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The planetary increase of brightness during retrograde motion: An explanandum constructed ad explanantem

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ABSTRACT

In Ancient Greek two models were proposed for explaining the planetary motion: the homocentric spheres of Eudoxus and the Epicycle and Deferent System. At least in a qualitative way, both models could explain the retrograde motion, the most challenging phenomenon to be explained using circular motions. Nevertheless, there is another explanandum: during retrograde motion the planets increase their brightness. It is natural to interpret a change of brightness, i.e., of apparent size, as a change in distance. Now, while according to the Eudoxian model the planet is always equidistant from the earth, according to the epicycle and deferent system, the planet changes its distance from the earth, approaching to it during retrograde motion, just as observed. So, it is usually affirmed that the main reason for the rejection of Eudoxus' homocentric spheres in favor of the epicycle and deferent system was that the first cannot explain the manifest planetary increase of brightness during retrograde motion, while the second can. In this paper I will show that this historical hypothesis is not as firmly founded as it is usually believed to be.

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1. Introduction

According to Simplicius (On the Heaven, 2, 12; [Bowen, 2013](#page-10-0): 136), Plato put forward to astronomers the following question: "By hypothesizing which smooth and orderly motions will the phenomena of the motions of the wandering [stars] be saved?" Most scholars call into question that Plato really put forth this demand ([Knorr, 1991:](#page-11-0) 319-320). The demand, however, authentically reflects the bases of any research program on astronomy in Ancient Greece. The first complete planetary system that fulfilled the Platonic request emerged almost immediately after: Eudoxus of Cnidus, one of Plato's most prominent disciples proposed the model of homocentric spheres. This model was universally known since it was proposed and partially modified by another of Plato's disciple, Aristotle (On the Heavens II,12, 291b-293a, [Allan, 1955](#page-10-0); Metaphysics Λ , [Tredennick, 1935](#page-11-0)). Eudoxus managed to explain the movements of the planets, with its variations only using spheres, all rotating with uniform motion and concentric to the Earth. The secret was to give to each sphere a certain angular speed and to attach the axis of each sphere to the immediately outer sphere at a certain angle: the combination of the movements of the spheres could produce the non-uniform motions of the Sun, Moon and planets. The system was so complex that it was completely understood only in the nineteenth century. It was Giovanni [Schiaparelli \(1875\)](#page-11-0) who had the merit of showing how the Eudoxian homocentric spheres could produce the retrograde motion of the planets, producing the famous hipopede.¹ See [Fig. 1.](#page-1-0)

The homocentric sphere model was not only the brilliant idea of an isolated genius; it was a real research program with important improvements made by Callippus and Aristotle [\(Mendell, 1998;](#page-11-0) [Schiaparelli, 1875\)](#page-11-0). At some point in the century after the death of Aristotle, however, the theory was abandoned. A new model, based on epicycles and deferents, appeared probably within a few

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Newer proposals compete today with that of Schiaparelli. See [Mendell, 1998](#page-11-0) and [Yavetz, 1998.](#page-11-0)

Fig. 1. Retrograde motion in Eudoxian model.

decades after or before 200 BC [\(Evans et al., 2013:](#page-10-0) 149-151). This model was a different research program, probably proposed by Apollonius but improved by Hipparchus and Ptolemy. It became the reigning astronomical paradigm until the Copernican Revolution. According to the epicycle and deferent simplest version, a planet revolves in a small circle called epicycle, the center of which revolves around the Earth in another circle called deferent. In the case of the planets, the deferent is responsible for the position of the planet on the Zodiac, while the epicycle is responsible for the retrograde motion: the planet retrogrades once per turn of the epicycle. Both the epicycle and deferent rotate in the same direction, therefore, the retrograde motion is produced when the planet is closest to the earth, because at that point the tangential speed of the epicycle and deferent go in opposite directions. See Fig. 2.

At least in a qualitative way, both the homocentric sphere and the epicycle and deferent models could explain the retrograde motion.² There was, however, another fact that remained unexplained, i.e., another explanandum: during their retrograde motion, planets increase their brightness. Thus, assuming that the absolute size of celestial bodies does not change, it was natural to interpret the change in brightness as a change in distance. Technically, a change in apparent size, and not a change in brightness, would imply a change in distance. Before the introduction of the telescope, however, brightness was mistaken with apparent size, because to the naked eye a brighter planet seems bigger. For us today, brightness and apparent size are different magnitudes, but for the Ancients a change in brightness was the same as a change in apparent size, which implied, consequently, a change in distance (cfr. [Goldstein, 1996:](#page-11-0) 1-2).

Whereas for the Eudoxian model planets are always equidistant from the Earth, for the epicycle and deferent model, planets change their distance from the Earth, getting closer during their retrograde motion. This difference is usually argued to be the main reason for the rejection of Eudoxus' homocentric spheres in favor of the epicycle and deferent model, because the former cannot explain the

Fig. 2. Retrograde motion in the simplest version of the epicycle and deferent system. The planet (P) rotates around the center of the epicycle, point C, which rotates around E, the center of the Earth in a deferent. Both P and C rotate on the same sense.

manifest planetary increase of brightness during retrograde motion, while the latter can.

Certainly, this difference is not the only one between these two models: the Eudoxian proposal had in its favor that it was by far more faithful to the idea of making the center of the universe the center of all celestial motions, given that for the epicycle and deferent model the planets revolved around a theoretical center which not only was not the Earth, but it was movable. On the contrary, the epicycle and deferent system had in its favor that it was much simpler to understand and by far more flexible and powerful than Eudoxian spheres.

Nevertheless, the main reason given for the rejection of Eudoxus' homocentric spheres in favor of the epicycle and deferent system is still the impossibility of the first to explain the patent change of brightness during retrograde motion. Many scholars who made important contributions to the history of Ancient astronomy, such as Thomas Heath, Giovanni Schiaparelli or John L. Dreyer, agree with this assertion. [Heath \(1913](#page-11-0): 221) affirms that "what was ultimately fatal to it [i.e., to the theory of concentric spheres] was of course the impossibility of reconciling the assumption of the invariability of the distance of each planet with the observed differences in the brightness, especially of Mars and Venus". Schiaparelli ([1926], [1998](#page-11-0): 122) is on the same path when states that: "of these difficulties, the most formidable was this: that according to the homocentric sphere hypothesis, the distance and the brightness (according to the ideas of that time) of each celestial body would have to remain absolutely invariable, because they are carried out over a spherical surface concentric with the Earth; whereas observations of the brightness of the planets appeared very different at different times, especially in the case of Mars and Venus." [Dreyer \(1953](#page-10-0): 141) also agrees when affirming that: "the homocentric system never received any further development or improvement, simply because, as Simplicius tells us, the great change in the brightness of the planets, especially Venus and Mars, rendered the idea of each planet being always at the same distance from the earth utterly untenable."

Furthermore, many philosophers of science offer the same explanation. For example, in his very influential (at least among philosophers of science and teachers) Copernican Revolution, [Thomas Kuhn \(1957:](#page-11-0) 58-59) explains that "...all homocentric systems have one severe drawback which in antiquity led to their early demise. Since Eudoxus' theory places each planet on a sphere concentric with the earth, the distance between a planet and the earth cannot vary. But planets appear brighter, and therefore seem closer to the earth, when they retrogress. During antiquity the homocentric system was frequently criticized for its failure to explain his variation in planetary brilliance, and the

 $\frac{2}{3}$ "At least in a qualitative way" because, as I will show later (Section [5\)](#page-6-0), the Eudoxian model (as well as some versions of the epicycle and deferent model) was not capable of making Venus and Mars move in retrograde motion at the periods that they actually do.

system was abandoned by most astronomers almost as soon as a more adequate explanation of the appearances was proposed [i.e., the epicycle and deferent system]." [Hanson \(1973](#page-11-0): 95) similarly states that "the deathblow to the Eudoxian-Aristotelian astronomy-cosmology, however, came from noting how enormously the brightness of Mars and Venus vary from one time to another. .Naked-eye astronomers of Greece encountered all this simply as a dramatic change in brightness, which was inconsistent with any geocentric, spherical model of the cosmos". Finally, more recently, [Musgrave \(1991:](#page-11-0) 259) says that "the most obvious difficulty [of the Eudoxus's system] was the variation in the brightness of the planets ... which showed that they were not always the same distance from the earth as the principle of concentrism demanded."

I will aim to show in this paper that the certainty of this claim, i.e., that the main reason for the rejection of the homocentric spheres in favor of the epicycle and deferent system was the change in brightness of the planets during retrograde motion, is unfounded. We do not really know what reasons forced the abandonment of the homocentric spheres in favor of the epicycle and deferent system.

In Section 2, I will show that it is simply false that the planets significantly change their brightness during their retrograde motion. This is only true for Mars, but not for the other planets. Particularly not for Venus, to which many authors, including Hanson, Heath, Dreyer and Schiaparelli, appeal. In Section [3,](#page-4-0) I will analyze the origin of the myth that the change of brightness of the planets, especially of Mars and Venus, during their retrograde motion was the main reason for rejecting the homocentric model. In Section [4,](#page-4-0) I will analyze Ancient testimonies on the change of planetary brightness, concluding that there is no strong evidence that the Ancients at the time when the Eudoxian model was abandoned were worried or even aware of planetary change of brightness, including Mars. The first testimonies are contemporary or posterior to Ptolemy, at least three centuries after the Eudoxian model had been completely abandoned. In Section [5,](#page-6-0) I will draw the reader's attention to the existence of what I shall call 'oppositesense epicycle models', i.e., planetary models in which the epicycle and the deferent rotate in opposite directions. In these well attested models, the retrograde motion is produced when the planet is farther away to the Earth and, consequently, they should appear to be less bright. Later, I will analyze the reason Ptolemy adduced for preferring the same-direction epicycle and deferent model for the planets, and I will show that it is not related to changes of planetary brightness. Finally, in Section [6](#page-7-0), I will consider the possibility that changes in the apparent size of the Sun and Moon could have played a role in the rejection of the homocentric spheres. I will finish the paper enumerating some interesting epistemological lessons that we can learn from this particular case.

2. Do the planets increase their brightness during retrograde motion?

The brightness of a planet depends mainly on four factors: 1) its size, 2) its distance from the Earth, 3) its phase, and 4) the darkness of the environment when the planet is seen. This last factor implies that planets usually are not visible during daylight, because the sun-light is enough for making them disappear. But even when the Sun is below the horizon, a planet is not visible if it is above the horizon but too close to the Sun, because it disappears in the glare of the Sun. Medieval astronomers call this phenomenon the arc visionis. The arc visionis for all planets is around 10° , but it is only 5° for Venus which is so brilliant that it is visible even when it is very close to the Sun. The arc visionis also implies that the planet would look less bright if the sky is brighter and the other way around. Certainly, ancient astronomers were aware of that.

The size of a planet is constant, so the change in brightness does not depend on that. What about the other two factors: the distance from the Earth and the phases? Brightness is proportional to apparent area or the planet's disk and so, a change in brightness is inversely proportional to the square of a change in distance. In addition, the more complete the phase is, the brighter the planet would appear. These two factors are thus interconnected.

Fig. $3(A)$ and (B) represent the maximum and minimum distance from the Earth that an inner and outer planet could reach respectively. In both cases, V_1 and M_1 represent the minimum distance and V_2 and M_2 the maximum distance. The square of the proportion between these maximum and minimum distances represents how much a planet changes its distance from the Earth. The proportions are roughly [\(Van Helden, 1985:](#page-11-0) 44), Mercury: 2.2; Venus: 6.1; Mars: 4.8; Jupiter: 1.5 and Saturn: 1.2. Only Mars and Venus vary greatly their distances from the Earth. This fact could explain why many texts explicitly mention the change in brightness of Mars and Venus. The retrograde motion of outer planet is produced when the planet is in opposition (M_1) , i.e., when the planet is both at its minimum distance from the Earth and in a full phase, having the maximum possible brightness.

[Fig. 4](#page-3-0) plots the magnitude of the planets known in Ancient times during one synodic period. Because negative magnitudes mean that the planet is brighter, I reversed the order of the values in axis X, so that the brighter the planet, the higher in the graph. The magnitude is in function of the elongation from the Sun, so that it could be imagined that the sun is in axis Y. The points that are inside the dotted box (representing the arc visionis) are invisible because the planet could never be seen so close to the Sun, even if the Sun is below the horizon (except, maybe in the extraordinary case of a total solar eclipse in which the planet is close enough to the Sun). In the case of the outer planets, the retrograde motion is at oppositions, i.e., at the extremes of the graph. It is clear that outer planets increase their magnitude during retrograde motion. It is also clear, however, that the growth of Mars is considerably greater than those of the other two planets. In fact, the change in distance of Mars is big enough for rendering its change of brightness (around three magnitudes) visible to the naked eye. The situation is, however, different for the other two superior planets: their change in brightness is hardly detectable for the necked eye. Saturn's magnitude difference is only of 0.6 and Jupiter's around $0.8³$ Typically, the difference of 1 magnitude is detectable to the naked eye, comparing both celestial bodies at the same time.

The case of the inner planets is different because they retrograde at (inferior) conjunction (V_1 of [Fig. 3\)](#page-3-0). It is true that at inferior conjunction they are closest to the Sun, but they are usually not visible because the glare of the sun makes them disappear. Even more important, during retrograde motion, they are in new phase, reaching their minimum possible brightness.

Mercury is only visible on greatest elongation so there is not much variation in distance during the intervals of visibility. Moreover, the small apparent size of Mercury prevents the detection of any variance in its apparent size. Venus's brightness also shows relatively little change, because the enormous variation in its

³ The values correspond to one synodic period starting at 1995. Data taken form Espenak, F. NASA reference publication 1349, Twelve Year Planetary Ephemeris: 1995-2006 [\(http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html\)](http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html). These values are representative, they could vary from one synodic month to the next, but not significantly.

Fig. 3. The distance from the Earth of an inner (A) and outer (B) planet during retrograde motion. S is the center of the Sun, E, the center of the Earth and V₁ and V₂ in figure A represent two different positions of an inner planet (Venus) and M_1 and M_2 in figure B, two different positions of an outer planet (Mars).

Fig. 4. The planetary change of magnitude. The values represent the planetary magnitude in function of the elongation during one synodic period starting at 1995. Data taken form Espenak, F. NASA reference publication 1349, Twelve Year Planetary Ephemeris: 1995-2006. [\(http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html](http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html)).

distance from the Earth is largely canceled by the effect of the phases: the more Venus increases its size, the more its phase decreases, arriving at new phase at its minimum distance. The change in brightness of Venus is only of around 0.8, a magnitude difference hardly detectable, similar to that of Jupiter. This very small change is partially due also to Venus's atmosphere: the presence of sulfuric acid droplets in the high atmosphere of the planet scatters an important amount of light [\(Mallama, Wang, & Howard, 2006](#page-11-0): 10), so even at a new phase is still shining. Moreover, as Fig. 4 shows, the interplay of distance and phase makes Venus to reach its maximum magnitude at about 39° of elongation from the Sun and, therefore, very far from the middle of the retrograde motion; actually, very close to the maximum elongation.

The fact that Venus does not change significantly in brightness is a very well-known fact today (see [Goldstein, 1996](#page-11-0): 1). It is even more surprising, then, to find scholars seemingly forgetting this fact when they say, in Hanson's words for example, "how enormously the brightness of Mars and Venus vary from one time to another".

It was only in medieval time when an author correctly described the phenomenon of the apparent size of Venus. The Jewish philosopher Levi ben Gerson (1288–1344) says: "it is also became clear to us on the basis of observation that the apparent size of the diameter of Venus is greater at greatest elongation from the Sun than at 0° or 180 $^{\circ}$ of anomaly [i.e., at conjunctions]. On the other hand we did not observe it to be greater at 180° of anomaly than at 0° of anomaly. All this is at variance with what follows from Ptolemy's model, for according to it the diameter of Venus should appear to be greater at 180 $^{\circ}$ of anomaly than at 0 $^{\circ}$ of anomaly by more than 6 times" [\(Goldstein, 1996](#page-11-0): 6; [Goldstein, 1985](#page-11-0): 105). Levi ben Gerson goes further and emphasizes that the observed difference in size of Mars is just 1:2, while the Ptolemaic model requires

1:7, and that he cannot perceive any changes of brightness in Mercury, Jupiter or Saturn ([Goldstein, 1985](#page-11-0): 105-106), as required by Ptolemaic models.

In conclusion, Venus's changes of brightness are hardly detectable to the naked eye, but even if they were, we would only see that it diminishes its brightness during retrograde motion, exactly the opposite of Mars. Therefore, why did so many renowned scholars affirm that both planets increase enormously or drastically (for using, again, Hanson's own words) their brightness during retrograde motion?

3. The origin of the myth: Simplicius

As I will show in the next section, there are few Ancient testimonies of the change of brightness of the planets. Nevertheless, the most ancient testimony that mentions the change in brightness of the planets as an argument for rejecting homocentric spheres comes from Simplicius. Many scholars quoted him explicitly from his commentary to Aristotle's De Caelo, which is undoubtedly the source of this assertion. It is important to bear in mind that Simplicius's epistemic reconstruction is not too trustable in general ([Bowen, 2002](#page-10-0)) and that he is describing situations which took place seven centuries before him. Simpicius affirms that the observed change in brightness of Venus and Mars was one of the reasons for the rejection of the homocentric spheres in favor of epicycle models. He says:

This is indeed obvious to the eye in some cases. That is, the star said to belong to Venus and, moreover, the [star belonging to] Mars appear many times greater at the middle of their retrogradations, with the result that, on moonless nights, the [star] of Venus for its part causes shadows to fall from bodies. (De Caelo 2.12, [Bowen, 2013:](#page-10-0) 165).

Simplicius is right about Mars, but doubly wrong about Venus: first it is impossible to see Venus in the middle of its retrograde motion because of the glare of the Sun, and second because, even if the sunlight would not make it disappear, it would still be at a new phase and so not in its maximum brightness!

Fig. 5. All the epicycles on the same deferent. It is clear that Venus and Mars have the greater variation of distance from the Earth.

Why did, then, Simplicius affirm this? In Ptolemy's epicycle and deferent model, Mars and Venus are the planets with the bigger epicycles, and, therefore the planets that vary their distance from the Earth the most. In Fig. 5 the epicycles of all the planets have been superimposed to the same deferent, so that their sizes are comparable. It is clear that Venus and Mars vary their distance from the Earth the most. That is how Simplicius (or his source) could have deduced (not observed) that Venus and Mars should "appear many times greater at the middle of their retrogradations". Therefore, as [Goldstein \(1996](#page-11-0): 4) says, "this observational claim is to be understood as a 'reconstruction' based on a consequence of Ptolemaic theory" and not as an observation that the theory should explain.

As [Kuhn \(1962\)](#page-11-0) and [Hanson \(1958\)](#page-11-0) highlighted, observation is usually theory-laden. This is particularly true in Simplicius's case, because ancient Greeks granted a much more important role to theory in detriment of observation than we do today, both in the building and confirming of their models. It is true that there was an "empiricist" evolution even inside Ancient astronomy. The relationship between theory and observation in Aristarchus of Samos is not the same in Ptolemy, for bringing up the extremes of this evolution. Aristarchus of Samos puts the empirical data that he used as input in his calculation of the sizes and distances of the Sun and Moon among the hypotheses and he never explains how he obtained these values [\(Heath 1913](#page-11-0): 352-353). Actually, many have conjectured that they were not observed at all [\(Tannery,](#page-11-0) [1883:](#page-11-0) 241, [Evans, 1998:](#page-10-0) 72) or that they were "the result of the complex interaction of some qualitative observations and theoretical considerations" ([Carman, 2014](#page-10-0): 55). The situation is clearly more refined in Ptolemy who in many cases describes the instruments and its degree of accuracy. For example, he describes and compares two different instruments for measuring the obliquity of the ecliptic (Almagest I,12; [Toomer, 1998](#page-11-0): 61-62) or explains in detail the degree of accuracy of the equatorial ring (Almagest I,3; [Toomer, 1998:](#page-11-0) 134). Nevertheless, it is undisputable that Ptolemy selected or altered the data in many cases in order to fit his theoretical requirements. One clear example is the observation of the lunar parallax: the value obtained is clearly wrong but extraordinary good for obtaining the lunar distance he is looking for at syzygies (Almagest V,12; [Toomer, 1998](#page-11-0): 61-62). Ptolemy explicitly recognizes a complex interplay between theory and observation in On Criterion of Truth ([Long, 1989\)](#page-11-0) and Harmonics ([Barker, 2000](#page-10-0)), in which he says that, even if the astronomer start from the data of the senses, the theory judges the data and helps the senses to improve their accuracy in perception. It is of great importance, then, always to bear in mind this complex relationship between theory and observation when one reads in an Ancient source that something-like an increasing in the brightness of a planet—has been observed.

4. Ancient testimonies of the change of planetary brightness

As I have already noted, the only planet with a clearly perceivable change of brightness is Mars. Thus, one could expect to find many references to it during ancient times. The textual evidence, however, is very limited. There are five observations of Mars in the Almagest, three of them are oppositions at years 130, 135 and 139, which Ptolemy used for determining the values of eccentricity and apogee of Mars model, but without mentioning its brightness (Almagest X, 7; [Toomer, 1998](#page-11-0): 484). In the Planetary Hypotheses ([Goldstein, 1967:](#page-11-0) 8), a later work, Ptolemy lists the value of the apparent diameters of each planet in terms of the apparent diameter of the Sun. Ptolemy says that these apparent sizes correspond to mean distances. Knowing the mean distance of the planets, Ptolemy is able to calculate the absolute size of the planets. Again,

there is no mention at all of changes in apparent size or brightness of the planets (but, obviously it is implicit in the fact that Ptolemy cares for deciding at which distance the apparent size will correspond). The value for Venus is taken from Hipparchus. Ptolemy says that, even if Hipparchus did not specify if this apparent size corresponded to the minimum, mean or maximum distance, 4 "we consider this amount to be its apparent diameter at mean distance, where the planet is usually seen, for at apogee and perigee it is hidden by the rays of the Sun" [\(Goldstein, 1967:](#page-11-0) 8). Ptolemy knew that Venus is not visible at the middle of a retrograde motion, probably because Ptolemy was a better observer than Simplicius!

In Ancient times-leaving Simplicius aside-Proclus and Theon of Smyrna are the only other authors who referred to the changes of planetary brightness (or apparent sizes), both after Ptolemy. Theon of Smyrna, 5 who is probably one generation younger than Ptolemy [\(Jones, 2015:](#page-11-0) 3) offers a very basic astronomical view of the time (sourced mainly from Adrastus) in the third part of his The Mathematics Useful for Reading Plato, fully devoted to astronomy. He sometimes describes arguments based on observations (for example, for proving the spherical shape of the Earth), but very often he mixes theoretical inferences and observational data in these descriptions. When he describes the planets (i.e., the five planets plus the Sun and Moon), he says that they move in longitude, in latitude, and also in depth. According to Theon, they "vary in size, being sometimes more distant, sometimes more close to the earth in the depths of space. This is why the speed of their movement through the Zodiac seems uneven: they do not travel equal distances in equal time; they go faster when they appear larger, because of their lesser distance from earth and they go more slowly when they appear smaller, because of their greater distance" [\(Dupuis, 1892](#page-10-0): 221-223). Theon does not explicitly mention the change of brightness during retrograde motion, but he does it implicitly when relating the change of sizes to the change of distances and the change of distances to the change of speeds. He seems to be saying, thus, that the change in size of the planets is an observational datum. Taking into account that Theon does not distinguish between what happens with the Sun and Moon (which do not retrograde) and the planets (which retrograde) and, among them, which one have a perceivable change in size (Mars) and which ones not (the rest), however, it seems reasonable to suppose that he is not talking of a "real" observation but of a "deduced observation": if the change of speed corresponds to a change in distance, then there would be also a change in apparent size. This case is a good example of the not-soclear distinction between theory and observation in Ancient authors.

The case of Proclus (who died around the year Simplicius was born) is more interesting because he refers to an unambiguously observational fact. Proclus gives the list of planetary phenomena that had to be explained by Ancient astronomers (the motion in longitude, in latitude, the retrogradations, etc.). The crucial point for Proclus was the observation that the planets "are sometimes observed bigger, sometimes smaller, as if they were moving in depth and sometimes they were closer to us and sometimes further away. So, many times there is not a perceivable difference in size between Mars and Jupiter or between Mercury and Venus, and they are only distinguishable through their color" (i.18; [Manitius, 1909:](#page-11-0) 10-11). Proclus introduces a new fact: Mars and Jupiter on the one hand, and Venus and Mercury on the other, sometimes have the same apparent size. Of considerable importance, he is not explicitly relating this fact to a change in speed, as Theon did. It is not clear, however, how Proclus could suggest that Mercury and Venus had the same apparent size making them undistinguishable, since Venus is always more than two magnitudes brighter than Mercury. Nevertheless, if Venus were close to the Sun and Mercury at its maximum elongation, the glare of the Sun would affect differently each planet, making Venus to diminish it brightness, even being as bright as Mercury. Proclus's observation, however, is certainly true for Mars and Jupiter. Jupiter is usually brighter than Mars, even when Mars is in the middle of its retrograde motion. Nevertheless, when Mars retrogrades close to its perigee, because of its large eccentricity, it becomes as bright as Jupiter, making it very difficult to distinguish each other, if not for Mars color. This impossibility of distinguishing them is almost exclusively due to changes in Mars, not in Jupiter. Proclus observation could thus be explained simply postulating a change in the brightness of Mars. This observation re-appears in Copernicus, who asserts that when Mars "shines all night, seems to equal Jupiter in size, being distinguished only by its reddish color. Yet in the other configurations it is found barely among the stars of the second magnitude, being recognized by those who track it with assiduous observations" (Copernicus I, X; [Rosen, 1992](#page-11-0): 22).

Therefore, the historical evidence seems to suggest that the pre-Ptolemaic astronomers did not care about the change of brightness (of Mars), even if it was perceivable. Bowen affirms that "no one before Ptolemy appears to have paid any attention to the fact that the stars (both fixed and wandering) differ in size (brightness), if they noticed it at all" [\(Bowen, 2013](#page-10-0): 289), and that "it is important to realize that, though there is some recognition in the period before Ptolemy that the stars (both fixed and wondering) differ in size or brightness form one another, there is no evidence that anyone noticed a variation in the size or brightness of any star, or that they thought such a variation important". [\(Bowen, 2002](#page-10-0): 161). The first clear testimonies that we have are contemporary to Ptolemy, in the case of Theon-even it seems a theory-laden observation-or around three centuries after him, in the case of **Proclus**

I have so far shown three reasons for putting into question the certainty with it is usually affirmed the idea that the change in brightness of the planets during retrograde motion played an important role in the abandonment of homocentric spheres in favor of epicycles and deferents. First, I showed that the change of brightness during retrograde motion is not clearly perceivable to the naked eye, except for Mars. Then, I identified the source of the myth, Simplicius's text, and showed that Simplicius's explanation is not trustable, not only because he is reconstructing an epistemic situation which occurred seven centuries before him, but also because his reconstruction suffers from serious difficulties: mainly that he (or his source) probably constructed the observations that the theory supposedly explained from the theory itself. Finally, I argued that there are no pre-Ptolemaic references to the change of brightness of the planets, even of Mars. Furthermore, even by Ptolemaic times when these references started to appear, the homocentric sphere model had been abandoned many centuries before.

Evidently, lack of evidence does not imply lack of existence. It is still possible that, even if Simplicius is not trustable, even if only Mars's change of brightness is perceivable, and even if there are no testimony about it, Ancients introduced epicycles for explaining the change of brightness of Mars. One could still argue that the change of brightness in Mars during retrograde motion is enough for affirming that it played an important role in the abandonment

Moreover, the fact that Hipparchus didn't specify the distance implies according to [Bowen, 2002:](#page-10-0) 162 that he "did not recognize any variation in the apparent diameter of Venus".

 5 I thank one of the anonymous referees for making me notice this text, as well as Calcidius's text mentioned in Section [6.](#page-7-0)

Fig. 6. Two possible epicycle and deferent models. In both figures E is the Earth, C is the center of the epicycle and P is the planet. Fig. 6(A) represents the same-sense model, in which the epicycle and deferent go on the same direction, Fig. 6(B) represents the opposite-sense model, in which the epicycle and deferent go on contrary directions.

of the Eudoxian spheres in favor of epicycles and eccentrics. After all, Ancient astronomers could have argued from analogy and, having noticed that Mars has a bigger epicycle, concluded that it is reasonable not to detect any variation in the brightness of the other two outer planets. Venus would be an exception, an anomaly to be explained but, still, the change in brightness could have been used as an experimentum crucis in favor of epicycles. This is explicitly suggested by Simplicius when, after claiming that the change of brightness in Mars and Venus is observable to the naked eye, he adds: "it is reasonable that [the same] happens in the others as well even if it is not manifest to sight. Indeed, not only is reasonable, it is in fact true, since the daily motion of the [other planets] appears unsmooth. But, concerning their apparent sizes, no difference is noticed because their variation in altitude and its opposite (which scientists used also to call month in depth) is negligible" ([Bowen, 2013:](#page-10-0) 167). The argument by analogy used by Simplicius seems plausible: since Mars (and Venus) has a nonuniform motion that could be explained by epicycles and shows variation in brightness, it is natural to think that the other planets (with a small epicycle), having a non-uniform motion too, would also have some variation in brightness, even if unperceivable to the naked eye.

In the next section, I will offer some other, stronger arguments to hold that the change of brightness was probably ignored when the epicycle and deferent system was proposed.

5. The opposite-sense epicycle model

As I mentioned earlier, in the Ptolemaic model for the five planets, the epicycle and the deferent rotate in the same sense. Therefore, the retrograde motion is produced when the planet is inside the deferent when the tangential speeds of epicycle and deferent go on contrary directions. It is also possible, however, to produce retrograde motion inverting the sense of the motion of the planet on the epicycle. In this case, the retrograde motion is produced when the planet is at its maximum distance from the center of the deferent. Fig. 6(A) represents the same-sense epicycle model: the retrograde motion is produced at the minimum distance; Fig. 6(B) represents the opposite-sense epicycle model: the retrograde motion is produced at the maximum distance.

The opposite-sense epicycle model is not simply a geometrical possibility; there is strong evidence that it was proposed in Ancient times. The first evidence is in Keskintos Inscription [\(Jones, 2006](#page-11-0)), a fragmentary astronomical inscription accidentally found in the Island of Rhodes and now stored at the Staatliche Museen of Berlin, dated from about 100 BCE. It contains a set of numerical data with some astronomical explanation that could only be understood if one assumes the opposite-sense epicycle model ([Jones, 2006:](#page-11-0) 28; [Neugebauer, 1975:](#page-11-0) 702-704).

The second evidence comes from an astrological papyrus of the second century A.D. (contemporary to Ptolemy), known as Papyrus Michigan 3.149. This papyrus is oriented mainly towards the astrological influences of the planets on the parts of the human body, but in its astronomical part it describes epicycle models. Among them, the papyrus describes some kind of mixed system: the same-sense model is attributed to the inner planets and the opposite-sense model to the outer ones [\(Neugebauer, 1972, 1975](#page-11-0): 805-808; [Robbins, 1936\)](#page-11-0). This is especially relevant to our topic for it shows that at least in this case the change in brightness is not related to the selection of the epicycle direction: the inner planets do not show change in brightness and, nevertheless, are carried in the same-sense epicycle model, making the planets to be closer to the Earth during their retrograde motion; the outer planets are carried in the opposite-sense model making the planets farther away from the Earth during their retrograde motion, even if one of them shows a clear increase of brightness during retrograde motion. If the epicycle and deferent system was proposed for explaining the change of brightness, this mixed system is meaningless.

The third evidence comes from Pliny the Elder, also contemporary to Ptolemy, who in his Natural History (II, 64-75; [Neugebauer, 1975](#page-11-0): 802-805) referred to this mixed model, although his discussion of planetary motion is very muddled and it is doubtful how much he really understood of it (see [Neugebauer,](#page-11-0) [1975](#page-11-0): 802). In particular, Pliny affirmed that when the outer planets are at opposition and, therefore, in the middle of retrograde motion, "they appear very small because they are at the distance of their greatest altitude and are moving with their smallest velocity" (Pliny II, 70; [1962](#page-11-0): 216). So, while Simplicius, who defended the same-sense model, claimed that he observed Venus to be bigger at the middle of the retrograde motion (which is patently false), Pliny, commenting the opposite-sense model, affirmed that he observed the outer planets (including Mars) to be smaller at retrograde motion (which is also patently false)!

Unlike the same-sense epicycle model, the opposite-sense has important restrictions for producing retrograde motion. In particular, it cannot make Venus and Mars to move in retrograde motion at the periods that they actually do. Furthermore, [Asger Aaboe](#page-10-0) [\(1963\)](#page-10-0) showed that the opposite-sense model does not produce retrograde motion when the speed of the deferent is greater than the speed of the epicycle, i.e., if for a certain period of time the planet makes more turns in longitude than retrograde loops. If the epicycle radius is made to be smaller than the deferent radius, then the model would not produce retrograde motion if the speed of the

Fig. 7. The time from greater speed to mean and from mean to minimum speed in both the same sense—Fig. 7(A)—and opposite-sense—Fig. 7(B)—epicycle models. In both figures E is the Earth, C is the center of the epicycle, A is the apogee and P is the perigee. The planet is moving at mean speed at M_1 and at M_2 .

epicycle is smaller than the double of the speed of the deferent. But this is precisely what happens with Venus (1 turn in the deferent in 1.598 turns in the epicycle) and Mars (1 turn in the deferent in 1.135 turns in the epicycle). Therefore, the opposite-sense model could not be used for predicting the motion of Mars or Venus. It seems that the opposite-sense model defenders were aware of this problem, for, in the Keskintos inscription [\(Jones, 2006](#page-11-0): 31), which shows parameters of the speeds of the planets, the speed of the epicycle of Mars is triplicated presumably in order to making it faster than the double of the speed of the deferent $(3 > 1.135^*2 = 2.27)$.⁶ In this way, the model is able to produce retrograde motion for Mars, but it produces three retrograde loops when Mars actually has only one, generating two spurious loops in each synodic cycle. This fact, again, is relevant for our argument for it shows that, if the change of brightness was an important reason for rejecting homocentric spheres in favor of the epicycle models, it is unlikely that some astronomers proposed an opposite-sense model for Mars that produces the contrary effect in the change of brightness, making Mars less bright during retrograde motion, even at the cost of a serious empirical anomaly, making 2/3 of the predicted retrograde motions spurious.

When Ptolemy introduced in the Almagest the planetary models, he explicitly said that there are two possibilities for producing retrograde motion (i.e., same-sense and opposite-sense models), and that he chose the same-sense model, making his reason explicit, without making any reference to the change in brightness of the planets at all. For correctly understanding Ptolemy's text it is important to realize that when he talks about the eccentric hypothesis he is talking also about the opposite-sense model, because they are equivalent, and when he talks about the epicyclic hypothesis, he is talking about the same-sense model (see Almagest III, 3, [Toomer, 1998](#page-11-0): 146). Ptolemy says:

"For [the retrograde motion] we find... that in the case of the five planets the time from greatest speed to mean is always greater than the time from mean speed to least. Now this feature cannot be a consequence of the eccentric hypothesis [i.e., opposite-sense model], in which exactly the opposite occurs, since the greatest speed takes place at the perigee in the eccentric hypothesis [i.e., opposite-sense model], while the arc from the perigee to the point of mean speed is less than the arc from the latter to the apogee in both [i.e., same- and opposite-sense] hypotheses. But it can occur as a consequence of the epicyclic hypothesis [i.e., same-sense model], however, only when the greatest speed occurs not at the perigee, as in the case of the Moon, but at the apogee; that is to say, when the planet, starting from the apogee, moves, not as the Moon does, in advance [with respect to the motion] of the universe, but instead towards the rear. Hence we use the epicyclic hypothesis [i.e., samesense model] to represent this kind of anomaly" (Almagest IX,5; [Toomer, 1998](#page-11-0): 442).

Fig. 7(A) and (B) help to understand this passage. Point A represents in both situations the apogee, i.e., the maximum distance from the Earth, and point P the perigee, i.e., the minimum distance. At points M_1 and M_2 the planet moves at mean motion, i.e., at the motion of the deferent because the epicycle tangential speed is perpendicular to the speed of the deferent. The planet turns at a constant angular speed around the epicycle. Therefore it takes more time to go from A to M_1 (or from M_1 to A) than from M_1 to P (or from P to M_1). Fig. 7(A) represents the same-sense model. In it, the maximum speed of the planet is reached at A, where the tangential speeds of the epicycle and deferent go in the same direction. In P, the tangential speeds go on contrary directions and, therefore, P represents the minimum speed. Fig. 7(B) represents the opposite-sense model, in which the maximum and minimum speeds are inverted: the planet reaches its minimum speed at apogee (A) and the maximum at perigee (P). Ptolemy says that he observed that for the planets, it takes more time to go from maximum speed to mean, than from mean to maximum. Therefore, the same-sense model must be chosen.

Consequently, as far as Ptolemy's explicit reasons are concerned, the change in brightness played no role in his choice of the samesense epicycle model.

6. The change of the lunar apparent size

I have already considered and discussed the evidence for affirming that the change of planetary brightness during retrograde motion was the main reason for the acceptance of the epicycle and deferent model and the rejection of homocentric models, and concluded that the evidence is not enough for holding the case.

⁶ The values related to Venus did not survive in the Keskintos Inscription, so whether it asserted a same-sense or an opposite-sense model for the inner planets remains unknown. The triplication of Mars anomalistic period is also attested by Simplicius ([Bowen, 2013:](#page-10-0) 151), but attributed to the homocentric spheres, which have a similar geometrical restriction (see [Aaboe, 2001](#page-10-0): 71-72; Heath, 1913: 209- 210).

Fig. 8. The Moon is farther away when it is rising than when it is at zenith. M is the Moon and A and B are two locations at the Earth's surface. At A the Moon is rising, while at B the Moon is at the zenith. CM is equidistant with BM.

Nevertheless, given that the motions of the Sun and Moon were also represented with epicycles and deferents (or eccentrics), whether changes in their apparent size could have played any role still remains unanalyzed.

The most obvious change in the apparent size in both the Sun and Moon is the sometimes huge increase of size that is observed when they are near the horizon, at rising or setting. When referred to the Moon, this phenomenon is known as "Moon illusion". Even if the reason of the illusion is still disputed today [\(Hershenson, 1989;](#page-11-0) [Ross & Plug, 2002](#page-11-0)), it is clear that (even for Ancient astronomers) this fact is not related to changes in distance. When the Moon (or the Sun) is near the horizon at some place, it would be at the zenith (or near it) at another longitude. Therefore, from two different places of the Earth the distance would be different, which makes no sense. As Cleomedes said referring to the Sun, "since it can be both rising and culminating at different places, it will be in total both larger and smaller: larger for those for whom it is rising, smaller for those for whom it is culminating $-at$ the same hour of the day! Nothing is more absurd than this." (Cleomedes II,I; [Todd & Bowen,](#page-11-0) [2004](#page-11-0): 100-101).

Moreover, the Moon is farther away when it is close to the horizon than when it is at zenith by around one Earth radius, so it should be smaller. In Fig. 8, the Earth is the bigger circle and the Moon the small one, M. The Moon is seen at the zenith at point B, but at the horizon at point A. The distance MA is bigger than distance MB (by CA). Therefore, taking into account the distance variation, the Moon should be seen smaller and not bigger at horizon.

Ptolemy offered two different explanations for this change in apparent size. In the first one, he followed the traditional opinion, also held by Aristotle (Meteor. 373b b12-13, [Lee, 1952:](#page-11-0) 252-253), Posidonius (fragment 119, [Kidd, 1999:](#page-11-0) 174-175) and Cleomedes ([Todd & Bowen, 2004:](#page-11-0) 101), appealing to atmospheric changes to explain the illusion. In the Almagest (I,3, [Toomer, 1998](#page-11-0): 39) he said that "the apparent increase in their sizes at horizons is caused, not by a decrease in their distances, but by the exhalations of moisture surrounding the earth being interposed between the place from which we observe and the heavenly bodies, just as objects placed in water appear bigger than they are, and the lower they sink, the bigger they appear". In his later work Optics (59; [Smith, 1996](#page-11-0): 151), however, he abandoned this incorrect physical explanation and claims that the phenomenon is due to a psychological illusion.⁷ He probably measured the apparent size of the moon closer to the horizon and to the zenith with the dioptra and realized that the apparent size is the same, even if it does not appear to be.

Leaving aside, therefore, the psychological illusion produced when the luminaries are near the horizon, I will consider whether other changes in the apparent size are detectable. Changes in the apparent size of the Sun are difficultly detected. Ptolemy said that using a dioptra, designed by Hipparchus but constructed by himself, he "found that the sun's diameter always subtends approximately the same angle, there being no noticeable difference due to [the variation in] its distance" (Almagest V,14; [Toomer, 1998](#page-11-0): 252). Therefore, Ptolemy could not perceive any change in the apparent size of the Sun. Calcidius, in his Commentary to the Timaeus, described the solar anomaly and said that the "Sun appears to move slower and be smaller to our eye when it is in Gemini [its apogee], and the biggest and fastest when it is in Sagittarius [its perigee]" (I, 82; [Moreschini, 2003:](#page-11-0) 271-274). This is the only Ancient author attesting a change in the apparent size of the Sun. This change in size could not be detected by the naked eye, not only because the change is very small (a variation between maximum and minimum distance of $0.016⁸$), but mainly because the brightness of the Sun would prevent any clear observation of its circumference. This is another case of mingling theory with observation: the change of speed is observable; the change of size (due to the change of distance) is theoretical.

This is not the case for the Moon. First of all, its changes of apparent size are highly more significant, with a variation of 0.157 between maximum and minimum apparent size, around 10 times bigger than solar variation.⁹ Furthermore, its circumference is usually clear enough to be observed directly. This difference could be measured by reasonably simple instruments, like the dioptra that, according to Ptolemy, was accurate enough for detecting changes in the lunar apparent size (Almagest V, 14; [Toomer, 1998](#page-11-0): 252). There is another method, however, to detect these changes: solar eclipses make it possible to detect changes in the size of the Moon. As I have already noted, the solar apparent size is undetectable. Therefore, if we perceive any difference in the relative sizes of the Sun and Moon in solar eclipses, it must be due to changes in the Moon.

Usually, when the eclipse is total, the Moon covers the totality of the Sun, showing that both luminaries have precisely the same apparent size. Sometimes, however, there are annular eclipses, i.e., some rim of the Sun shines around the Moon, implying that in those eclipses, the apparent size of the Moon is a bit smaller than the apparent size of the Sun. If we interpret a change of apparent size as a change in distance, we can conclude that the Moon changes its distance from the Earth. Proclus referred to annular and total eclipses just after his mention of the changes of sizes of the planets, in his list of phenomena. Simplicius added this kind of evidence to that of the changes in brightness of the planets in favor of epicycles and against homocentric spheres. In this case, Simplicius seems to be correct [\(Bowen, 2013](#page-10-0): 167-168). One cannot be sure, for there is no testimony (except, again, that of Simplicius himself), that the change in the apparent size of the Moon played a role as an objection against homocentric spheres, but at least there is no doubt that annular eclipses were an attested fact in Ancient times.

⁷ For a different interpretation of Ptolemy's text see [Ross and Ross \(1976\)](#page-11-0).

 $^8\,$ The variation is expressed as (maximum value $-$ minimum value)/(maximum value $+$ minimum value). I thank Hernán Greco for recommending me this formula. Data taken form Espenak, F. NASA reference publication 1349, Twelve Year Planetary Ephemeris: 1995-2006 [\(http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html](http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html)). I took the difference between the maximum and minimum apparent diameter of the sun during year 2006 and it is just 64", being the maximum diameter 1887.9" and the minimum 1951.9 ".

⁹ Data taken form Espenak, F. NASA reference publication 1349, Twelve Year Planetary Ephemeris: 1995-2006 [\(http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.](http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html) [html](http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html)). I took the difference between the maximum and minimum apparent diameter of the moon during years $1995-1998$ and it is 247.5 , being the maximum diameter 2010.7 \degree and the minimum 1463.2 \degree .

Even if annular eclipses are more common than total eclipses, they are not too frequent and one particular city might not have seen an annular eclipse for centuries. For example, Alexandria saw, from the date of its foundation (332 BCE) to the death of Ptolemy (168 CE), only two annular eclipses, one at 234 BCE and another at 80 CE. None of them were visible to Ptolemy, who was born around 10 years after the second eclipse. Athens saw, from the date of its foundation (508 BCE) to the death of Ptolemy, as well two annular eclipses, one in 478 BCE and the other in 164 CE, leaving more than 640 years without annular eclipses.¹⁰ There are ten more annular eclipses during these seven centuries that were visible from somewhere in Greece or close to it. In addition, there are some testimonies in Ancient times that astronomers were probably aware of annular eclipses. It might be the case, however, that some purportedly observed annular eclipses where actually total eclipses that showed a solar corona, i.e., the sun-light that sometimes appears around the Moon during total solar eclipses ([Bowen, 2013:](#page-10-0) 250). There are not enough elements to be certain that Ancient astronomers actually observed annular eclipses, 11 because the Ancient observer cannot be expected to distinguish simply by observation an annular eclipse from a total-with-corona eclipse, and there is not enough data for identifying the particular eclipses mentioned in some ancient texts. This, however, is not relevant for my argument, for the fact is that if they thought that during some total solar eclipses all the Sun was covered and sometimes not, then, assuming that changes in the solar apparent size are not perceptible, the lunar apparent size must change, and, consequently, its distance from the Earth must change too.

There are at least two descriptions of what could have been annular or a total-with-corona solar eclipses. Plutarch, in his On the Face in the Moon, discussing why during a total solar eclipse the sky was not as dark as during night, said that "even if the moon, however, does sometimes cover the sun entirely, ... a kind of light is visible about the rim which keeps the shadow from being profound and absolute." ([Cherniss & Helmbold, 1957:](#page-10-0) 122). There is no agreement about to which historical eclipse Plutarch is referring, but it seems clear that he is describing a real eclipse.¹² Philostratus, in the Life of Apollonius of Tyana, said that the death of the emperor had been previously announced by a "remarkable portent ... in the heavens. The orb of the sun was surrounded by a wreath which resembled a rainbow, but dimmed the sunlight." (VIII, chap. 23; [1912](#page-11-0): 387). [Grant \(1852:](#page-11-0) 376-377) affirms that Philostratus is describing a solar total-with-corona eclipse, but it is really doubtful even whether he is talking about an eclipse or not (see [Newton,](#page-11-0) [1970:](#page-11-0) 114).

These are the only two attested observations of annular or totalwith-corona solar eclipses during ancient times, but the discussion about the possibility of annular eclipses in astronomical context is also documented. A papyrus of around 190 BCE-usually called Ars Eudoxi (Art of Eudoxus) (P. Louvre 2388 Ro + P. Louvre 2329 Ro) because the first letters of each of the first 12 lines of a poem form an acrostic TEXNH EU Δ OEOU, meaning "Art of Eudoxus"—contains some kind of rudimentary astronomy handbook that deals with some features of solar and lunar eclipses. The papyrus refers that solar eclipses can never be total but must remain at most annular, because the apparent size of the Sun is always greater than the apparent size of the Moon [\(Tannery, 1893:](#page-11-0) 292-293; [Blass, 1887:](#page-10-0) 23- 24; see also [Neugebauer, 1975](#page-11-0): 668, 688-687). The author of the papyrus claims, therefore, that all total solar eclipses are annular.

The same opinion is recorded by Cleomedes (II, 4, [Todd &](#page-11-0) [Bowen, 2004](#page-11-0): 142-143) who said that "some predecessors believed that in total [solar] eclipses when the centers of the deities are in a straight line, the rim of the Sun is observed encircling the Moon by protruding in all directions"; but Cleomedes himself denied the possibility of them, because "this is not part of what we detected". So, Cleomedes seems to affirm that there are not annular eclipses. A similar attitude towards annular eclipses could be inferred from the Almagest. Actually, according to Ptolemy, the lunar and solar apparent sizes are the same only "when [the Moon] is at its greatest distance from the earth" (Almagest V, 14; [Toomer,](#page-11-0) [1998:](#page-11-0) 252), avoiding the possibility of annular eclipses.

So far, thus, there is only one testimony affirming that all total eclipses are annular, and two saying that annular eclipses are impossible. Nevertheless, what is required for claiming that annular eclipses could have played a role in the acceptance of epicycles and deferents is the acknowledgment of the possibility of both kinds of eclipses. There is only one testimony of that kind in ancient sources: that of Sosigenes, the Peripatetic, contemporary to Ptolemy.

Sosigenes is usually attributed with the observation of an annular eclipse ([Neugebauer, 1975](#page-11-0): 104, n. 4; 668). Even if this is not certain, it seems that he was at least aware of them. Actually, according to Proclus, Sosigenes thought "that during eclipses near the Earth the Sun was observed not to be entirely covered but to extend beyond the disk of the Moon with the extremes of its own circumference and to cast light unimpeded [by the Moon]" (Hyp. 4.97-99; [Manitius, 1909:](#page-11-0) 130.9-26, translation taken from [Bowen,](#page-10-0) [2013](#page-10-0): 249). This is the only reference that links the lunar distance from the Earth with the nature (total or annular) of solar eclipses.¹³

In sum, even if the testimonies are clearly not too many, one cannot discard the possibility that the acknowledgment that sometimes total eclipses are annular, but not always, played a role in the recognition of the changes of lunar apparent size and consequently on the changes of distance of the Moon from the Earth. This, in turn, could have been used against Eudoxus's homocentric spheres and in favor of epicycles and deferents.

¹⁰ Espenak, Fred. NASA Eclipse Web Site. Javascript Solar Eclipse Explorer. [http://](http://eclipse.gsfc.nasa.gov/JSEX/JSEX-index.html) [eclipse.gsfc.nasa.gov/JSEX/JSEX-index.html.](http://eclipse.gsfc.nasa.gov/JSEX/JSEX-index.html)

 11 According to [Grant \(1855](#page-11-0): 371) the first eclipse that "is unequivocally asserted to have been annular" was the eclipse of April 9th, 1567 observed by Clavius at Rome. Clavius says "although the Moon was placed between my sight and the Sun it did not obscure the whole Sun as previously but (a thing which perhaps never before occurred at any other time [quod nunquam fortassis alias euenit]) a certain narrow circle was left on the Sun, surrounding the whole of the Moon on all sides" ([Clavius 1906:](#page-10-0) 508, translation taken from [Stephenson 1997](#page-11-0): 347). But, according to NASA Eclipse web site ([http://eclipse.gsfc.nasa.gov/SEcat5/SE1501-1600.html\)](http://eclipse.gsfc.nasa.gov/SEcat5/SE1501-1600.html), even if it was a hybrid eclipse, i.e., that it appears annular and total along different sections of its path, it was total from Rome. Kepler'[s \(1604:](#page-11-0) 297-299) calculations of the Sun distance at the moment of the eclipse showed also that it must be total, and not annular. See [Stephenson, Jones, and Morrison \(2007\)](#page-11-0) for an accurate analysis of this eclipse. This confirms how difficult it is to distinguish an annular from a total eclipse simply from observation.

 12 There is no total agreement about what eclipse Plutarch could have been referring to. See [Cherniss and Helmbold \(1957:](#page-10-0) 9-13). [Ginzel \(1899](#page-11-0): 202-204) says that it should be the eclipse on 20 March 71 CE, but [Sandbach \(1929:](#page-11-0) 15-16) supposes that it could be the eclipse of 5 January 75 CE or that of 27 December 83 CE, preferring the first one. [Stephenson \(1997:](#page-11-0) 364), like Ginzel, prefers that of year 71 and supposes that it was a total-with-corona and not an annular eclipse, because Plutarch's description implies a reduction in daylight (even if not as profound as during night) but during annular eclipses "there is hardly any noticeable reduction in daylight" [\(Stephenson, 1997:](#page-11-0) 364). Moreover, according to NASA Eclipse web site ([http://eclipse.gsfc.nasa.gov/SEcat5/SE1501-1600.html\)](http://eclipse.gsfc.nasa.gov/SEcat5/SE1501-1600.html), none of the three candidates was an annular eclipse, so he must have seen a total eclipse.

¹³ Proclus also says that "some astronomers" reported annular eclipses, without giving more details (Proclus, Hyp. I,19; [Manitius, 1909:](#page-11-0) 10,11). [Neugebauer \(1975](#page-11-0): 668) says that Polemarchus of Cyzicus observed an annular eclipse, but there are not enough reasons for this assertion [\(Bowen, 2013](#page-10-0): 145).

7. Conclusion

It is usually affirmed both in books and in history of astronomy or epistemology courses and seminars that the main reason for the rejection of Eudoxus' homocentric spheres in favor of the epicycle and deferent system is the impossibility of the former to explain the patent change of brightness during retrograde motion. I showed that holding this position is not so simple. The change of brightness of Mars together with the change of apparent size of the Moon could have played a role, but to say that "the most obvious difficulty [of the Eudoxus's system] was the variation in the brightness of the planets", as [Musgrave \(1991](#page-11-0): 259) states, is certainly oversimplifying the matter. There is not enough evidence for holding with certainty that this was the case. Rather, the evidence points in the opposite direction: it seems that the change of brightness during retrograde motion has been a theoretical observation: i.e., a fact first derived from the theory and then introduced in the set of phenomena that the theory successfully explained.

In this situation, there is no other position but to accept that we do not know why the epicycle and deferent system was preferred over the homocentric spheres. We can certainly speculate on the possible reasons for the abandonment of homocentric spheres in favor of epicycles. First, while the homocentric spheres cannot make Venus and Mars have retrograde loops at the correct period, this is possible for the (same-sense) epicycle and deferent model, which turns to be a very strong empirical reason. Second, the epicycle and deferent system is by far simpler to understand than the homocentric spheres. It is enough to recall that Eudoxus's proposal was not understood for many centuries, and probably would have remained so if the great Schiaparelli had not unveiled its mystery to us. Third, the epicycle and deferent model is a more flexible and fruitful model. That it is extraordinary fruitful has been shown by [Hanson \(1960\),](#page-11-0) who proves, relating epicycles and deferents with Fourier series, that it is possible to reconstruct any possible continuous, delimited and periodic orbit with epicycles and deferents. Certainly, this is not what the Ancients did, for they used other tools besides the epicycles like eccentrics and equant points to face the anomalies, but Hanson's paper shows the mathematical power of the model.¹⁴ Nothing similar can be said about homocentric spheres. Finally, its flexibility is related to the possibility of working together with other tools, like the abovementioned eccentrics and equant points. The combination of epicycles with eccentrics and equant points made the model incredibly accurate.

Finally, I want to argue that the fact that the change of brightness probably did not play a main role in the acceptance of epicycles offers some useful epistemological lessons. First, one should never underestimate the power that simple but false explanations have, making them extraordinarily immune to refutation. One can easily show that according to homocentric spheres the planets cannot change its geocentric distance, and that according to (same-sense) epicycle and deferent model the planets approach the Earth when they retrograde and that an increase of brightness would be naturally interpreted as a decrease of distance that it is hard to resist to such an explanation. Something similar happens with other widespread false but simple explanations, like that the reason for Columbus to turn around the Earth was to prove that the Earth was spherical, 15 or that during winter the weather is cold and during summer hot, because of the elliptical orbit of the Earth that makes it to be farther away from the Sun at winter and closer at summer. In the same way that it is easy to realize that the Earth could not be at the same time closer and farther from the Sun, when it is summer in one hemisphere and winter in the other one, it should not be difficult to realize (at least for scholars) that it is impossible to see Venus in the middle of its retrograde motion because it is in conjunction with the Sun. But the simple-false explanations are so powerful that they seem to block this kind of simple objection.

Second, this is a fascinating case of theory-laden observation, which could be used for illustrating this idea in textbooks. On the one hand, Simplicius says that Venus and Mars are seen brighter at the middle of their retrograde motion, exactly as the theory he is defending, the same-sense epicycle and deferent model, asserts. On the other hand, Pliny affirms that the outer planets are seen less bright at the middle of their retrograde motion, exactly as the theory he is defending, the opposite-sense epicycle and deferent model, asserts. And they are both wrong.

Finally, this case could be used against some scientific realism positions, because the (same-sense) epicycle and deferent system was not conceived for explaining the change of brightness, and nevertheless it successfully explains the changes of brightness of Mars. Therefore, we have here a successful and novel prediction of a clearly false theory that the realists should face (Carman & Díez, 2015).

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 14 Actually, what Hanson shows is that the epicycle and deferent system is simply irrefutable if you are ready to add as many epicycles as necessary. See Carman, 2010. See as well the animation of a very complex orbit reproduced using 10.000 epicycles and deferents: [https://www.youtube.com/watch?v](https://www.youtube.com/watch?v=QVuU2YCwHjw)=[QVuU2YCwHjw](https://www.youtube.com/watch?v=QVuU2YCwHjw) (copyright: Carman, C. & R. Serra, 2005).

¹⁵ On the source of this error see [Gingerich, 1992](#page-11-0) and [Russell, 1991.](#page-11-0)

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