Peirce's diagrammatic logic and the opposition between logic as calculus vs. logic as universal language

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Abstract

In the last century Jaakko Hintikka tried to determine Peirce's *locus* within the framework of the "Logic as Calculus vs. Logic as Universal Language" opposition in the history of mathematical logic, placing it in the former tradition. For this purpose Hintikka reformulated the opposition devised earlier by Jean van Heijenoort in order to investigate not only the development of notations and formal languages in the origins of mathematical logic but also the very original ideas in them. The aim of this paper is to show some difficulties in placing Peirce's diagrammatic conception of deductive logic inside this opposition. Firstly, Hintikka's distinction presupposes a *linguistic* conception of logic by the founders of mathematical logic. However this was *not Peirce's own ultimate conception*. Secondly, there is now enough evidence (provided recently by Francesco Bellucci and Ahti-Veikko Pietarinen) that Peirce conceived his diagrammatic system of the Existential Graphs mainly as a tool for logical analysis. This analysis is not of linguistic nature but rather of a *semiotic* one.

Keywords: DIAGRAMMATIC REASONING – HISTORY OF LOGIC – C. S. PEIRCE

1. Introduction

This paper discusses the place of Peirce's diagrammatic conception of deductive reasoning in the history and philosophy of mathematical logic as it was exemplified through his system of Existential Graphs. At the end of the last century Jaakko Hintikka tried to determine and describe Peirce's *locus* within the framework of the traditions of "Logic as Calculus" and "Logic as Language" in the history of mathematical logic, placing it in the former tradition[1]. Hintikka had reformulated the distinction devised earlier by Jean van Heijenoort in order to analyze not only the development of notations and formal languages in the origins of symbolic logic but the very original

[1] see Jaakko Hintikka "The Place of C. S. Peirce in the History of Logical Theory". In *The Rule of Reason: The Philosophy of Charles Sanders Peirce*, edited by Jacqueline Brunning and Paul Forster, Toronto: University of Toronto Press, 1996, pp. 13–33.

ideas that were decisive in these origins[2]. Hintikka widened the scope of van Heijenoort's distinction in order to include the "ultimate presuppositions" concerning *the nature of language*, which were essential for him in 20th Century philosophy. Hintikka's claim was mainly guided by the anticipation of "model-theoretic" features in Peirce's work.

One of the claims of this paper is that Hintikka's distinction presupposes a *linguistic* conception of logic by the founders of mathematical logic (ultimately related to ordinary language). However this was *not Peirce's own ultimate conception*. When Peirce claimed that the Existential Graphs were "not intended to serve as a universal language for mathematicians or other reasoners like that of Peano"[3], he was ruling out *in toto* this linguistic conception of logic. Thus, it can be suggested that Peirce's ideas cannot be adequately captured by this distinction. Moreover, there is now enough evidence (provided recently by Francesco Bellucci and Ahti-Veikko Pietarinen) to believe that Peirce conceived his system of the Existential Graphs not only as a diagrammatic proof procedure for deductive logic, but also (and mainly) as a tool for logical analysis. This analysis is not of linguistic nature but of a semiotic one.

In the introduction of his book *Logical Forms Part I*, Oswaldo Chateaubriand refers to the "current linguistic view of logic" in relation to the mainstream ideas in mathematical logic as developed in last century[4]. According to Chateaubriand, this view "explained away" the "abstract content" of logic "in linguistic terms", that is, in syntactic or semantic terms. Even if Chateaubriand talked in fact about *formal* languages, logical concepts are usually elucidated as features of the language. The specific languages are analyzed in order to identify words and expressions or phrases representing logical concepts, and thus logical analysis is applied to specific languages without being a theory of specific languages.

The relation between logic and language was previously discussed by Jean van Heijenoort in his seminal paper of 1967 in a more specific way. In it, two fundamental lines of thought in the history of modern logic were distinguished: *logic as language* (represented by Frege's conceptual notation) and *logic as calculus* (represented by the algebra of logic). He took this distinction from Frege's own opposition between *lingua characterica* and *calculus ratiocionator*, expressed in various papers to the effect of sketching two different goals of formalism. Thus, he stated: "I was trying, in fact, to create a 'lingua characterica' in the Leibnizian sense, not a mere 'calculus ratiocinator'"[5]. Hintikka reformulated the distinction conceived by van Heijenoort in order to analyze not only the development of notations and formal languages in the origins of symbolic logic, but also the very original ideas that were decisive in these

[2] see Jean van Heijenoort. "Logic as Calculus and Logic As Language". *Synthese* 24 (1967), pp. 324-330. DOI: https://doi.org/10.1007/BF00485036. (Reprinted in *Selected Essays*. Naples, Bibliopolis, 1985, pp. 11-16.)

[3] Peirce, CP 4.424.

[4] see Chateaubriand, Oswaldo: *Logical Forms Part I* 2001, p. 14 *passim*.

[5] Frege, Gottlob: "Über den Zweck der Begriffsschrift" (original from 1883). Translated as "On The Aim of The Conceptual Notation" in Frege, Gottlob: *Conceptual Notation and related articles*, ed. by Terrel Ward Bynum, Oxford, Oxford University Press, 1972, pp. 90-100.

origins. Hintikka widened the scope of van Heijenoort's distinction in order to understand the "ultimate presuppositions" concerning *the nature of language* that were essential for him in 20th Century philosophy.

2. The search for traditions

From the very beginning there was an interest in understanding the roots of the striking rise of mathematical logic in the second half of 19th Century. Said interest was motivated by philosophical reasons: This "new logic" extended the scope of deductive reasoning and introduced a new methodology. As a consequence of this search, different intellectual and methodological traditions were discovered in the evolution of mathematical logic, which in turn led to historical and historiographical reflection.

In his book on the algebra of logic, Louis Couturat distinguished between *logique mathématique* and *logique des mathématiques*[6].This distinction was later reformulated as being between the tradition of *algebra of logic* and the tradition of *mathematical logic*[7]. Now then, in his Preface to the English translation of Couturat´s book, published in 1914, Philip E. Jourdain characterized the developments in the emergence of symbolic logic in terms of the differentiation between *rational calculus* and *universal language*. Jourdain stated explicitly:

"The *calculus ratiocinator* aspect of symbolic logic was developed by Boole, de Morgan, Jevons, Venn, C.S. Peirce, Schröder, Mrs. Ladd Franlin, and others; the *lingua characteristica* aspect was developed by Frege, Peano and Russell. Of course there is no hard and fast boundary-line between the domains of those two parties. Thus Peirce and Schröder early began to work at the foundations of arithmetic with the help of the calculus of relations; and thus they did not consider the logic calculus merely as an interesting branch of algebra. Then Peano paid particular attention to the calculative aspect of his symbolism. Frege has remarked that his own symbolism is meant to be a *calculus ratiocinator* as well as a *lingua characteristica*, but the using of Frege's symbolism as a calculus would be rather using a three-legged stand-camera for what is called 'snap-shot' photography." [8]

This quotation contains many issues that would be intensively discussed later on. Jourdain took both expressions from the attempts Frege had madeto clarify the system of logic he had devised in his 1879 book *Begriffsschrift*. The notions *calculus ratiocinator* and *lingua characteri[sti]ca* that Frege used originated in G. W. Leibniz's reflections concerning the role of scientific systems of signs in formal sciences as well as his projects of scientific universal languages. In some methodological and philosophical circles in Germany Leibniz's ideas were discussed and interpreted during the 19th Century.[9]

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[6] See Couturat, Louis: *L´Algebre de la logique*. Paris, Gauthier-Villars, 1905.

[7] See Ivor Grattan-Guinness, Ivor: "Living together and living apart. On the interactions between mathematics and logics from the French Revolution to the First World War", *South African Journal of Philosophy* 7 (1988), no. 2, pp. 73-82.

[8] Jourdain, Philip E. 1914: "Preface" of In Couturat, Louis *The Algebra of Logic*. English translation by Lydia Gillingham, Chicago and London, Open Court, 1914, pp. iii-x, p. viii.

[9] (A paper presented in 1845 by the philosopher Adolf Trendelenburg was very influential, see v.g. Peckhaus, Volker. *Logik, Mathesis universalis und allgemeine Wissenschaft. Leibniz und die Wiederentdeckung der formalen Logik im 19. Jahrhundert*, Berlin, Akademie Verlag, 1997 ISBN 9783050031118, pp. 130 ff.).

Fifty years later and in a different philosophical context, Jean van Heijenoort conceived his influential distinction using the two leibnizean labels. A *lingua characteristica* implied a language with a fixed interpretation in order to express content. Conversely, *a calculus ratiocionator* was conceived only as a system of signs with many possible interpretations.[10] In this way he initiated an influential perspective on the historiography of symbolic logic and, as Volker Peckhaus stated, "was to a great extent responsible for today's common understanding of the early directions in modern logic." [11]This perspective motivated further discussions about the criteria used to establish the very origins and nature of symbolic logic, and raised many questions concerning the original formulation of quantification, the distinction between subject and predicate and above all the problem of interpretation of logic systems and the birth of metalogical research.

The historical accuracy of van Heijenoort's distinction has been called into question[12], and it can be claimed that it was not a merely a descriptive distinction, but that it also included an important *normative* aspect. In fact, the prevalent ideas in van Heijenoort's milieu (mainly influenced by W. V. O. Quine) imply the conception of a main historical school of thought that has its point of departure in Frege´s work and continues through the *Principia Mathematica* of Whitehead and Russell. According to this school, logic systems aim to capture the logic underlying ordinary language, so they should express its deep logical structure. In any case, it is not difficult to argue that behind this understanding underlay a "linguistic" conception of logic: every logic system would be conceived as a "language" in some sense of the word.

3. Hintikka's distinction

Following the ideas of Richard Collingwood in his *Essay on Metaphysics*, Jaakko Hintikka dug up the tacit and implicite "ultimate presuppositions" of philosophy in 20th century Philosophy. This "Collingwoodian question" posed by Hintikka concerned *the nature of language*, as he formulated it in the introduction to a volume containing his main papers on this topic:

"This Collingwoodian question can be formulated in a first rough approximation by asking whether language – our actual working language, Tarski's "colloquial language" – is *universal* in the sense of being inescapable."[13]

In order to find an accurate framework for this question, Hintikka firstly reinterpreted van Heijenoort's distinction in terms of "language as universal medium" (or "the

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[10] See van Heijenoort op. cit.

[11] Peckhaus, Volker. 2004. "Calculus Ratiocinator vs. Characteristica Universalis? The Two Traditions in Logic, Revisted", *History and Philosophy of Logic* **25** (2004), 3-14, p. 5); doi 10.1080/01445340310001609315

[12] See *inter alia* Peckhaus op. cit.

[13] Hintikka, Jaakko. *Lingua Universalis vs. Calculus Ratiocinator: An Ultimate Presupposition of Twentieth-Century Philosophy* Dordrecht et al., Springer, 1997, p. ix. ISBN 978-0-7923-4246-5 DOI 10.1007/978-94-015-8601-6.

universality of language”, and “language as calculus” [14](or “the model-theoretic tradition”), and secondly he applied it to the analysis of many cases in 20 century logic and philosophy. A clear shift of focus can be perceived here. Originally, the distinction was related to the *historical* development of symbolic logic in the second half of the 19th century. Hintikka used it to show the different philosophical assumptions in current works in philosophy and logic, thus giving the distinction a broader scope.

Hintikka’s approach opened a rich discussion within the areas of philosophy of language and philosophy of logic, promoting the study of the philosophical roots of modern logic. The distinction would then be applied to plenty of notable authors in 20th century logic and philosophy: Ludwig Wittgenstein, Rudolf Carnap, Alfred Tarski and David Hilbert, among others. As it was already noticed in the introduction, Hintikka dedicated a whole paper to apply the distinction to his ideas on logic.

The distinction led to the analysis of discussion about different semantic frameworks, the philosophical relevance of Model Theory, the admissibility of modal concepts, the notion of logical truth, and, in general, the ontological suppositions underlying in modern logic. In the paper devoted to the case of Quine, Hintikka summarized his idea through the following table[15]):

UNIVERSALITY OF LANGUAGE	LANGUAGE AS CALCULUS
Semantics is ineffable	Semantics is possible
Interpretation cannot be varied	Interpretation can be varied
Model Theory impossible (or irrelevant)	Model Theory possible (and important)
Only one world can be talked about	Possible worlds are possible
One domain of quantification in the last analysis	Ranges of fully analysed quantifiers can be different
Ontology is the central problem	Ontology is conventional
Logical truths are about this world	Logical truths are in all possible worlds

According to Hintikka, in the universalistic conception of language, semantics cannot be defined in *our only language* without falling into a vicious circle. So, semantics cannot be expressible in the language. This fact motivated Hintikka to speak of the “ineffability of semantics”.

4. Hintikka on Peirce

Hintikka tried to determine and describe Peirce’s *locus* in the framework of the two traditions. He rightly maintained that Peirce’s work in logic did not represent the

[14] See Hintikka, Jaakko. “On the Development of the Model-Theoretic Viewpoint in Logical Theory”. *Synthese* 77 (1988), 1,pp. 1-36 DOI: https://doi.org/10.1007/BF00869545
[15]Hintikka, Jaakko: "Quine as a Member of The Tradition of The Universality of Language". In Robert B. Barrett and Roger Gibson (eds.), *Perspectives on Quine*, Oxford, Blackwell, 1990, pp. 227 f.

dominant perspective on the philosophy of logic around 1950, when Frege was established as the "grounding father" of mathematical logic. (see the famous statement by Quine in the preface of his book *Methods of Logic*: "Logic is an old subject, but since 1879 is a great one")[16]

Hintikka's interpretation was guided mainly by some specific aspects of Peirce's thoughts, namely, his interest in modal logic, and his willingness to apply logic to the study of logic itself, including a model-theoretic viewpoint. Now, as it has been remarked, Hintikka's distinction presupposes a *linguistic* conception of logic, *which is not Peirce's own ultimate conception*. Hintikka based his interpretation on what he called "Peirce's own testimony"[17], represented by this quotation from his *Minute Logic* from 1902:

"this system [the Existential Graphs] is not intended to serve as a universal language for mathematicians or other reasoners, like that of Peano."[18]

However, what Peirce states immediately after this claim should also be taken into account:

"… this system is not intended as a calculus, or apparatus by which conclusions can be reached and problems solved with greater facility than by more familiar systems of expression." [19]

So, Peirce seems to exclude also the possibility of interpreting his Existential Graphs as a "calculus". Hintikka understood that here "Peirce was dealing with interpreted logic, not merely formal inferences"[20]. Hence, he was proposing "unmistakably" a "model-theoretical" view of formal languages, remaining in the tradition of logic as calculus. Notwithstanding, this text calls for further examination. In fact, Peirce gives some justification of his claim:

"The principal desideratum in a calculus is that it should be able to pass with security at one bound over a series of difficult inferential steps. What these abbreviated inferences may best be, will depend upon the special nature of the subject under discussion. But in my algebras and graphs, far from anything of that sort being attempted, the whole effort has been to dissect the operations of inference into as many distinct steps as possible." [21]

According to Peirce, the main feature of a calculus is its inferential power in a computation sense. That was an important point in the algebra of logic. However,

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[16] Quine, W.V.O. *Methods of Logic Methods of Logic*. Cambridge, Mass., Harvard Univ. Press, 2nd. Ed. 1956. ISBN-13: 978-0674571761, p. vii))

[17] Hintikka, Jaakko: *Lingua Universalis vs. Calculus Ratiocinator: An Ultimate Presupposition of Twentieth-Century Philosophy* Dordrecht et al., Springer, 1997, pp. 142 ff.

[18] Peirce, Charles Sanders. *Collected Papers*. 8 vols., vols. 1- 6 ed. by Charles Hartshorne & Paul Weiss, vols. 7-8 ed. by Arthur W. Burks. Cambridge (Mass.), Harvard University Press, 1931-1958, 4.424

[19] loc. cit.

[20] Hintikka op. cit., p. 143

[21] *loc. cit*.

Peirce adds the idea of “dissecting the operations of inference” as an essential goal of his work in logic. This idea should receive an accurate examination and be connected with his conception of deduction.

5. Peirce’s ultimate philosophy of logic

Peirce’s move into a diagrammatic system at the end of the 19th century was not merely a change of notation for the usual logical notions, but rather it was based on his idiosyncratic conception of deduction. For him, deduction was a process accomplished by means of *diagrams*. The following text, from his main paper on the algebra of logic from 1885, gives evidence of his position:

“The truth, however, appears to be that all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts.” [22]

Diagrammatic reasoning is, according to Peirce, the best method for representing the “course of thinking”. This diagrammatic conception of logic was a direct consequence of his conception of mathematics: mathematical thought was for him essentially diagrammatic[23]. As he stressed in a text published in volume IV of *New Elements of Mathematics*, *mathematical proof* is characterized as the process of construction and transformation of diagrams. In general, diagrammatic reasoning is any form of ‘valid necessary reasoning’[24]

A proper understanding of Peirce´s idea of logic and mathematics must take into account the fact that simultaneously with his research in logic, Peirce developed his *theory of signs* (or *semiotics*). This theory was conceived as a logic of science, a theory of knowledge and a theory of mind, and sometimes Peirce referred to it as ‘Logic’ in a broad sense.[25] As it is known, Peirce´s theory of signs was based on the triadic relation between the sign-vehicle (the *representamen*), the designatum and the interpreter producing the semiosis. He conceives the relation of being a sign as *triadic*: *X is a sign of Y for Z*, as he expressed in this famous passage in the *Collected Papers*:

“A sign, or representamen, is something which stands to somebody for something in some respect or capacity.”[26]

As well as defining sign as a triadic relation, Peirce classified signs by means of many triadic distinctions according to different aspects of signs. One of them results from the analysis of the relation between the sign-object and the *designatum*. This is the

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[22] Peirce, *op. cit*. 3.363.)
[23] See, v. g. Peirce, *op. cit*., 3.429.
[24] Peirce, *op. cit*., 1.54 and CP 5.162.
[25] See Peirce, *op. cit*. 1.444 and 2.93.
[26] Peirce, *op. cit*. 2.228.

classification in *icon*, *index*, and *symbol*. Peirce considered it to be "very important": There are "three kinds of signs which are all indispensable in all reasoning". [27] They are typically characterized in the following way.[28] Icons represent their object according to their form or structure. It is said that they *resemble* their objects. Indices are directly connected to objects. An index "refers to the Object that it denotes by virtue of being really affected by that Object"[29]. This means that indices designate in virtue of an *actual* connection between the sign and its object (for example, the footprint on the beach or also proper names in language). Finally, a symbol refers "to the Object that it denotes by virtue of a law, usually an association of general ideas, which operates to cause the Symbol to be interpreted as referring to that Object."[30] It should be noted that, in fact, the three kinds are *aspects* of signs: every sign belongs to a certain extent to the three kinds; one of them tends to prevail in one case or another.

Now then, as Peirce wrote in his MS 293, a Diagram is an Icon of a set of rationally related objects. In a diagram the *analytical* role of icons turns out to be essential. (Peirce writes explicitly: "icon [or analytic picture]"[31].) A diagram is decomposed in parts, so that has an important role in knowledge acquisition, for it gives information about the designated object. Peirce wrote around 1895:

"For a great distinguishing property of the icon is that by the direct observation of it other truths concerning its object can be discovered than those which suffice to determine its construction".[32]

This fact has been characterized as *operational iconicity* by Friedrich Stjernfelt:

"The icon consists of parts whose relations mirror the relations between the corresponding parts of the object, and the sign is used to gain information". (Stjernfelt 2006, p. 71)

Furthermore, the icon "is also the only sign by the contemplation of which more can be learnt than lies in the directions for the construction of the sign." (*loc. cit*.)

Summarizing, there are three main actions related to diagrams: constructing, experimenting and observing. Deduction is defined as operating with diagrams and diagrams are *structural* representations on the basis of visual properties. These operations have an *experimental* function: Modifications in the diagrams are made in order to arrive to the goal. A handy example of a diagrammatic deduction should be the manipulation of a geometric figure according to a set of rules in order to obtain a theorem in classical Euclidean geometry.

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[27] Peirce, *op. cit*. 1.369.

[28] See Peirce, *op. cit*. 2.247.

[29] Peirce, *op. cit*. 2.248.

[30] Peirce, *op. cit*. 2.249.

[31] Peirce, *op. cit*. 1.275.

[32] Peirce, *op. cit*. 2.279.

On the verge of the 20th century, Peirce conceived a diagrammatic formulation of logic, corresponding at least to First Order Logic, which he called *Existential Graphs*. In a letter he wrote to Philip Jourdain from 1908, he regarded it as his *chef d'oeuvre* in logic and one year later he wrote to William James that it "ought to be the logic of the future".[33] Peirce was convinced that the Existential Graphs were adequate to express the form of every deductive argument or mathematical reasoning in general.

"This, then, is the purpose for which my logical algebras were designed but which, in my opinion, they do not sufficiently fulfill. The present system of existential graphs is far more perfect in that respect, and has already taught me much about mathematical reasoning". (Peirce: CP 4.429).

Thus, he worked intensively on them until his death in 1914, that is, simultaneously with the development of the most complex chapters of his theory of signs.

Existential Graphs had both an *analytical* and an *operational* function in the study of deductive reasoning. The analytic function is the philosophically prevailing one (see next section). Peirce accomplished to conceive two systems. The Alpha system is devoted to the logic of propositional connectives, whereas the Beta system corresponds to the logic of quantifiers and identity. Furthermore, he sketched the Gamma and Delta systems in order to capture modal logic and higher order logic. Peirce achieved a full account of the first two systems, while the third remained in a fragmentary form.[34]

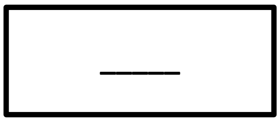
This is not the place to provide a full description of the systems or to describe their evolution and the different versions of them. Research carried out in many unpublished manuscripts provides a more accurate idea about them. However, it is important to recall the basic ideas behind them in order to understand their proper semiotic nature. The system of Existential Graphs starts with the "sheet of assertion", a blank surface.

Every graph written ("scribed") in this sheet is understood as the assertion of a true sentence. In order to express elementary predicate logic, the usual presentation of the Alpha and Beta systems includes two basic signs to be considered: the *line of identity* and the *cut*.

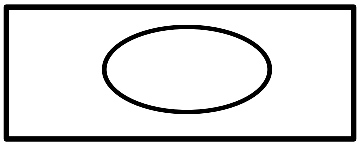
The *line of identity*

[33] See Roberts, Don: *The Existential Graphs of Charles. S. Peirce*. La Haya, Mouton, 1973. p. 11.

[34] See *inter alia* Roberts *op. cit*., and Hilpinen, Risto. 2004: "Peirce's Logic", in *Handbook of the History of Logic*, vol. 3: *The Rise of Modern Logic: From Leibniz to Frege*, Amsterdam, Elsevier North Holland et al., 2004, pp. 623-670.

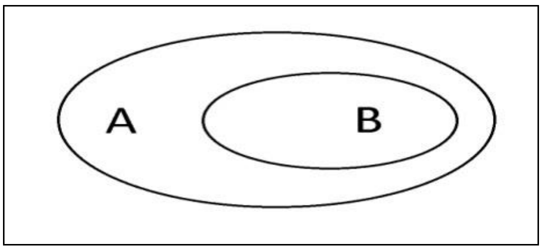


expresses the existence of something in the universe of discourse. If a predicate P is attached to a line, the diagrammatic expression of the sentence “There is a P” is obtained. So, it corresponds to the notion of the existential quantifier[35]. A cut is a closed line drawn on the assertion sheet.



It cuts its content off from the whole sheet. Hence, it expresses the notion of (classical) negation. A sentence **A**, written within a cut in the assertion sheet is understood as the assertion of “It is not the case that **A**” (classically equivalent to the assertion that **A** is false). Besides, two sentences inscribed in the assertion sheet can be understood as the conjunction of both.

Thus, the classical quantifiers and connectives are diagrammatically expressed by means of the combination of the two preceding signs, and the recursive application of the basic signs gives rise to the expression of sentences of arbitrary logical complexity. For example the conditional sentence “If **A**, then **B**” is expressed using two cuts, one enclosing the other, in this form:



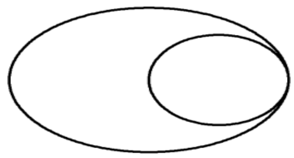
Clearly, this complex graph can be also understood as “It is not the case that **A** and not **B**”.

That is the standard and usual presentation of the basic signs of both the Alpha and Beta systems. However, the Alpha system has been conceived in a different way too. The *scroll* was a choice made by Peirce between 1900 and 1909 as a basic sign in Alpha instead of cut. It corresponds to the *illation* (interpretable also as material implication):

“I thought I ought to take the general form of argument as the basal form of composition of signs in my diagrammatization; and this necessarily took the form of a ‘scroll,’ that is a curved line without contrary flexure and returning.”[36]

[35] The line of identity expresses also the identity relation. Furthermore, the system Beta has no individual variables.
[36] Peirce, *op. cit.*, 4.564.

Thus, the scroll can be described as a *sole* continuous line, giving rise to two attached or connected ellipses, one inside the other (*loops)*, both drawn on the sheet of assertion.[37] According to Ms 455 (1903), it consists of "two closed lines one inside the other". In the MS 693 (1904) Peirce provided instructions for drawing the scroll with *only one line*, *without lifting the pencil from the paper*. The result was the following graph consisting of an *outer loop* and an *inner loop*. So, the scroll corresponds to the following graph (or alternative topological variants):



By means of it every propositional connective could be defined, which shows a decisive feature of Peirce's Existential Graphs.[38]

6. The analytical nature of the Existential Graphs

It has been remarked already that the Existential Graphs provide an analysis of logical concepts. In the Ms 455, p. 2 (1903), The purpose of the EG was"to enable us to separate reasoning into its smallest steps so that each one may be examined by itself".

In fact, this diagrammatic system is the *continuation* of Peirce's previous algebraic study of logic but carried out more accurately. It has been mentioned before that through his contributions both in algebra and in diagrammatic logic Peirce tried "to dissect the operations of inference into as many steps as possible".[39] Hence, the "exact" or mathematical logic had mainly an analytical role. In 1893 he wrote that the aim of logic was "to analyze reasoning and see what it consists in" and related to his exposition of the Existential Graphs he argued that it is "the business of logic to be analysis and theory of reasoning, but not the practice of it." [40]

According to Peirce, *analysis* meant the decomposition of something into its basic elements. It applied above all to concepts in general:

"[I]f one concept can be accurately defined as a combination of others, and if these others are not of more complicated structure than the defined concept, then the defined concept is regarded as ***analyzed*** into these others. Thus A is grandparent of B, if and only if A is a parent of somebody who is a parent of B, therefore grandparent is

[37] See Roberts *op*. *cit*., p. 34.
[38] The parallelism to the algebraic system formulated by Peirce in his "On the Algebra of Logic" from 1885 is noteworthy. There he used the "copula of inclusion" as the primitive sign for propositional logic. Different versions of the Existential Graphs occurring in unpublished manuscripts show that the Existential Graphs reveal more subtle than the standard presentation. Some of these details will be examined in the next section (see Roberts *op*. *cit*., for further details).
[39] Peirce, *op*. *cit*., 4.424.
[40] Peirce, *op*. *cit*., 2.532 and CP 4.134.

analyzed into parent and parent. So stepparent, if taken as not excluding parentage, is analyzed into spouse and parent; and parent-in-law into parent and spouse.” [41]

Here Peirce seems to present the usual and basic idea of analysis without further specifications. So the problem is to determine why the system of the existential graphs would provide the *right* analysis of logical concepts. Obviously, the existential graphs are icons, and icons are “analytic signs” for Peirce in the sense that they always show a decomposable structure. However, the same could be said about other notations (that are also iconic in Peirce’s general sense).

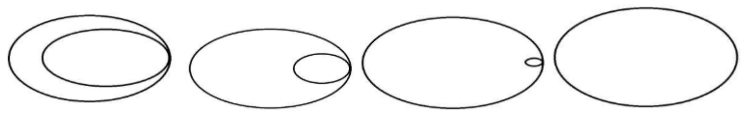
Bellucci and Pietarinen try to provide an answer to the aforementioned question about this special role of the Existential Graphs which implies a particular idea of analysis. The key idea can be understood as *uniqueness of decomposition*:

“A system, constructed so as to employ the least amount of logical machinery and the least number of logical objects, forces us to the correct analysis of propositions. To say that an analysis is correct can in the first place mean nothing more than this.” [42]

As a justification for this claim they quote passages from Peirce’s unpublished manuscripts. According to Peirce, the system of the Existential Graphs provides “the only method by which all connections of relatives can be expressed by a single sign”, as “the System of Existential Graphs recognizes but one mode of combination of ideas” (MS 482, 1897 and MS 490, 1906)
[43]

Hence, the conclude that “Alpha is more analytic than other systems because, at bottom, it employs one single logical conception, that of consequence *de inesse*, or material implication. To express the material conditional, the system employs one single logical symbol”. [44] This single sign is the aforementioned “scroll” used by Peirce in early versions of the EG.

The case of the negation operator is a good example of this kind of analysis. It can be analyzed in a simple way by means of the scroll. Under the condition that nothing is inscribed within the inner loop of the scroll[45], the scroll can be shrunk continuously until it is finally transformed into a cut. This procedure can be reproduced through the following sequence of graphs:



[41] Peirce, *op*. *cit*., 1.294.

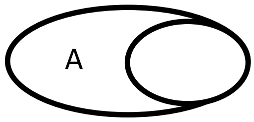
[42] Bellucci, Francesco & Ahti-Veiko Pietarinen. 2016. “Existential Graphs as an Instrument for Logical Analysis. Part 1: Alpha”, *Review of Symbolic Logic*. 9 (2016) no. 2. ISSN 1755-0203 2016, DOI: https://doi.org/10.1017/S1755020315000362, p. 210

[43] Bellucci & Pietarinen 2016, *op*. *cit*., p. 211

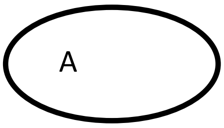
[44] Bellucci & Pietarinen 2016, loc. cit.

[45] It is a *pseudograph*, see Peirce *op*. *cit*., 4.455 and Roberts *op*. *cit*., p. 36.

This fact has interesting consequences in relation to Peirce's philosophy of logic. The *pseudograph* represents a contradiction (provided that in the inner loop nothing can be inscribed). For example, the graph



means 'A implies a contradiction' and the final graph



should be read as 'not A'.

From this, Peirce follows in MS 481, p. 8, from 1896 that certain signs must express negation, and that the concept of negation has been "virtually analyzed". This means that the scroll is for him the *accurate semiotic device* for *analyzing* the concept of negation. If the cut is not a basic graph, negation would not be a basic notion. It should be characterized derivatively by means of the scroll. Moreover, the scroll satisfies the condition of being not only a sign for a basic logical notion but also of playing the role assigned to auxiliary signs or 'punctuation marks' (like brackets, etc.), called by Peirce 'Collectional Signs'[46].

7. Concluding remarks

Peirce's systems of Existential Graphs are formulated *prima facie* as diagrammatic proof systems for logic; the validity of logical arguments and logical principles can be proven in these systems. Notwithstanding, the preceding discussion provides enough evidence to argue that the analytical role of the Existential Graphs challenges the place of Peirce's ideas in the context of the logic as calculus vs. logic as language opposition. Undoubtedly, analysis is a core notion in philosophy, having been interpreted in many ways along history and deserving a deeper discussion. In any case, analysis is for Peirce a more fundamental task than the construction of a calculus with multiple interpretations (as it is in the tradition of logic as calculus), and it is not "linguistic" as far as it is carried out in a diagrammatic system quiet different from ordinary language or a regimented version of it.[47]

Furthermore, diagrammatic systems are not absolutely different from "sentential" or "linguistic" notations, because *all notations are iconic*, that is, their signs are "analytic signs". Logical analysis could be carried out *prima facie* in any suitable notation, and the Existential Graphs are the "most correct" or best system of signs according to

[46] See Bellucci & Pietarinen *op*. *cit*., pp. 221 ff. for further details.

[47] It can be useful to compare this idea of analysis in the system of Existential Graphs with an analogous idea of analysis from the other side of the Atlantic. It can be found in Gottlob Frege's *Begriffsschrift* or conceptual notation. The importance of the notion of analysis in Frege's thought is very well known too. Some hints to its role in logic can be found in Legris, Javier. 2012. "Between Calculus and Semantic Analysis. Symbolic Knowledge in the Origins of Mathematical Logic". In *Symbolic Knowledge from Leibniz to Husserl*, ed. by Abel Lassalle Casanave. London, College Publications, 2012, pp. 1-49.

Peirce´s notion of analysis. Bellucci and Pietarinen understand it in the sense that "no system is more analytical than a single-sign notation"[48]. The Alpha system differs to other notations of propositional logic, due to the fact that it consists of the fewest signs capable of expressing propositional logic. Peirce's idea of logical analysis brings to mind the well-known reduction of propositional connectives to one single operator such as the Sheffer's stroke.[49]Notwithstanding, the scroll represents the case of an absolute single sign (brackets being dispensable). A particular (semiotic) notion of analysis is behind this reductive procedure that would introduce changes in the very notion of icon. This fact deserves a more detailed discussion.[50]

Therefore, the conception of logic as calculus implies two different features: (i) It is a "blind procedure" in order to make computations, and (ii) a variety of interpretations of the symbolic system is conceivable (it is a "model-theoretic" view)[51]. It is evident that the system of Existential Graphs does not imply *prima facie* these features. At the same time, it does not seem reasonable to think of it as a "universal language" (even in the sense of a universal system of signs with a fixed designation). It can be argued that we are now investigating the nature of logic from a different perspective. If this is the case, then Peirce was in many respects beyond the logic as calculus vs. logic as language opposition, leading to a different viewpoint.

[48] Bellucci & Pietarinen *op*. *cit*., p. 221.

[49] Peirce had already conceived this kind of reduction for the algebra of logic; see Bellucci & Pietarinen *op*. *cit*.

[50] Furthermore, this situation gives rise to a discussion of the usual idea of *multiple readings* of diagrams. This idea is related to the typical feature of *productive ambiguity* of them. If the uniqueness of analysis led to the impossibility of 'multiple readability', then Peirce's notion of icon would be problematic.

[51] A further discussion of these features can be read in Legris *op*. *cit*.