific institutions, governments, different type of businesses, the world wide web, etc. All this available data can be stored and studied, but for some of them the underlying dynamics is unknown and a first-principle approach is far from being achieved. The term data mining refers to the extraction and recognition of patterns in large data sets. In this sense, data mining has two primary goals[36]: Prediction and description. To achieve them, data mining uses the following task: Classification, regression, clustering, summarization, modeling, and deviation detection. The importance of data mining in the future scientific theory of cities is vital. As we will discuss, the appearance of patterns in urban data can tell us something on the most probable microscopic dynamics of these social systems.

The previous mentioned methods are used in advanced applications such as Intelligent transportation systems (ITS), which is related to transport and traffic in general. The basic idea behind ITS is to provide services to make transport safer and more coordinated[37]. The technologies used in ITS varies and evolve very fast. Some of them are speed cameras and automatic number plate recognition and traffic signal control systems, etc. More sophisticated applications, very commons nowadays, integrate live information coming from various sources, like users using smart-phone or alike devices, parking information, weather information, official information reporting different events, as, for example, accidents, traffic jam, etc. The relevance of systems like the ITS increase notoriously if we consider networks of flowing information as the so-called Internet of Things (IoT). Objects connected to this network can be sensed and controlled, and the information of these can be processed and used as a feedback loop to increase the accuracy of the live information send to any user of the IoT network[38].

3 The Complex Systems Approach

In the last decade there have been lots of efforts to quantify urban variables in order to have a more comprehensive and scientific understanding of cities: How they grow, internal dynamics, etc[39]. The goal of these lines of research is to try to obtain a kind of first principles laws to describe cities, to generate predictions with the least possible degree of uncertainty. In order to do so, this approach consists in analyzing a huge amount of information, using data mining among other techniques, and then trying to discover patterns, or if possible a kind of universal laws.

These universalities could be emergent macroscopic features: A city can be defined as a place where individuals interactions occur at many levels, and probably some of its quantitative characteristics simply scale with some level of organisation, size, number of individuals, etc. Then, a possible ansatz will be the following: In a city some properties will scale, in the same way as in biology using the Kleiber's law, where we can apply allometric scaling properties of the metabolic rate with respect to mass[40, 41]. The search for this kind of relations are extremely important since, as mentioned before, can be associated with the first principles that generate them. In a way, there could be a kind of self-similarity in urban systems, so "cities are a scaled version of one another"[42] (it is obvious that we are not taking here of the uniques characteristics that each city has). The mathematical formulation of this ubiquitous behavior can be written as

$$Y(t) = Y_0(t)N(t)^{\beta}, \tag{1}$$

where $Y(t)$ represents some urban variable such as road surface, total employment, new patents, total wages, etc., $Y_0(t)$ is a normalized constant, which must be clearly time-dependent following the evolution of the city, and $N(t)$ is its size[43]. The applicability of the previous ideas is nowadays a topic of important controversies, something that we will comment in detail below. Independently of the result of these controversies, it is without any doubt a big step to put the discussion on quantitative measures and to search first principle laws in urban systems. As we will see later it looks like there is an important issue on the definition of the city size per se, and on what are actually the boundaries of a city.

3.1 Controversies on Scaling

In these paragraphs we will address the current discussion on scaling theories applied to cities. There is a clear body of work that supports the idea that cities scale on size[42]. These types of ideas are in some sense supported by the metaphors and successful previous works coming from biology[40–42]. In recent works these ideas have been questioned. To cite two of the works explicitly: "...as the previous discussion illustrates, it is dangerous to interpret empirical results without any mechanistic insight...It should therefore be obvious that, until they have a satisfactory understanding of the mechanisms responsible for the observed behavior, scientists should refrain from giving policy advice that might have unforeseen, disastrous consequences. If they choose to do so anyway, policy makers should be worried about what is, at best, a shot in the dark"[44], and: "The analysis shows that population size alone does not provide us enough information to describe or predict the state of a city as previously proposed, indicating that the expected scaling laws are not corroborated. All this indicates that a theory of cities cannot rest simply on a relationship like Equation (1), because relevant patterns pertaining to social behavior, such as the well-known Pareto distribution of wealth, cannot be grasped if only aggregated values are considered. A theory of cities needs therefore to reproduce the main relevant emergent behaviors that are encoded in the diversity and heterogeneities of cities. It is only through this perspective that city planning and policy making can be effective"[43]. From our point of view these kind of worries on the "scientific theory of cities" based on scaling are making a strong statement: Cities remain as complex objects that are not well understood, and given the present state of the world there is an urgent need to study, comprehend, manage and explore different futures scenarios using, for example, agents based simulations applied in urban systems.

4 Transdisciplinary Exploration on Cities, a Case of (Possible) Study

In these final words, we want to stress the idea of the urgent need for a scientific understanding on cities. As mentioned, we agree with the idea that cities can and must be planned taking into account our best knowledge on these cooperative human systems. The last comment is due to the fact that in the present literature there is a tension and there are different points of view on if cities should be planned or if at the end these social cooperative systems should be thought as self-evolutionary systems. In relation with the last, there is an important component in the dynamics of cities, which sometimes shapes cities in almost an irreversible way: public policy.

We want to mention a current example of the kind of permanent types of bifurcation that a city can suffer. The city of Valparaiso, in Chile, after experimenting a declined economy for several decades, was declared a UNESCO World Heritage Site in 2003, and it was entitled Chile's Cultural Capital; it is also the legislative, naval and regional capital at the same time. For economical reasons, Valparaiso is now facing at least two different future scenarios: One is the imminent transformation of its coastline to extend the port activities (named Terminal 2) and its capacity, which will imply the final loss of a posible public sea border for the city and basically to focus the development investments in only one economic activity in this vital part of the city, as the UNESCO report indicates: "The Expansion of Terminal 2 at the port must be executed without altering the city's heritage and environmental values and its Outstanding Universal Value (OUV)"[45]. On the other hand, Valparaiso could be protected and enhanced by considering its Outstanding Universal Value (OUV), "its attributes related to the cultural landscape aspects of the city (such as the shoreline, the geographical amphitheatre)"[45], and go for a more diversified and broader approach in the formulation of planning tools, to formulate integrated policies for conservation and development which could increase social value and cultural diversification.

At the same time, due to a catastrophic fire out of control destroying almost three thousand houses in April 2014, Valparaiso also needs reconstruction of some of its urban and public areas. These are the types of accidents, we previously mentioned, which a city could suffer during its evolution. A type of permanent accident that shapes a city. What we believe is important here, and given the previous discussion, is to realize the impact that public policies has on city planning, like the future of Valparaiso. A naïf self-organized dynamic point of view can be disastrous. We should avoid short-viewed economic and political driven decisions. Moreover methodological scientific tools should be in hand to give a bigger picture to different scenarios and probable final states of different urban planning. Without any doubt, and among other traditional methodologies, more data mining, agent-based simulations, structural scaling, etc. are needed in the present planning of urban environment, thinking not only in the future decades but much beyond that. Focusing on Valparaiso: "A broader approach is needed for the management of the property, particularly in relation to the reconciling of development needs of a port city with its heritage"[45].

Given the state of the art and the present knowledge on the evolution of the urbanization of the planet, and the imminent consequences, there is an enormous responsibility on public policy to, at least, do not practice what we know now for sure is wrong for human interaction in this artificial environment. It is time to sum efforts to understand the next step of human social living, the time where our planet will be urbanized. A transdisciplinary view is of mayor importance. Not only scientific transdisciplinary collaboration is needed, it is important to obviously add urban planning, public policy, cultural and governmental forces, among others, to transform cities in future places for human cooperation.

The methods shown in this comunication are some of the ones used in complexity science, which, at the same time, is probably the best scientific methodological way to deal with the kind of problems concerning urban dynamics and design. It is very important to realise that none of the methods included in this chapter define complexity science, neither the agglomeration of them. Beyond the concepts, tools and methods presented, complexity science offers a new way of thinking on the issues discussed here. It focuses attention on dynamic connections, evolution and interdisciplinary thinking.

We are sure the next decades will be times in which humans cooperation in cities, and the studies on how we can enhanced it, will be one of the dominant topics in complexity science, and certainly time is shrinking.

Acknowledgements The authors thanks Conicyt Fondecyt 1140278.

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